Non-perturbative properties of high-T QCD from lattice calculations

Frithjof Karsch, Brookhaven National Laboratory & Bielefeld University

- Introduction:

  thermal scales in QCD: T, gT, g^2T,...

- Heavy quark free energies

  screening and running couplings

- Bulk thermodynamics

  the equation of state: QCD and SU(3)

- Hadronic fluctuations

  quark number and charge fluctuations

- Conclusions

Thermal scales in QCD

- the hard scale: $p \sim T$
  thermal modes, bulk thermodynamics, eg. pressure

$$\frac{p}{T^4} = a_{SB} f_p(g(T))$$

- the soft scale: $p \sim gT$
  static color-electric modes, eg. Debye screening

$$\frac{m_D(T)}{g(T)T} = \sqrt{\frac{N_c}{3} + \frac{n_f}{6} + \frac{n_f}{2\pi^2}\left(\frac{\mu_q}{T}\right)^2} \cdot f_E(g(T))$$

- the ultra-soft scale: $p \sim g^2 T$
  static color-magnetic modes, eg. spatial string tension

$$\frac{\sqrt{\sigma_s}}{g^2(T)T} = c_M f_M(g(T))$$

Non-thermal scales in thermal QCD

- even harder scales: $p \gg T$, $r^{-1} \gg T$, $M \gg T$

  short distance physics, eg. quarkonium

  $$g^2(r,T)$$

- quantitative questions, eg.

  When does T become the dominant scale?

Hierachy of scale ?

- Perturbation theory provides a hierachy of length scales

  $T \gg gT \gg g^2T ... \Rightarrow$ guiding principle for effective theories,

  resummation, dimensional reduction...

  These scales are not well separated close to T_c !!

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  resummation, dimensional reduction...

  These scales are not well separated close to T_c !!

- Early lattice results show that $g^2(T) > 1$ even at $T \sim 5T_c$

  G. Boyd et al, NP B469 (1996) 419: SU(3) thermodynamics..

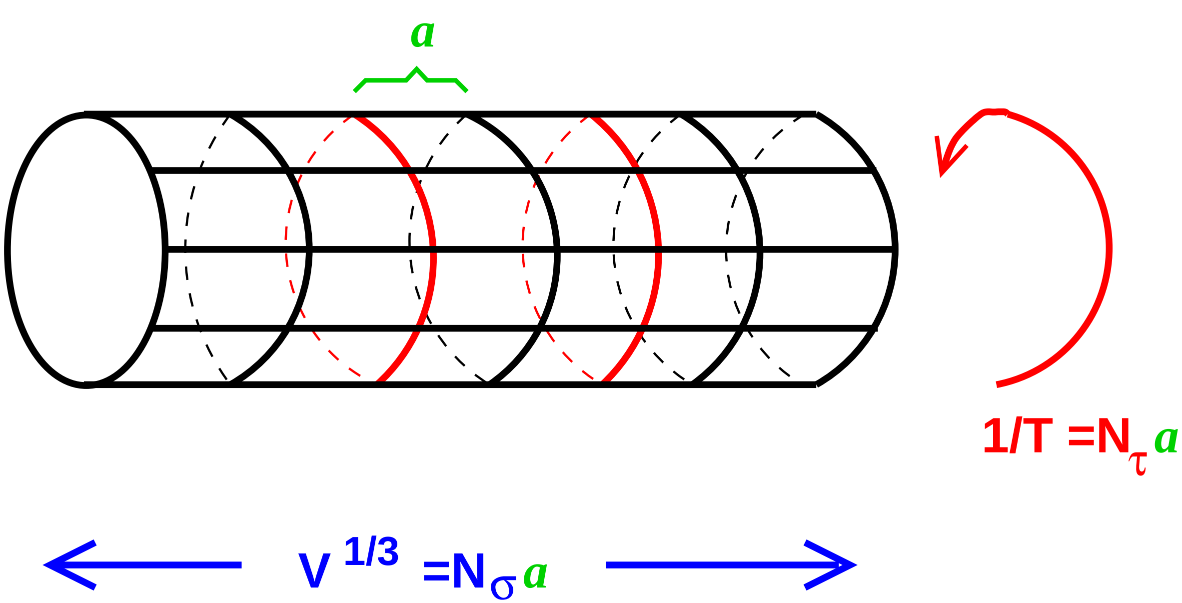
  ...one has to conclude that the temperature dependent running coupling has to be large, $g^2(T) \simeq 2$ even at $T \simeq 5T_c$

- the Debye screening mass is large close to T_c

- the spatial string tension does not vanish above T_c

  $\sqrt{\sigma_s} \neq 0 \Rightarrow$ the QGP is "non-perturbative" up to very high T

Analyzing hot and dense matter on the lattice: $\mathbf{N}_{\sigma}^{3} \times \mathbf{N}_{\tau}$



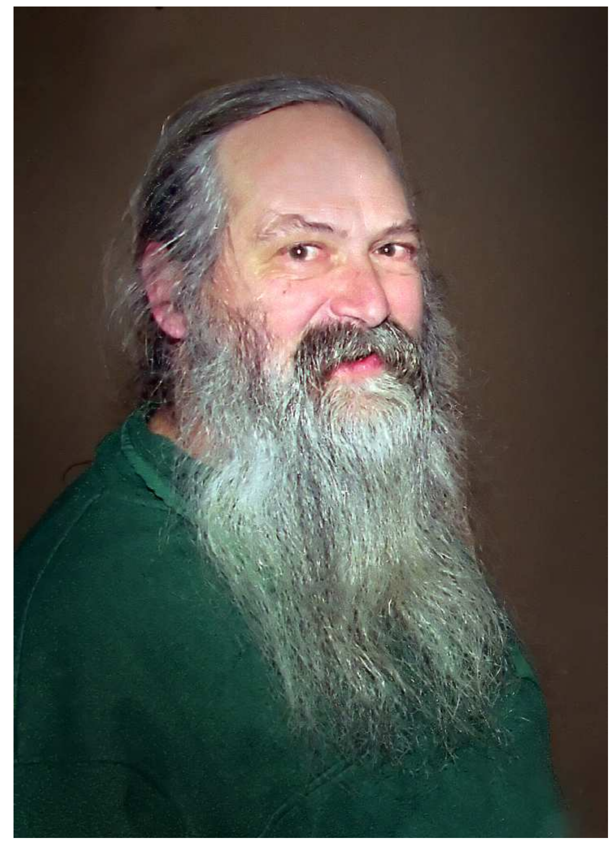
Quantum Chromo Dynamics

partition function: $Z(V, T, \mu) = \int \mathcal{D}\mathcal{A}\mathcal{D}\psi\mathcal{D}\bar{\psi}\ \mathrm{e}^{-S_E}$

$$\boxed{S_E = \int_0^{1/T} dx_0 \int_V d^3x\ \mathcal{L}_E(\mathcal{A}, \psi, \bar{\psi}, \mu)}$$

temperature volume chemical potential

Michael Creutz



Phys. Rev. D21 (1980) 2308

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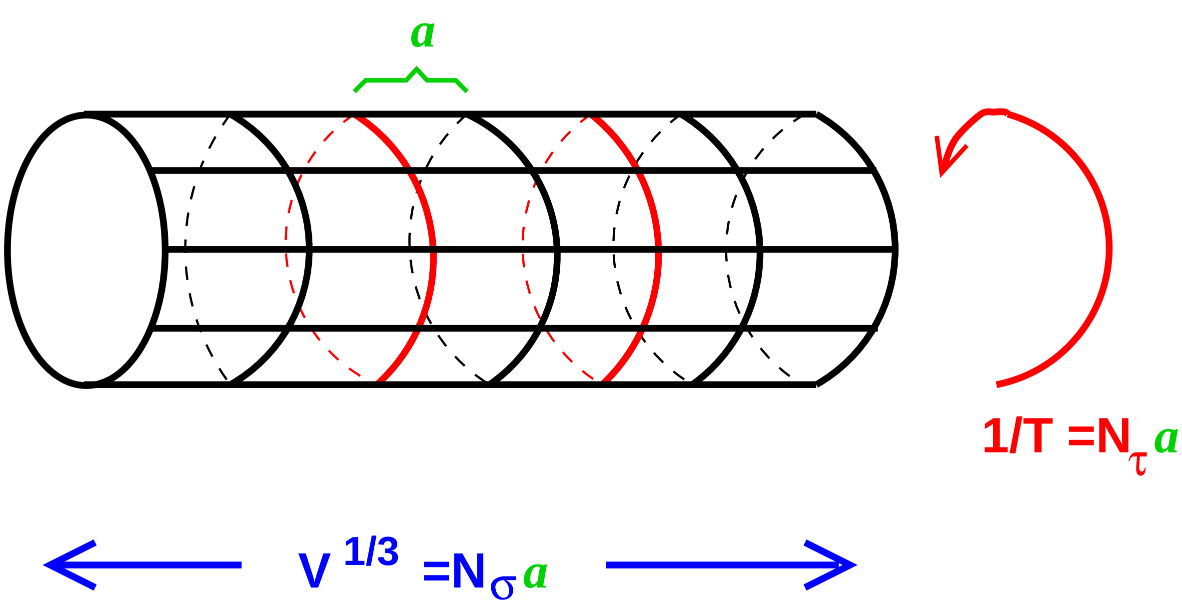

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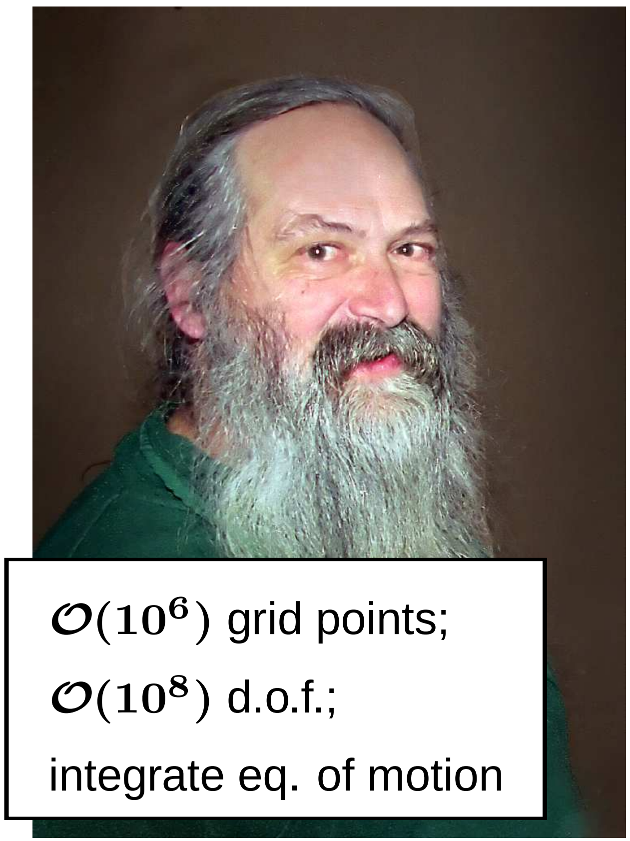

$\mathcal{O}(10^6)$ grid points;

$\mathcal{O}(10^8)$ d.o.f.;

integrate eq. of motion

Phys. Rev. D21 (1980) 2308

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Screening of heavy quark free energies – remnant of confinement above T_c –

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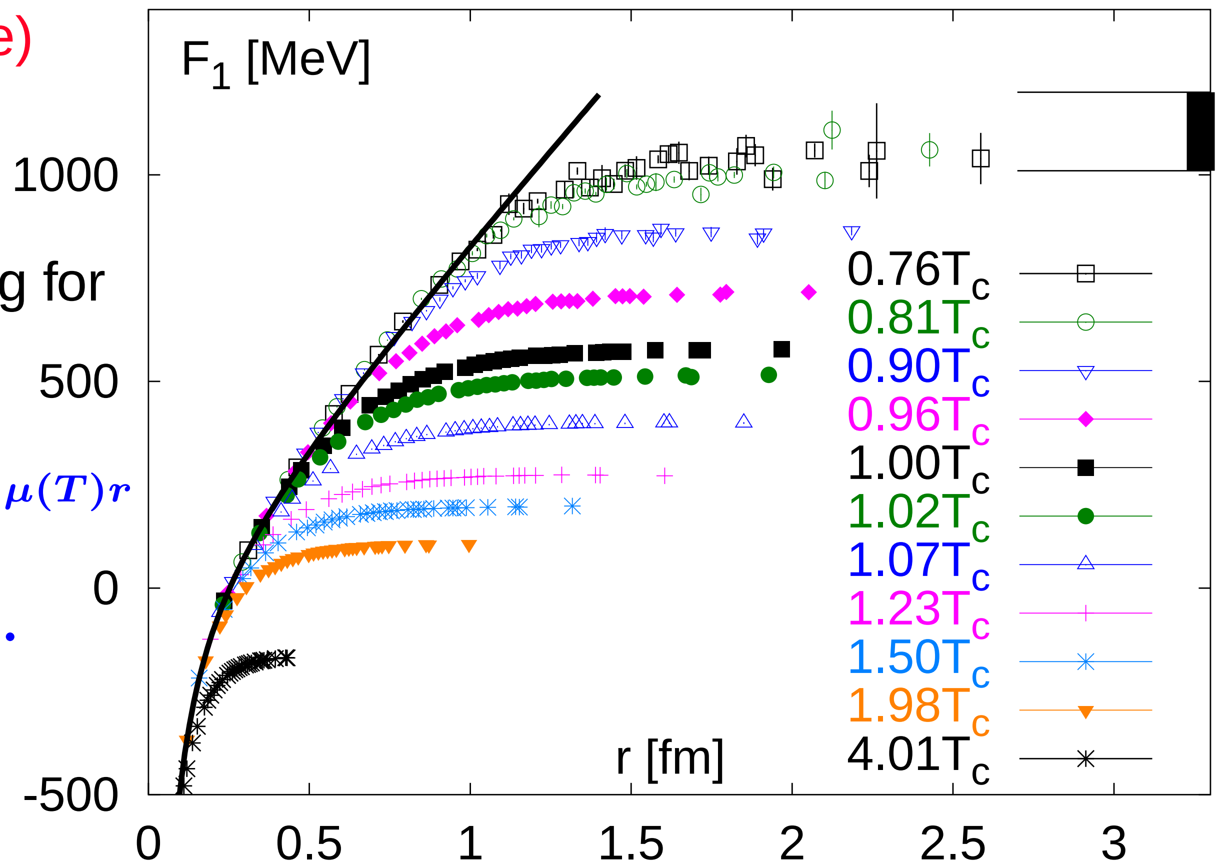
- singlet free energy
  (in Coulomb gauge)



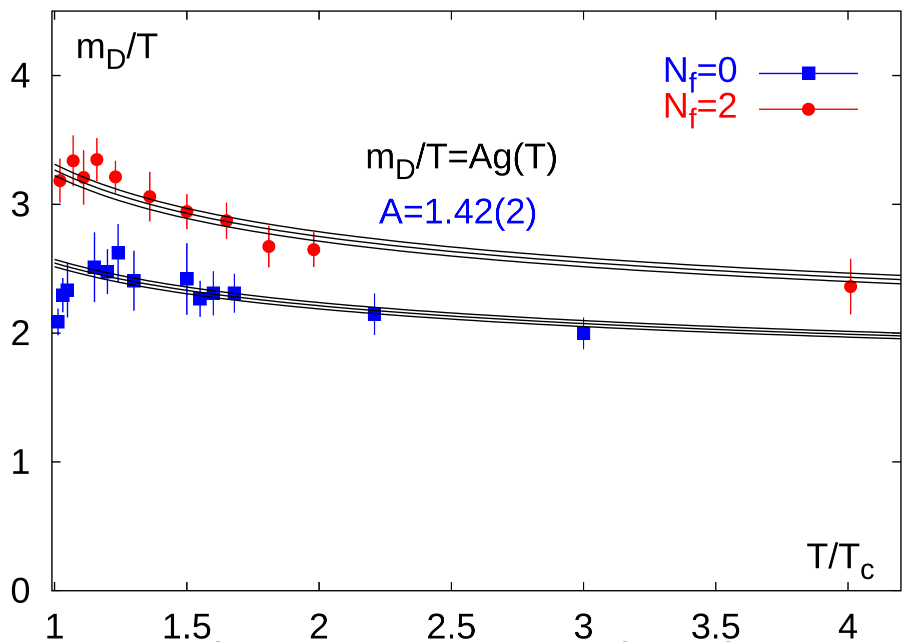
- $T \simeq T_c$: screening for $r \gtrsim 0.5$fm

$$F_1(r,T) \sim \frac{\alpha(T)}{r} e^{-\mu(T) r} + const.$$

- $F_1(r,T)$ follows linear rise of $V_{\bar{q}q}(r, T=0) = -\dfrac{4\alpha(r,T=0)}{3r} + \sigma r$

Non-perturbative Debye screening

- leading order perturbation theory: $m_D = g_D(T)T\sqrt{1+\frac{n_f}{6}}$
- $T_c < T \lesssim 10T_c$: non-perturbative effects are well represented by an "A-factor": $m_D \equiv A g_D(T)T,\ A \simeq 1.5$
- perturbative limit is reached very slowly (logarithms at work!!)



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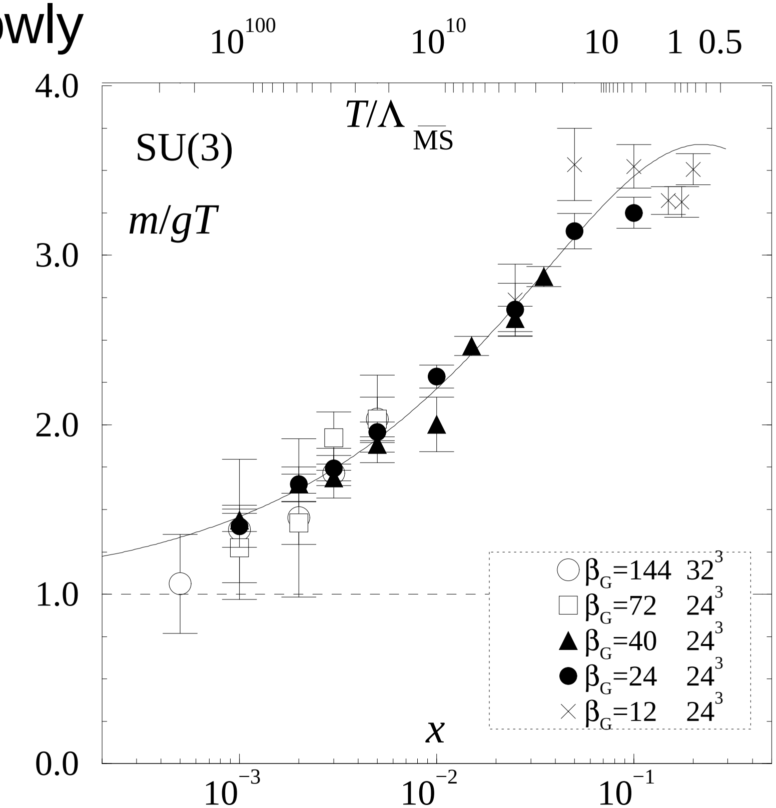

K.Kajantie et al, PRL 79 (1997) 3130

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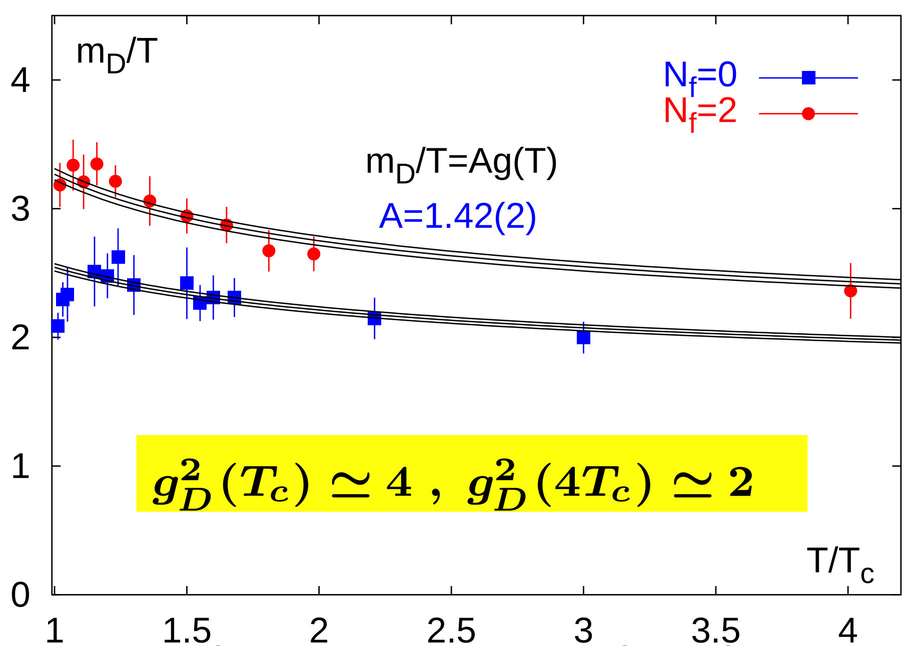

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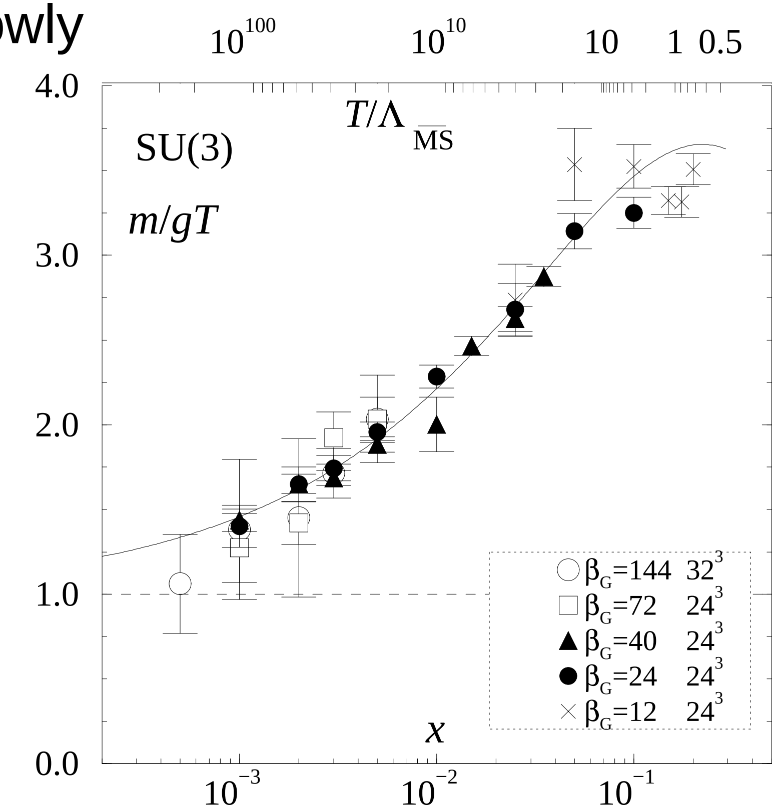

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Non-perturbative Debye screening
μ_q-dependence

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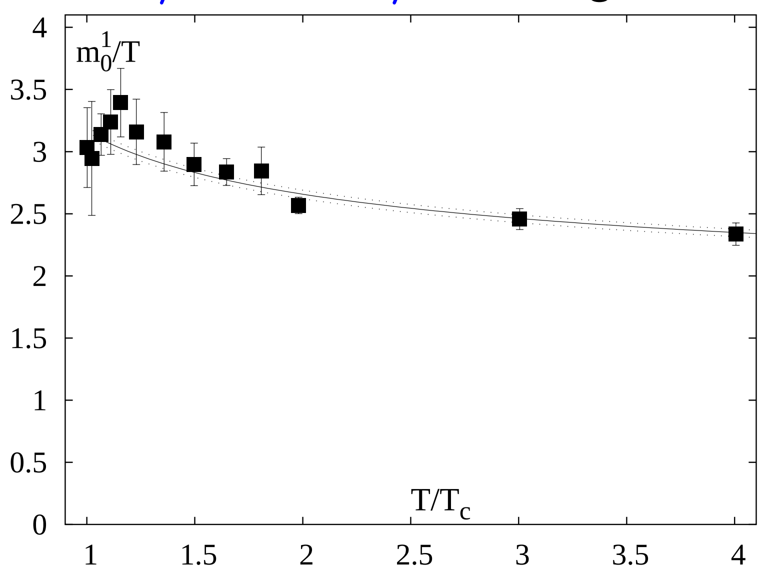
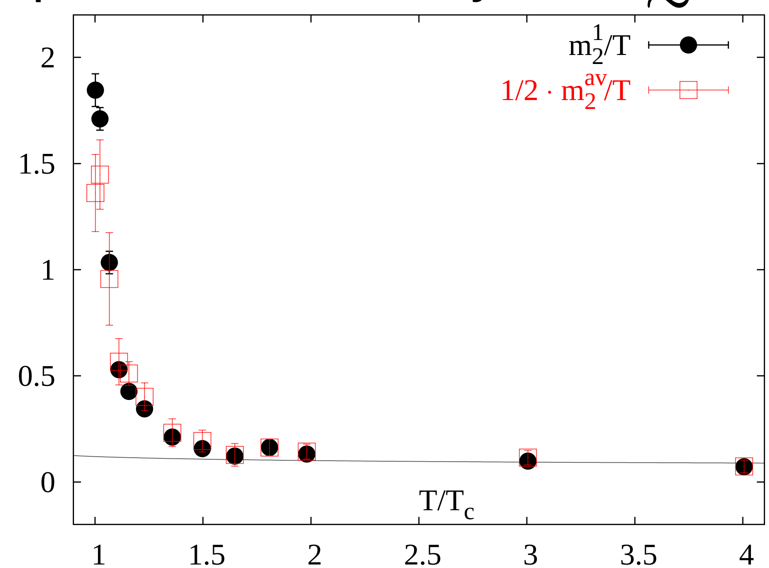
$$m_D = g(T)T\sqrt{1 + \frac{n_f}{6} + \frac{n_f}{2\pi^2}\left(\frac{\mu_q}{T}\right)^2}$$

- Taylor expansion, 2-flavor QCD:

$$m_D(T) = m_0(T) + m_2(T)\left(\frac{\mu_q}{T}\right)^2 + \mathcal{O}(\mu_q^4)$$

$m_2/m_0 = 3/8\pi^2$: agrees with perturbation theory for $T \gtrsim 1.5T_c$





M. Döring et al., Eur. Phys. J. C46 (2006) 179

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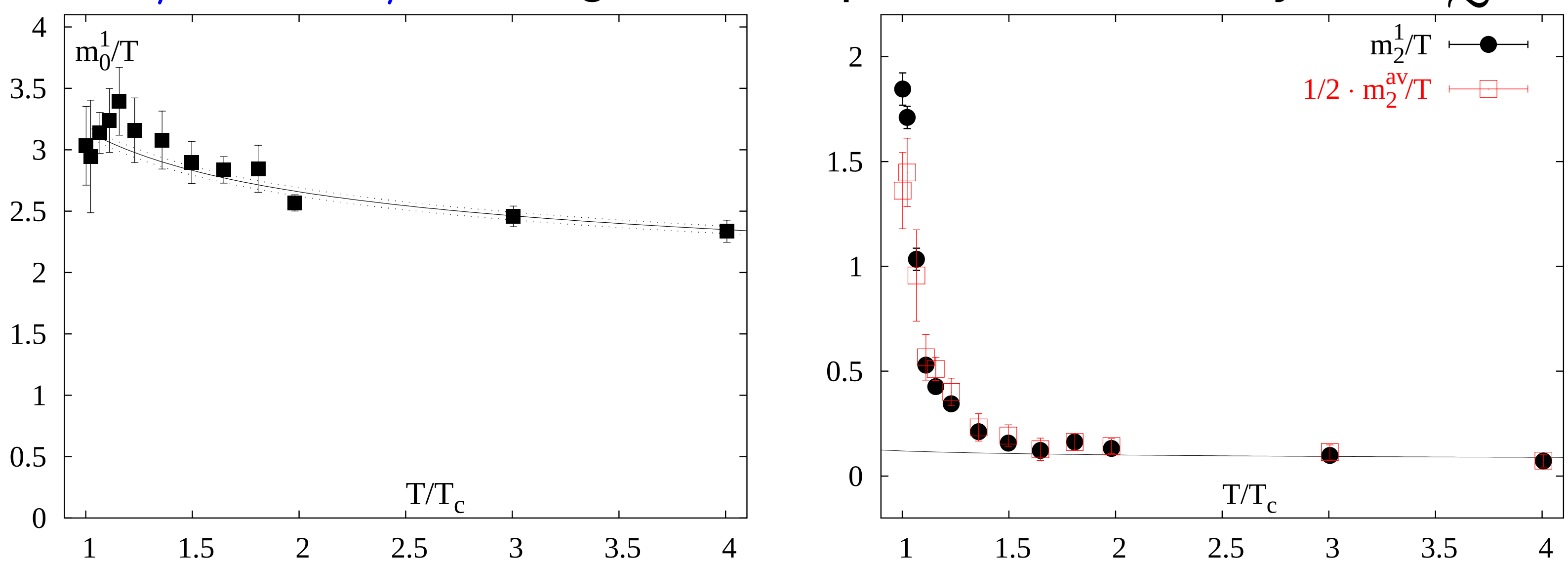
non-perturbative effects are in the glue

quark sector "perturbative" above $T \gtrsim 1.5T_c$?



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String breaking and screening
Does a heavy quark bind a light (anti) quark?

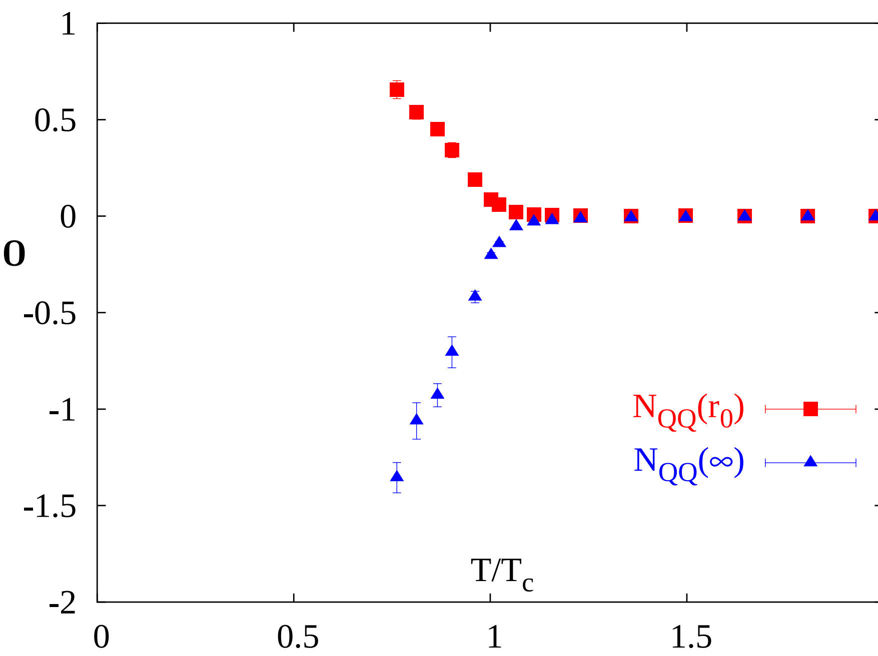
- Static quark-quark source in a thermal heat bath

  triality = 0: medium provides additional quark or 2 anti-quarks

- average quark number in the presence of two static quark sources

$$Z_{QQ}(T,\mu,r) = \int \mathrm{d}U \ \mathrm{Tr L_0 Tr} L_r \ \mathrm{det D}(m_q,\mu) \ \mathrm{e}^{-\beta S_G}$$

$$N_{QQ}(T,r) = \frac{\partial \ln Z_{qq}(T,\mu,r)}{\partial \mu/T}\bigg|_{\mu=0}$$



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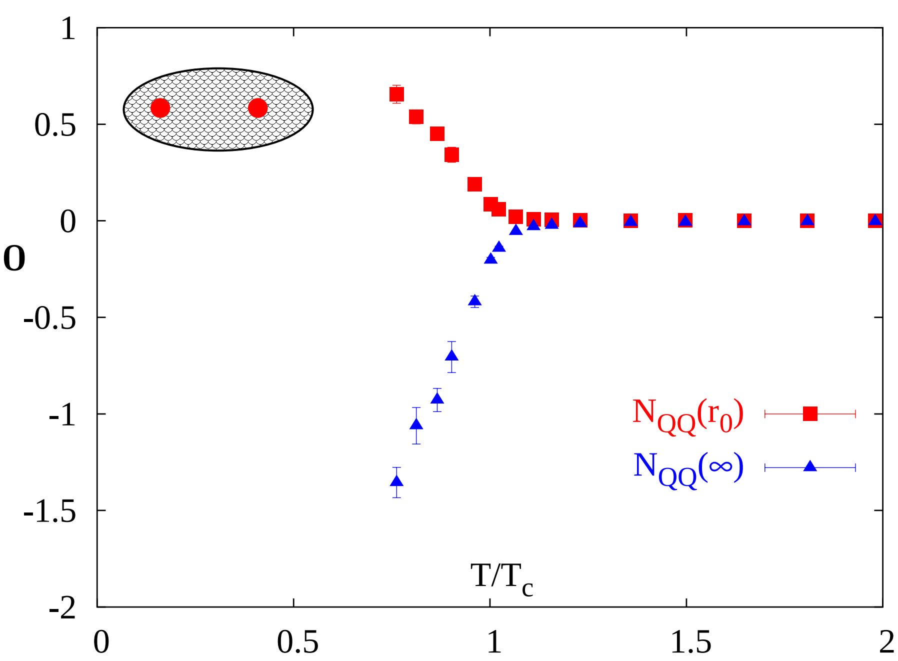

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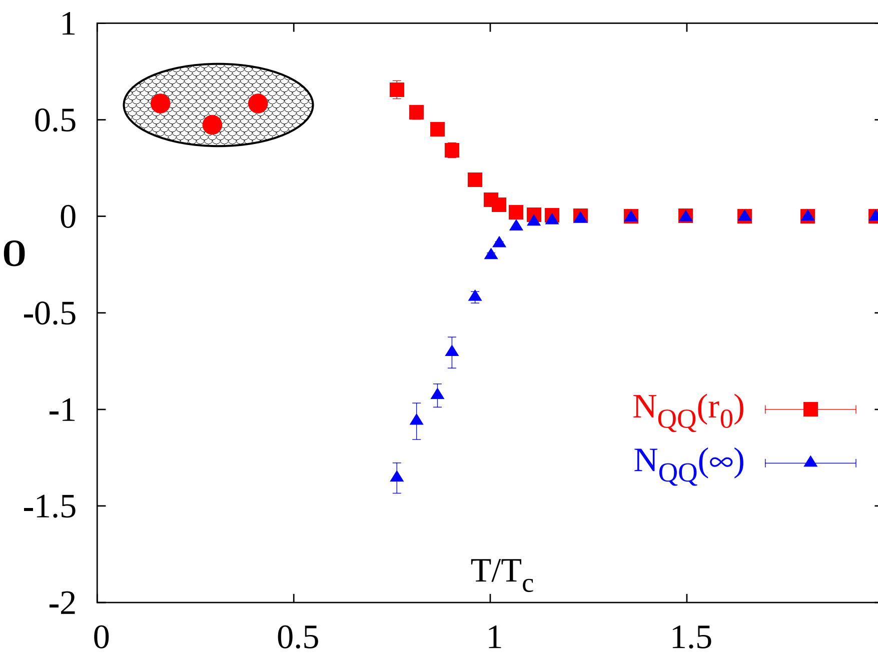

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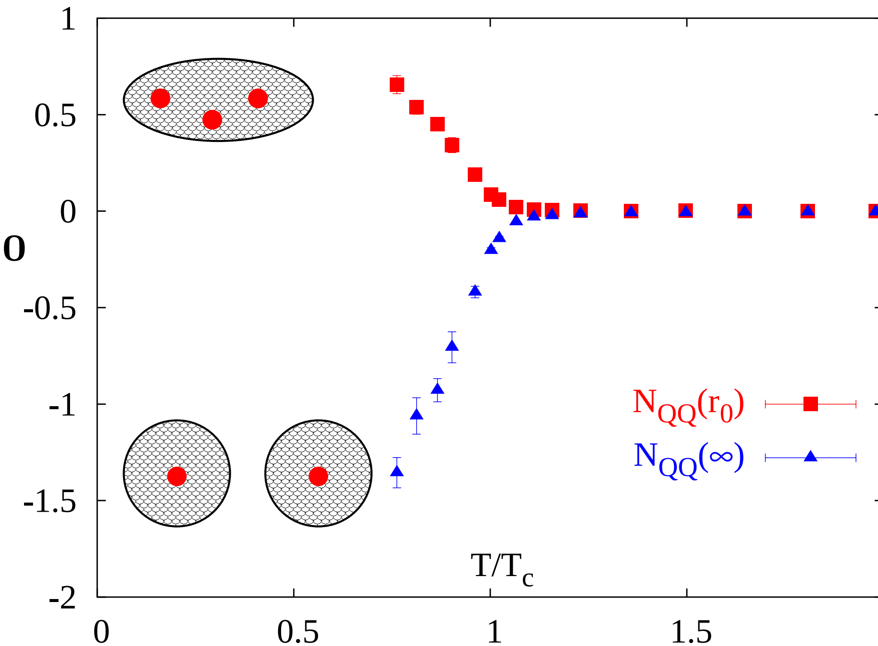

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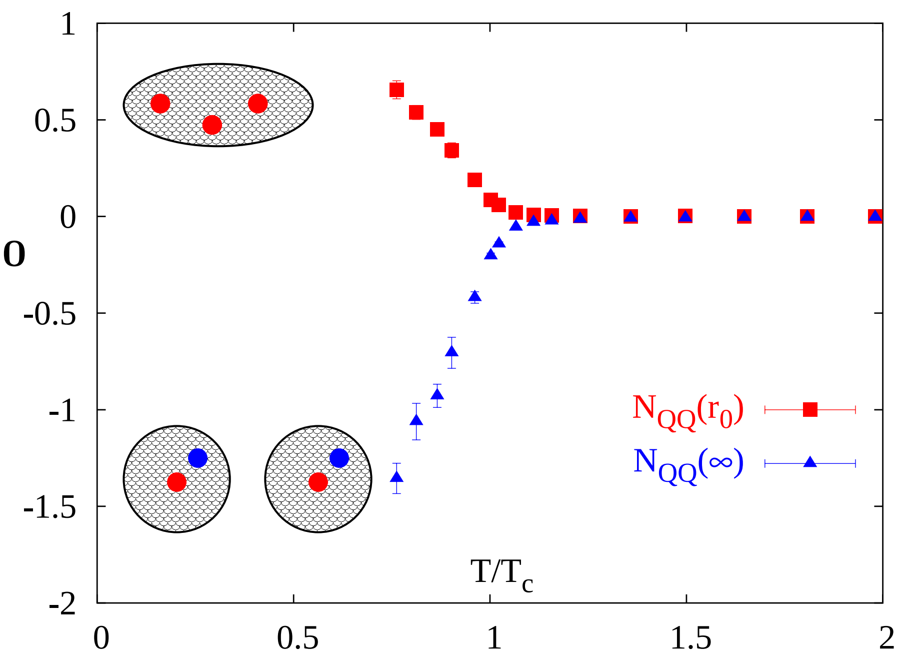

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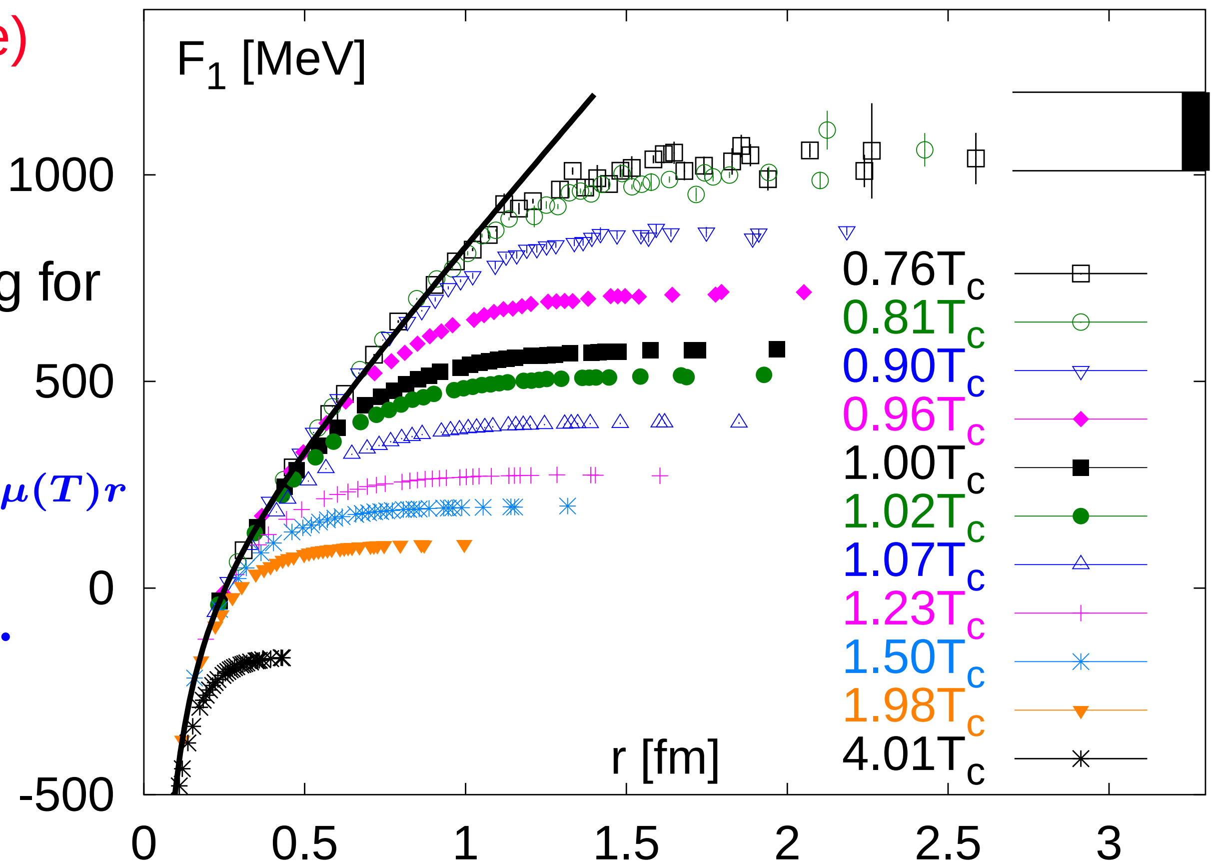

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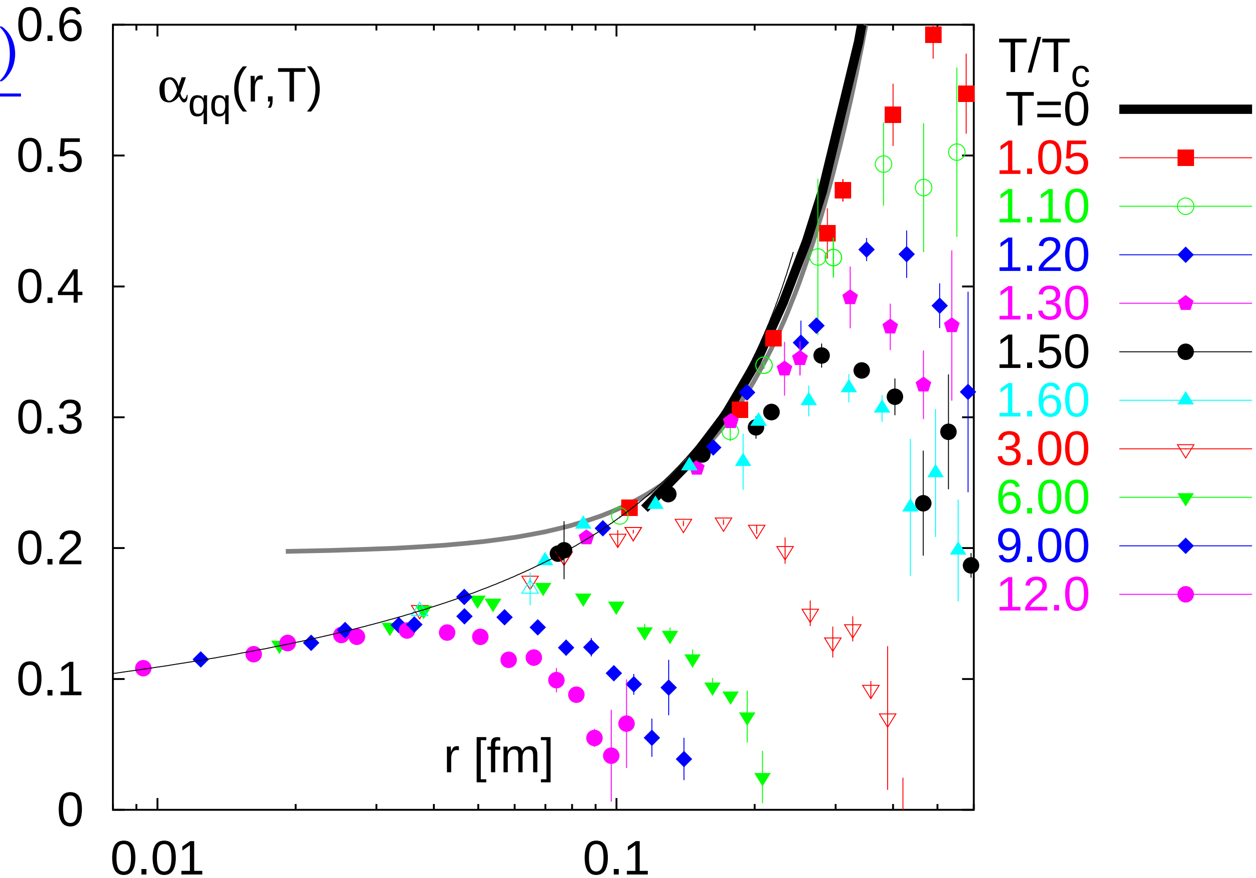

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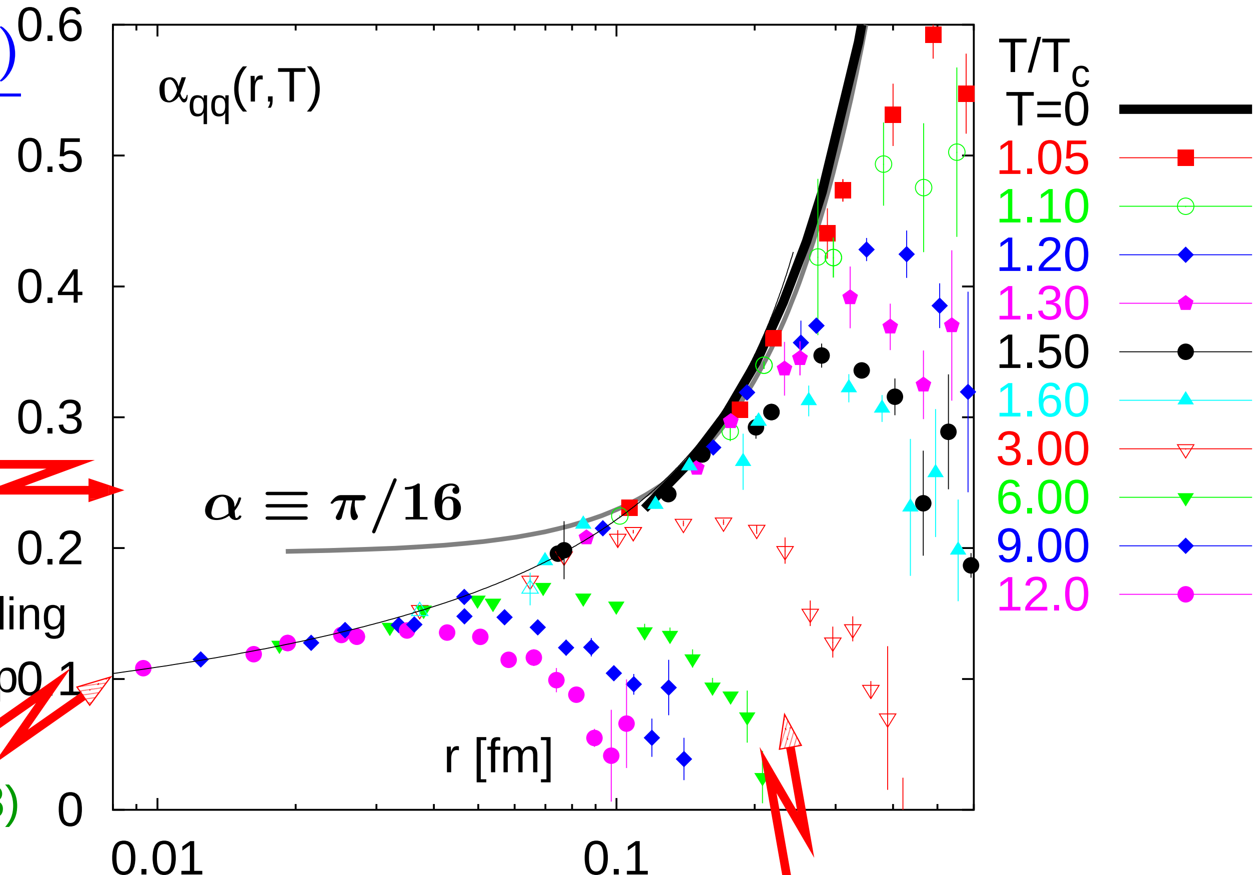

large distance: constant Coulomb term (string model)

short distance: running coupling $\alpha(r)$ from $(T=0)$, 3-loop
(S. Necco, R. Sommer, Nucl. Phys. B622 (2002) 328)

T-dependence starts in non-perturbative regime for $T \lesssim 3\,T_c$

- short distance physics ⇔ vacuum physics

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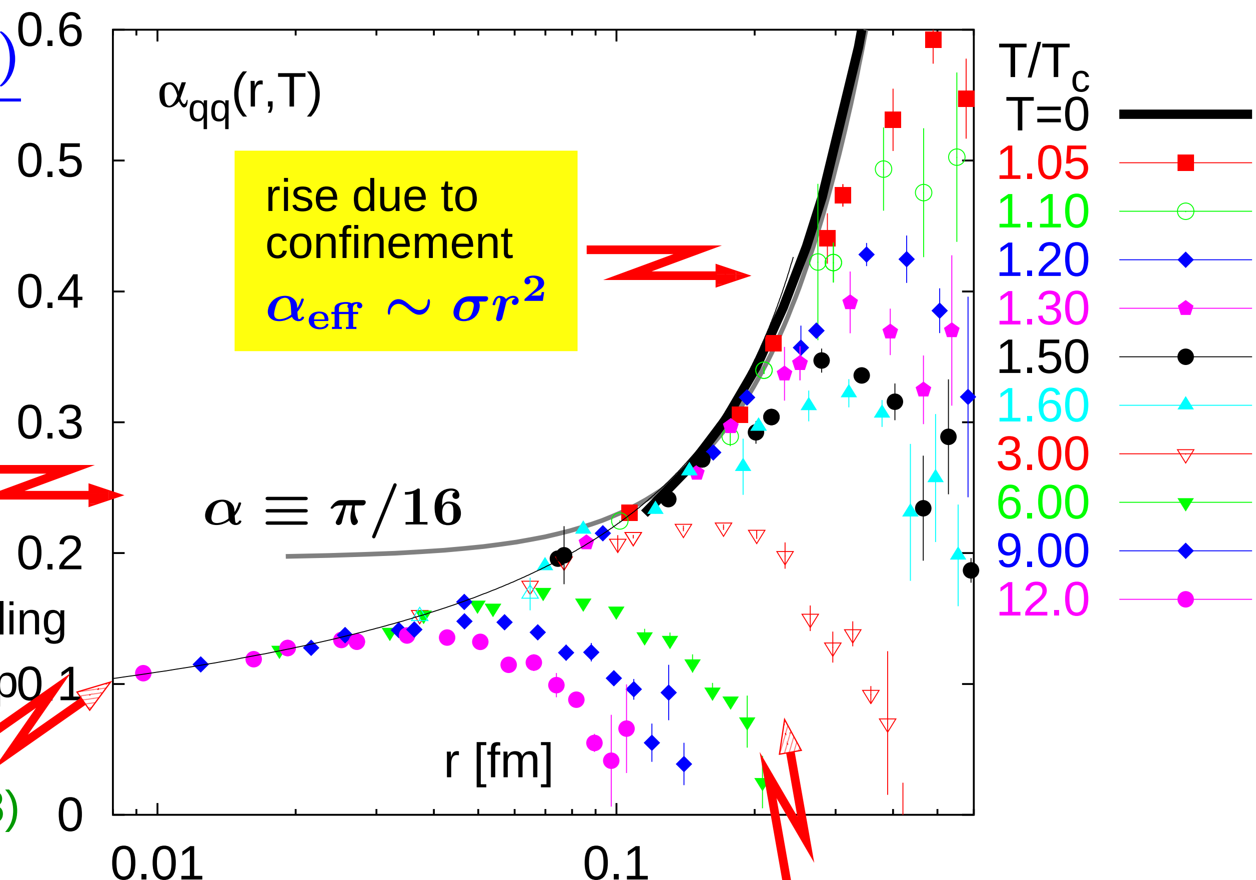

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- short distance physics $\Leftrightarrow$ vacuum physics

The spatial string tension

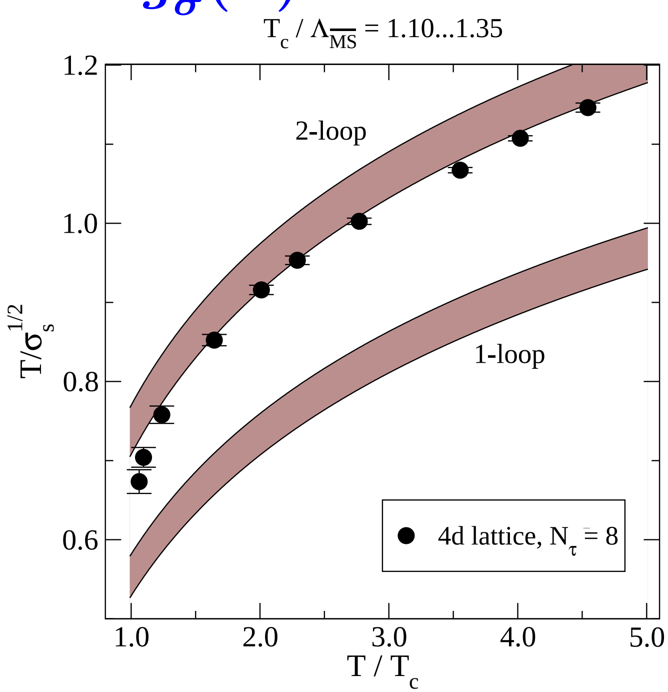
Does dimensional reduction work with light quarks?

- Non-perturbative, vanishes in high-T perturbation theory:

$$\sqrt{\sigma_s} = - \lim_{R_x, R_y \to \infty} \ln \frac{W(R_x, R_y)}{R_x R_y}$$

- $\frac{\sqrt{\sigma_s}}{g_\sigma^2(T)T} = c_\sigma \ , \ c_\sigma \equiv c_3 \ , \ g_E^2 \equiv g_\sigma^2 T$

c_3: 3-d SU(3), LGT
g_σ^2: 2-loop dim. red. pert. th.



4-d SU(3), LGT

M. Laine, Y. Schröder, JHEP 0503 (2005) 067

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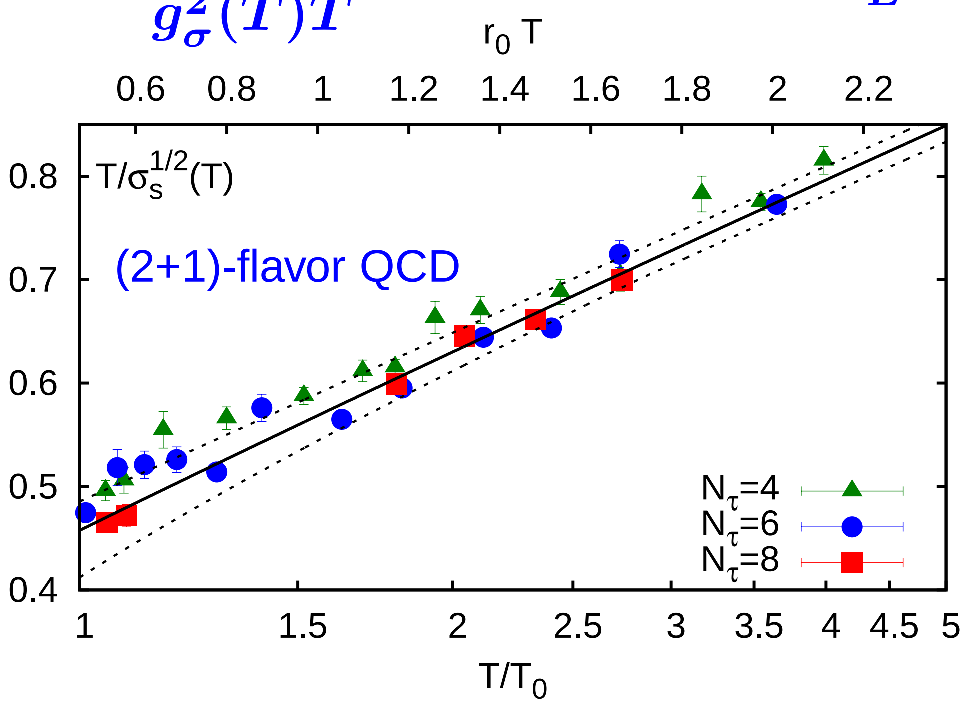


$g_\sigma^2(T_c) \simeq 3.7 \ , \ g_\sigma^2(5T_c) \simeq 2$

dimensional reduction works for $T \gtrsim 2T_c$

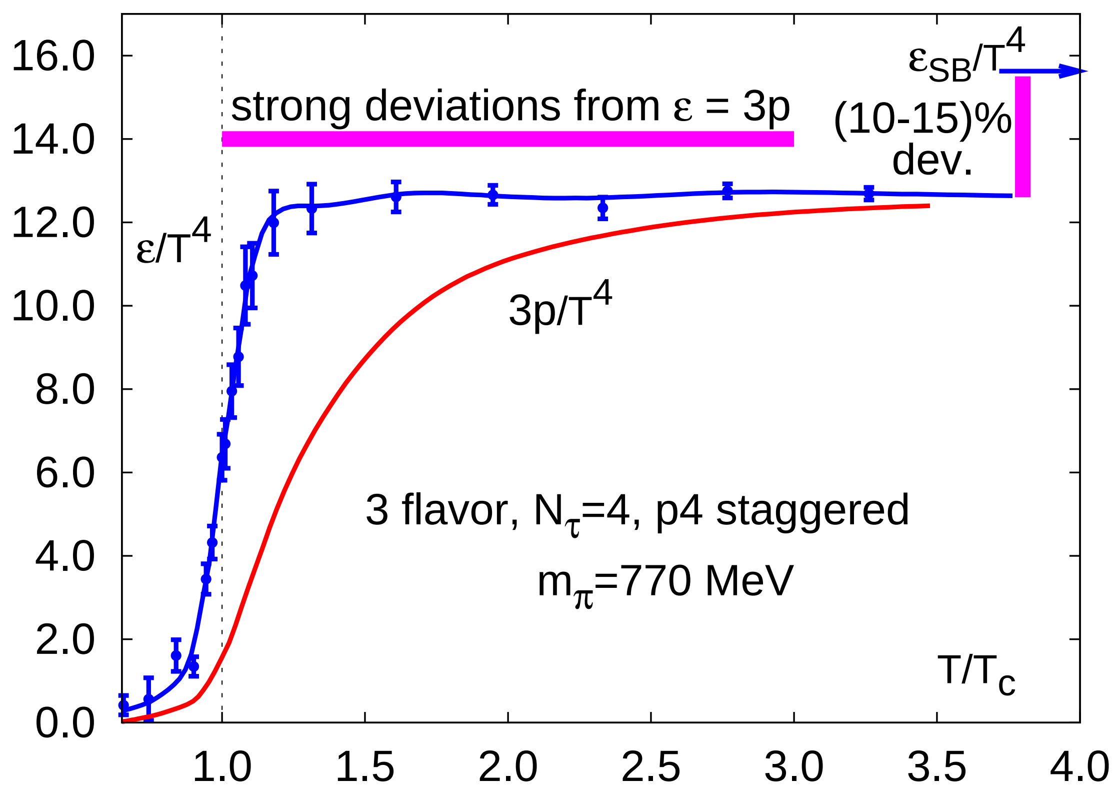
- c_M (almost) flavor independent
- $g_\sigma^2(T)$ shows 2-loop running

$c_\sigma = 0.553(1)$ [SU(3)]
$c_\sigma = 0.54(1)$ [QCD]

RBC-Bielefeld, in preparation

QCD equation of state

- two prominent features of EoS that characterize the non-perturbative structure of QCD at high temperature
  - strong deviations from ideal gas behavior ($\epsilon = 3p$) for $T_c \leq T \lesssim 3T_c$
  - deviations from Stefan-Boltzmann limit persist even at high T



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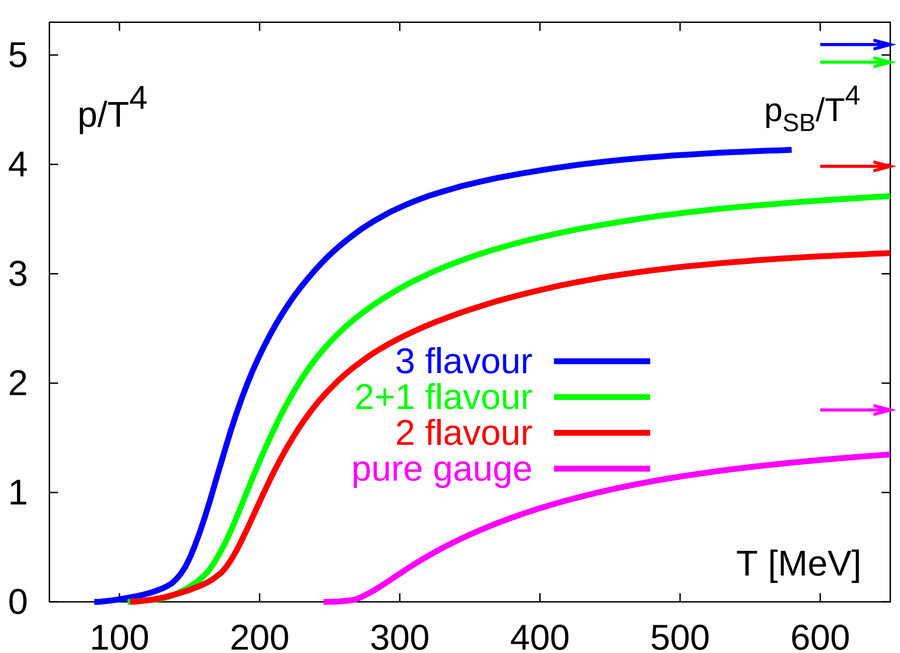
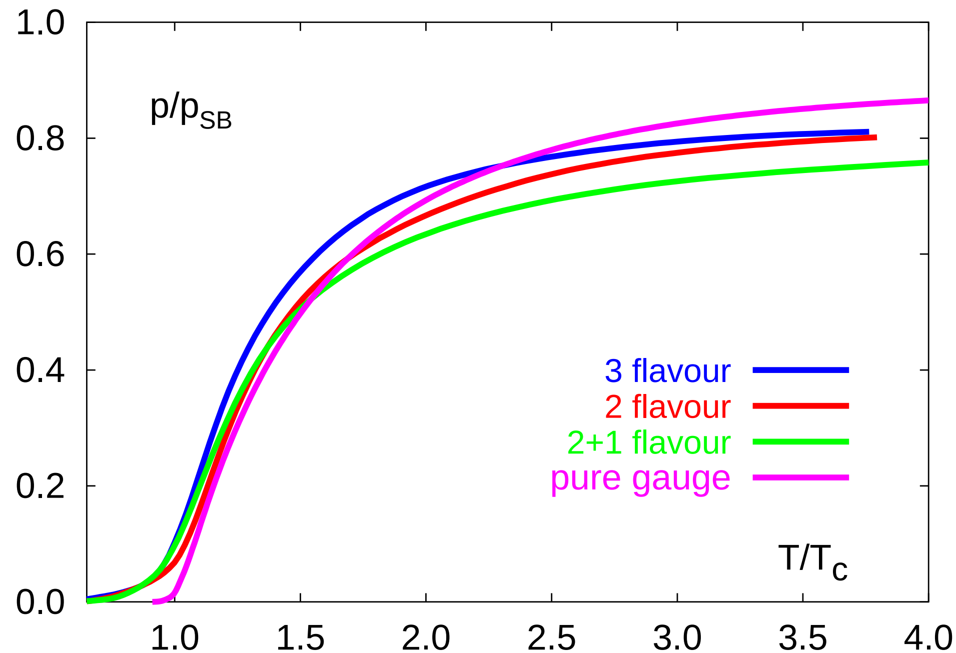
- structure of EoS is 'universal', i.e. shows little quark mass dependence in ϵ/ϵ_{SB} vs. T/T_c
- quark content changes only 'details'

The pressure revisited

- $T \gtrsim (2\text{-}3)T_c$: deviations from ideal gas understood in terms of HTL-resummed perturbation theory
- $T \lesssim 2T_c$: strong deviations from ideal gas
- deviations from p_{SB} almost flavor independent

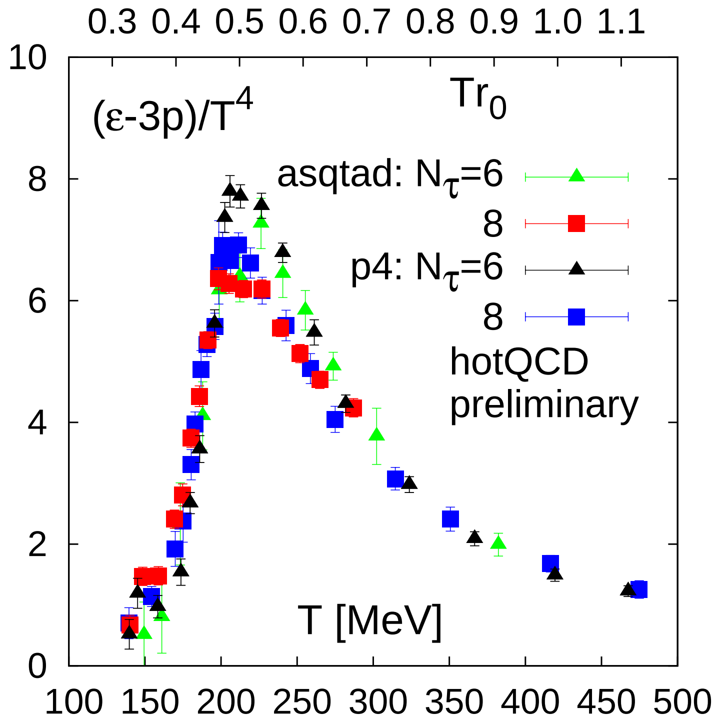




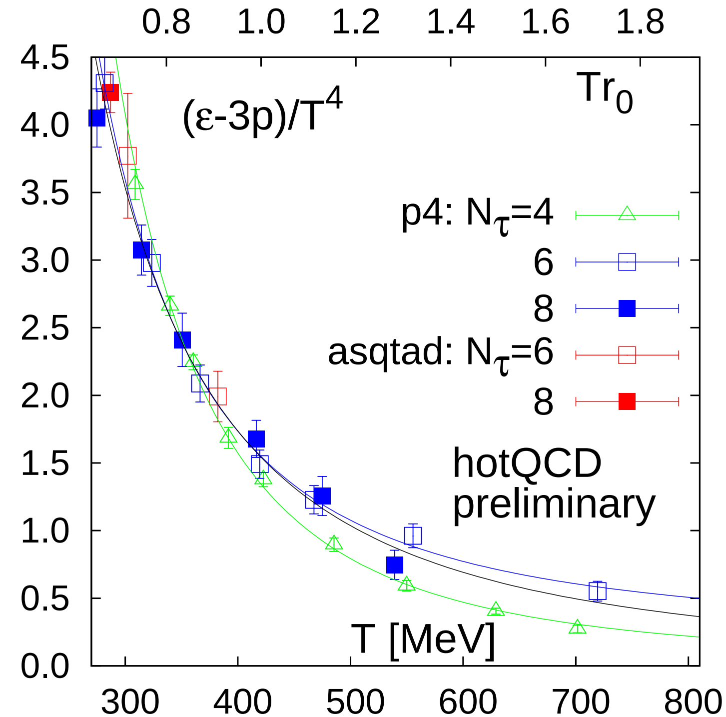
FK, E. Laermann and A. Peikert, Phys. Lett. B478 (2000) 447

(2+1)-flavor QCD: ...towards the cont. limit ($N_\tau = 8$)

with light quarks ($m_\pi \simeq 220$ MeV)



trace anomaly

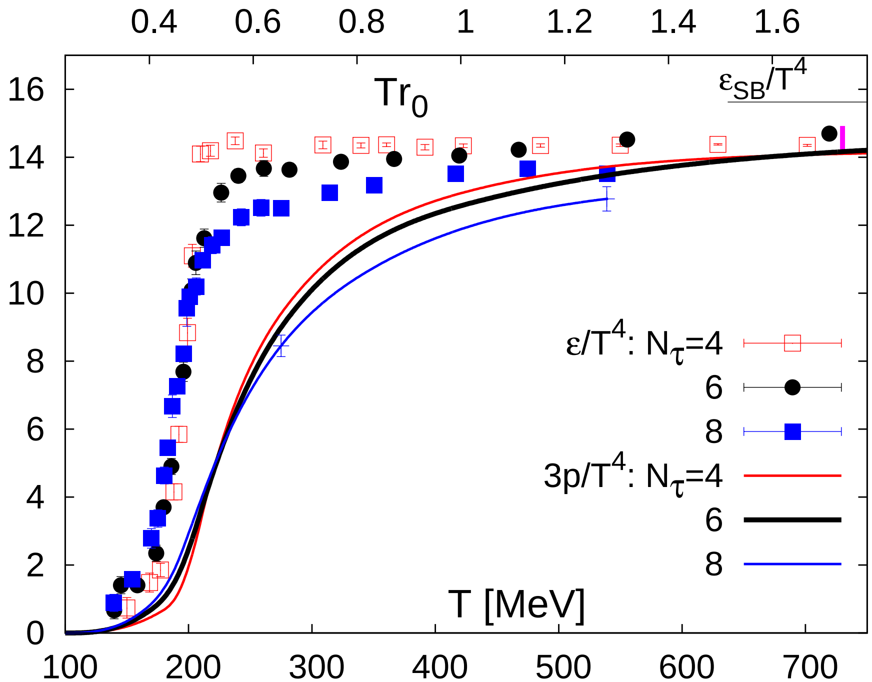


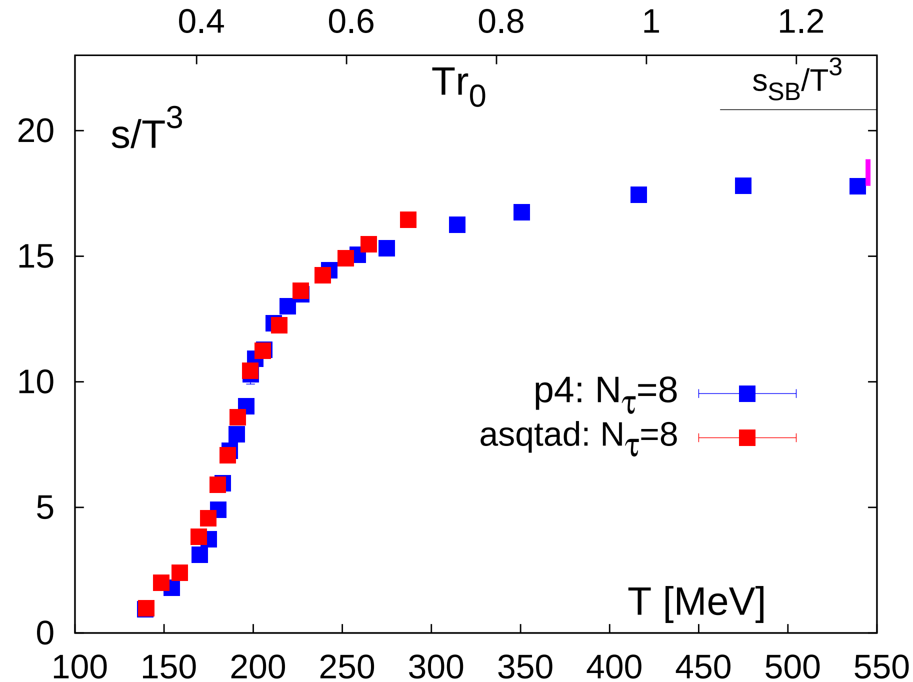
high-T behavior

- non-conformal:
  $[(\epsilon - 3p)/T^4]_{max} \simeq 7$
  at $T_{max} \simeq 200$ MeV
  ($\sim$ softest point of EoS)
- some cut-off effects in the peak region; p4 and asqtad agree
- $T \gtrsim 300$ MeV: good agreement between $N_\tau = 6$ and 8 results
- non-perturbative:
  $(\epsilon - 3p)/T^4 \sim A/T^2 + B/T^4$
  for $1.5T_c \lesssim T \lesssim 4T_c$

Pressure, Energy and Entropy

- p/T^4 from integration over $(\epsilon - 3p)/T^5$;
  using piecewise quadratic fit with $T_0 = 100$ MeV with $p(T_0) = 0$;
- systematic error on $3p/T^4 \simeq 0.33$
- good scaling behavior; good agreement between different discretization schemes

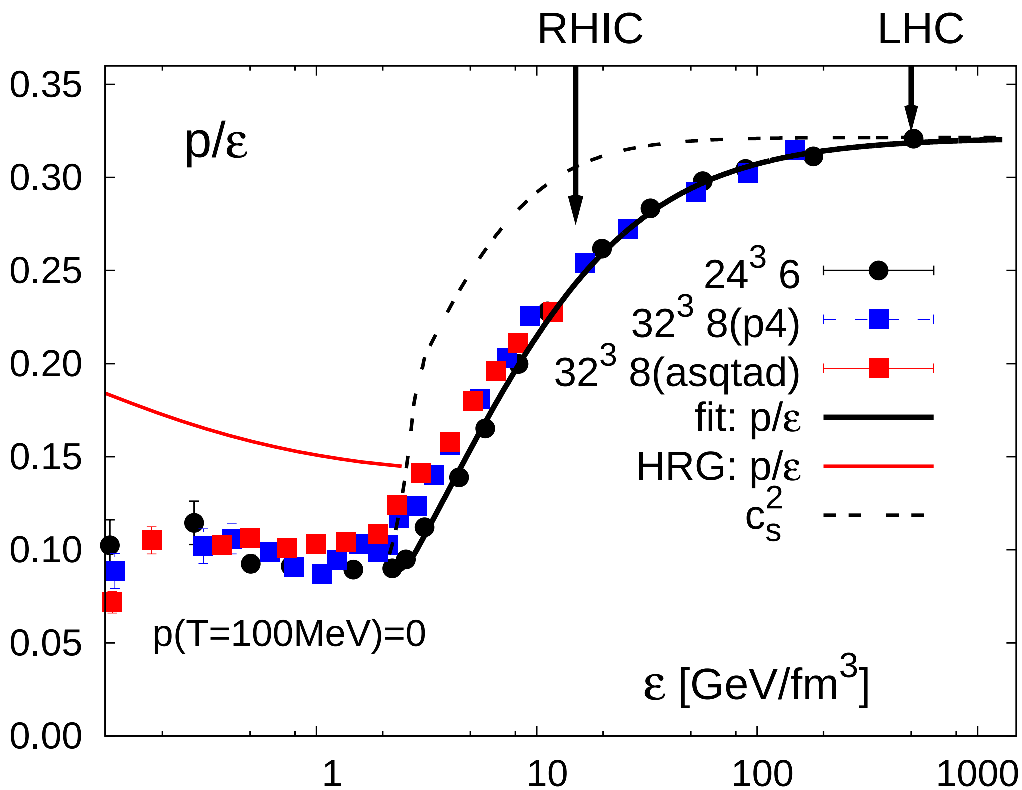




EoS and velocity of sound

- $p/\epsilon \ \Rightarrow$ velocity of sound:

$$c_s^2 = \frac{\mathrm{d}p}{\mathrm{d}\epsilon} = \epsilon \frac{\mathrm{d}(p/\epsilon)}{\mathrm{d}\epsilon} + \frac{p}{\epsilon} \equiv \frac{s}{c_V}$$



pressure set to zero
at $T = 100$ MeV

hotQCD preliminary

fit: $p/\epsilon = c - a/(1 + b\epsilon)$ for $\epsilon \gtrsim 4$ GeV/fm^3

SU(3) Thermodynamics - revisited: $\langle G^2 \rangle_T$

- SU(3) EoS deviates from ideal gas by about 15% at $4T_c$
- slow approach to the high temperature limit
- consistent with logarithmic running of the coupling (cf. 4d vs. 3d)

trace anomaly

$T = 0$: non-vanishing gluon condensate

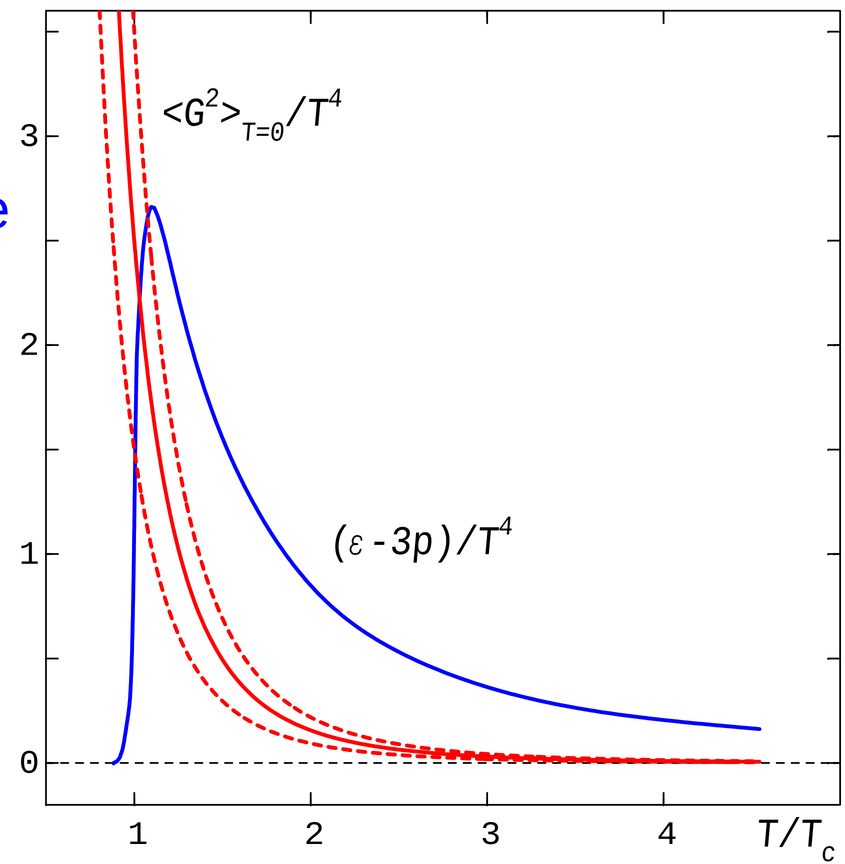
$$\epsilon - 3p = \langle G^2 \rangle_{T=0} - \langle G^2 \rangle_T$$

curve: $\langle G^2 \rangle_{T=0} = (1-2)$GeV/fm^3

non-pertubative vacuum properties show up at high-T, but die out early

$\epsilon - 3p \sim T^2$;

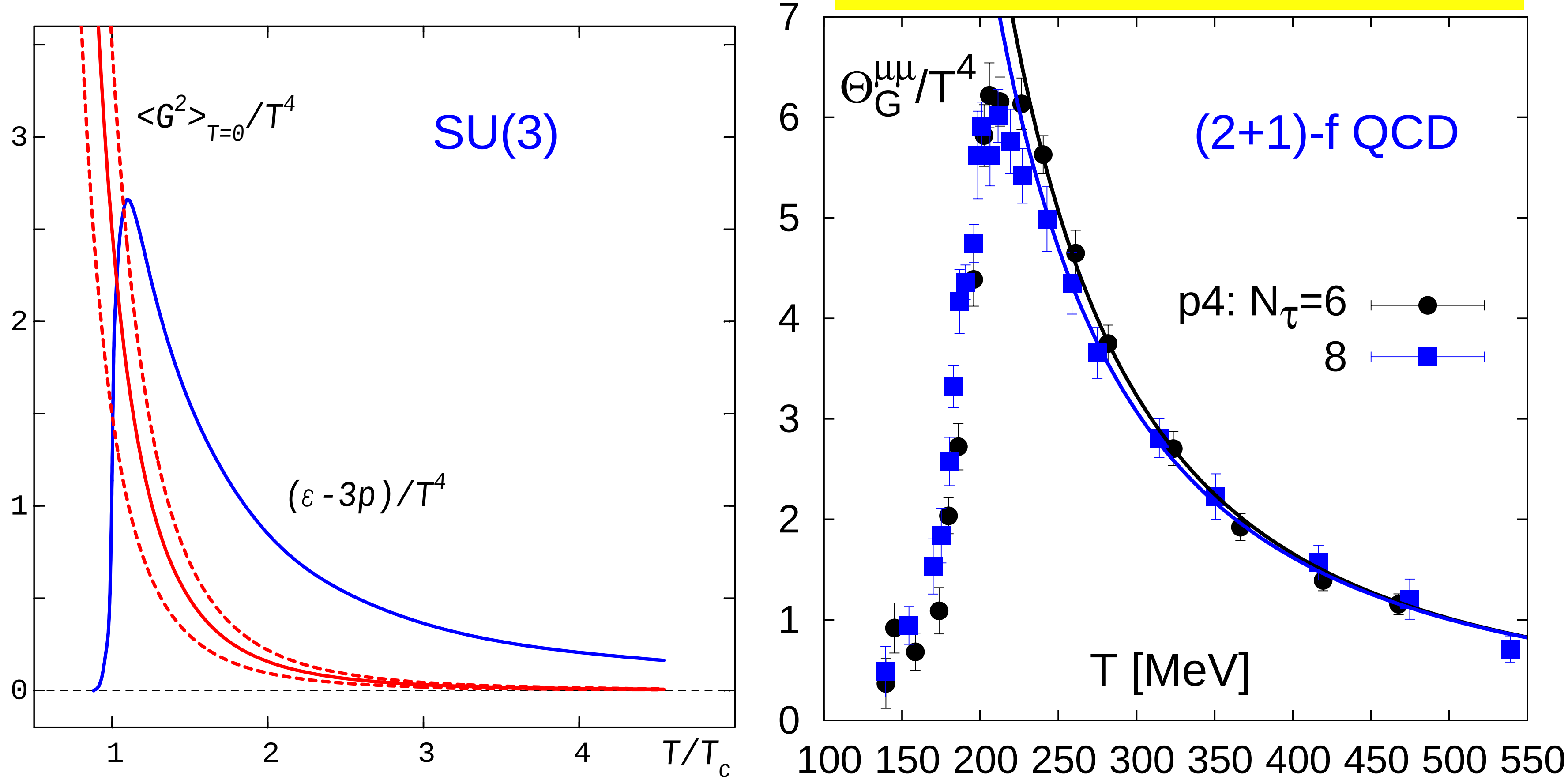
fuzzy bag: R. Pisarski (2007)



(2+1)-flavor QCD: Gluon condensate: $\langle G^2 \rangle_T$

- $(\epsilon - 3p)/T^4 = \Theta_G^{\mu\mu}/T^4 + \Theta_F^{\mu\mu}/T^4$
- $\Theta_F^{\mu\mu}/T^4 = \sum_f m_f \left(\langle \bar{\psi}\psi \rangle_{T=0} - \langle \bar{\psi}\psi \rangle_T \right)$
- $\Theta_G^{\mu\mu} = \langle G^2 \rangle_{T=0} - \langle G^2 \rangle_T$

$$\Theta_G^{\mu\mu}/T^4 = A/T^2 + 4B/T^4$$



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$$\Theta_G^{\mu\mu}/T^4 = A/T^2 + 4B/T^4$$

fits:

bag model: $B \equiv$ bag const.

$B = (0.001 - 0.002)$GeV^4

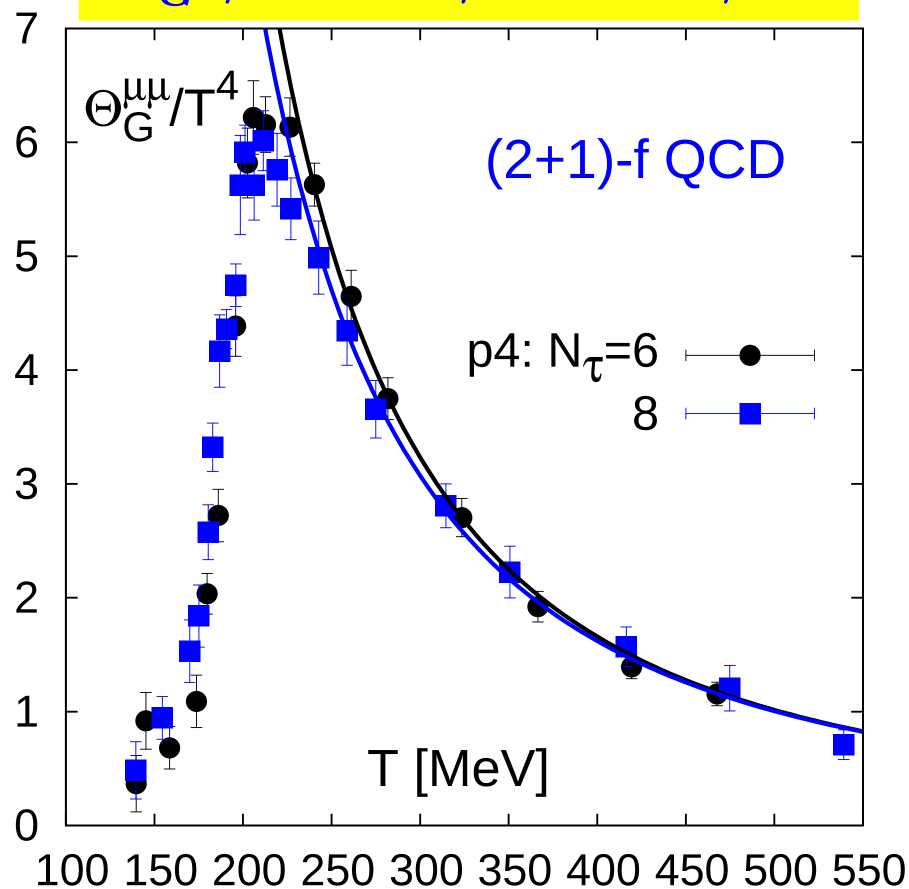
$\Rightarrow \; B^{1/4} = (180 - 300)$ MeV

$A = 0.24(2)$GeV^2

$\Rightarrow \; A^{1/2} \simeq 500$ MeV

slow approach to

perturbative regime



Hadronic fluctuations at $\mu = 0$ from Taylor expansion coefficients for $\mu > 0$

$n_f = 2,\ m_\pi \simeq 770$ MeV: S. Ejiri, FK, K.Redlich, PLB633 (2006) 275
$n_f = 2+1,\ m_\pi \simeq 220$ MeV: RBC-Bielefeld, preliminary

- Taylor expansion of bulk thermodynamics in terms of $\mu_{q,s}$

$$\begin{aligned} \frac{p}{T^4} &\equiv \frac{1}{VT^3} \ln Z(V,T,\mu_q,\mu_s) \\ &= \sum_{i,j} c_{i,j} \left(\frac{\mu_q}{T}\right)^i \left(\frac{\mu_s}{T}\right)^j \end{aligned}$$

- expansion coefficients evaluated at $\mu_{q,s} = 0$ are related to fluctuations of $B,\ S,\ Q$ at $\mu_{B,S,Q} = 0$:

$\Uparrow$ baryon number, strangeness, charge fluctuations

event-by-event fluctuations at RHIC and LHC

Hadronic fluctuations at $\mu = 0$ from Taylor expansion coefficients for $\mu > 0$

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- higher derivatives $\Rightarrow$ higher moments
- mixed derivatives $\Rightarrow$ correlations

$$
\begin{aligned}
2c_2^x &= \frac{\partial^2 p/T^4}{\partial(\mu_x/T)^2} = \frac{1}{VT^3}\langle(\delta N_x)^2\rangle_{\mu=0} = \frac{1}{VT^3}\langle N_x^2\rangle_{\mu=0} \\
24c_4^x &= \frac{\partial^4 p/T^4}{\partial(\mu_x/T)^4} = \frac{1}{VT^3}\left(\langle(\delta N_x)^4\rangle - 3\langle(\delta N_x)^2\rangle^2\right)_{\mu=0} = \frac{1}{VT^3}\left(\langle N_x^4\rangle - 3\langle N_x^2\rangle^2\right)_{\mu=0} \\
4c_{22}^{xy} &= \frac{\partial^4 p/T^4}{\partial(\mu_x/T)^2\partial(\mu_y/T)^2} = \frac{1}{VT^3}\left[\langle N_x^2 N_y^2\rangle - 2\langle N_x N_y\rangle^2 - \langle N_x^2\rangle\langle N_y^2\rangle\right]_{\mu=0}
\end{aligned}
$$

$$\mathbf{with}\quad x = q,\ s$$

Hadronic fluctuations and chiral symmetry restoration

- expect 2^{nd} order transition in 3-d, O(4) symmetry class;

  scaling field: $t = \left| \frac{T-T_c}{T_c} \right| + A \left(\frac{\mu_q}{T_c} \right)^2$, $\mu_{crit} = 0$

  singular part: $f_s(T, \mu_u, \mu_d) = b^{-1} f_s(t b^{1/(2-\alpha)}) \sim t^{2-\alpha}$

$$\frac{\partial^2 \ln \mathcal{Z}}{\partial \mu_q^2} \sim t^{1-\alpha} \qquad , \qquad \frac{\partial^4 \ln \mathcal{Z}}{\partial \mu_q^4} \sim t^{-\alpha} \qquad (\mu = 0)$$

- O(4)/O(2): $\alpha < 0$, small $\Rightarrow$

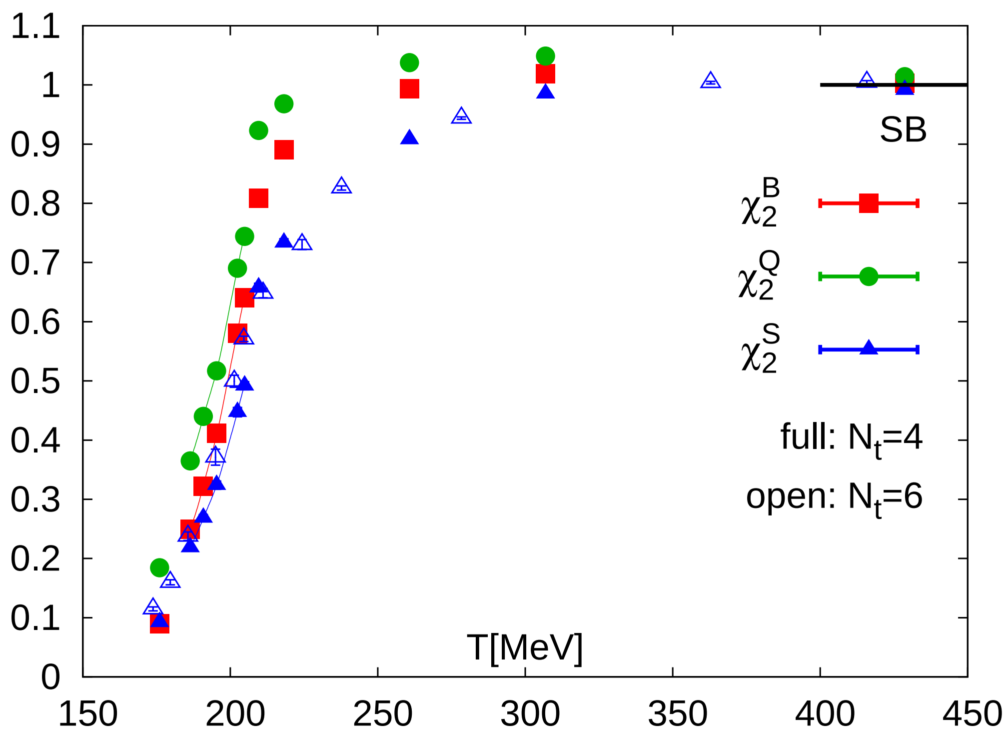
  $\langle (\delta N_q)^2 \rangle$ dominated by T-dependence of regular part

  $\langle (\delta N_q)^4 \rangle$ develops a cusp

Quadratic fluctuations of baryon number charge & strangeness in (2+1)-flavor QCD

RBC-Bielefeld, preliminary

vanishing chemical potentials:



$$\chi_2^Q = \frac{1}{VT^3}\langle Q^2\rangle$$

$$\chi_2^B = \frac{1}{VT^3}\langle N_B^2\rangle$$

$$\chi_2^S = \frac{1}{VT^3}\langle N_S^2\rangle$$

rapid approach to SB limit

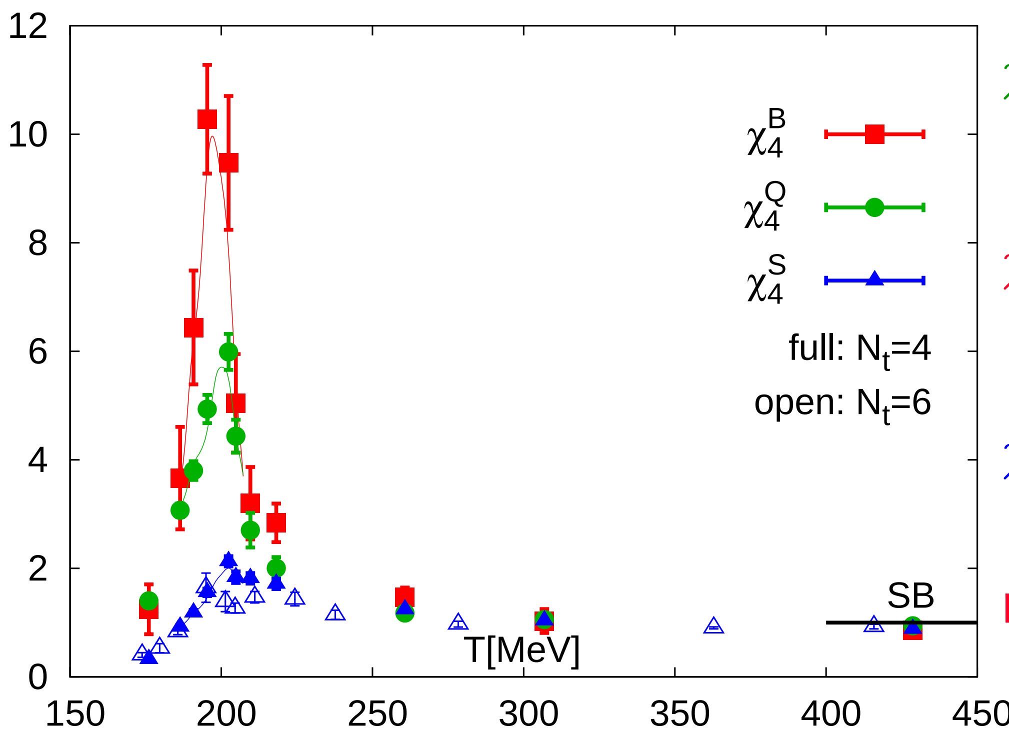
$\Rightarrow$ smooth change of quadratic fluctuations across transition region

chiral limit: $\chi_2^B,\ \chi_2^Q \sim |T-T_c|^{1-\alpha} + \mathbf{regular}$

Quartic fluctuations of baryon number charge & strangeness in (2+1)-flavor QCD

RBC-Bielefeld, preliminary

vanishing chemical potentials:



$$\chi_4^Q = \frac{1}{VT^3}\left(\langle Q^4\rangle - 3\langle Q^2\rangle^2\right)$$

$$\chi_4^B = \frac{1}{VT^3}\left(\langle N_B^4\rangle - 3\langle N_B^2\rangle^2\right)$$

$$\chi_4^S = \frac{1}{VT^3}\left(\langle N_S^4\rangle - 3\langle N_S^2\rangle^2\right)$$

rapid approach to SB limit

$\Rightarrow$ large light quark number & charge fluctuations across transition region

chiral limit: $\chi_4^B,\ \chi_4^Q \sim |T-T_c|^{-\alpha} + \mathbf{regular}$

Quark number in Boltzmann approximation

$$\begin{aligned} p_m/T^4 &= F(T,m,V)\cosh(B\mu_q/T) \\ d_2^q &\equiv \frac{\partial^2 p_m/T^4}{\partial(\mu_q/T)^2} = B^2 F(T,m,V)\cosh(B) \\ d_4^q &\equiv \frac{\partial^4 p_m/T^4}{\partial(\mu_q/T)^4} = B^4 F(T,m,V)\cosh(B) \end{aligned}$$

ratio of fourth (d_4^q) and second (d_2^q) cumulant of quark number fluctuation gives "unit of charge" carried by the particle with mass "m":

$$m \gg T \quad \Rightarrow \quad R_{4,2}^q \equiv \frac{d_4^q}{d_2^q} = B^2$$

Charge fluctuations in Boltzmann approximation

- hadronic resonance gas: contributions from isosinglet ($G^{(1)}:\ \eta,\ldots$) and isotriplet ($G^{(3)}:\ \pi,\ldots$) mesons as well as isodoublet ($F^{(2)}:\ p,n,\ldots$) and isoquartet ($F^{(4)}:\ \Delta,\ldots$) baryons

$$\begin{aligned}\frac{p(T,\mu_q,\mu_I)}{T^4} &\simeq G^{(1)}(T)+G^{(3)}(T)\frac{1}{3}\left(2\cosh\left(\frac{2\mu_I}{T}\right)+1\right)\\ &+F^{(2)}(T)\cosh\left(\frac{3\mu_q}{T}\right)\cosh\left(\frac{\mu_I}{T}\right)\\ &+F^{(4)}(T)\frac{1}{2}\cosh\left(\frac{3\mu_q}{T}\right)\left[\cosh\left(\frac{\mu_I}{T}\right)+\cosh\left(\frac{3\mu_I}{T}\right)\right]\end{aligned}$$

- charge fluctuations at $\mu_q=\mu_I=0$;
  isospin quartet $F^{(4)}$ contains baryons carrying charge 2

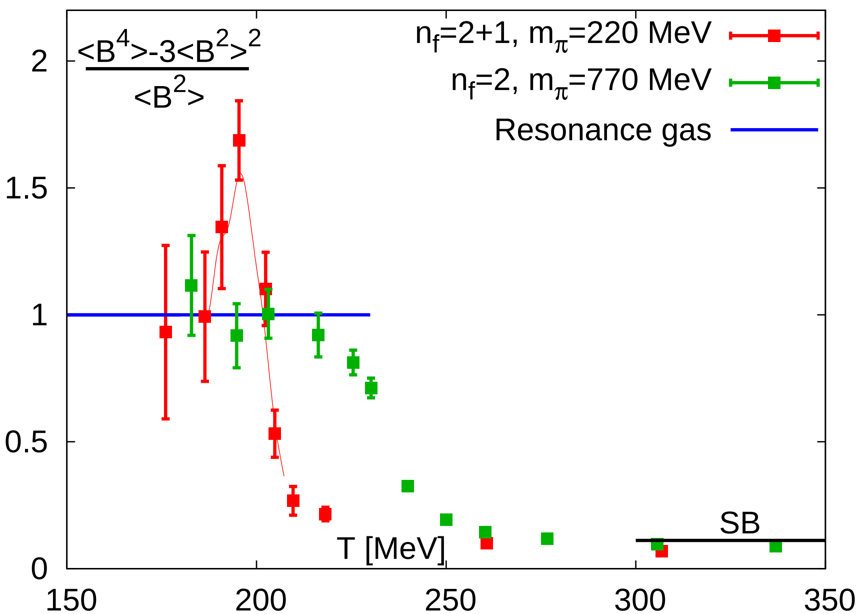
$$R^Q_{4,2}\equiv\frac{d^Q_4}{d^Q_2} = \frac{4G^{(3)}+3F^{(2)}+27F^{(4)}}{4G^{(3)}+3F^{(2)}+9F^{(4)}} \rightarrow 1 \text{ for } T\rightarrow 0$$

  contribution of doubly charged baryons increases quartic relative to quadratic fluctuations

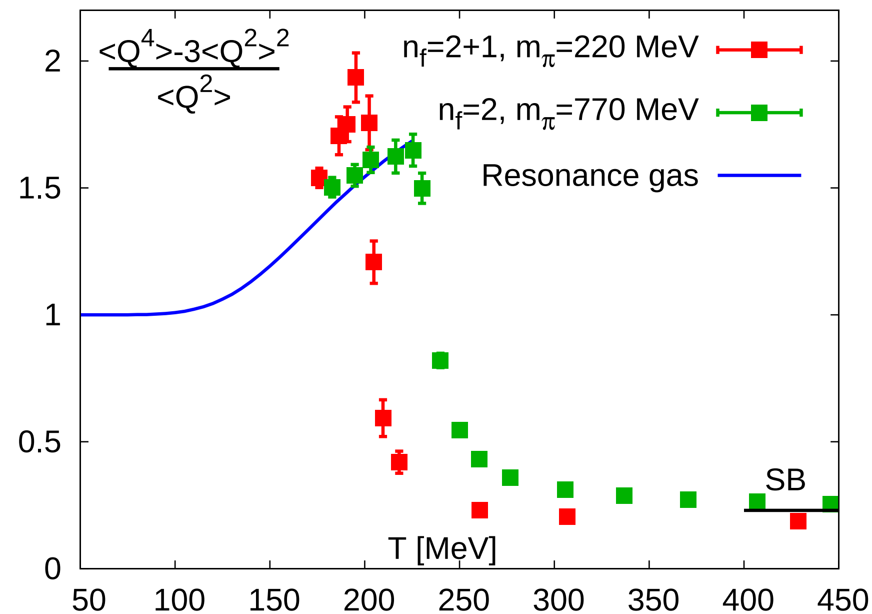
Ratios of quartic and quadratic fluctuations of charges in (2+1)-flavor QCD

RBC-Bielefeld, preliminary

baryon number fluctuation



charge fluctuation



chiral limit: ratios $\sim |T - T_c|^{-\alpha} + \mathbf{regular}$

$\Rightarrow$ enhancement over resonance gas values

$\Rightarrow$ may be observable in event-by-event fluctuations

(valence) quark sector quickly ($T \gtrsim 1.5T_c$) behaves perturbative

Deconfinement and χ-symmetry

- The chiral phase transition (i.e. at $m_q = 0$) is deconfining
  - true in QCD, i.e. SU(3) + fermions in the fundamental representation
  - SU(3) + fermions in the adjoint representation: $T_{deconf} < T_\chi$
- The transition in QCD with physical quark masses is a crossover

  In which sense is the transition

  deconfining and chiral symmetry restoring?

  - deconfinement: heavy hadrons $\Rightarrow$ light quarks and gluons; liberation of many new light degrees of freedom
    $\Rightarrow$ rapid change in ϵ/T^4, s/T^3, ...
  - chiral symmetry restoration: vanishing mass splittings, no new degrees of freedom
    $\Rightarrow$ minor effect on bulk thermodynamics, but
    rapid change of chiral condensate

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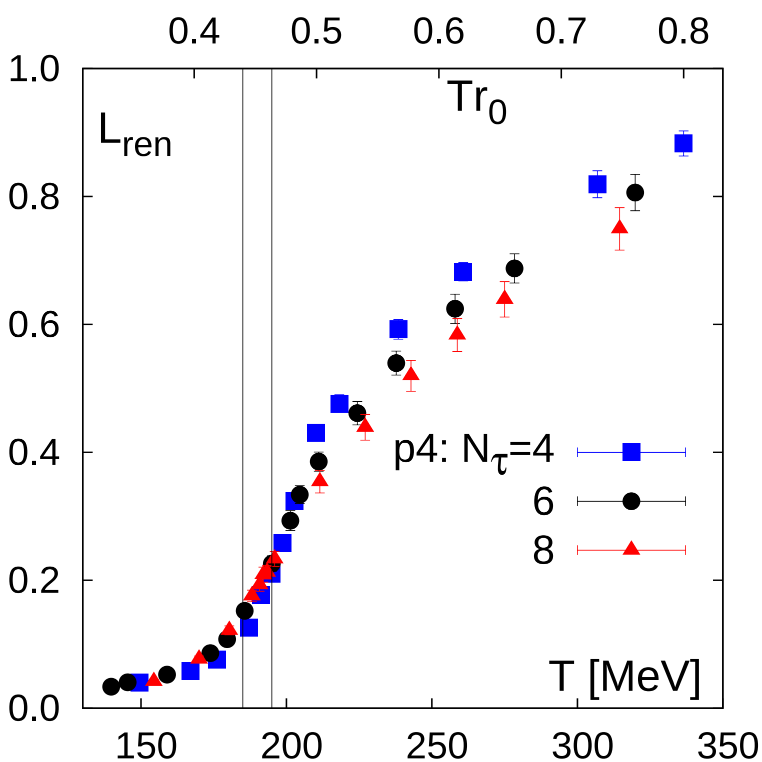
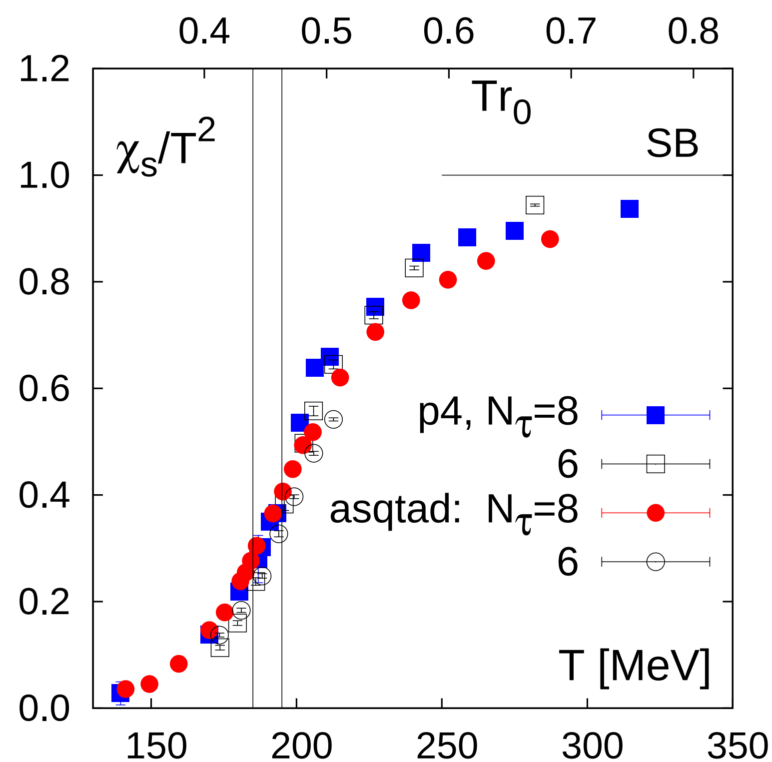
Deconfinement

- renormalized Polyakov loop and strange quark number susceptibility

$$L_{ren} \sim \mathrm{e}^{-F_Q(T)/T}, \qquad \chi_s/T^2 \sim \langle N_s^2 \rangle$$

band: $185\mathrm{MeV} \leq T \leq 195\mathrm{MeV}$





$N_\tau = 4,\ 6$ (p4): RBC-Bielefeld, PRD77, 014511 (2008)

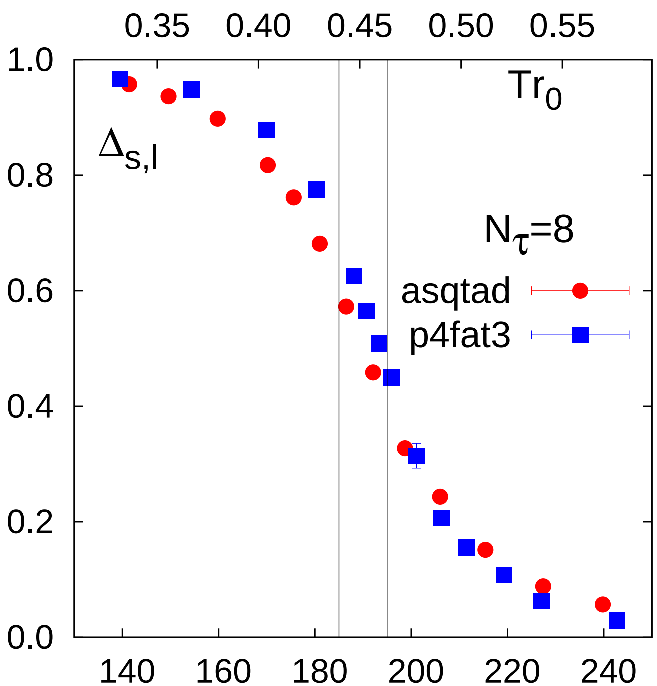
$N_\tau = 8$, and $N_\tau = 6$ (asqtad): hotQCD, preliminary

χ-condensate and susceptibility

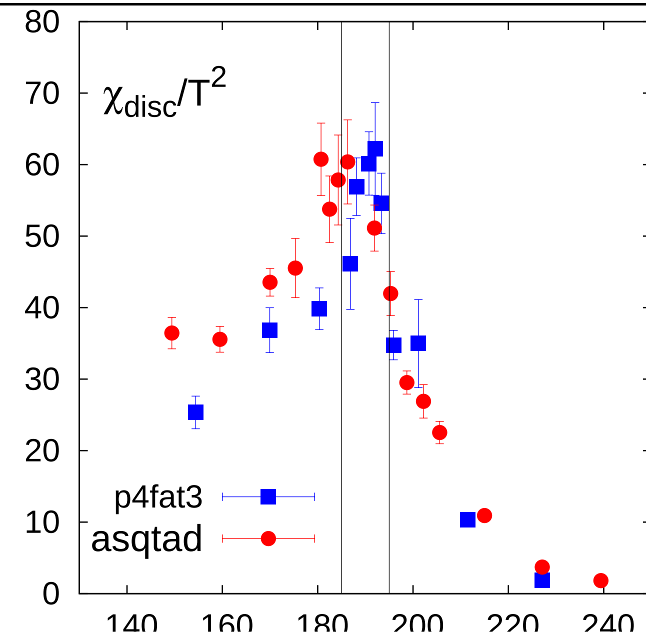
- sudden change in chiral condensate is, of course, related to peaks in the (singlet) chiral susceptibility

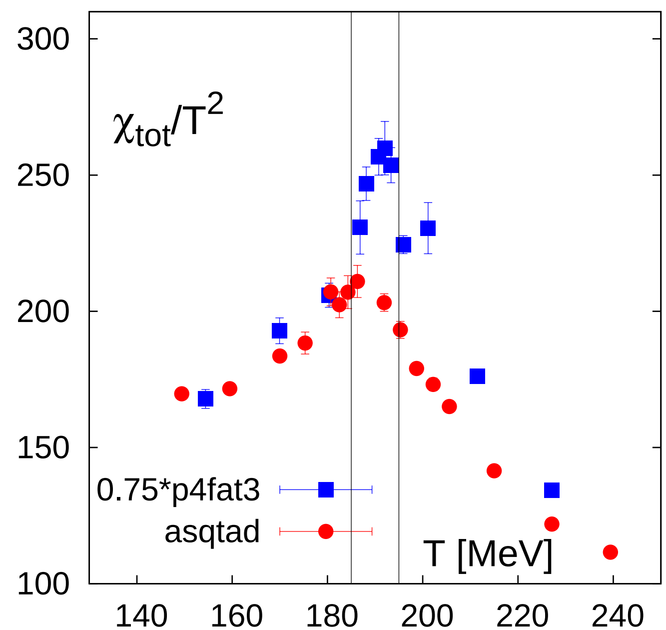
$$\chi_{tot}/T^2 = 2\chi_{dis}/T^2 + \chi_{con}/T^2$$

$$\Delta_{l,s}(T) = \frac{\langle\bar{\psi}\psi\rangle_{l,T} - \frac{m_l}{m_s}\langle\bar{\psi}\psi\rangle_{s,T}}{\langle\bar{\psi}\psi\rangle_{l,0} - \frac{m_l}{m_s}\langle\bar{\psi}\psi\rangle_{s,0}}$$



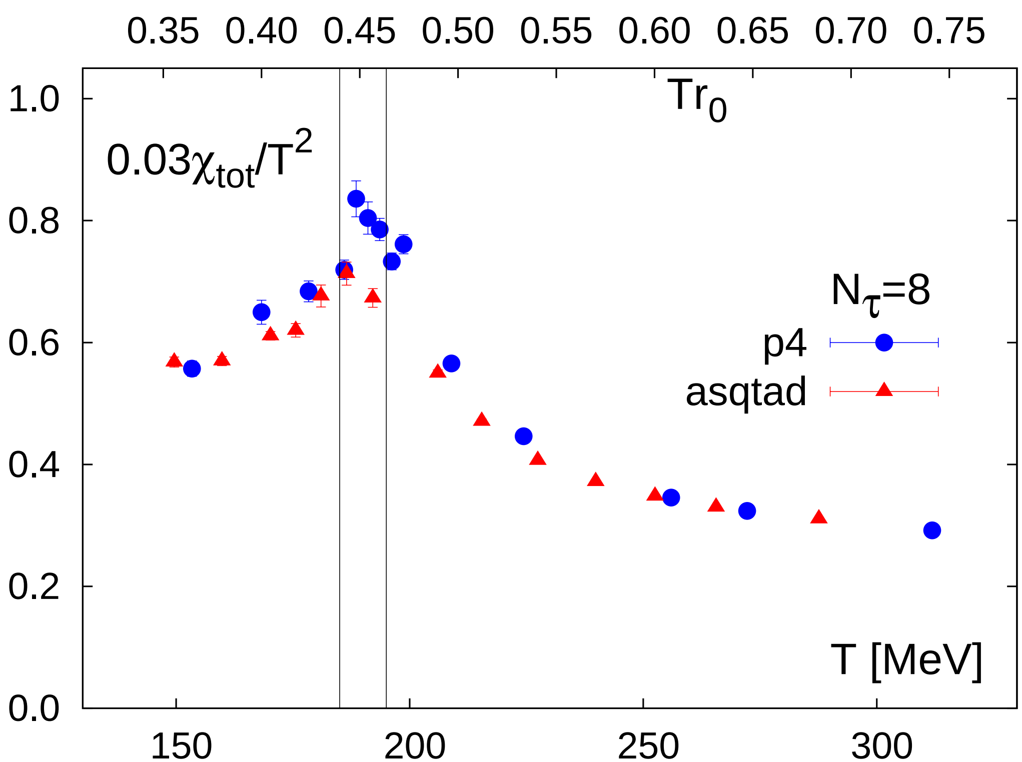
p4 and asqtad: hotQCD, preliminary





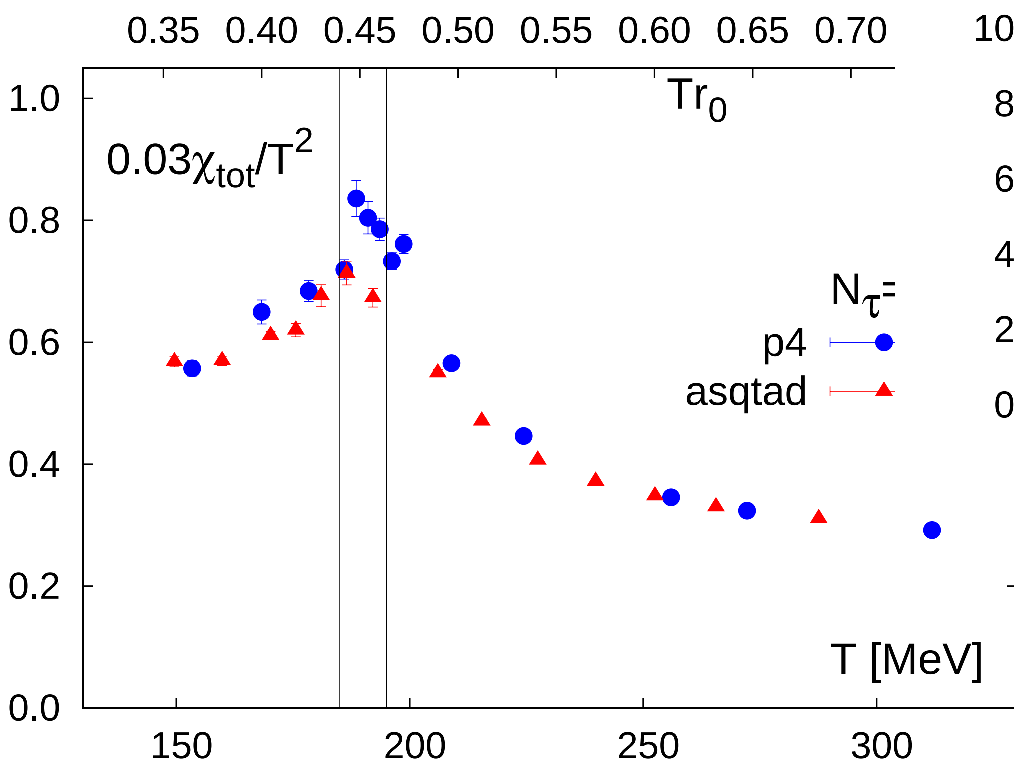
χ-symmetry restoration

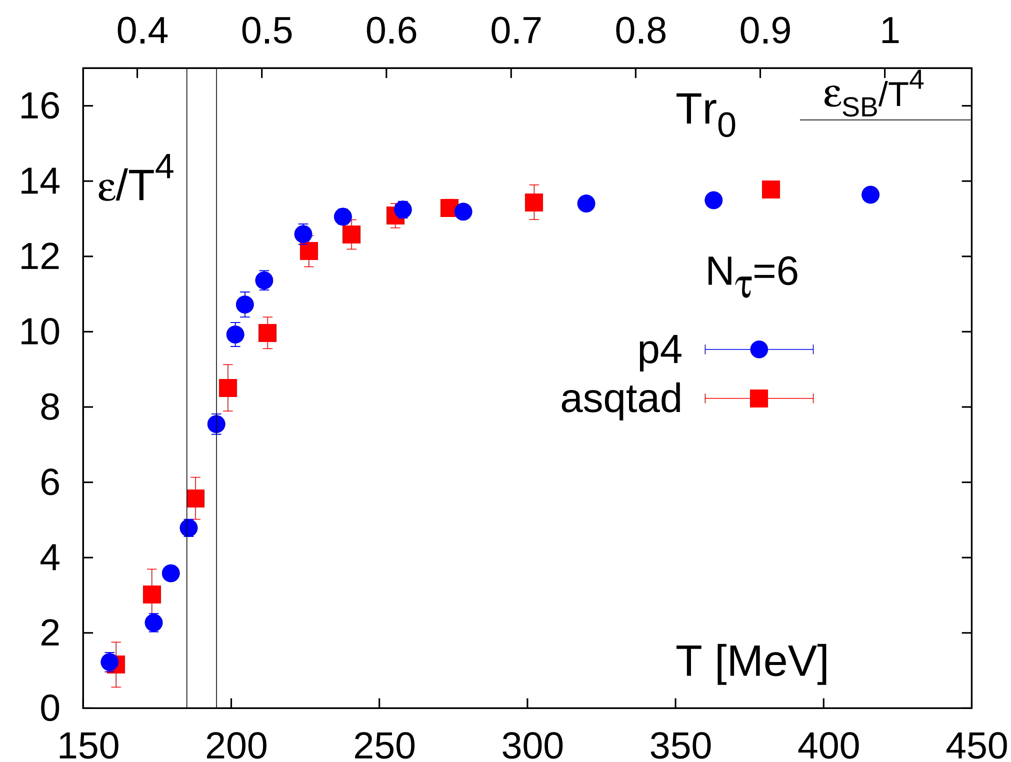
- χ-symmetry restoration:
  drop in condensate;
  peak in susceptibilities



Deconfinement and χ-symmetry and bulk thermodynamics

- most prominent features of bulk thermodynamics are related to deconfinement
- χ-symmetry restoration:
  drop in condensate;
  peak in susceptibilities





do they stay closely related
in the continuum limit?

Conclusions

- glue sticks

  the interesting non-perturbative physics in QCD happens in the gluon sector

- quarks add flavor

  quarks add to the picture by 'modifying prefactors' (in accord with dimensional reduction approach)

- no glue $\Rightarrow$ no binding

  'glue-free' observables show early onset of perturbative behaviour

Conclusions

- non-perturbative QCD-EoS $\sim$ pure gauge theory EoS

  the interesting non-perturbative physics in QCD happens in the gluon sector

- nothing qualitatively new in QCD with light quarks

  quarks add to the picture by 'modifying prefactors' (in accord with dimensional reduction approach) except close to T_c!!

- quantum numbers are carried by "quarks" already close to T_c

  'glue-free' observables show early onset of perturbative behaviour

Finally...

- the regime $T_c \leq T \lesssim (1.5 - 2.0)T_c$ differs from the regime $T \gtrsim (1.5 - 2.0)T_c$

  It is more difficult (impossible?) to describe it quantitatively in terms of conventional theoretical high-T concepts: perturbation theory, resummation, dimensional reduction

Finally...

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  It is more difficult (impossible?) to describe it quantitatively in terms of conventional theoretical high-T concepts: perturbation theory, resummation, dimensional reduction

- Do we see new physics? $\Rightarrow$ Quark Gluon Liquid
- or, remnants of old physics? $\Rightarrow$ confinement