Lattice gauge theories, large N and QCD strings

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GGI 08

Preamble

Is $N = 3$ close to $N = \infty$? Is large-N confining?

Hot SU(N) gauge theory

The closed string spectrum in $D = 3$ and $D = 4$

also – given time

Topology and interlacing θ-vacua

$D = 3$: comparing with Karabali-Nair

Twisted Eguchi-Kawai : space-time reduction

Large N – Preamble

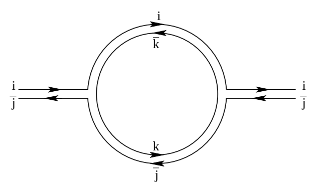
• QCD : $g^2 \leftrightarrow$ scale, so no small expansion parameter

$\overset{'tHooft}{\Rightarrow}$

try

$$\mathrm{SU}(N) \simeq \mathrm{SU}(\infty) + \mathrm{O}(1/N^2)$$

•



$$\Rightarrow \quad g^2 N \quad \text{constant at large } N$$

• $N \to \infty$ colour singlet phenomenology

't Hooft, Witten, Veneziano, Manohar, ...

zero decay widths; no mixing; exact OZI, η'; SU(6) for baryons; ...

• no scattering of colour singlets – integrability?

but strongly interacting bound states

• factorisation colour singlet operators: e.g.

$\langle \Phi_1(x_1)\Phi_2(x_2)\rangle = \langle \Phi_1\rangle\langle \Phi_2\rangle \left\{1 + O\left(\frac{1}{N^2}\right)\right\}$

$\Rightarrow$ Witten's Master Field $\rightarrow$ translation invariant $\rightarrow$ Eguchi-Kawai single point reduction

• Feynman diagrams on 2D surfaces :

$g^2N \rightarrow \infty \rightarrow$ vertices dense $\rightarrow$ stringy sheets

$\Rightarrow$

$N = \infty$ gauge theory $\sim$ a string theory 't Hooft

$N = \infty$ gauge theory $\sim$ dual to a string theory

Maldacena

Lattice – Preamble

Euclidean $R^D \longrightarrow$ hypercubic lattice on T^D

$x_\mu \bullet - \bullet\, x_\mu + \widehat{\mu}\delta x\ :\ A_\mu(x) \in$ SU(N) Lie Algebra

$\longrightarrow$

$x_\mu \bullet - - - \bullet\, x'_m u\ :\ P\left\{e^{\int_x^{x'} A.dx}\right\} \in$ SU(N) group

$\xrightarrow{x_\mu = a n_\mu}$

$an_\mu \bullet - - - \bullet\, an_\mu + a\widehat{\mu} \quad : \quad U_\mu(n) \in$ SU(N) group

i.e. SU(N) matrices U_l on each link l

gauge transformation: $U_\mu(n) \to g(n)U_\mu(n)g^\dagger(n+\widehat{\mu})$

$\longrightarrow$ gauge invariant action?

Tr$\prod_{l\in\partial c} U_l$ gauge invariant for any closed curve c

$\longrightarrow$ so

$Z = \int \prod_l dU_l e^{-\beta S}$ where $S = \sum_p \left\{1 - \frac{1}{N}\mathrm{ReTr}u_p\right\}$
where u_p is product links around the plaquette p is a suitable, although not unique, SU(N) lattice gauge theory

$\longrightarrow$ symmetries ensure that:

$$\int \prod_l dU_l e^{-\beta S} \quad \stackrel{a\to 0}{\longrightarrow} \quad \int DA e^{-\frac{4}{g^2}\int d^4x TrF_{\mu\nu}F_{\mu\nu}}$$

with

$$\beta = \frac{2N}{g^2(a)} \stackrel{a\to 0}{\longrightarrow} \infty$$

and we vary the parameter β in order to vary the lattice spacing a

$\longrightarrow$ Monte Carlo:

$$Z^{-1}\int \prod_l dU_l \Phi(U) e^{-\beta S} = \frac{1}{n}\sum_{I=1}^{n} \Phi(U^I) + O(\frac{1}{\sqrt{n}})$$

• calculating masses from Euclidean correlators:

$\Phi(t)$ a gauge invariant operator

$$\langle \Phi^\dagger(t = an_t)\Phi(0)\rangle = \sum_i |c_i|^2 e^{-aE_i n_t} \stackrel{t\to\infty}{\simeq} |c|^2 e^{-man_t}$$

where am is lightest mass with quantum numbers of Φ in lattice units

• continuum limit :

$$\frac{am(a)}{a\sqrt{\sigma(a)}} = \frac{m(a)}{\sqrt{\sigma(a)}} = \frac{m(0)}{\sqrt{\sigma(0)}} + ca^2\sigma + O(a^4)$$

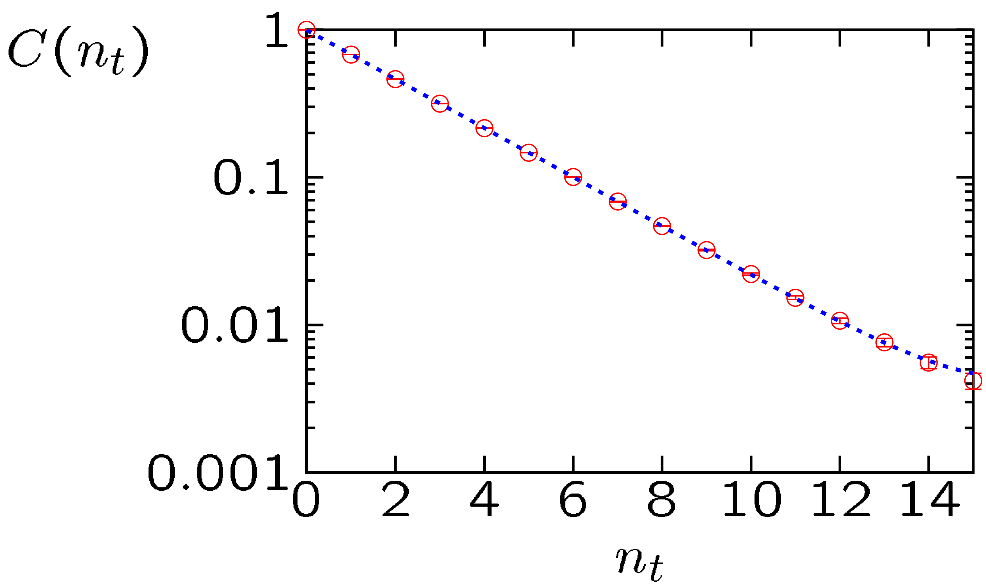
• large N limit :

$$\frac{m(N)}{\sqrt{\sigma(N)}} = \frac{m(\infty)}{\sqrt{\sigma(\infty)}} + \frac{c}{N^2} + O\left(\frac{1}{N^4}\right)$$

Can we do accurate calculations?

SU(3), 32^4, $a \simeq 0.046$ 'fm'

best blocked/smeared glueball operator

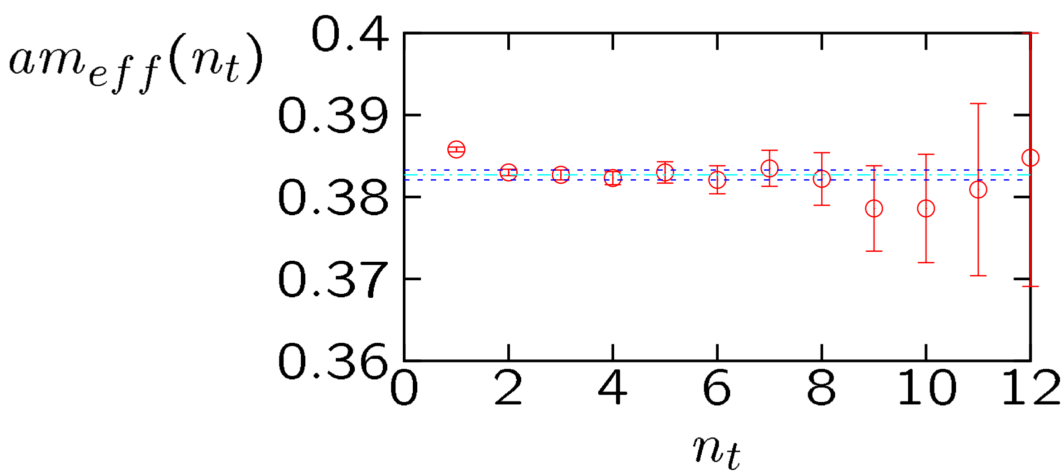


$$C(t = an_t) \stackrel{t\uparrow}{\simeq} |c|^2 e^{-man_t}$$

$\Rightarrow$

fit : $am_{0^{++}} = 0.330(7)$

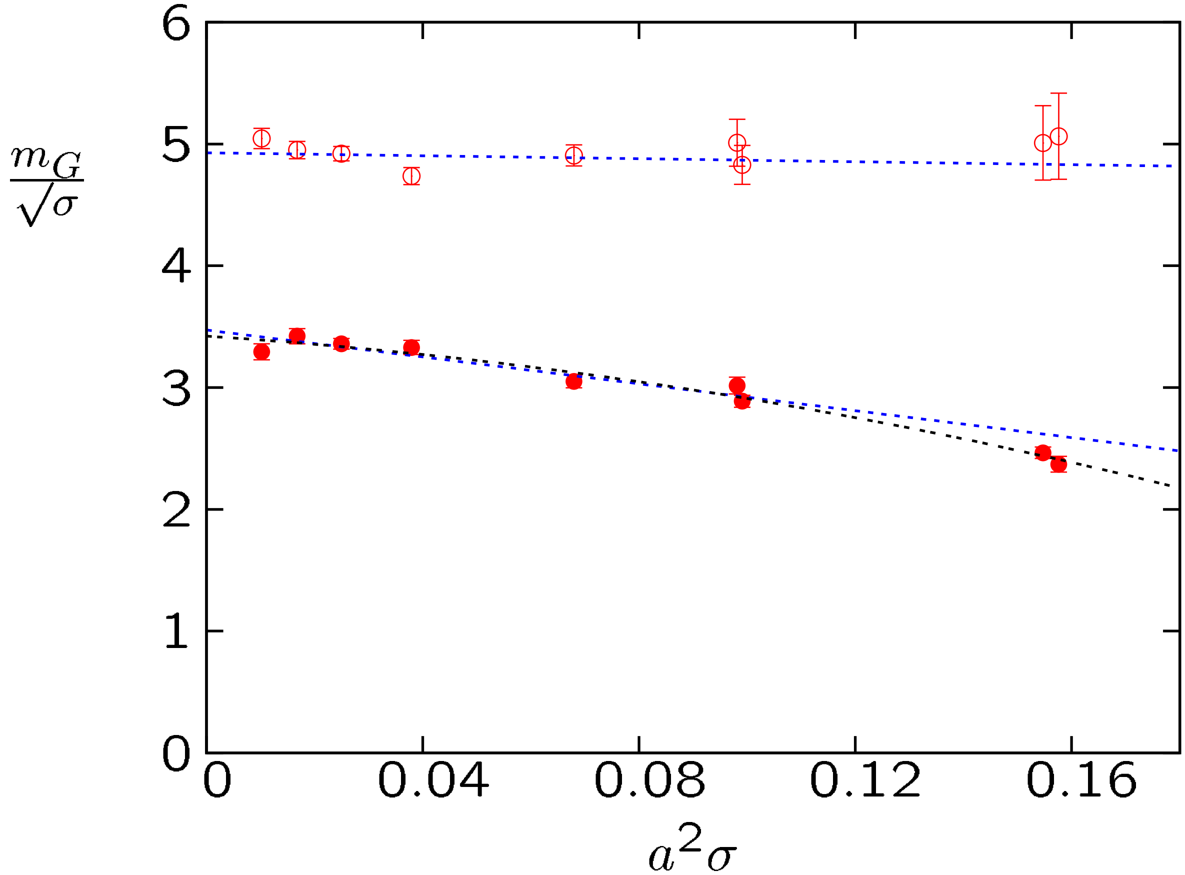
$$am_{eff}(n_t) = -\ln \frac{C(n_t)}{C(n_t - 1)} \xrightarrow{n_t \to \infty} am_{0^{++}}$$



$\Rightarrow$

fit : $am_{0^{++}} = 0.3312(67)$

Continuum limit mass spectrum: SU(3)



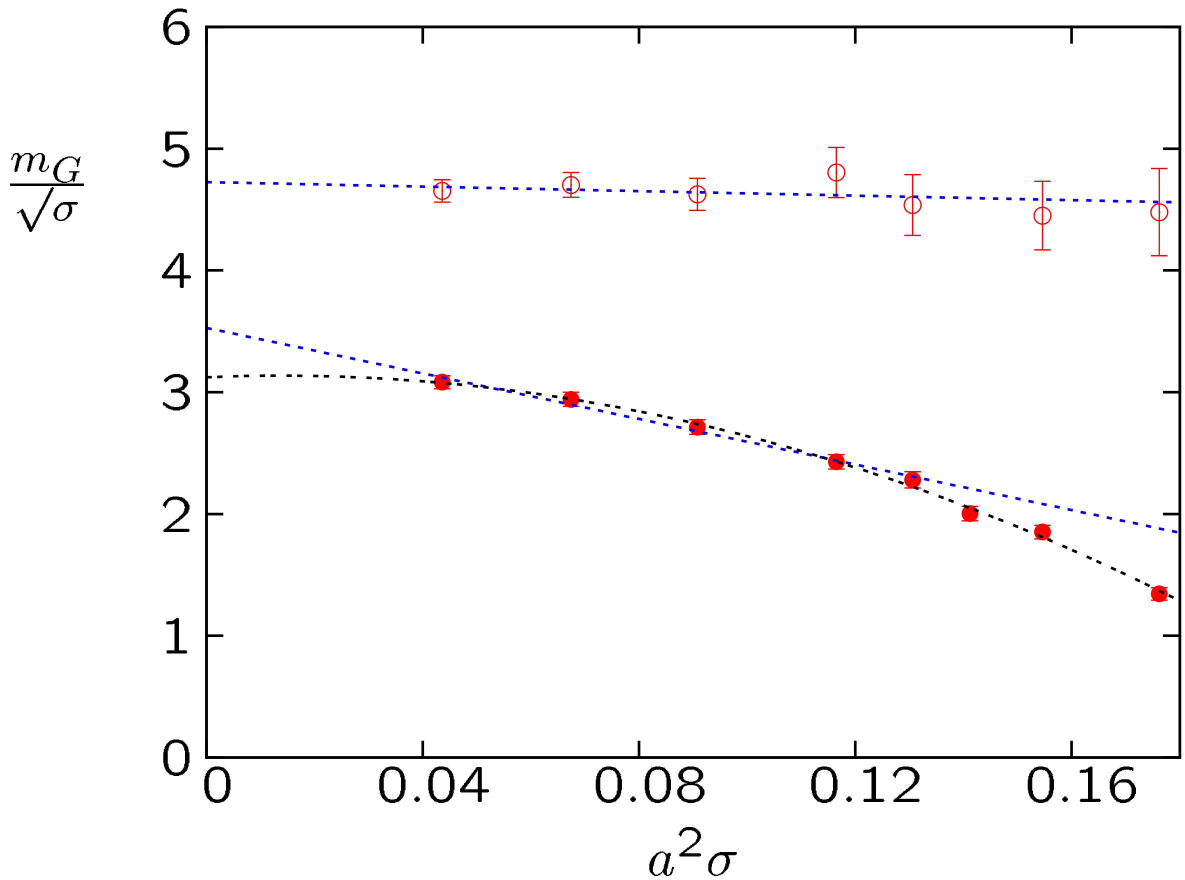
$O(a^2)$ continuum extrapolations:

$$\frac{m_{0^{++}}}{\sqrt{\sigma}} = 3.47(4) - 5.52(75)a^2\sigma$$

$$\frac{m_{2^{++}}}{\sqrt{\sigma}} = 4.93(5) - 0.61(1.36)a^2\sigma$$

$O(a^4)$ continuum extrapolation very similar

Continuum limit mass spectrum: SU(8)



$O(a^2)$ continuum extrapolation:

$$\frac{m_{0^{++}}}{\sqrt{\sigma}} = 3.53(8) - 9.3(1.0)a^2\sigma$$

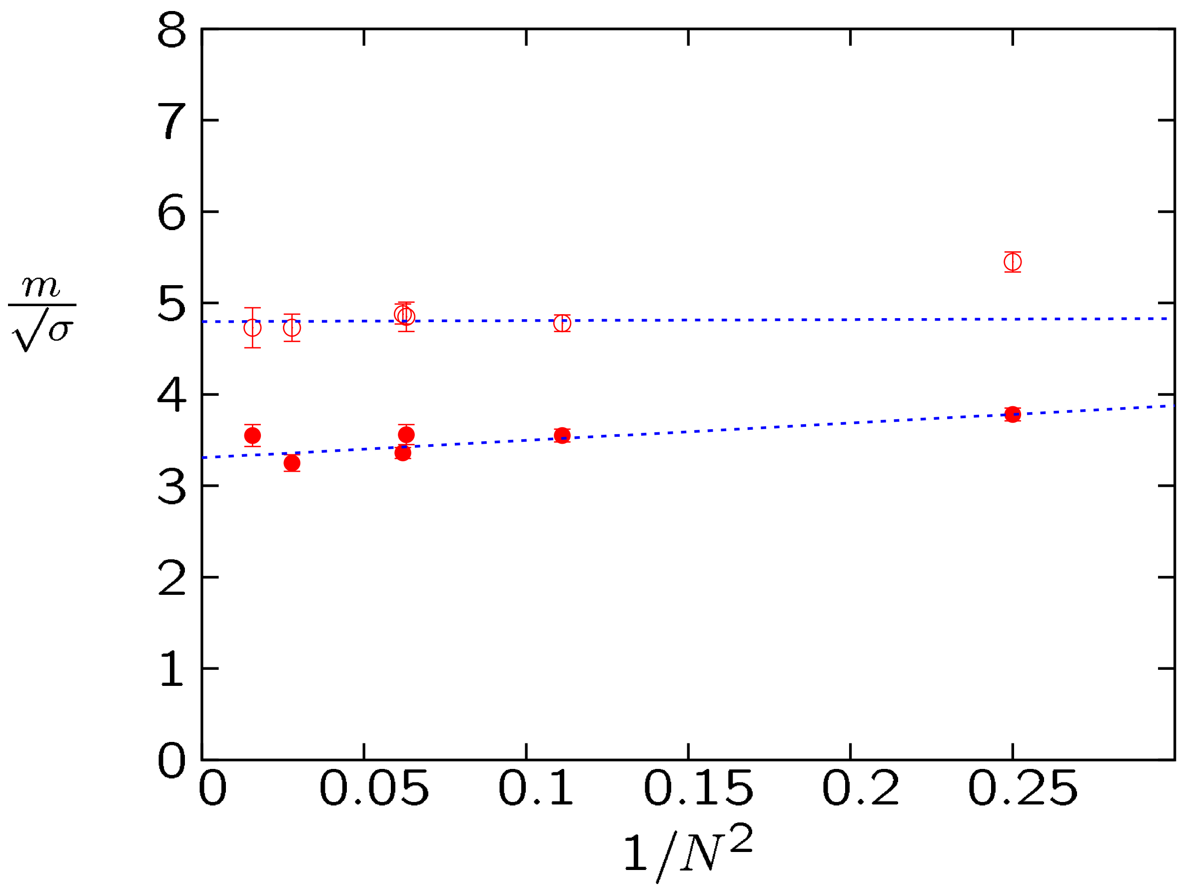
$O(a^4)$ extrapolation

$$\frac{m_{0^{++}}}{\sqrt{\sigma}} = 3.13(25) + 1.66a^2\sigma - 66.0(a^2\sigma)^2$$

this systematic error $\sim 5 \pm 3\times$ naive $O(a^2)$ statistical error !

Mass spectrum: large-N limit

B.Lucini, M.Teper, U.Wenger: hep-lat/0404008



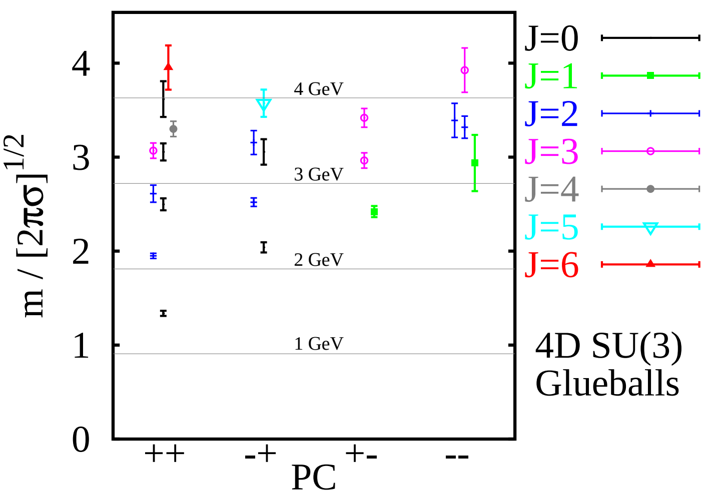
$O(1/N^2)$ extrapolations to $N = \infty$:

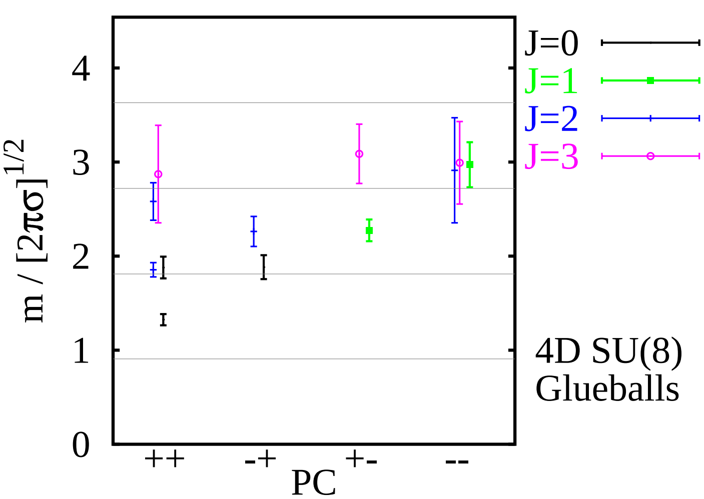
$$\frac{m_{0^{++}}}{\sqrt{\sigma}}|_N = 3.31 + \frac{1.90}{N^2}$$

$$\frac{m_{2^{++}}}{\sqrt{\sigma}}|_N = 4.80 + \frac{0.11}{N^2}$$

spectrum : SU(3) vs SU(8)

H. Meyer, M. Teper: hep-th/0409183



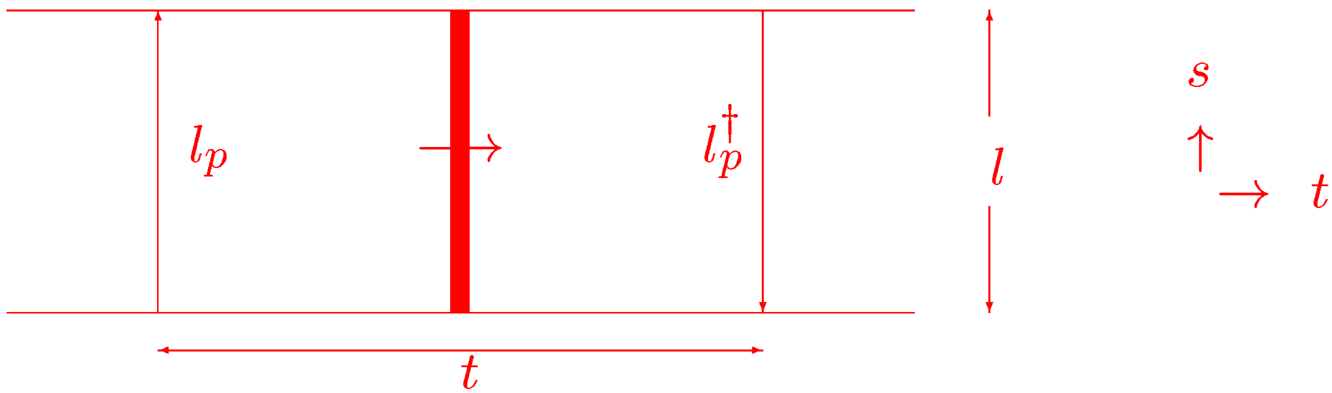


Linear confinement in SU($N \to \infty$)?

Calculate the mass of a confining flux tube winding around a spatial torus of length l, using correlators of Polyakov loops:

$$\langle l_p^\dagger(t) l_p(0) \rangle \overset{t\to\infty}{\propto} \exp\{-m_p(l)t\}$$

in pictures



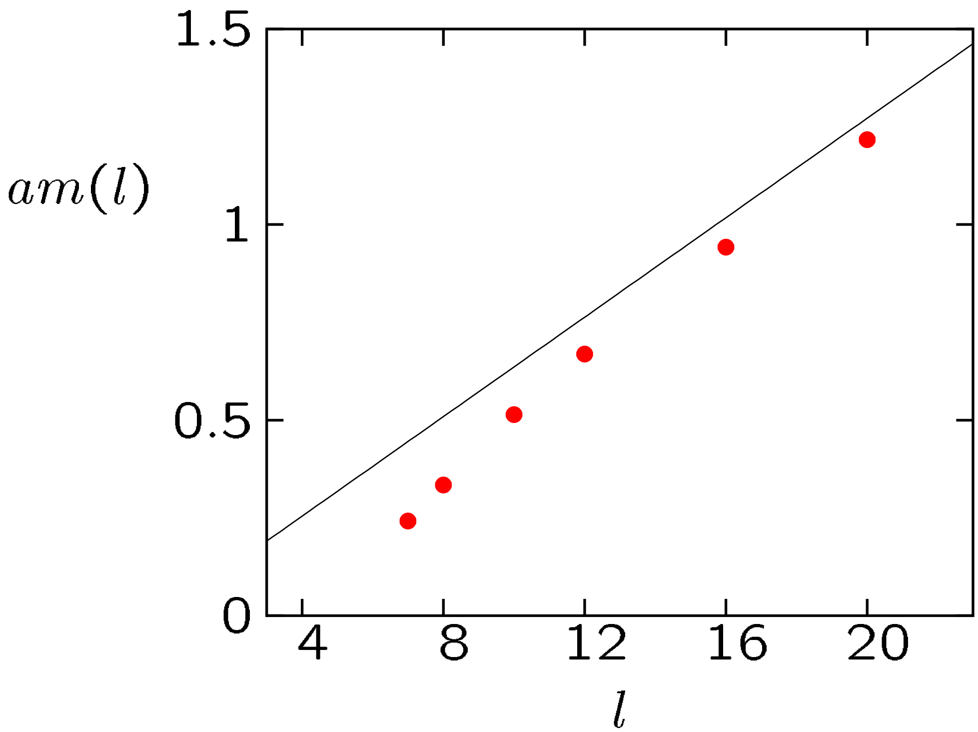
where we expect, for linear confinement,

$$m_p(l) \overset{l\to\infty}{=} \sigma l - \frac{\pi(D-2)}{6l} + O\left(\frac{1}{l^3}\right)$$

- no sources, no Coulomb terms, flux tubes for $l \geq 1/T_c$

SU(6)

H. Meyer, M. Teper: hep-lat/0411039



indeed we find

$$am(l) \simeq \sigma l$$

over a range of 'string' lengths up to

$$l \simeq 5.0 \times \frac{1}{\sqrt{\sigma}}$$

surely large enough to be asymptotic ...

So :

- SU(3) $\sim$ SU(∞) for many quantities

- linear confinement persists at large N

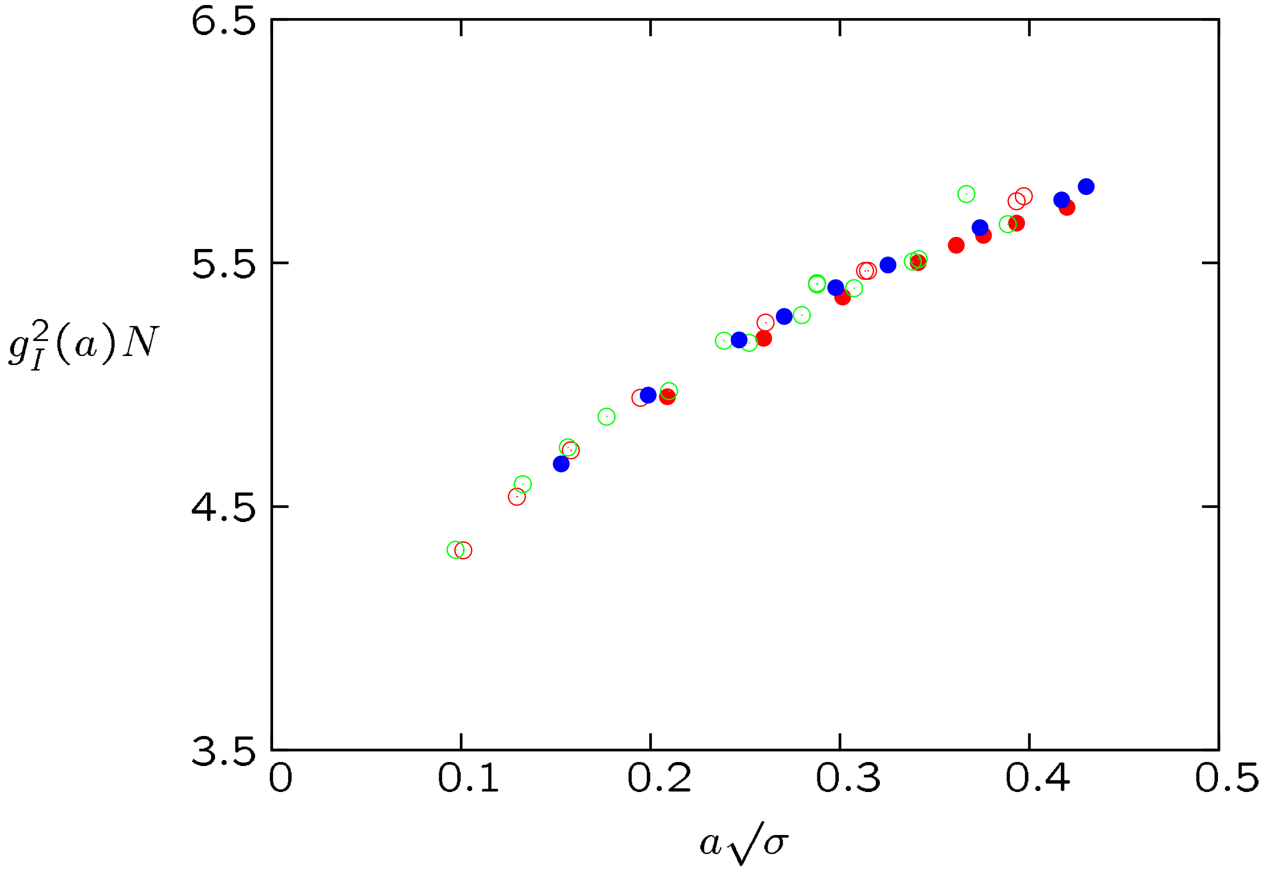
the apparent phenomenological relevance of large-N, provides the motivation for pursuing further the properties of this theory ...

$g^2 N$ fixed as $N \to \infty$?

Lucini, Teper, Wenger: hep-lat/0502003

- $g^2(l)$ versus $\frac{l}{\xi}$ with $\xi = \frac{1}{\sqrt{\sigma}}$, $l = a$

and using $\beta = 2N/g_L^2(a)$ $g_I^2 = g_L^2/u_p$

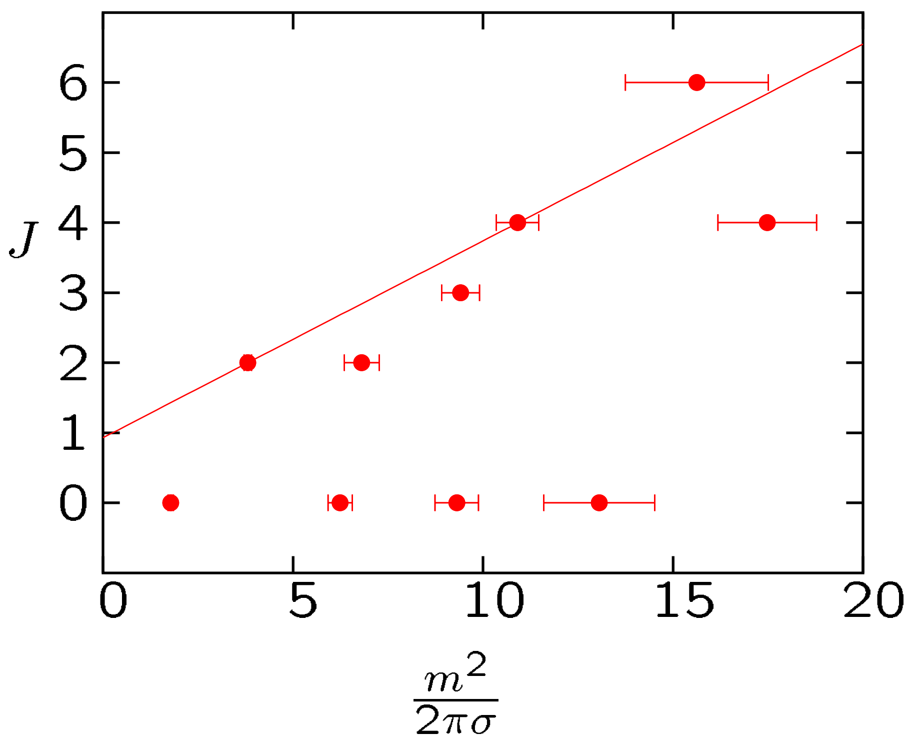


SU(2) ○ ; SU(3) ○; SU(4) ● ;
SU(6) ○ ; SU(8) ●

Pomeron: the leading glueball Regge trajectory?

H. Meyer, M. Teper: hep-th/0409183

Chew-Frautschi plot: $PC = ++$ states in SU(3) gauge theory



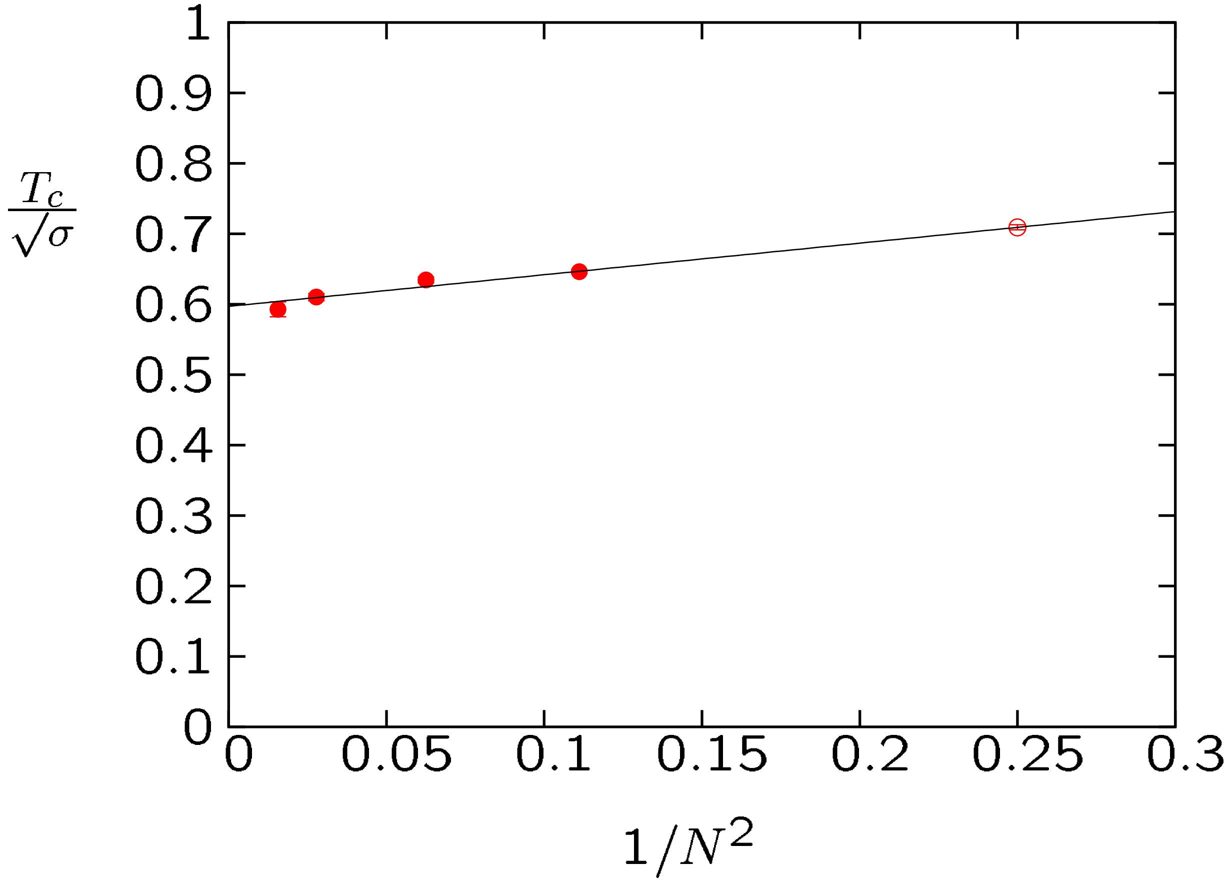
$$\alpha(t) = 0.93(24) + 0.28\alpha'_R t$$

$$\alpha'_R = \frac{1}{2\pi\sigma} \simeq 0.9\text{GeV}^{-2}$$

Deconfining temperature in D=3+1

B.Lucini, M.Teper, U.Wenger: hep-lat/0307017,0502003

$L_s^3 L_t \Rightarrow T = \frac{1}{a(\beta)L_t}$ if $L_s \gg L_t$



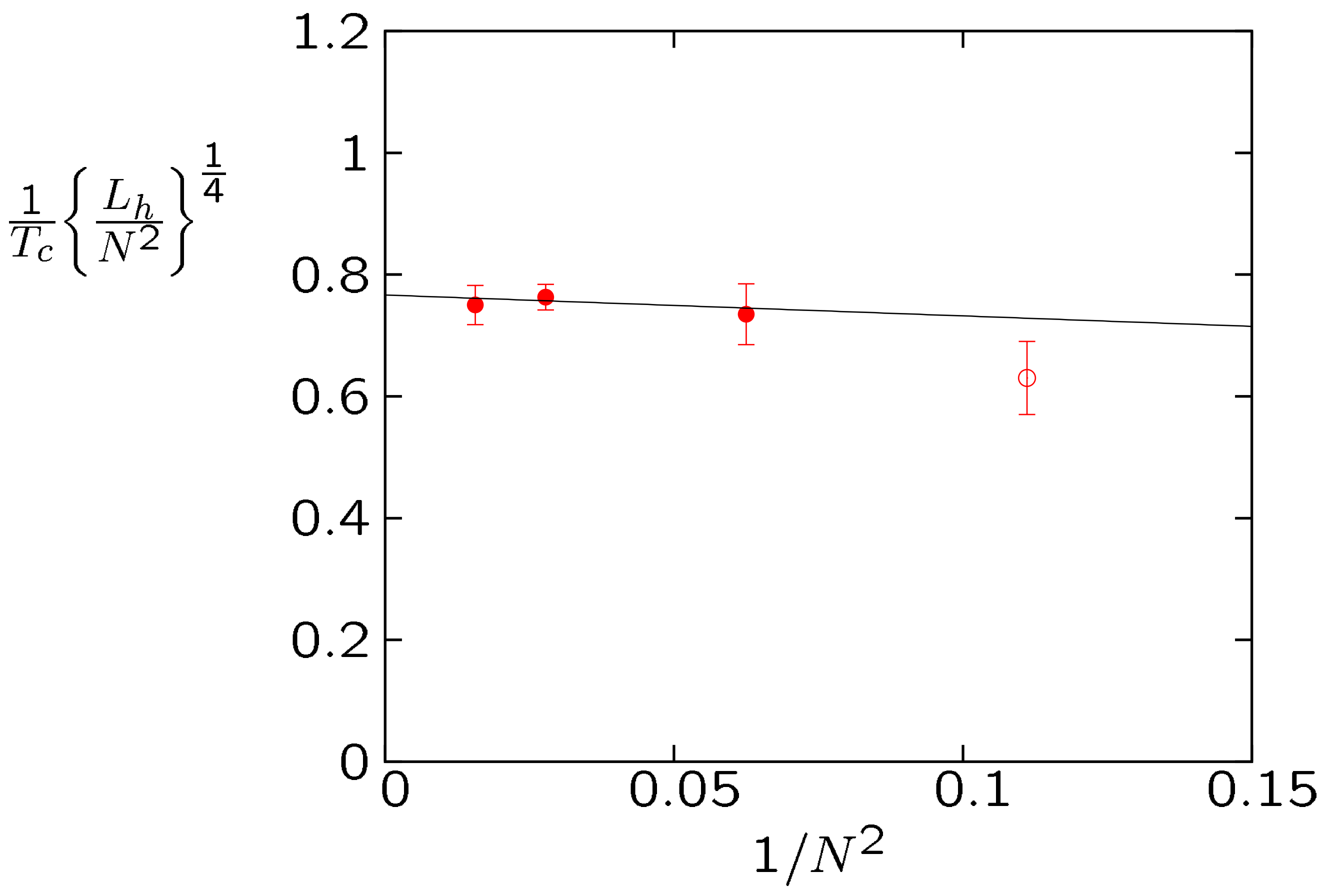
2nd order ∘ ; 1st order •

$\Rightarrow$

$$\frac{T_c}{\sqrt{\sigma}} = 0.597(4) + \frac{0.45(3)}{N^2}$$

Confinement-deconfinement latent heat

B.Lucini, M.Teper, U.Wenger: hep-lat/0307017,0502003



$\Rightarrow$

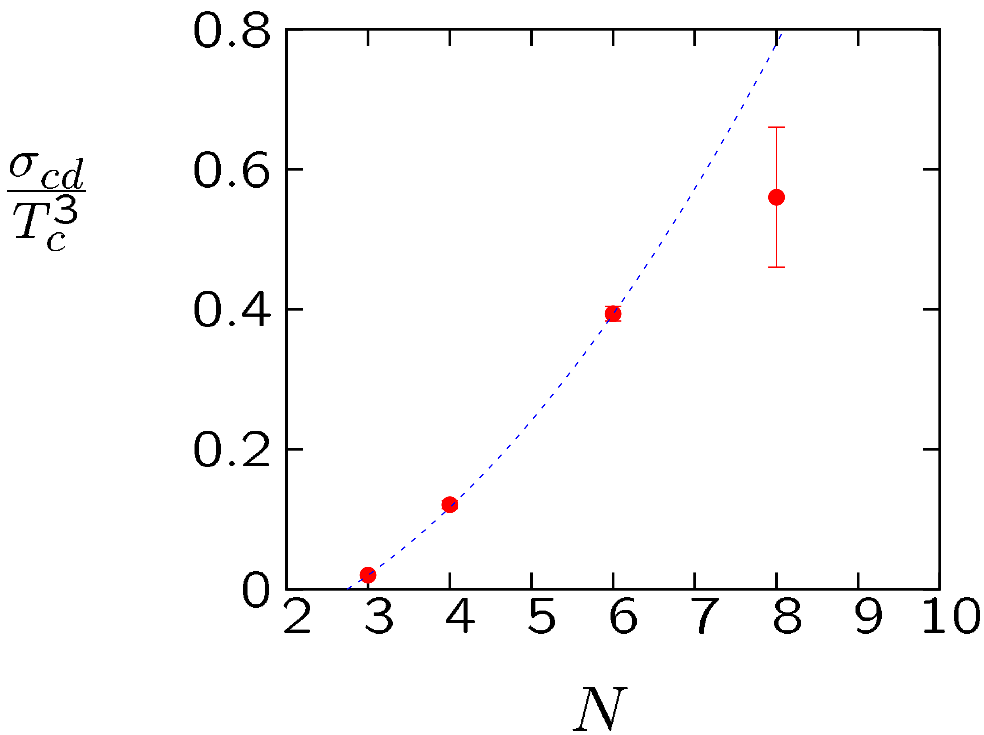
large-N deconfinement is 'normal' first order

$N = 3$ 'weakly' first order

Confinement-deconfinement wall tension

B.Lucini, M.Teper, U.Wenger: hep-lat/0502003

aT=0.2 :



fit :

$$\frac{\sigma_{cd}}{T_c^3} = 0.0138N^2 - 0.104$$

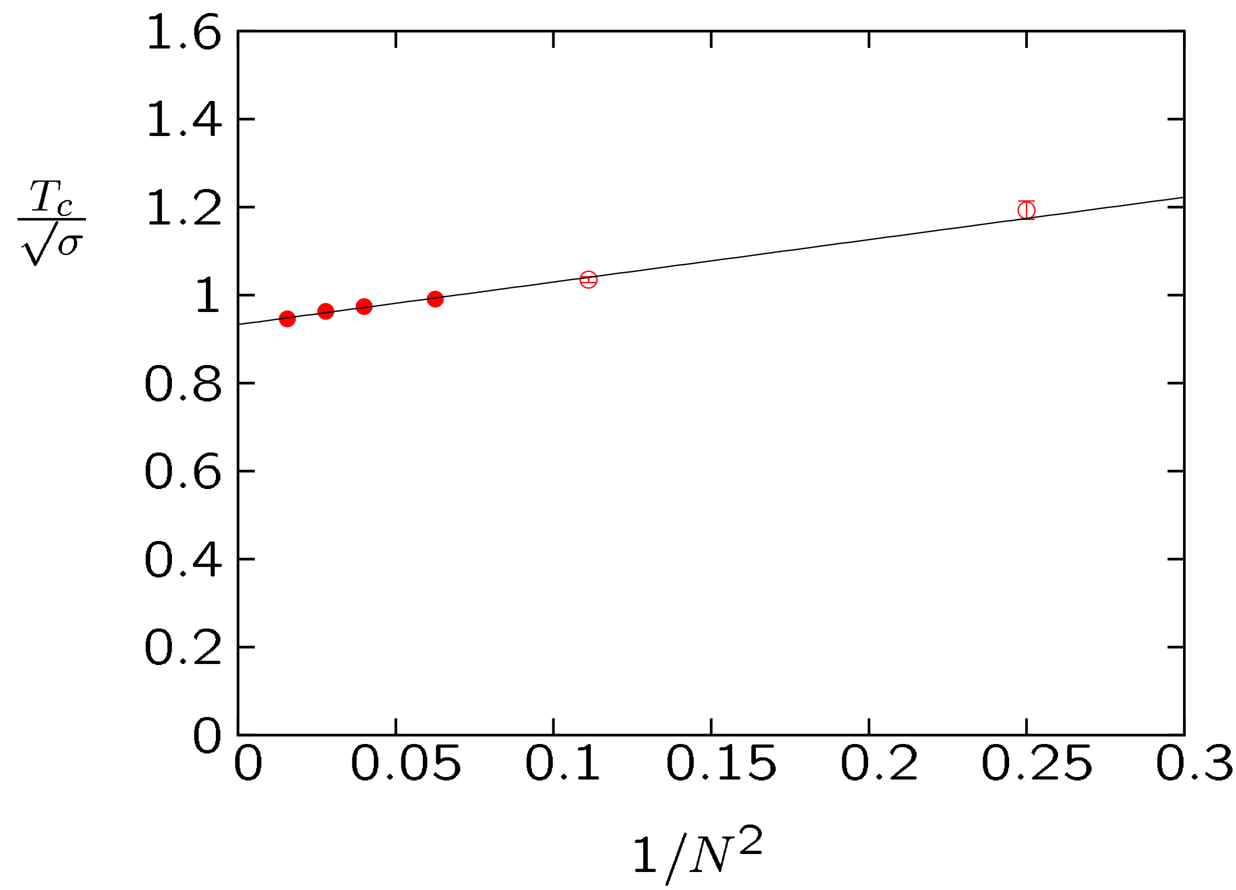
$\Rightarrow$

interface tension small

D=3+1 $\longrightarrow$ D=2+1

$\frac{T_c}{g^2 N}$ J. Liddle, M.Teper : hep-lat/0509082; in preparation

$\frac{\sqrt{\sigma}}{g^2 N}$ B. Bringoltz, M.Teper : hep-th/0611286



2nd order ○ ; 1st order ($N = 4?$) ●

$\Rightarrow$

fit : $\frac{T_c}{\sqrt{\sigma}} = 0.933(4) + \frac{0.96(8)}{N^2}$ preliminary

Single or multiple deconfining transitions?

B.Lucini, M.Teper, U.Wenger: hep-lat/0502003

let l_p be the Polyakov loop (fund repn), then

$\langle l_p \rangle = 0$; confined

$\langle l_p \rangle = z \in Z_N$; deconfined

i.e. deconfinement $\leftrightarrow Z_N$ ssb

$\Rightarrow$

is there one transition or several?

e.g.

$$SU(4): \qquad Z_4 \stackrel{T=T_c}{\longrightarrow} Z_2 \stackrel{T=T_{c'}}{\longrightarrow} 1$$

corresponding to

$T = T_c$: k=2 strings break – but not k=1

$T = T_{c'}$: k=1 strings break

NO : there is only one transition

N counting of free energies (heuristic)

$$Z = e^{-\frac{F}{T}} = \sum_n e^{-\frac{E_n}{T}} \quad (1)$$

$$= \sum_{c=singlet} e^{-\frac{E_c}{T}} + \sum_{g=gluons} e^{-\frac{E_g}{T}} \quad (2)$$

$$= e^{-\frac{F_c}{T}} + e^{-\frac{F_g}{T}} \quad (3)$$

and at $T = T_c$ we have

$$F_c = F_g$$

but

$F_g \sim N^2$ colour singlet entropy $\sim N^0$

so reason that $T_c \not\to 0$ as $N \to \infty$ is that

$$E_c = hadron\, masses + E_{vac}$$

and

$$E_{vac} \sim -N^2 \sim gluon\, condensate$$

so

$$F_g = -E_{vac} \text{ at } \; T = T_c$$

Strongly Coupled Gluon Plasma - at large N?

B. Bringoltz, M.Teper: hep-lat/0506034

Consider

$$Z(T,V) = \exp\left\{-\frac{F}{T}\right\} = \exp\left\{-\frac{fV}{T}\right\} = \int DU \exp\left(-\beta S_{\mathrm{W}}\right).$$

now $p = T\frac{\partial}{\partial V}\log Z(T,V) = \frac{T}{V}\log Z(T,V) = \frac{T}{V}\int_{\beta_0}^{\beta} d\beta' \frac{\partial \log Z}{\partial \beta'}$

but $\frac{\partial \log Z}{\partial \beta} = -\langle S_W \rangle = N_p \langle u_p \rangle$

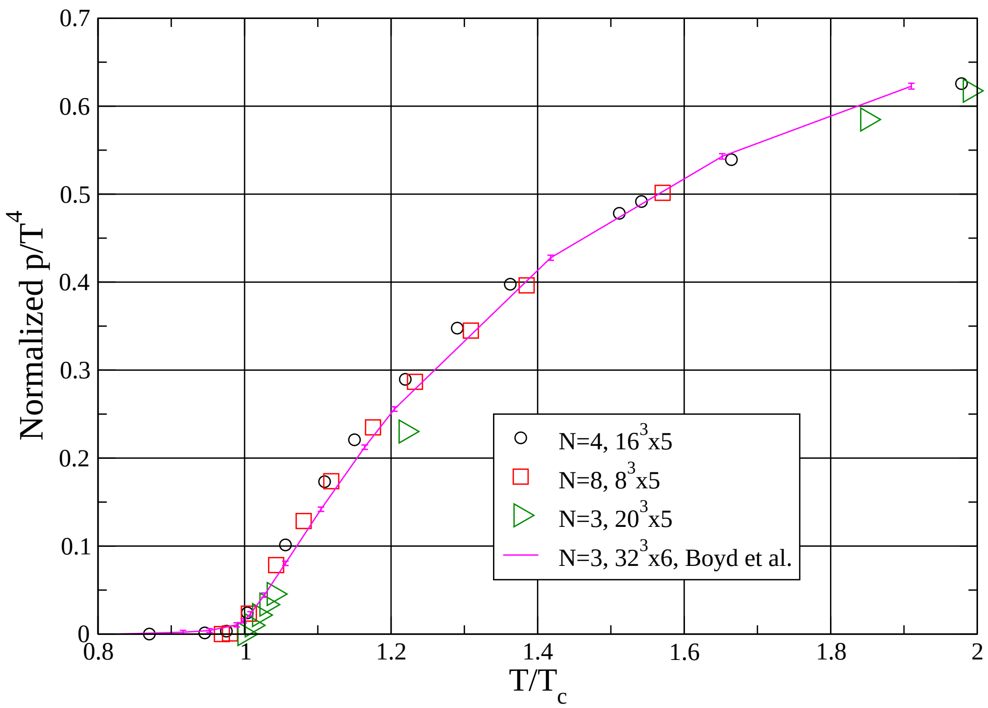
so $a^4[p(T) - p(0)] = 6\int_{\beta_0}^{\beta} d\beta' (\langle u_p \rangle_T - \langle u_p \rangle_0)$.

i.e. $\frac{p(T)}{T^4} = 6L_t^4 \int_{\beta_0}^{\beta} d\beta' (\langle u_p \rangle_T - \langle u_p \rangle_0)$.

similarly $(\epsilon - 3p)/T^4 = 6L_t^4(\langle u_p(\beta)\rangle_0 - \langle u_p(\beta)\rangle_T) \times \frac{\partial \beta}{\partial \log(a(\beta))}$.

Strong Gluon Plasma - high-T pressure anomaly

B. Bringoltz, M.Teper: hep-lat/0506034



$\Rightarrow$

SGP is a large-N phenomenon: dynamics must survive at $N = \infty$

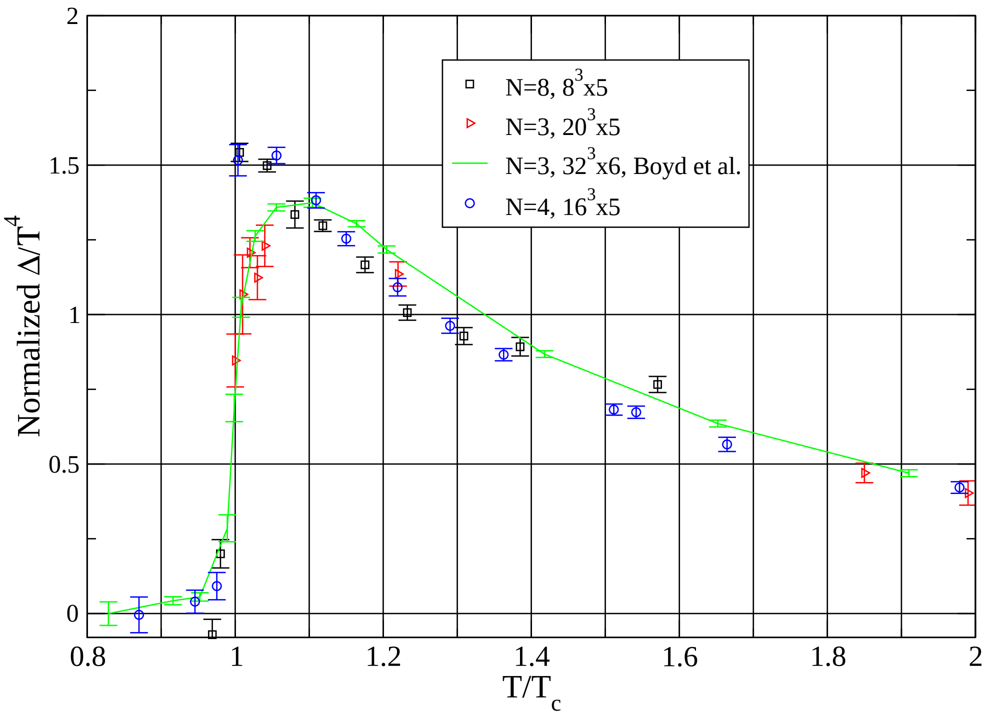
$\Rightarrow$

- not (colour singlet) hadrons above T_c
- not topology (instantons)

$\Delta \equiv \epsilon - 3p$

B. Bringoltz, M.Teper: hep-lat/0506034

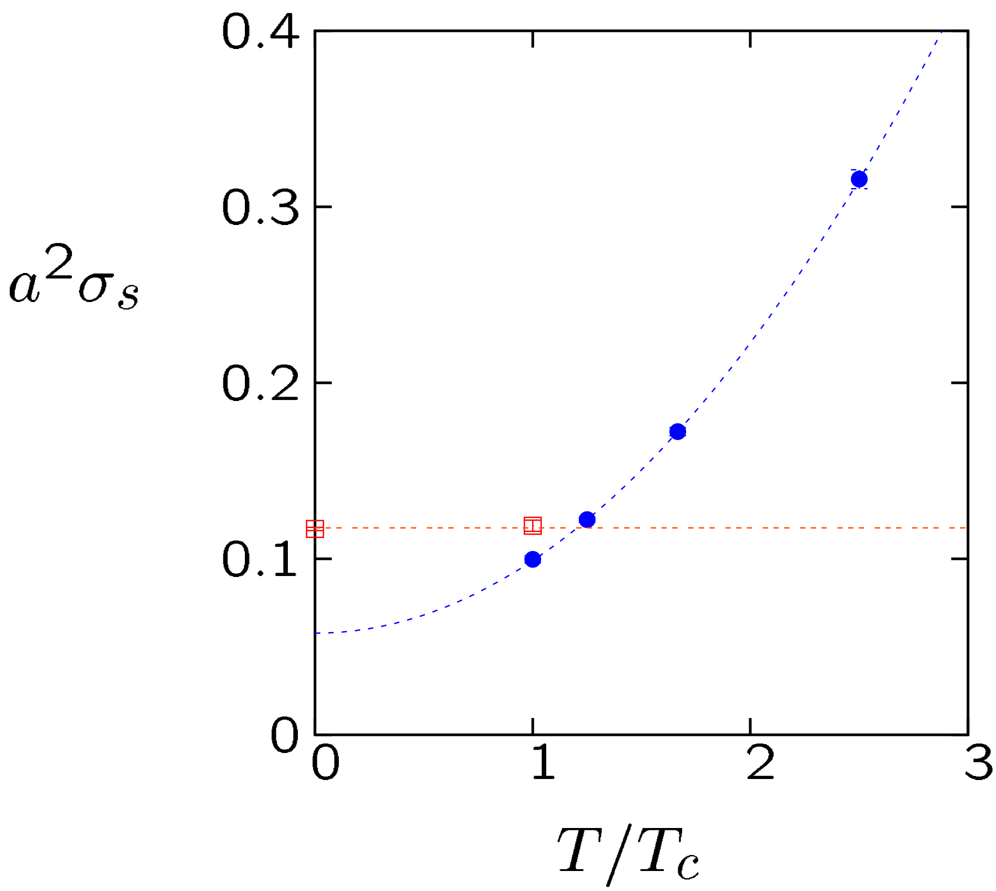
$\Delta = 0$ in Stefan-Boltzman gas



$N = 8$ spatial string tension

Lucini,Teper,Wenger: hep-lat/0502003

$a = 1/5T_c$:



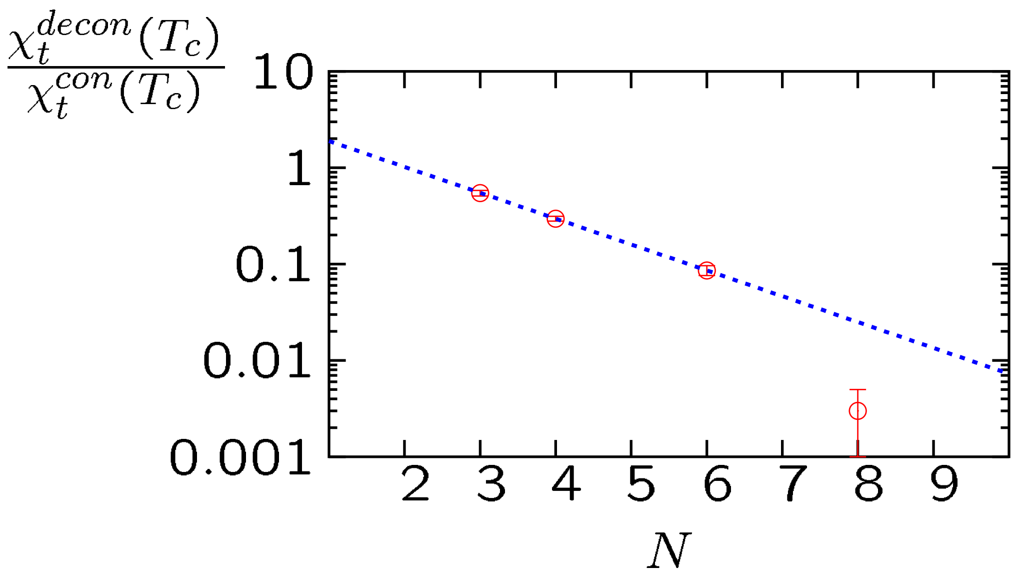
quadratic and constant fits shown

no topological fluctuations in deconfined phase ...

Lucini,Teper,Wenger: hep-lat/0401028

(Del Debbio,Panagopoulos,Vicari: hep-lat/0407068)

$\chi_t \equiv \langle Q^2 \rangle / V$ in confining/deconfining phases at $T = T_c$



$\Rightarrow$

deconfined topological fluctuations

vanish with N exponentially fast

In Summary:

• $SU(\infty)$ has a 'strong' first order deconfining transition in both $D = 4$ and $D = 3$.
It is second order for $N < 3$ in $D = 4$ and for $N < 4$ in $D = 3$

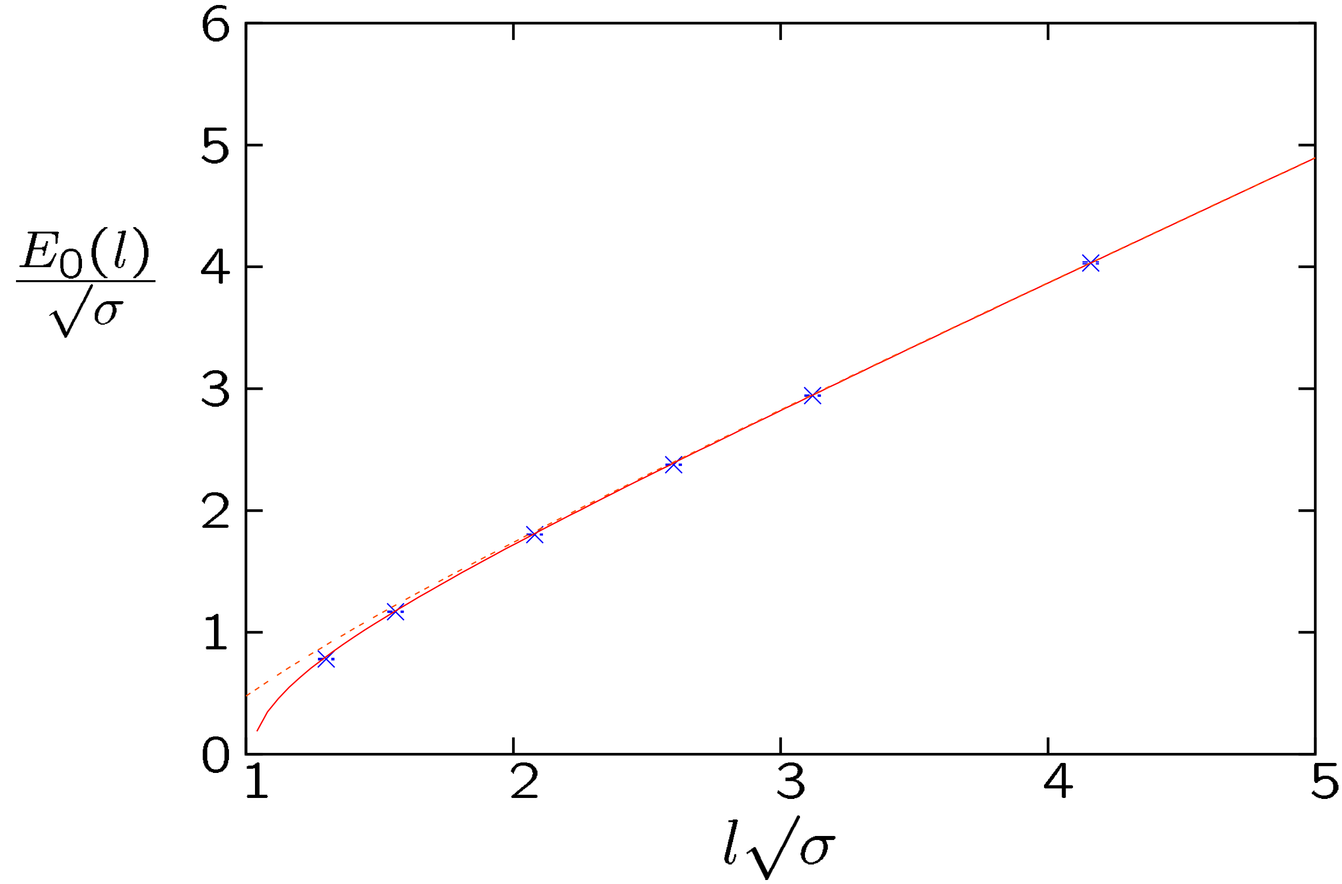
• the 'strong coupling gluon plasma' (e.g. pressure anomaly) above T_c is a large-N phenomenon
$\Rightarrow$
certain explanations are excluded (e.g. some bound states surviving close to T_c)
AdS/CFT has hope of being relevant

The spectrum of flux tubes that are closed around a spatial torus of length l : SU(N) $D = 2+1$

- linear confinement?
- bosonic string universality class?
- $E_n(l)$: expansion in $1/l$ or covariant Nambu-Goto?

A.Athenodorou, B.Bringoltz, M.Teper arXiv:0709.0693
B.Bringoltz, M.Teper arXiv:0802.1490, hep-th/0611286
A.Athenodorou, B.Bringoltz, M.Teper in progress

D=2+1 ; SU(5)



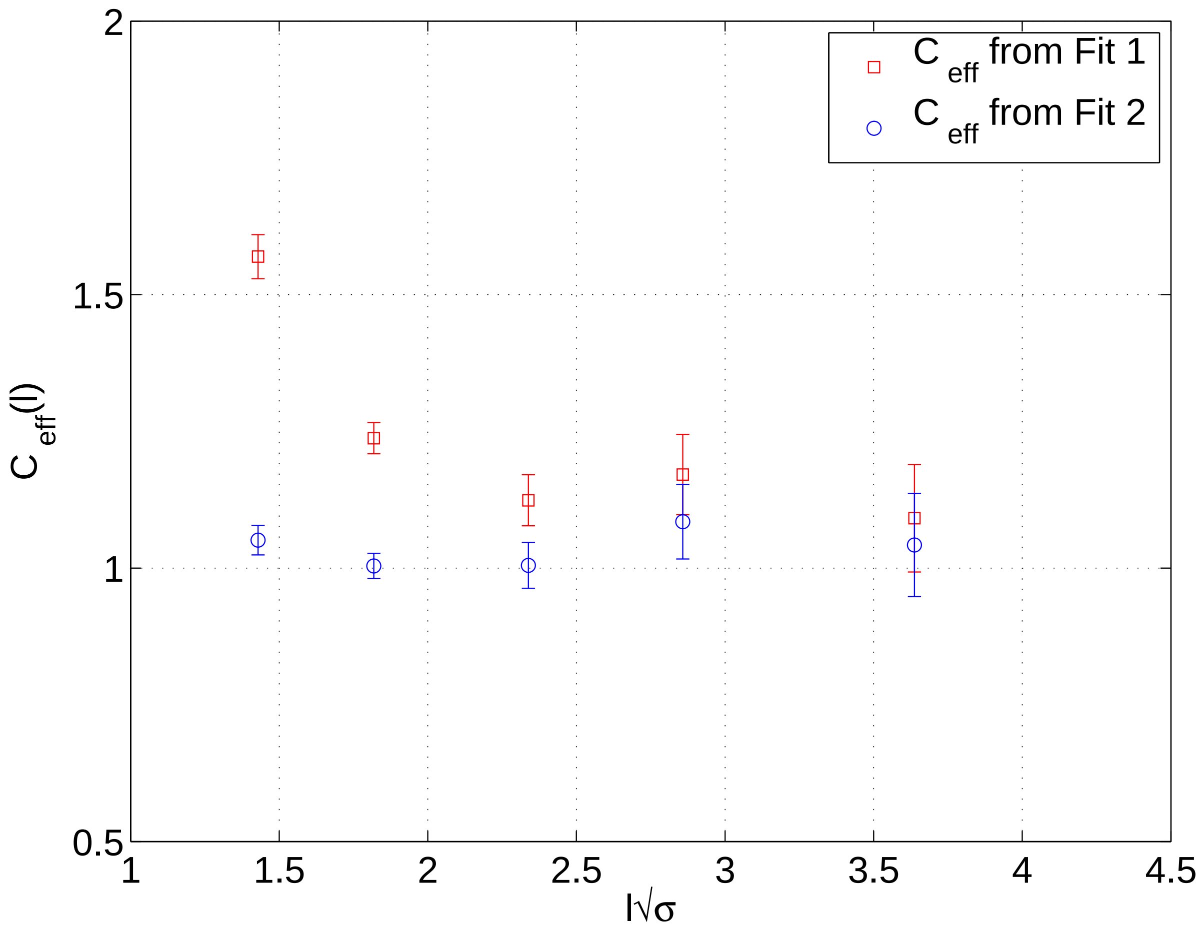
Luscher:...

$$E_0(l) = \sigma l - \frac{\pi}{6l}$$

Nambu-Goto:-

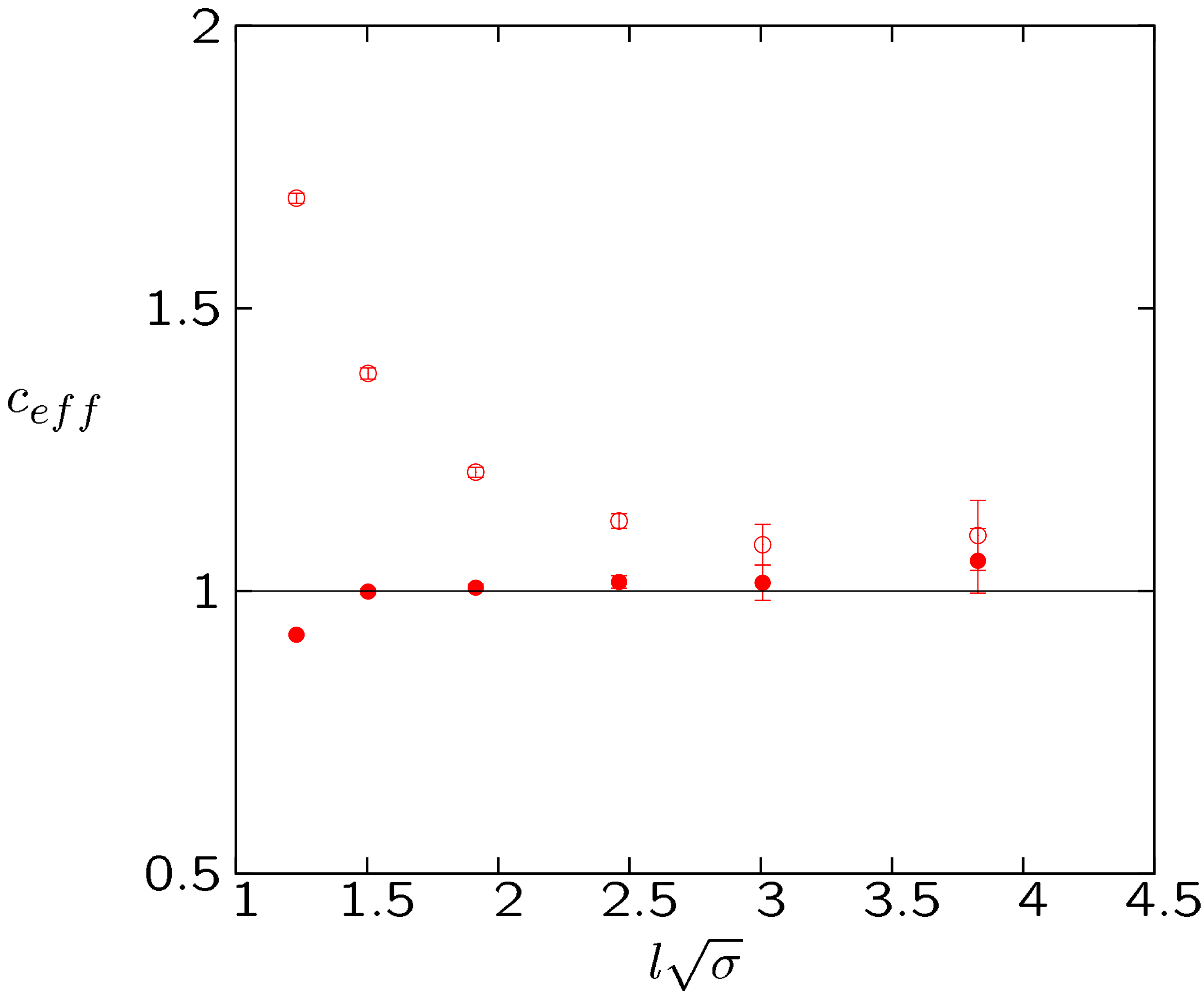
$$E_0(l) = \sigma l \left(1 - \frac{\pi}{3\sigma l^2}\right)^{\frac{1}{2}}$$

fitting the 'central charge'
SU(5) : $l_c\sqrt{\sigma} \simeq 1.07$



○ : c_{eff} from Luscher
○ : c_{eff} from Nambu-Goto

SU(2) : $l_c\sqrt{\sigma} \simeq 0.94$

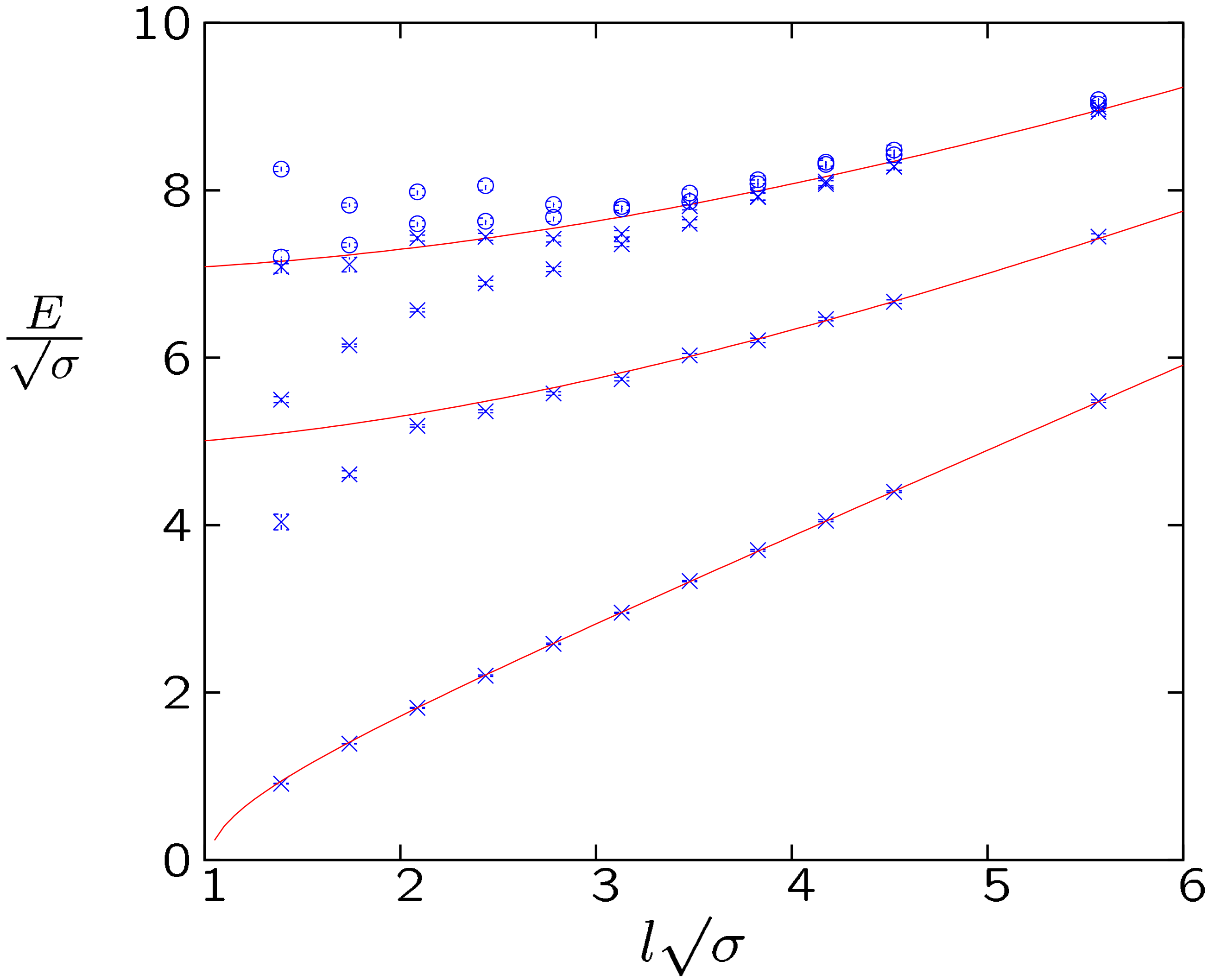


○ : c_{eff} from Luscher

● : c_{eff} from Nambu-Goto

SU(3) : closed string spectrum

$a\sqrt{\sigma} = 0.17395(7)$; $l_c\sqrt{\sigma} \simeq 1.0$



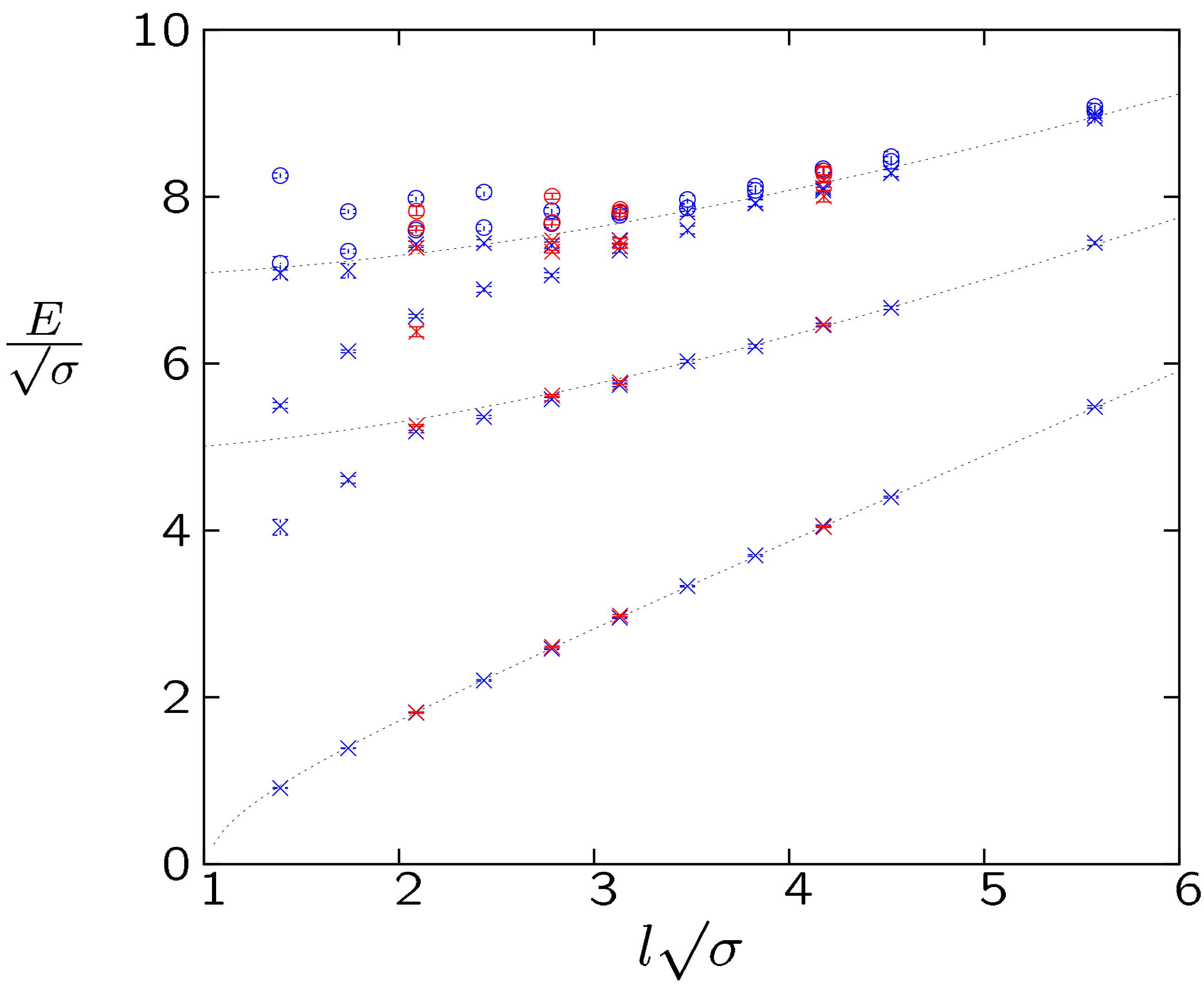
- : Nambu-Goto (σ from ground state)

$\times$: +ve parity

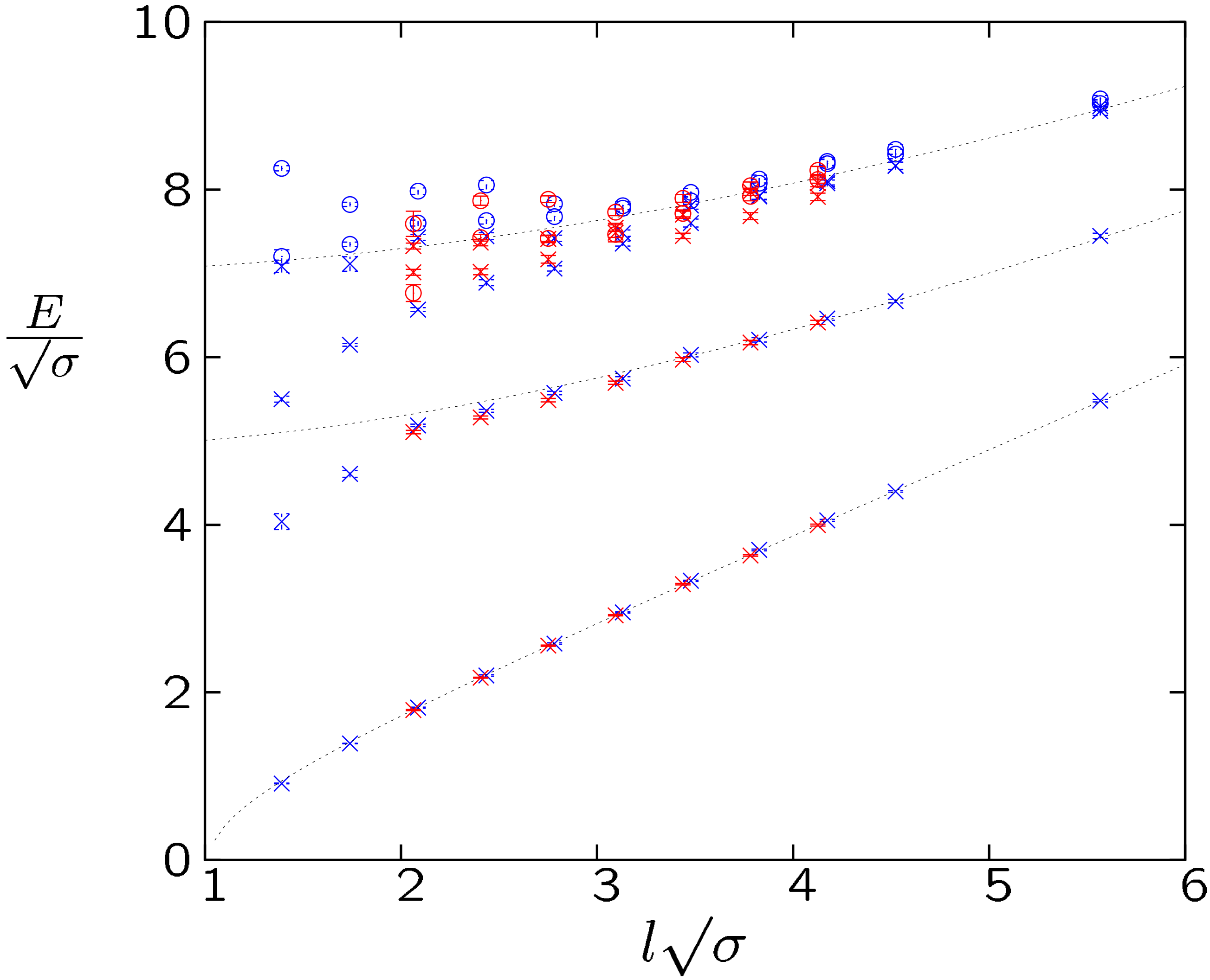
$\circ$: -ve parity

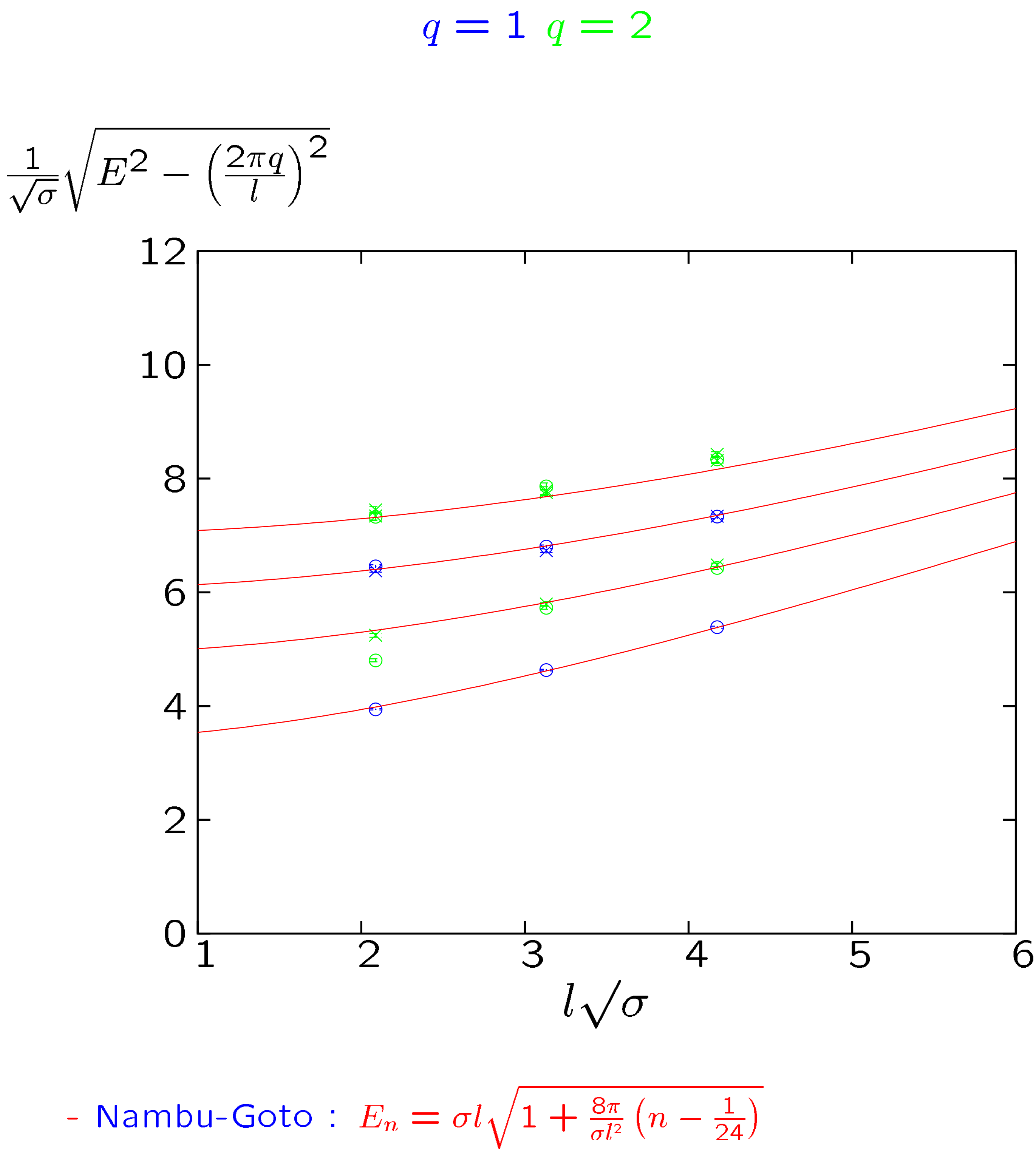
SU(3) : continuum limit?

$a\sqrt{\sigma} \simeq 0.174 \quad vs \quad a\sqrt{\sigma} \simeq 0.087$

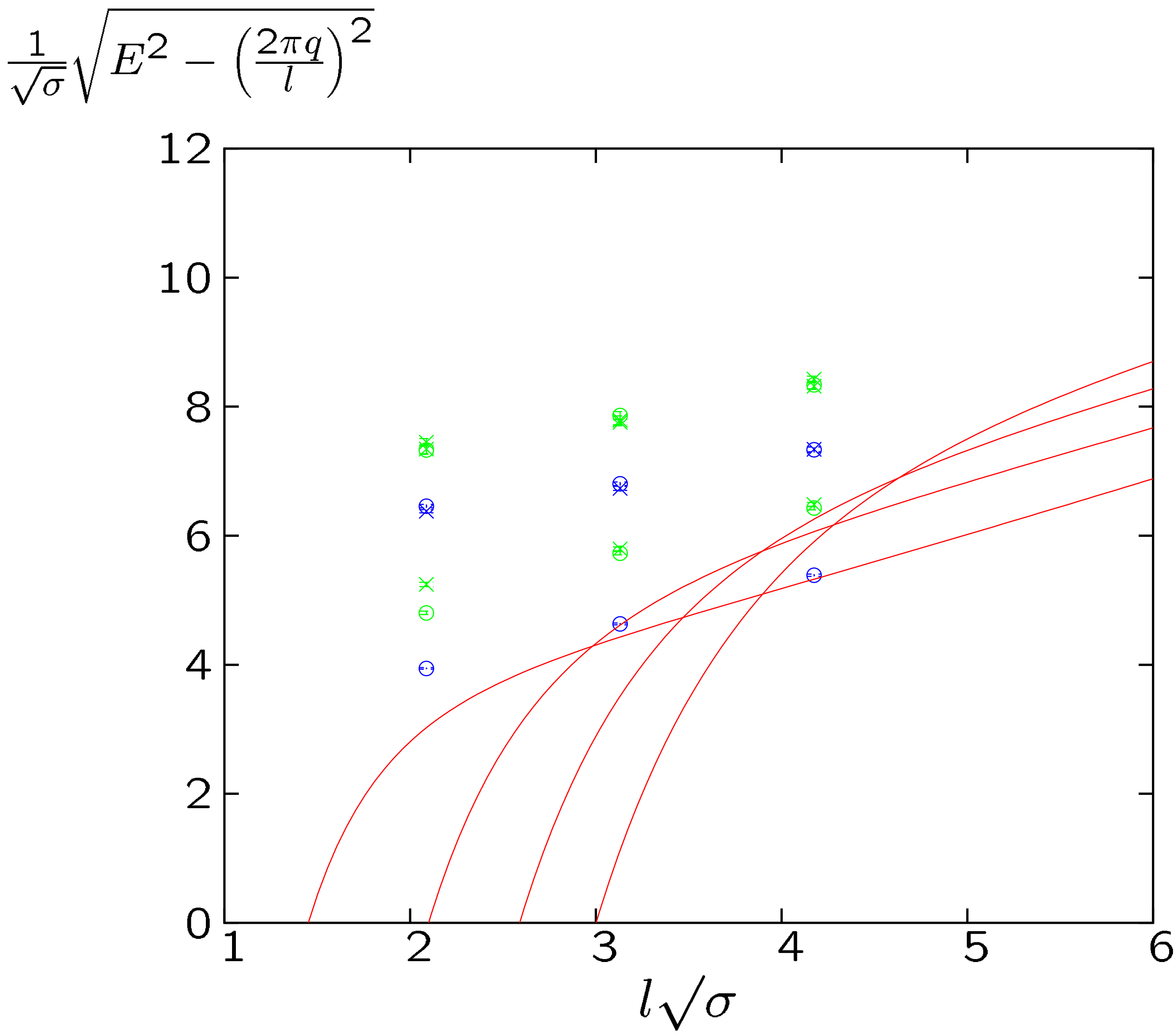


SU(3) vs SU(6) : same a





$q = 1$ $q = 2$
$\frac{1}{\sqrt{\sigma}}\sqrt{E^2 - \left(\frac{2\pi q}{l}\right)^2}$
12
10
8
6
4
2
0
1
2
3
4
5
6
$l\sqrt{\sigma}$
- Nambu-Goto : $E_n = \sigma l \sqrt{1 + \frac{8\pi}{\sigma l^2}\left(n - \frac{1}{24}\right)}$



- Luscher-Weisz 2004: $E_n = \sigma l + \frac{4\pi}{l}\left(n - \frac{1}{24}\right) - \frac{8\pi^2}{\sigma l^3}\left(n - \frac{1}{24}\right)^2$

Why ?

the covariant Nambu-Goto expression e.g. for $q = 0$,

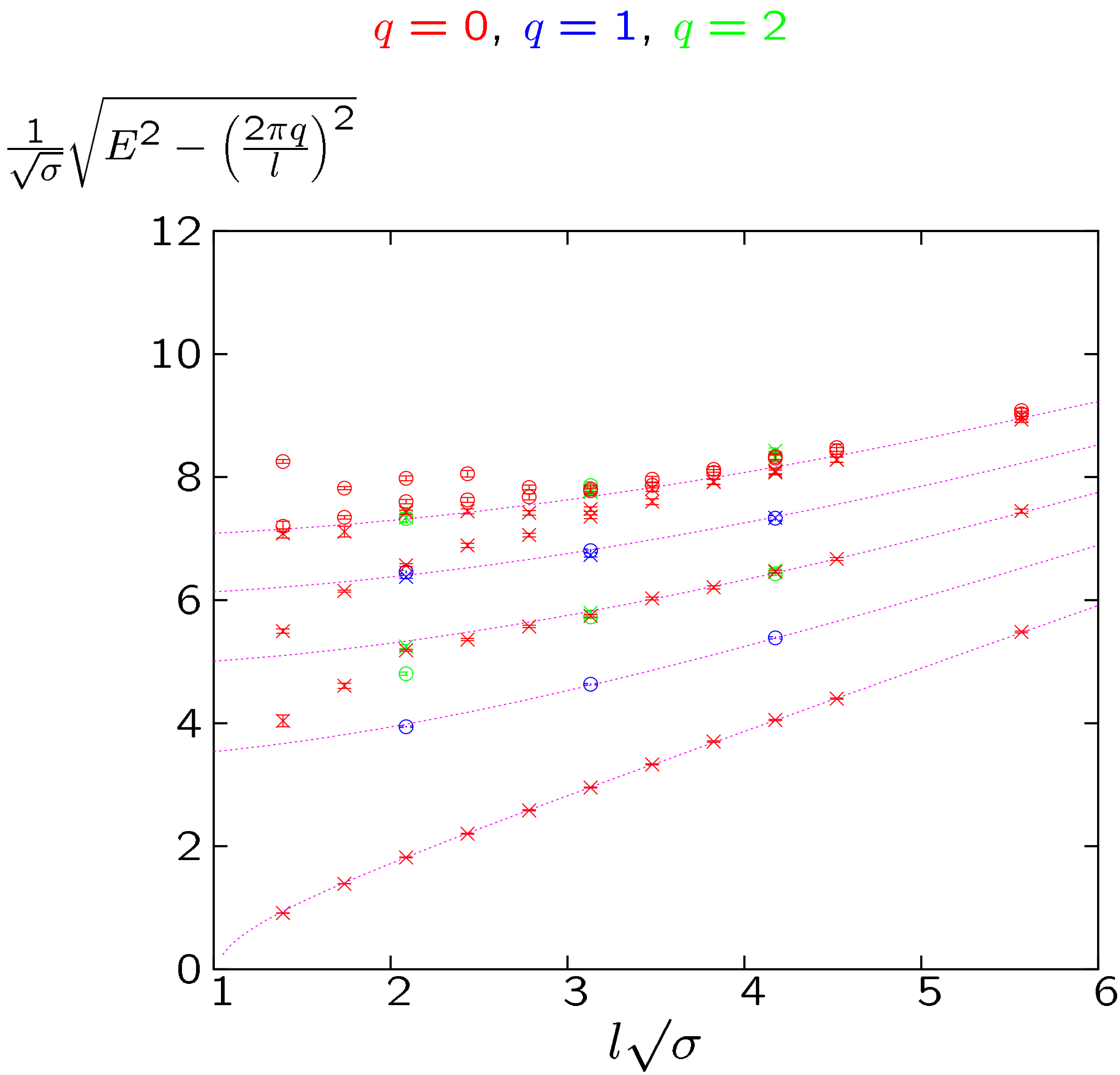
$$E(l) = \sigma l \left(1 + \frac{8\pi}{\sigma l^2}\left(n - \frac{D-2}{24}\right)\right)^{\frac{1}{2}}$$

can only be expanded as a power series in $1/l\sqrt{\sigma}$ when

$$x \equiv \frac{8\pi}{\sigma l^2}\left(n - \frac{1}{24}\right) \leq 1$$

whereas in practice we have a very good fit by Nambu-Goto even down to

$$x \sim 12 \qquad : \qquad l\sqrt{\sigma} \sim 2\ ,\ n = 2$$



$q = 0$, $q = 1$, $q = 2$
$\frac{1}{\sqrt{\sigma}}\sqrt{E^2 - \left(\frac{2\pi q}{l}\right)^2}$
12
10
8
6
4
2
0
1
2
3
4
5
6
$l\sqrt{\sigma}$

content of NG states:

$\vert 0\rangle$	P=+, q=0
$a^R(k=1)\vert 0\rangle$	P=−, q=1
$a^R(k=1)a^L(k=1)\vert 0\rangle$	P=+, q=0
$a^R(k=2)\vert 0\rangle$	P=−, q=2
$a^R(k=1)a^R(k=1)\vert 0\rangle$	P=+, q=2
$a^R(k=2)a^L(k=1)\vert 0\rangle$	P=+, q=1
$a^R(k=1)a^R(k=1)a^L(k=1)\vert 0\rangle$	P=−, q=1
$a^R(k=2)a^L(k=2)\vert 0\rangle$	P=+, q=0
$a^R(k=1)a^R(k=1)a^L(k=2)\vert 0\rangle$	P=−, q=0
$a^R(k=2)a^L(k=1)a^L(k=1)\vert 0\rangle$	P=−, q=0
$a^R(k=1)a^R(k=1)a^L(k=1)a^L(k=1)\vert 0\rangle$	P=+, q=0
$a^R(k=3)a^L(k=1)\vert 0\rangle$	P=+, q=2
$a^R(k=2)a^R(k=1)a^L(k=1)\vert 0\rangle$	P=−, q=2
$a^R(k=1)a^R(k=1)a^R(k=1)a^L(k=1)\vert 0\rangle$	P=+, q=2

observed near-degeneracies for $l \geq 2/\sqrt{\sigma} \sim 1\text{fm} \sim$ width flux tube!

• in D=2+1 SU(N) gauge theories, confining flux tubes belong to the universality class of a simple bosonic string theory

• more than that, the Nambu-Goto covariant free string spectrum

$$E^2(l) = (\sigma\, l)^2 + 8\pi\sigma\left(\frac{N_L+N_R}{2} - \frac{D-2}{24}\right) + \left(\frac{2\pi q}{l}\right)^2 .$$

is very accurate down to values of $l\sqrt{\sigma}$ where an effective string theory expansion, $x = l\sqrt{\sigma}$,

$$\frac{E_n}{\sqrt{\sigma}} = x\left(1+\frac{c}{x^2}\right)^{\frac{1}{2}} = x + \frac{c}{2x} - \frac{c}{8x^3} + \cdots$$

makes no sense (is far past its range of convergence)

• So, since in the range of $l\sqrt{\sigma}$ where such a power expansion is relevant, any difference with Nambu-Goto is totally negligible, it is clear that there is a challenge here to incorporate string corrections to Nambu-Goto in some 'resummed' or 'non-perturbative' way ...

• It is hard to understand how a short 'blob-like' flux tube can look so much like an elementary string unless it has a description as some kind of string, even when short – as perhaps in some kind of gravity duals ...

What about $D = 3 + 1$?

earlier work has provided good evidence that for SU(2) and SU(3) the universality class of the effective string theory describing long flux tubes is bosonic e.g. for the ground state

$$E_0(l) = \sigma l - \frac{\pi}{3l} + O(l^{-2})$$

and there is some evidence that excited states tend towards the corresponding behaviour:

$$E_n(l) = \sigma l + \frac{4\pi}{l}\left(n - \frac{1}{12}\right) + O(l^{-2})$$

but only when they are very long (and hard to calculate!):

$$l\sqrt{\sigma} \gg 1$$

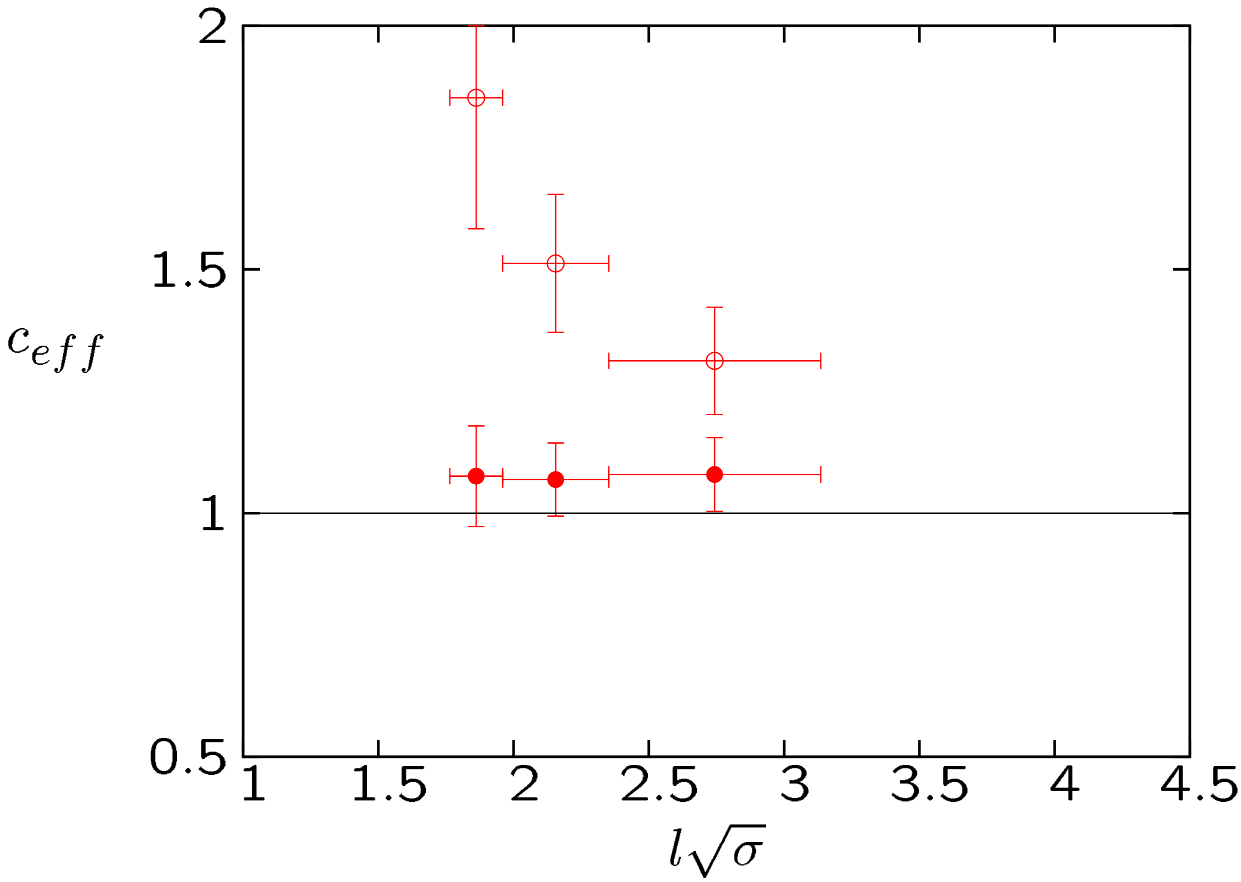
here I show some preliminary indications that what we found for $D = 2 + 1$ SU(N) gauge theories i.e. an amazingly precocious onset of Nambu-Goto behaviour:

$$E_n(l) = \sigma l(1 + \frac{8\pi}{\sigma l^2}\left(n - \frac{1}{12}\right)^{\frac{1}{2}}$$

is also the case in $D = 3 + 1$ – but only an indication so far ...

D=3+1 ; SU(3) ; $l_c \simeq 1.6$

B.Bringoltz, A.Athenedorou, M.Teper: in progress



Luscher:○

$$E_0(l) = \sigma l - c_{eff}\frac{\pi}{3l}$$

Nambu-Goto:●

$$E_0(l) = \sigma l \left(1 - c_{eff}\frac{2\pi}{3\sigma l^2}\right)^{\frac{1}{2}}$$