p_T fluctuations and multiparticle clusters in heavy-ion collisions

("CLUSTER SCALING")

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Based very closely on WB, Hiller, Florkowski, Bożek, nucl-th/0510033,
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A simple way of doing the statistics

• - variable dividing into classes (multiplicity n), dynamical variable $p_i = |\vec{p}_{T,i}|$

• - divide the events into classes of the same n, $P(n) = N(n)/N_{\rm all}$ is the probability of obtaining event of multiplicity n

• - n and $p_1, p_2, \ldots, p_n$ vary from event to event. The probability of a given configuration is $P(n)\rho_n(p_1, \ldots, p_n)$, where $\rho_n(p_1, \ldots, p_n)$ is the conditional probability distribution of occurrence of $p_1, \ldots, p_n$ provided we have multiplicity n. Note ρ_n depends functionally on n. The normalization is

$$\sum_n P(n) = 1, \quad \int dp_1 \ldots dp_n \rho_n(p_1, \ldots, p_n) = 1$$

The *marginal* probability densities are defined as

$$\rho_n^{(n-k)}(p_1, \ldots, p_{n-k}) \equiv \int dp_{n-k+1} \ldots dp_n \rho_n(p_1, \ldots, p_n),$$

with $k = 1, \ldots, n-1$. These are also normalized to 1

• We introduce

$$\langle p \rangle_n \equiv \int dp \rho_n(p) p, \quad \mathrm{var}_n(p) \equiv \int dp \rho_n(p) \left(p - \langle p \rangle_n\right)^2,$$

$$\mathrm{cov}_n(p_1, p_2) \equiv \int dp_1 dp_2 \left(p_1 - \langle p \rangle_n\right) \left(p_2 - \langle p \rangle_n\right) \rho_n(p_1, p_2).$$

The subscript n indicates that the averaging is taken within samples of a given multiplicity n

• - Remark: note that I am not using the the inclusive quantities, defined through the *inclusive* probability distributions related to the marginal probability distributions in the following way:

$$\rho_{\mathrm{in}}(x) \equiv \sum_n P(n) \int dp_1 \ldots dp_n \sum_{i=1}^{n} \delta(x - p_i) \rho_n(p_1, \ldots, p_n) = \sum_n n P(n) \rho_n(x),$$

$$\rho_{\mathrm{in}}(x, y) \equiv \sum_n P(n) \int dp_1 \ldots dp_n \sum_{i,j=1, j \neq i}^{n} \delta(x - p_i) \delta(y - p_j) \rho_n(p_1, \ldots, p_n)$$

$$= \sum_n n(n-1) P(n) \rho_n(x, y)$$

which are normalized to $\langle n \rangle$ and $\langle n(n-1) \rangle$, respectively

- - Define wider classes, $1 \leq n_1 \leq n \leq n_2$, $\sum_n = \sum_{n=n_1}^{n_2}$
- - For the variable $M = \sum_{i=1}^{n} p_i/n$ we find immediately an *exact* result

$$\langle M \rangle = \sum_n P(n) \int dp_1 \ldots dp_n M \rho_n(p_1, \ldots, p_n) = \sum_n P(n) \langle p \rangle_n,$$

$$\langle M^2 \rangle = \sum_n P(n) \int dp_1 \ldots dp_n M^2 \rho_n(p_1, \ldots, p_n)$$

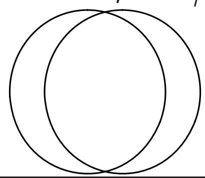
$$= \sum_n \frac{P(n)}{n} \langle p^2 \rangle_n + \sum_n \frac{P(n)}{n^2} \left[\sum_{i,j=1, j\neq i}^{n} \mathrm{cov}_n(p_i, p_j) + n(n-1)\langle p \rangle_n^2 \right]$$

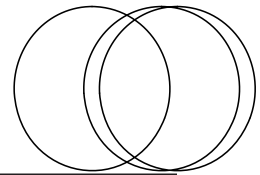
$$\sigma_M^2 = \sum_n \frac{P(n)}{n} \sigma_{p,n}^2 + \sum_n P(n) \left(\langle p \rangle_n \right)^2 - \left(\sum_n P(n) \langle p \rangle_n \right)^2 +$$

$$+ \sum_n \frac{P(n)}{n^2} \left[\sum_{i,j=1, j\neq i}^{n} \mathrm{cov}_n(p_i, p_j) \right]$$

A look at some data:PHENIX @ 130 GeV

$|\eta| < 0.35$, $0.2 < p_T < 1.5$ GeV, $\Delta\phi = 45^o$





centrality	0-5%	0-10%	10-20%	20-30%
$\langle n \rangle$	59.6	53.9	36.6	25.0
σ_n	10.8	12.2	10.2	7.8
$\langle M \rangle$	523	523	523	520
σ_p	290	290	290	289
σ_M	38.6	41.1	49.8	61.1
$\langle M \rangle^{\mathrm{mix}}$	523	523	523	520
σ_M^{mix}	37.8	40.3	48.8	60.0

PHENIX, PRC66 (2002) 024901, nucl-ex/0203015

$\langle M \rangle$ and σ_p are practically constant in the "fiducial" centrality range $c = 0 - 30\%$ **(1)**

(1) allows us to replace $\langle p \rangle_n$ with $\langle M \rangle$ and $\sigma_{p,n}^2 = \langle p^2 \rangle_n - \langle p \rangle_n^2$ with σ_p^2:

$$\sigma_M^2 = \sigma_p^2 \sum_n \frac{P(n)}{n} + \sum_n \frac{P(n)}{n^2} \left[\sum_{i,j=1,j\neq i}^{n} \mathrm{cov}_n(p_i, p_j) \right]$$

[corrections can be worked out]

In mixed events, by construction, particles are not correlated, hence the covariance term vanishes and

$$\sigma_M^{2,\mathrm{mix}} = \sigma_p^2 \sum_n \frac{P(n)}{n} \simeq \sigma_p^2 \left(\frac{1}{\langle n \rangle} + \frac{\sigma_n^2}{\langle n \rangle^3} + \dots \right) \qquad \mathbf{(2)}$$

where we have used the fact that $P(n)$ is narrow and expanded $1/n = 1/[\langle n \rangle + (n - \langle n \rangle)]$ to second order in $(n - \langle n \rangle)$

centrality	0-5%	0-10%	10-20%	20-30%
$\langle n \rangle$	59.6	53.9	36.6	25.0
σ_n	10.8	12.2	10.2	7.8
σ_p	290	290	290	289
σ_M^{mix}	37.8	40.3	48.8	60.0
$\sigma_p \sqrt{\frac{1}{\langle n \rangle} + \frac{\sigma_n^2}{\langle n \rangle^3}}$	38.2	40.5	49.8	60.8

(2) works within 1% [which is a trivial statistical statement checking that our approximations work]

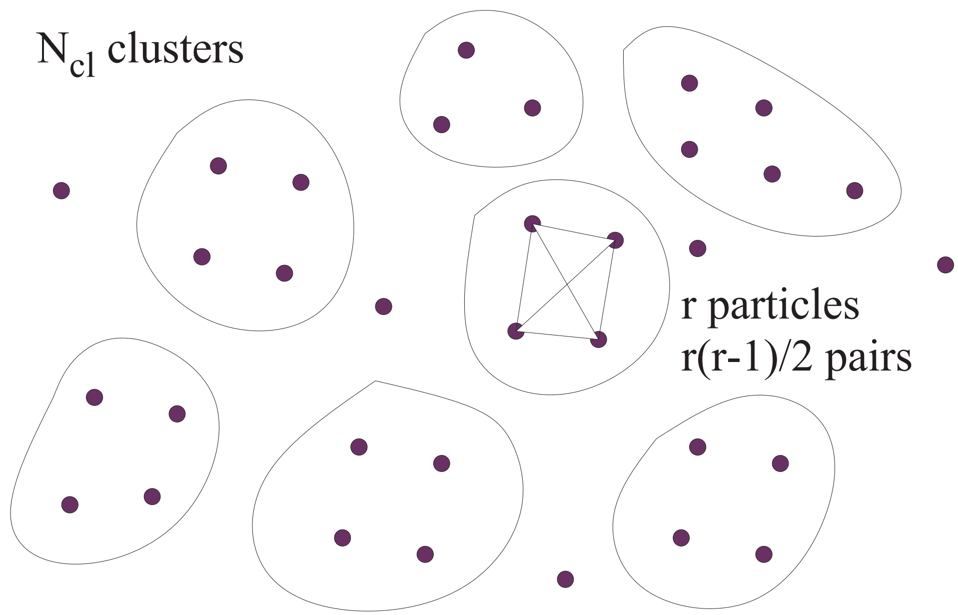
Subtracting the last two equations yields

$$\sigma_{\rm dyn}^2 \equiv \sigma_M^2 - \sigma_M^{{\rm mix},2} = \sum_n \frac{P(n)}{n^2} \sum_{i,j=1,j\neq i}^{n} {\rm cov}_n(p_i,p_j) \simeq \frac{1}{\langle n\rangle^2} \sum_{i,j=1,j\neq i}^{\langle n\rangle} {\rm cov}(p_i,p_j) \qquad \textbf{(3)}$$

centrality	0-5%	0-10%	10-20%	20-30%
$\langle n\rangle$	59.6	53.9	36.6	25.0
σ_M	38.6	41.1	49.8	61.1
$\sigma_M^{\rm mix}$	37.8	40.3	48.8	60.0
$\sigma_{\rm dyn}\sqrt{\langle n\rangle}$	60.3 ± 1.6	59.2 ± 1.5	59.8 ± 1.2	57.7 ± 1.1

$\sigma_{\rm dyn}^2 \sim 1/\langle n\rangle$ (within 2%, round-off errors!), which together with **(3)** places severe constraints on physics - not all particle can be correlated!

Multiparticle clusters (in momentum space)



The average number of correlated pairs within a cluster is $\langle r(r-1)/2\rangle$. Some particles may be unclustered, hence $\langle N_{\rm cl}\rangle\langle r\rangle/\langle n\rangle = \alpha$. Then

$$\sigma^2_{\rm dyn} = \frac{\alpha\langle r(r-1)\rangle}{\langle r\rangle\langle n\rangle}{\rm cov}^* = \frac{\alpha r^*}{\langle n\rangle}{\rm cov}^*, \quad r^* = \frac{\langle r(r-1)\rangle}{\langle r\rangle},$$

which complies to the scaling of $\sigma_{\rm dyn}$ if $\alpha r^*{\rm cov}^*$ is independent of $\langle n\rangle$ (in the fiducial centrality range). For a fixed number of particles in each cluster we have $r^* = \langle r\rangle - 1$, for the Poisson distribution $r^* = \langle r\rangle$, while for wider distributions $r^* > \langle r\rangle$.

Cluster scaling

$\sum_{i \neq j} \mathrm{cov}(p_i, p_j) \sim \langle n \rangle$

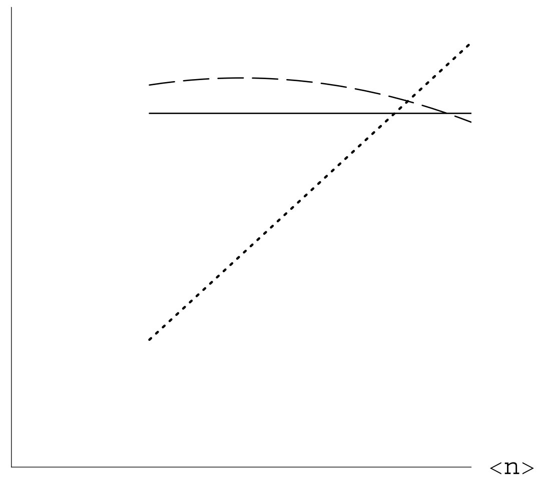
$\sigma^2_{\mathrm{dyn}} \sim \frac{1}{\langle n \rangle}$

$F_{p_T} \sim 1$

$\Phi_{p_T} \sim 1$

$\Sigma_{p_T} \sim \frac{1}{\sqrt{\langle n \rangle}}$

STAR $\langle \Delta p_i \Delta p_j \rangle \sim \frac{1}{\langle n \rangle}$



If the multiplicity of produced particles $\langle n \rangle$ is used - sensitive to final state, if $\langle n \rangle \to N_p$ - sensitivity to initial state

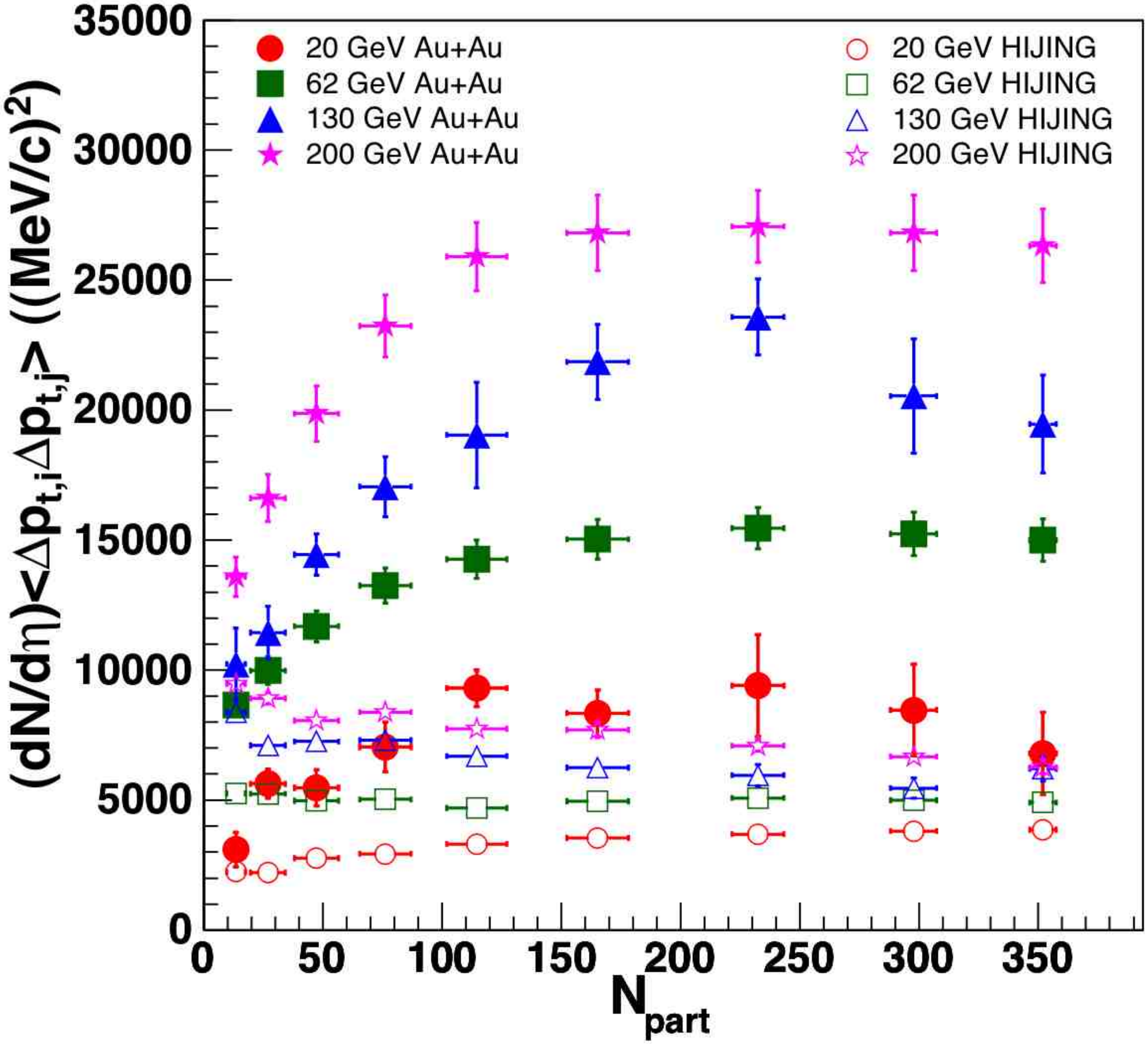
Try both, *i.e* use also just $\langle n \rangle$ to define the classes

Comply to scaling: PHENIX@130, STAR perhaps except for 130, CERES (within error bars), PHENIX@200 - fixed plane,

Scaling violations: PHENIX@200 (random), NA49 (try $\langle n \rangle$ instead of N_p)

[Paul Sorensen's plot]

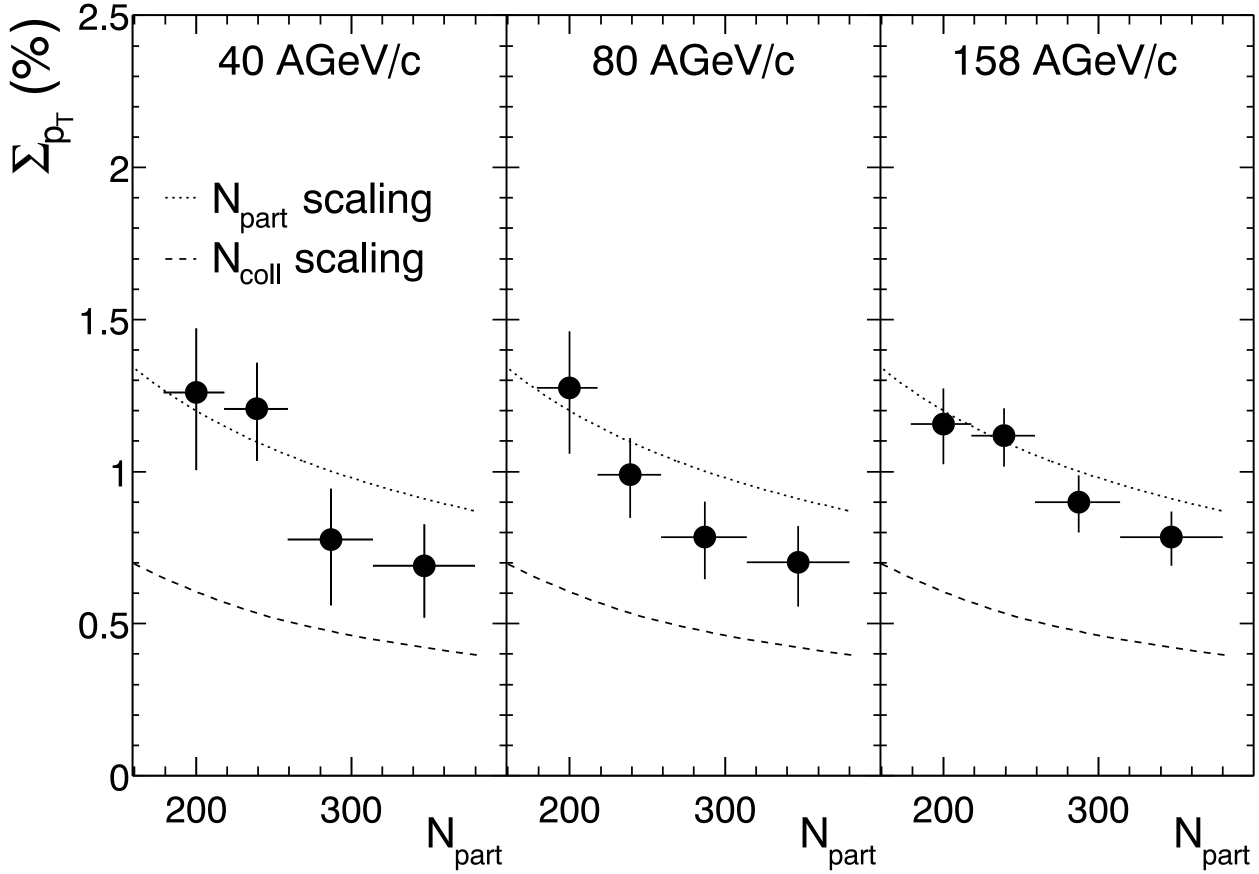
STAR



STAR, hep-ph/0504031

$$\frac{dN}{d\eta}\langle \Delta p_i \Delta p_j \rangle \simeq \sigma^2_{\rm dyn}\langle n \rangle / \Delta\eta \sim const.$$

CERES



$$\Sigma_{p_T} \equiv \frac{\sigma_{\rm dyn}}{\langle p_T \rangle} \sim \frac{1}{\sqrt{\langle n \rangle}}$$

Works approximately quite well! (errors large)

How strong are the correlations?

a - detector efficiency, number of observed particles $\sim a$, number of pairs $\sim a^2$. Thus

$$\mathrm{cov}^* = \sigma^2_{\mathrm{dyn}} \frac{\langle n \rangle}{a r^*}.$$

For PHENIX@130 $a \simeq 10\%$, which gives

$$\mathrm{cov}^* \simeq \frac{0.035 \ \mathrm{GeV}^2}{r^*}.$$

The natural scale set by $\sigma_p^2 \simeq 0.08 \ \mathrm{GeV}^2$ (recall that $| \, \mathrm{cov}^* \, | \leq \sigma_p^2$). For $r = 2$ the value of cov^* would assume 45% of the maximum possible value. This is unlikely, as argued from model estimates presented below, where cov^* at most $0.01 \ \mathrm{GeV}^2$. Thus a natural explanation of the above number is to take a significantly larger value of r^*. The higher r^*, the easier it is to satisfy the data even with small values of cov^*.

Same for STAR

Very similar quantitative conclusions from the STAR data [nucl-ex/0504031]. The measure used by STAR is the estimator for $\sigma^2_{\rm dyn}$:

$$\langle \Delta p_i \Delta p_j \rangle = \frac{N_{\rm event} - 1}{N_{\rm event}} \sigma^2_M - \frac{1}{N_{\rm event}} \sum_{k=1}^{N_{\rm event}} \frac{\sigma^2_p}{n_k} \simeq \sigma^2_{\rm dyn} \qquad (1)$$

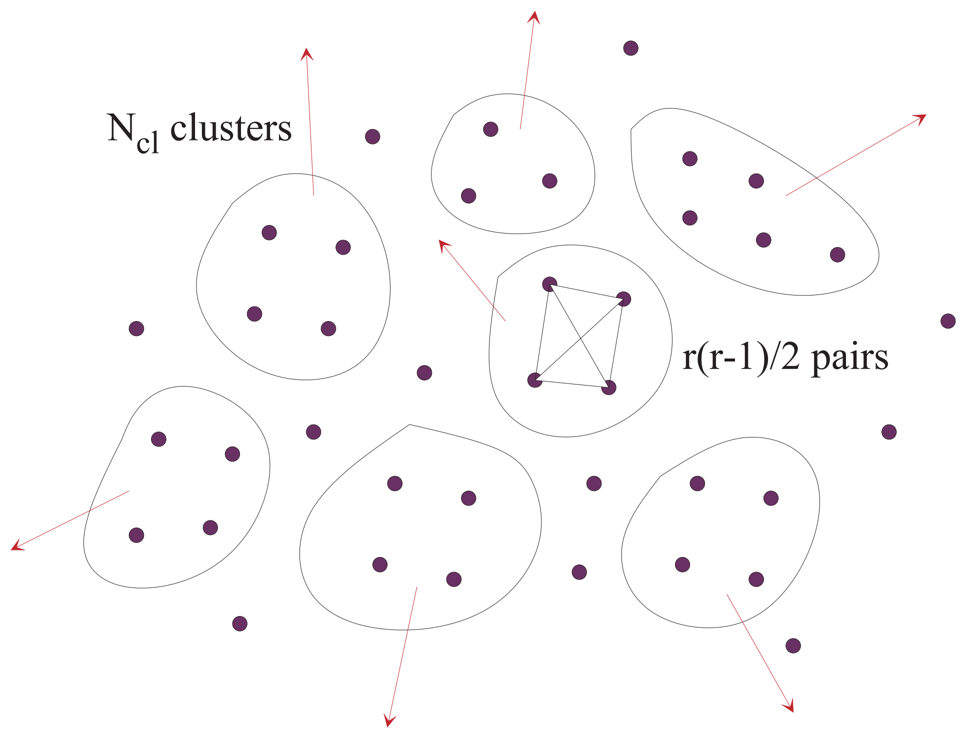
Assuming $a = 0.75$ we find

$$\begin{aligned} {\rm cov}^* r^* &= 0.058, 0.043, 0.035, 0.014 \ \text{GeV}^2 \\ \text{for } \sqrt{s_{NN}} &= 200, \quad 130, \quad 62, \quad 20 \ \text{GeV} \end{aligned}$$

The value at 130 GeV is close to PHENIX. Significant beam-energy dependence! This may be due to increase of the covariance per pair with energy, and/or increase of the number of clustered particles

What is the nature of clusters?

• - (mini)jets, resonances, droplets of matter receding at the same collective velocity

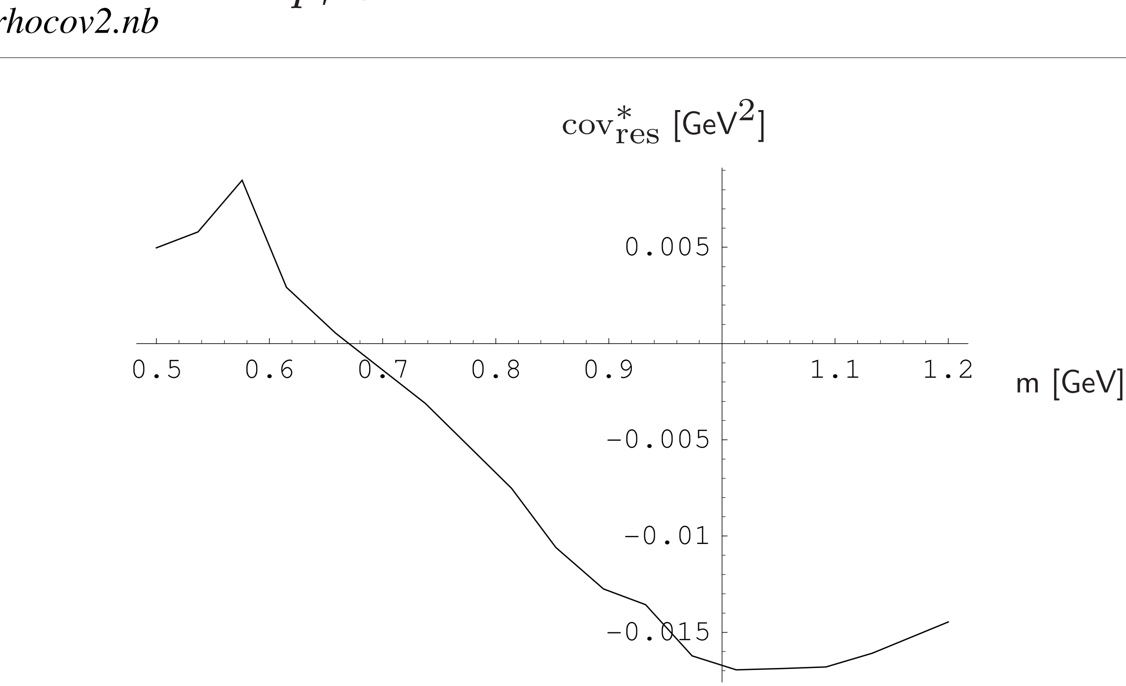


"Lumped clusters": lumps of matter move at some collective velocities, correlating the momenta of particles belonging to the same cluster

Covariance from the decay of resonances

$$\mathrm{cov}^*_{\mathrm{res}} = \frac{\int d^3p \int \frac{d^3p_1}{Ep_1} \int \frac{d^3p_2}{Ep_2} \delta^{(4)}(p-p_1-p_2) C \frac{dN_R}{d^3p} \left(p_1^\perp - \langle p^\perp \rangle\right) \left(p_2^\perp - \langle p^\perp \rangle\right)}{\int d^3p \int \frac{d^3p_1}{Ep_1} \int \frac{d^3p_2}{Ep_2} \delta^{(4)}(p-p_1-p_2) C \frac{dN_R}{d^3p}}$$

dN_R/d^3p - resonance distribution from the Cooper-Frye formula with the single freezeout model, p_1, p_2 - momenta of daughters, E_p - energy of the particle with momentum p, C - cuts



Cancellations between contributions of various resonances are possible; `Therminator` - negligible contribution of resonances to the p_T correlations. Accidental zero near the ρ-meson mass, need of accurate implementation of cuts.

Thermal clusters

Emission from local thermalized sources: each element of the fluid moves with its collective velocity and emits particles with locally thermalized spectra. The picture refelects charge conservation within the local source [Bożek, WB, Florkowski, Acta Phys. Hung. A22 (2005) 149].

$$\mathrm{cov}^*_{i,j} = \frac{\int d\Sigma_\mu u^\mu \int d^3p_1 (p_1^\perp - \langle p^\perp \rangle) f_i^u(p_1) \int d^3p_2 (p_2^\perp - \langle p^\perp \rangle) f_j^u(p_2)}{\int d\Sigma_\mu u^\mu \int d^3p_1 f_i^u(p_1) \int d^3p_2 f_j^u(p_2)}$$

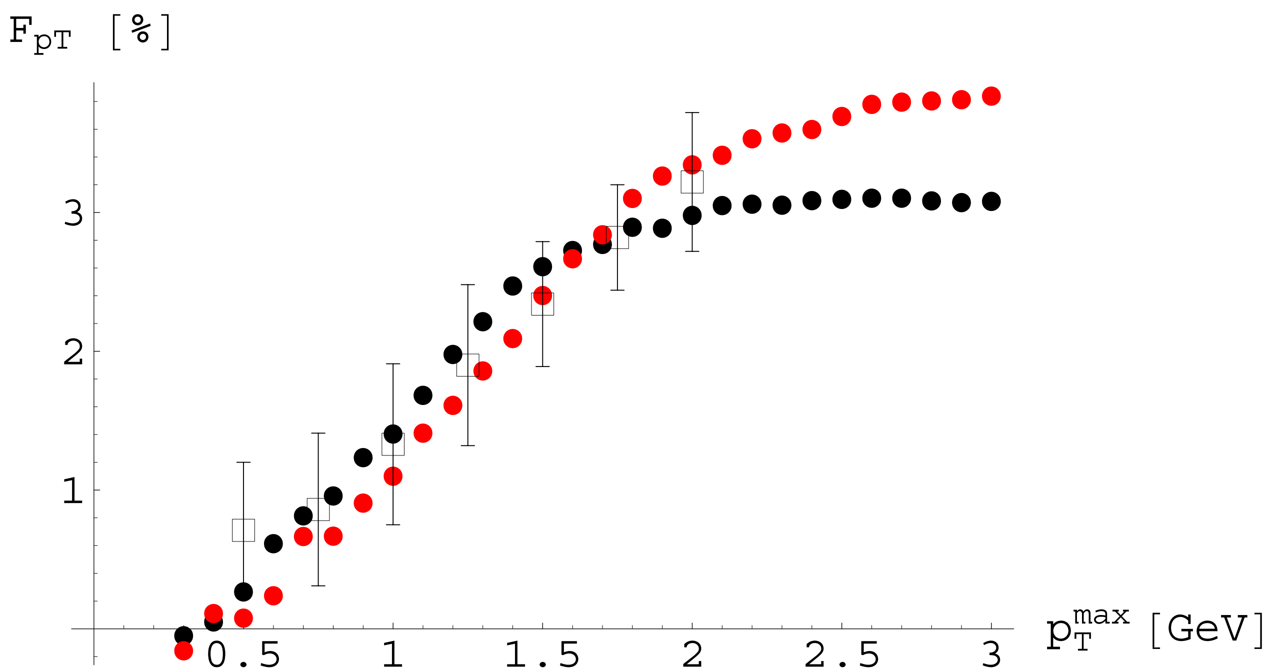
$f_i^u(p) = (\exp(p \cdot u/T) \pm 1)^{-1}$ - boosted thermal distribution, $u(x)$ -expansion velocity, $d\Sigma_\mu$ - integration over the freeze-out hypersurface. Fix flow such that $\langle M \rangle = 554$ MeV

T [MeV]	10	100	120	140	165	200
$\langle\beta\rangle$	0.94	0.72	0.69	0.58	0.49	0.31
σ_p^2 [GeV^2]	0.056	0.19	0.15	0.15	0.14	0.12
cov^* [GeV^2]	0.027	0.011	0.0088	0.0063	0.0034	0.0006

Results depend strongly on temperature. At realistic thermal parameters the experimental value of the covariance, $0.035\ \mathrm{GeV}^2/r^*$, cannot be accounted for unless the number of (charged) particles belonging to a cluster is sizeable, at least $5 - 10$

$p_\perp^{\max}$ dependence: data - PHENIX Au+Au @ 200, $c = 20 - 25\%$

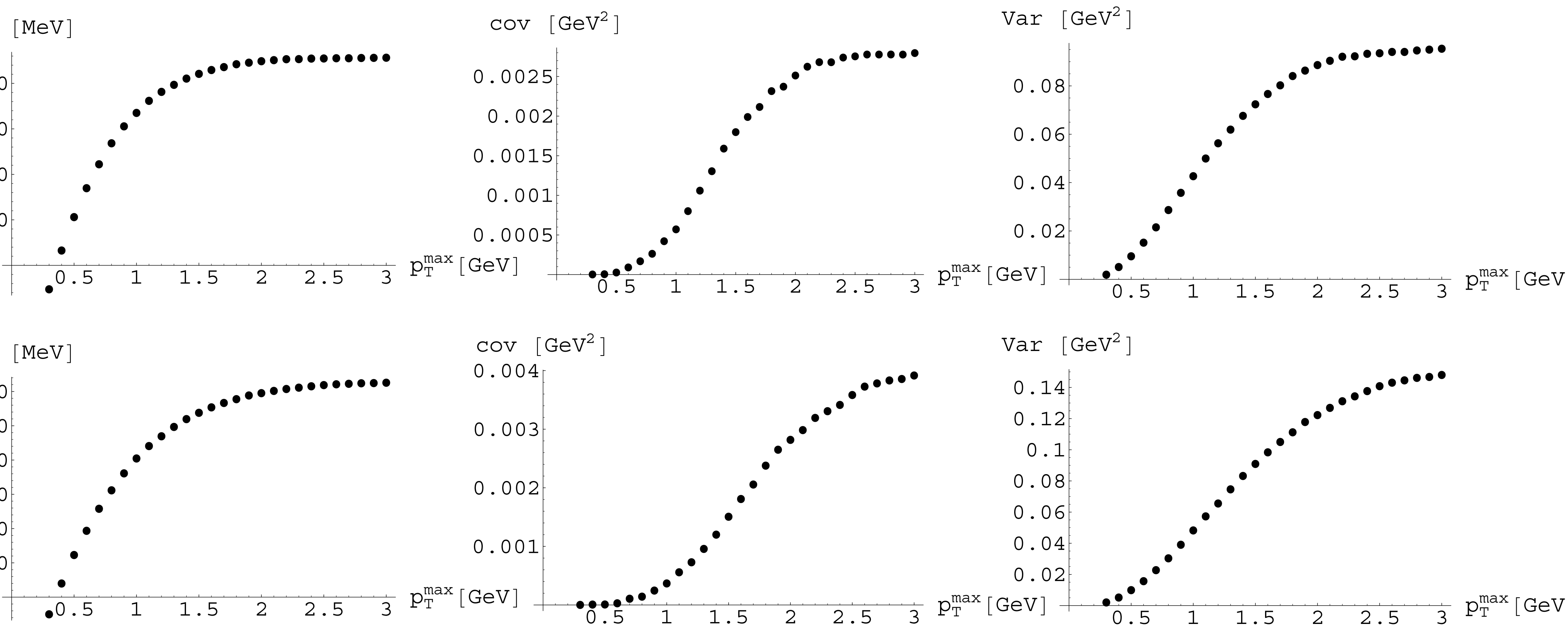
thermal clusters - red: $T = 165$ MeV, $ar^* = 2.1$, black: $T = 130$ MeV (T lowered due to resonance decays !!!), $ar^* = 2.9$



$$\omega = \frac{\sigma_M}{< M >}, \quad F_{p_T} = \frac{\omega_{\text{ data}} - \omega_{\text{ mix}}}{\omega_{\text{ mix}}} \simeq \frac{\text{cov}^* ar^*}{2\sigma_p^2}$$

Why soft physics goes to $p_T \sim 2$ GeV

top: $T = 130$ MeV, bottom: $T = 165$ MeV



Conclusion

1. A "combinatoric" attempt
2. Constant $\langle M \rangle$ and σ_p in the fiducial centrality range made the calculation easy, otherwise somewhat more involved formulas are needed but the analysis is straightforward. Need more experimental info.
3. Appearance of scaling of $\sigma^2_{\rm dyn}$ with $1/\langle n \rangle$ (and appropriately in other *equivalent* measures of correlations) suggest the cluster picture of the fireball
4. This cluster scaling can be also seen at STAR and at CERES
5. Use also $\langle n \rangle$, not N_p only.
6. The clusters may a priori originate from very different physics: (mini)jets, droplets of fluid formed in the explosive scenario of the collision, or other mechanisms leading to multiparticle correlations
7. The magnitude of the observed $\sigma_{\rm dyn}$ can be easier achieved when several (4-10 charged) particles are present in a cluster (thermal clusters estimate)
8. In the thermal clusters model F_{p_T} grows linearlyly with $p_\perp^{\rm max}$ and then saturates around 2 GeV
9. More detailed microscopic modelling necessary