Extra flavor in N=1 SYM and Baryonic Symmetry

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Gauge/Gravity duality for non-conformal theories

Gauge/Gravity duality of $N = 1$ theories

I. Klebanov and M. Strassler

- Confiment and chiral symmetry breaking
- IR phenomena: physics of glueballs, condensates etc.
- Models of SUSY breaking
- Field theory at finite temperature

Compact manifolds with conic singularities

- Models of early Universe and stringy inflation

Our goal is to understand gauge/gravity duality in a more general context

Plan of the talk

We focus on duality between $N = 1$ SYM and Klebanov-Strassler solution

- Family of IR vacua in field theory manifest through family of gravity backgrounds
  Flat valley of vacua is due to unbroken $U^*(1)$ symmetry
- Extra flavor in field theory corresponds to D7-brane
  Plan limit $M >> 1$ corresponds to probe regime (no backreaction of geometry)

We study impact of extra flavor on moduli space and show that the latter does not change

Geometry of Klebanov-Strassler solution

$$\boxed{ds^2 = h^{-1/2}dx^2 + h^{1/2}ds_6^2}$$

- Geometry is a warped product of a flat Minkowski space and non-compact CY $z_1^2 + z_2^2 + z_3^2 + z_4^2 = \epsilon^2$
  - geometrical realization of confiment $h(0) = const$
  - chiral symmetry $\partial/\partial\Psi$ is broken explicitly
  - non-trivial flux through S^3 cycle counts M fractional D5 branes
- background is ISD and dilaton is constant
- metric is Ricci-flat
- unbroken Z_2 symmetry

Field theory dual of KS solution

Field theory interpretation

- $SU(N) \times SU(N+M)$ theory
- fundamental supermultiplets A_α, B_β with $SU(2) \times SU(2)$ symmetry
- meson $2 \times 2 \cdot N \times N$ matrix (scalars) $\boxed{M_{\alpha\beta} = A_\alpha B_\beta}$
- baryon operators (scalars) $A =$ "A_α^{2M}", $B =$ "B_β^{2M}"

Theory experiences duality cascade behavior (Seiberg duality) which turns $SU(N) \times SU(N+M)$ gauge group into $SU(N-M) \times SU(N)$

Coupling constants never become small together superpotential at the

last step of the cascade $\boxed{W = X(\det M + AB - \Lambda_{2M}^{4M})}$

- baryonic branch $\boxed{AB = const}$

Baryonic branch of moduli space

- baryonic branch $\boxed{AB = const}$

Baryonic branch is one dimensional complex plane

- $\boxed{A = \zeta e^{i\varphi}}$ $\boxed{B = \zeta^{-1} e^{-i\varphi}}$

$U(1)_{\text{Baryon}}$ is a symmetry of moduli space

- $\boxed{\varphi \to \varphi + \text{const}}$

- Massless fluctuation of φ is Goldstone boson of $U(1)_{\text{Baryon}}$
- Massless fluctuation of ζ and φ form supermultiplet

Corresponding massless fluctuations of SUGRA for KS case:
φ can not condense! S. Gubser, C. Herzog, I. Klebanov

Baryonic branch of KS solution

- KS solution is a part of real one dimensional family of solutions (geometries)
  The family is naturally parametrized by U – asymptotic behavior of fields at "UV"
- $N = 1$ SUSY is unbroken for all U

Different geometries from the branch correspond to different IR vacua of field theory

$$\boxed{\zeta \sim U}$$

A. Butti, M. Grana, R. Minasian, M. Petrini, A. Zaffaroni

Connection between geometry and field theory, relation between U and baryonic condensate

A.D., I. Klebanov, N. Seiberg, M.Benna, A.D., I. Klebanov

D7 brane on KS and baryonic branch

Extra flavor in field theory

Probe D7 = extra flavor

- Classically extra flavor can not brake SUSY

$$\delta W = h_1 \tilde{q}(A_1 B_1 + A_2 B_2) q + h_2 \tilde{Q}(B_1 A_1 + B_2 A_2) Q + \alpha \tilde{q} q \tilde{q} q + \beta \tilde{Q} Q \tilde{Q} Q$$

Theory is never weakly coupled

Can SUSY breaking occur nonperturbatively?
K. Intriligator,, N. Seiberg, D. Shih

Field theory picture is undeveloped

We are going to use dual gravity to answer this question

Motivation: adding D7 to KS

Adding D7 to KS background

- Spectrum of mesons and IR physics
  S. Kuperstein
- Compactification and stringy cosmology
  D7-brane plays crucial role in stabilization of Kahler moduli

Motivation: adding D7 to baryonic branch

Unlike KS case D3-brane breaks SUSY on the bbranch

A.D., I. Klebanov, N. Seiberg

Phenomenology

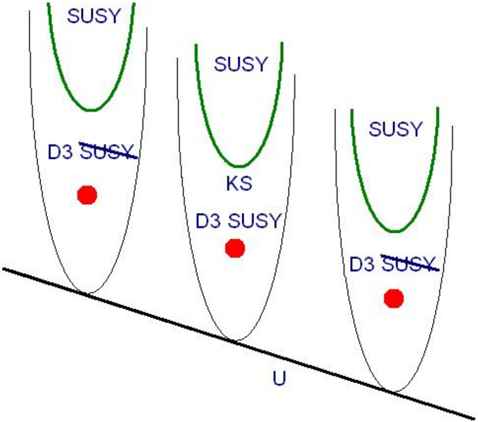
- Backreaction of D3 breaks SUSY on D7
  D-term induced gauge meditation
  Y. Nakayama

Motivation: adding D7 to baryonic branch

Cosmology

Compactification fixes some non-zero value of U...

- D3 breaks SUSY in a "controllable way" and uplifts the potential



Is D7-brane preserves SUSY on baryonic branch?

SUSY D-branes and Generalized Calibration

Action of D7 brane is calibrated by "closed" form W_C

L.Martucci, P.Smith

$$\boxed{S_{DBI} + S_{CS} \geq \mathrm{Vol}_4 \int_{\Sigma} W_C[F = dA]}$$

Calibration condition is saturated for SUSY embedding

- D5 wrapping minimal S^3 of conifold
  Constant tension along the branch
  $W_C = \Omega^{(3,0)}$
  $\phi(t) = \phi(a, v, t)$
  A.D., Y. Tachikawa, I. Klebanov

Calibration condition

$$\det(G+M) \geq (\mathrm{Pf}J - \mathrm{Pf}M)^2 + (J \cdot M)^2$$

Inequality is saturated when M is of $(1,1)$ type.

$$\sqrt{(\mathrm{Pf}J - \mathrm{Pf}M)^2 + (J \cdot M)^2} \geq \Re\left(e^{i\theta}\left(\mathrm{Pf}J - \mathrm{Pf}M + iJ \cdot M\right)\right)$$

Inequality is saturated when

$$\Im\left(e^{i\theta}\left(\mathrm{Pf}J - \mathrm{Pf}M + iJ \cdot M\right)\right) = 0$$

Kappa-symmetry equation for D7 mutually supersymmetric with background: $\cos\theta = e^{\phi}$

- M is of $(1,1)$ type
- $$\frac{1}{2}\left(J \wedge J - M \wedge M\right)\Big|_{\Sigma} = -\frac{e^{2A}}{U} J \wedge M\Big|_{\Sigma}$$

Calibration form

$$e^{4A}e^{-\phi}\sqrt{\det(G+M)} + C \wedge e^{M}\Big|_{\Sigma} \geq W_C\Big|_{\Sigma}$$

where

$$W_C = \frac{1}{2}(e^{2A}J)\wedge(e^{2A}J) - U(e^{2A}J)\wedge M + U^2 B\wedge M - \frac{U^2}{2}B\wedge B$$

Using that $e^{2A}J = UB - d\left[U(\pi+\chi)g_5\right]$ and $M = B + dA$ we get $dW_C = 0$

- On-shell action

$$S = U^2 \mathrm{Vol}_4 \int_{\partial\Sigma}\left[A(\pi+\chi)\wedge dg_5 + \frac{1}{2}(\pi+\chi)^2 g_5\wedge dg_5\right]$$

$\int_{\partial\Sigma} g_5 \wedge dg_5 = \mathrm{const}$ for large radius

When embedding Σ is SUSY?

- For which holomorphic Σ there is such A satisfying kappa-symmetry condition?
  - Field theory interpretation of arbitrary Σ is not clear
  - Geometrical interpretation of calibration does not provide an answer

Extra identity $d(e^{2A-\phi}J \wedge e^{2A-\phi}J) = 0$ guarantees that kappa-symmetry equation for D7 is integrable

Ouyang embedding

- An interesting example is “Ouyang” embedding
  $\Sigma: \ z_1 + iz_2 = \mathrm{const}$ in the KS case
  - $F = dA$ if of $(1,1)$ type
  - $M = B + F$ is primitive on Σ: $J \wedge M|_\Sigma = 0$

Different approaches to find A

- Use unbroken $U(1) \times U(1)$ to express general $F = dA$ $(1,1)$ form through one function $f(\theta_1, \theta_2)$
  The problem is to satisfy primitivity condition: second order differential equation for $f(\theta_1, \theta_2)$ this a non-trivial RHS
- Integrate kappa-symmetry condition
  $A \wedge e^{2A} J = -w_3 g(t) + d(..)$
  The problem is to make $F = dA$ of $(1,1)$ type

Kuperstein embedding

Kuperstein embedding $z_4 = \mu$

$A = 0$ satisfies kappa-symmetry in the KS case
S.Kuperstein
Generalization to the baryonic branch ($U \neq 0$)

- $\mu = 0$: D7 touches the tip. Usual conifld variables $\theta_i, \phi_i, \psi, t$ applicable
- $\mu \neq 0$: D7 stretches to $|\epsilon^2 + \mu^2| + |\mu^2|$

$z_4 = \mu$ preserves half of $SU(2) \times SU(2)$ symmetries

Kuperstein embedding

$SU(2) \times SU(2)$ invariant gauge field $A = \zeta(t) g_5$ solves kappa-symmetry equation

Kappa-equation can be integrated

$$\boxed{dw_\zeta|_\Sigma = 0}$$

For $A = \zeta(t) g_5$:

$$w_\zeta = \left[(\zeta+\chi)^2 + 2\pi(\zeta+\chi) + (h_2^2 \sinh^2(t) e^{-2\phi} - \pi^2(e^{-2\phi} - 1))\right] g_5 \wedge dg_5$$

implies $\boxed{\zeta + \chi \sim U t^2 e^{-2/3t}}$ and $\boxed{S \sim r^4 + U^2 \log^2 r}$

Outline

Extra flavor on bbranch

- D7 preserves SUSY for any U
- Nontrivial gauge field develops for $U \neq 0$
- Action does not depend on embedding μ

$$\boxed{S \cong r^4 + U^2 \log^2 r + 0 + O(r^{-1})}$$

Future directions

- Other embeddings
- Interpretation of $U^2 \log^2(r)$ term in action
- Stability of μ after compactification
- Phenomenological and cosmological models