Lower dimensional defects in lattice Yang-Mills theory

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Outline

1. Three dimensional domain-walls
2. Two dimensional surfaces (P-vortices)
3. One dimensional (monopoles)
4. Fine structure of QCD confining string
   - Setup and theoretical expectations
   - Transverse string profile
   - Direct approach
5. Conclusions

3D

Topological defects and the spectrum of Dirac operator

Topological defects – regions of space with large absolute value of topological charge density.

To uncover topology of gluonic fields one could study low-lying modes of the Dirac operator

$$D\psi_\lambda(x) = \lambda\psi_\lambda(x)$$

Exact zero modes

$$n_+ - n_- = Q_{top}$$

$$\chi = \langle Q_{top}^2\rangle / V \propto m_{\eta'}^2 f_\pi^2$$

Near-zero modes

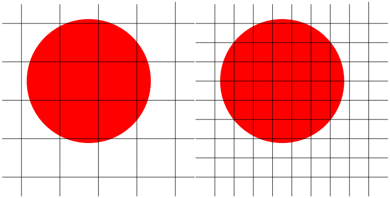
$$\langle\bar\psi\psi\rangle = -\pi \lim_{\lambda\to 0} \rho(\lambda)$$

Inverse Participation Ratio

- $I_\lambda = V \sum_x \rho_\lambda^2(x), \quad \rho_\lambda(x) = \psi_\lambda^\dagger(x)\psi_\lambda(x), \quad \sum_x \rho_\lambda(x) = 1\,.$
- IPR characterizes the inverse fraction of sites contributing to the support of $\rho_\lambda(x)$.
- For delocalized modes $\rho_\lambda(x) = 1/V$ and $I_\lambda = 1$.
- For extremely localized modes $\rho_\lambda(x) = \delta(x - x_0)$ and $I_\lambda = V$.
- For mode localized on a fraction f of sites support of $\rho_\lambda(x)$ occupies the volume $V_f = f\,V$ and $I_\lambda = V/V_f$.

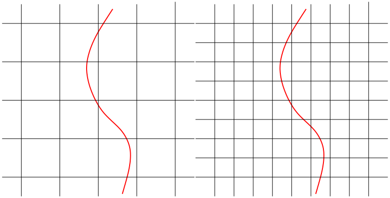
Dependence of IPR on lattice spacing

"Thick" object



$$I_\lambda \underset{a\to 0}{\longrightarrow} const$$

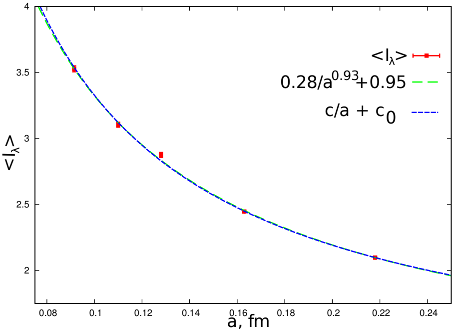
"Thin" object



$$I_\lambda \underset{a\to 0}{\longrightarrow} a^{d-4}$$

d – dimensionality of object.

Dependence of IPR on lattice spacing



Fit of lattice data with $I_\lambda = c_0 + c/a^{4-d}$ gives $d = 3$.

Low-lying modes are localized on a domain-walls, not conventional “thick” instantons.

Short summary on 3D defects

- The volume occupied by low-lying modes of Dirac operator being expressed in physical units tends to zero in the continuum limit of vanishing lattice spacing ($a \to 0$).
- Low-lying eigenmodes of Dirac operator exhibit fine-tuning: localization occurs at the UltraViolet scale but at the same time eigenmodes are responsible for the InfraRed physics.
- It seems, the vacuum is made of infinitely thin three-dimensional domain-walls.

2D

Definition of P-vortices

In $SU(2)$ lattice Yang–Mills theory P-vortices are defined in terms of projected fields which replace original $SU(2)$ fields by Z_2 fields:

- Use the gauge freedom to fix the maximal center gauge – maximize the squared trace of link variable:

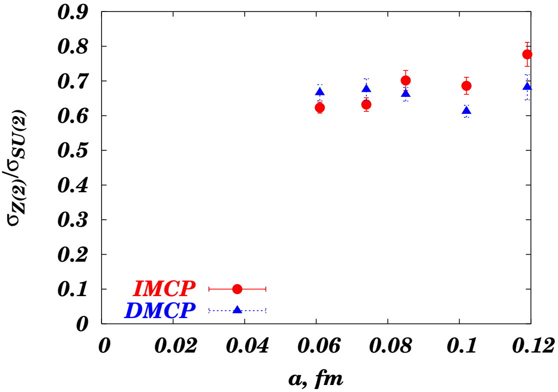
$$\max_{\Omega} F[U] = \max_{\Omega} \sum_{x,\mu} (\mathrm{Tr}\, U_{x,\mu})^2$$

- Z_2 gauge field is defined as:

$$Z_{x,\mu} = \mathrm{sign}\, \mathrm{Tr}\, U_{x,\mu}$$

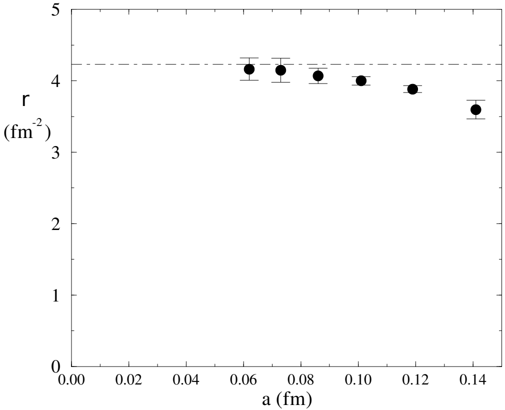
Vortices are closed surfaces constructed from plaquettes on a dual lattice dual to negative plaquettes on original lattice.

They provide nonzero string tension



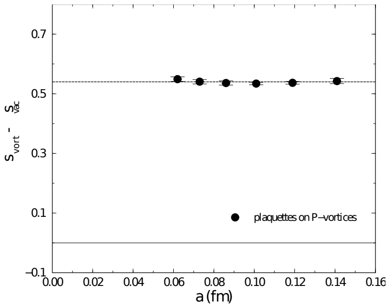
Scaling of P-vortices

Are vortices "physical" objects? To check this the scaling of total area of vortices with lattice spacing were studied.



- $A_{vort} = 6\rho_{vort} V_4$, V_4 – lattice volume in physical units.
- $\rho_{vort} \approx 4(\mathrm{fm})^{-2}$.
- Total area scales thus vortices are physical.

Divergent non-Abelian action of the vortex



Excess of non-Abelian action density on the vortex, s_{vort}, is independent on the lattice spacing:

- $\langle s_{vort} \rangle \approx 0.54$ (lattice units)

Thus total non-Abelian vortex action:

- $\langle S_{vort} \rangle \approx 0.54 \frac{A_{vort}}{a^2}$ (physical units)

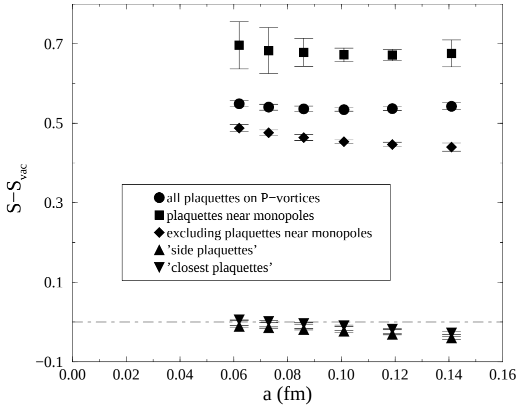
is divergent.

Two scales InfraRed and UltraViolet coexist (fine-tuned):

$$S_{vort} \propto (\Lambda_{QCD} a)^{-2}.$$

Infinitely thin “strings”

To probe the internal structure of vortices the average action density near vortex world-sheet was measured as a function of lattice spacing.



- Vortices appear as infinitely thin objects which populate vacuum with no sign of any internal structure.
- At presently available lattices the size of vortex is $R_{vort} \leq 0.06\,\text{fm}$.

Chirality and P-vortices

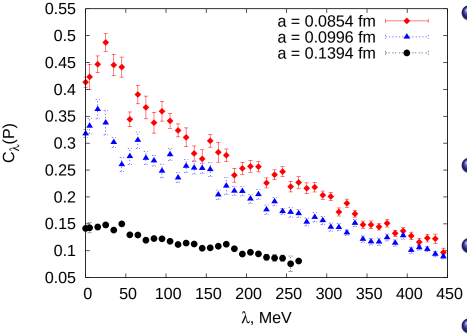
The next question is whether the surfaces carry chirality and explain topological defects. Are they related to fermionic low-lying modes?

To answer this question the correlator $C_\lambda(P)$ was studied:

$$C_\lambda(P) = \frac{\sum_{P_i} \sum_{x \in P_i} (\rho_\lambda(x) - \langle \rho_\lambda(x) \rangle)}{\sum_{P_i} \sum_{x \in P_i} \langle \rho_\lambda(x) \rangle},$$

where $\rho_\lambda(x)$ – scalar fermionic density normalized with $\sum_x \rho_\lambda(x) = 1$, V is a lattice volume. $\{P_i\}$ is a set of plaquettes on original lattice dual to a set of vortex plaquettes on the dual lattice $\{D_i\}$.

Vortices carry chirality



- There is strong positive correlation between intensities of topological modes and density of vortices nearby.
- Value of correlator depends on the eigenvalue.
- Correlation is strong only for topological fermionic modes.
- Data exhibit strong lattice spacing dependence.

Short summary on 2D defects

- With vortices it is possible to shed some light upon the confinement problem.
- Center vortices are infinitely thin surfaces which carry chirality and have UltraViolet divergent non-Abelian action.
- Their total area is in physical units (Λ_{QCD}) and scales in the continuum limit.
- Thus these defects are fine-tuned: two scales coexist.

1D

Definition of monopoles

In $SU(2)$ lattice Yang–Mills theory monopoles are defined in a three stage process:

- Use the gauge freedom to bring the non-Abelian fields as close to the Abelian ones as possible, i.e. fix maximal Abelian gauge (MAG):

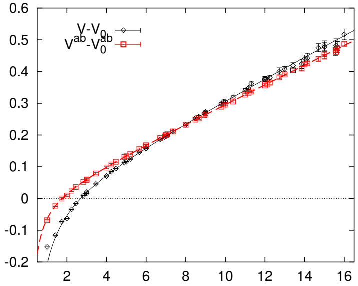
$$\min_{\Omega} F[A] = \min_{\Omega} \frac{1}{V} \int_V d^4x \, (A^1_\mu)^2 + (A^2_\mu)^2 \,.$$

- Project the non-Abelian fields into their Abelian part by putting $A^{1,2}_\mu \equiv 0$.

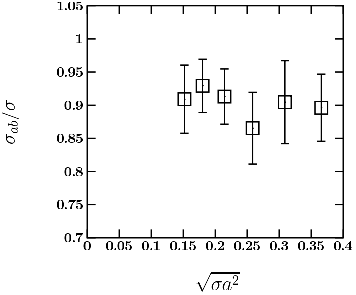
Monopoles are defined in each lattice cube using Gauss law for Abelian field. They form closed trajectories.

They also provide nonzero string tension

Non-Abelian and Abelian static potentials

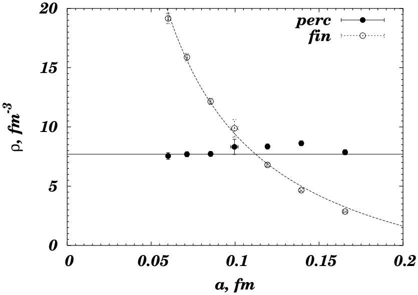


Ratio of Abelian and non-Abelian string tensions



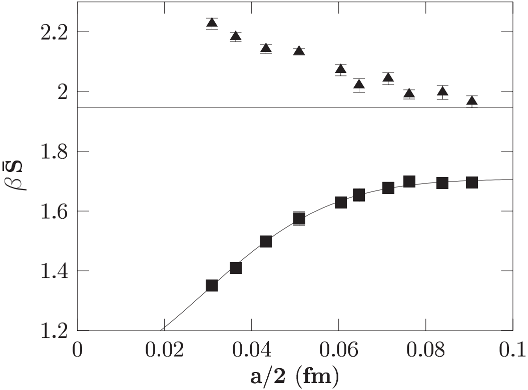
QCD vacuum as a dual superconductor (dual Abelian Higgs model).

Scaling of monopole densities



- There is always a single percolating cluster:
  $l_{perc} \propto V, \quad V \to \infty.$
- Density of percolating cluster scales:
  $\rho_{perc} \equiv \frac{\langle l_{perc} \rangle}{4L^4 a^3} = 7.70(8)\mathrm{fm}^{-3}.$
- There are a lot of finite clusters:
  $l_{fin} \propto O(a)$
- Their density is divergent:
  $\rho_{fin} \propto 1/a$

Divergent monopole action

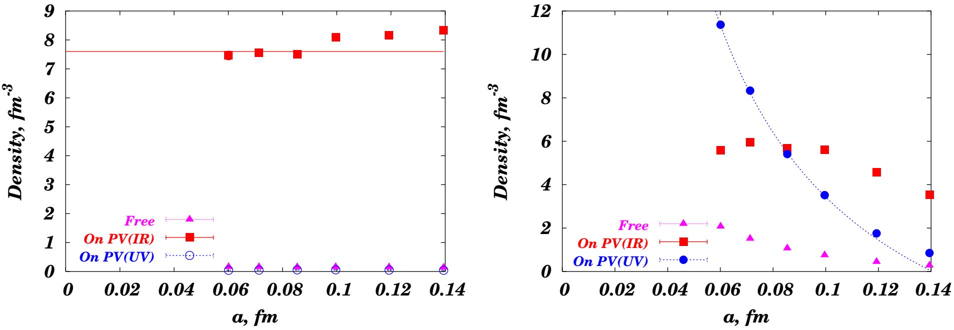


Again two scales InfraRed and UltraViolet coexist (fine-tuned):

$$S_{mon} \propto l_{perc} \cdot \bar{S} \propto (\Lambda_{QCD} a)^{-1}.$$

Interplay between monopoles and vortices

Are monopoles and vortices showing similar mixture of scales interrelated?



Data shows that monopoles populate infinitely thin P-vortices.

Short summary on 1D defects

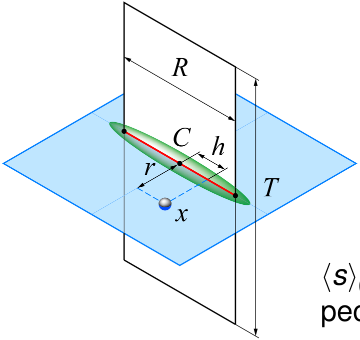
- It is possible to explain confinement in terms of monopoles (dual superconductor model) and what is why they are worth to study.
- Abelian monopoles carry divergent non-Abelian action.
- Density of percolating monopoles scales with lattice spacing.
- Monopoles are fine-tuned: IR and UV scales coexist.
- Monopoles live on infinitely thin 2D surfaces.

Fine structure of QCD string

based on arXiv:0704.1203 [hep-lat]

Geometrical setup

Static quark-antiquark pair separated by the distance R created at time $t = 0$ and annihilated at $t = T$ is represented by rectangular $T \times R$ Wilson loop.



$$\Delta s = \langle s \rangle_0 - \langle s \rangle_W = \langle s \rangle_0 - \lim_{T\to\infty} \frac{\langle s(h,r) W(R,T) \rangle_0}{\langle W(R,T) \rangle_0}$$

$\langle s \rangle_0$ – action density $s = \mathrm{Tr}\, F^2_{\mu\nu}$ vacuum expectation value

IR/UV "mixing" in vacuum action density

Conventional prediction (OPE): $s = \mathrm{Tr}\, F^2_{\mu\nu}$

$$\langle s \rangle_0 = \frac{\alpha_0}{a^4} + \gamma_0\, \Lambda^4_{QCD} \quad \text{[up to logarithms]}$$

However, it had long been discussed that this pattern is more involved

$$\langle s \rangle_0 = \frac{\alpha_0}{a^4} + \frac{\beta_0\, \Lambda^2_{QCD}}{a^2} + \gamma_0\, \Lambda^4_{QCD}$$

and includes explicit IR/UV "mixing" term. As for the difference Δs:

$$\Delta s = \frac{\beta\, \Lambda^2_{QCD}}{a^2} + \gamma\, \Lambda^4_{QCD}\,.$$

Note, leading divergence vanishes, as expected.

Theoretical expectations: string width

Regardless of how small the "mixing" term is, it has rather dramatic consequences. Rigorous action sum rules

$$\int d^3x\,\Delta s = V(R) \qquad \text{(up to logarithms)}$$

for $R \gg \Lambda_{QCD}^{-1}$ allow to estimate squared string width δ^2

$$\delta^2 \;\propto\; \sigma\cdot\Delta s \;\approx\; \sigma\cdot[\beta\,\Lambda^2_{QCD}/a^2 + \gamma\,\Lambda^4_{QCD}]^{-1} \;\xrightarrow{a\to 0}\; 0 \qquad [!]$$

Compare with effective string theory prediction:

- Gaussian profile

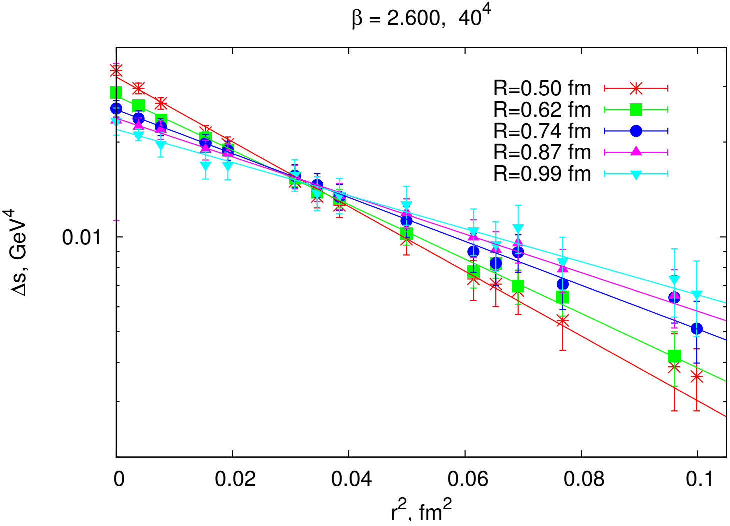
$$\Delta s(h=0) = C(R)\,\exp\{-r^2/\delta^2(R)\}$$

- Infinitely long QCD string does not exist

$$\delta^2(R) = \frac{1}{\pi\sigma}\ln[R/R_0] \;\xrightarrow{R\to\infty}\; \infty$$

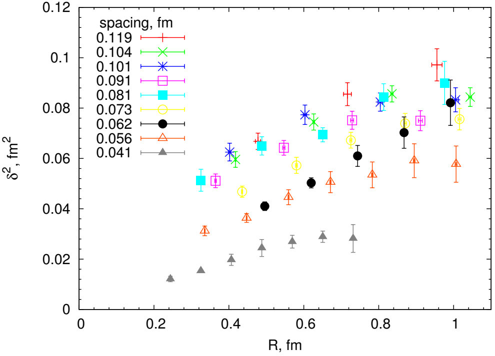
Transverse profile at $h = 0$

Transverse profile is Gaussian for $R \gtrsim 0.3$fm, width increases with R.



String width at $h = 0$

Squared string width $\delta^2(R)$ vs. R at various spacings.



- String widening with $R \to \infty$ (probably logarithmic) is observed.
- Systematic drop of δ^2 for $a \lesssim a_{cr} = 0.07\,\text{fm}$ is observed.
- Thus flux tube rapidly shrinks with $a \to 0$.
- If this is caused by quadratic divergence $\beta\Lambda^2_{QCD}/a^2$, which could be estimated:

$$\beta\Lambda^2_{QCD} \approx a^2_{cr} \cdot \gamma\Lambda^4_{QCD} \approx (50\ \mathrm{MeV})^2$$

On-axis ($r = 0$) action density difference

Return now to large R limit of (rigorous) action sum rules

$$\delta^2 \Delta s \approx \sigma = const \qquad [R \gg \Lambda_{QCD}^{-1}]$$

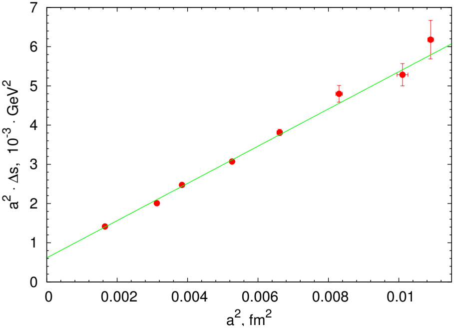
Measuring Δs at the string geometrical center

$$(h = r = 0)$$

allows to confirm string shrinkage independently.

Action density at the string center, $R \to \infty$

Plot of the product $a^2 \cdot \Delta s$ versus a^2.



Action density at the string geometrical center diverges quadratically in the continuum limit. Fit gives:

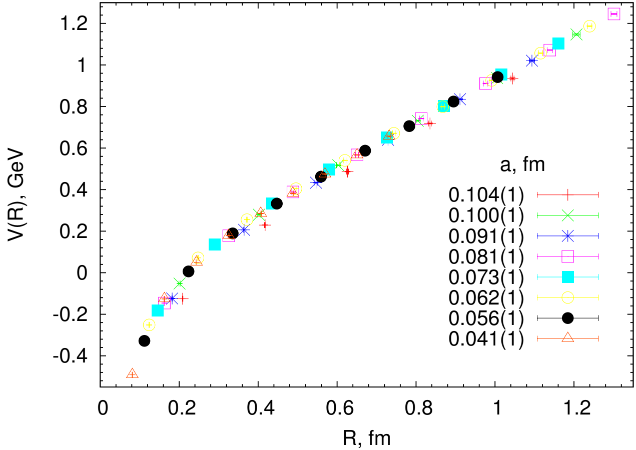
$$\beta\Lambda^2_{QCD} = (25(2)\,\mathrm{MeV})^2$$

Short summary on fine structure of QCD string

- String widening is seen at finite UV cutoff and is compatible with logarithmic law, however, this is a subleading effect.
- Width of the confining string shrinks almost linearly and its action density quadratically diverges in the limit $a \to 0$, so that the observable heavy quark potential remains physical:

$$\left.\begin{array}{c} \delta \sim a \\ \Delta s \sim a^{-2} \end{array}\right\} \to \delta^2 \cdot \Delta s \approx \sigma = const.$$

Heavy quark potential



There is no sign whatsoever of UV cutoff dependence.

Conclusions

- Topological fermionic modes live on three dimensional domain-walls and the volume occupied by them shrinks to zero in the continuum limit.
- It seems, QCD vacuum is populated with infinitely thin 2D surfaces (strings) with point-like particles (monopoles) living on them.
- All these defects exhibit power-like dependences on the lattice spacing and fine-tuning.
- QCD confining string connecting static quarks shrinks to infinitely thin line ($\delta \propto a$).
- There are only pieces of theory. Could AdS/QCD help?

Thanks for your attention.