Comments on holographic QCD

Based on:

* E. Antonyan, J. Harvey, S. Jensen, D.K., hep-th/0604017

* I. Klebanov, D.K., A. Murugan, 0709.2140

* O. Aharony, D.K., 0803.3547

The goal of holographic QCD is to find a string dual for $SU(N_c)$ YM theory with N_f flavors of quarks in the fundamental rep, in the limit $N_c \to \infty$, $N_f =$ fixed.

There are a number of constructions that have been proposed for this. I will focus here on a particular one, but much of what I'll say is applicable more generally.

Review of Witten's construction of pure $SU(N_c)$ Yang-Mills

Consider N_c D4-branes in IIA string theory stretched in (01234). The low energy theory on the branes is 4+1d SYM with 16 supercharges and coupling $\lambda = g_s N_c \ell_s = g^2 N_c$

4+1d gauge coupling

λ is a length scale. Effective coupling at energy E is $\lambda \cdot E$.

$E\lambda \ll 1$: gauge theory is weakly coupled.

$E\lambda \gg 1$: need UV completion (in string theory, one is provided by the (2,0) theory on M5's).

To reduce from 4+1 → 3+1 d and break SUSY, compactify $x^4 \sim x^4 + 2\pi R_4$ with the fermions in the 4+1 d gauge theory anti-periodic:

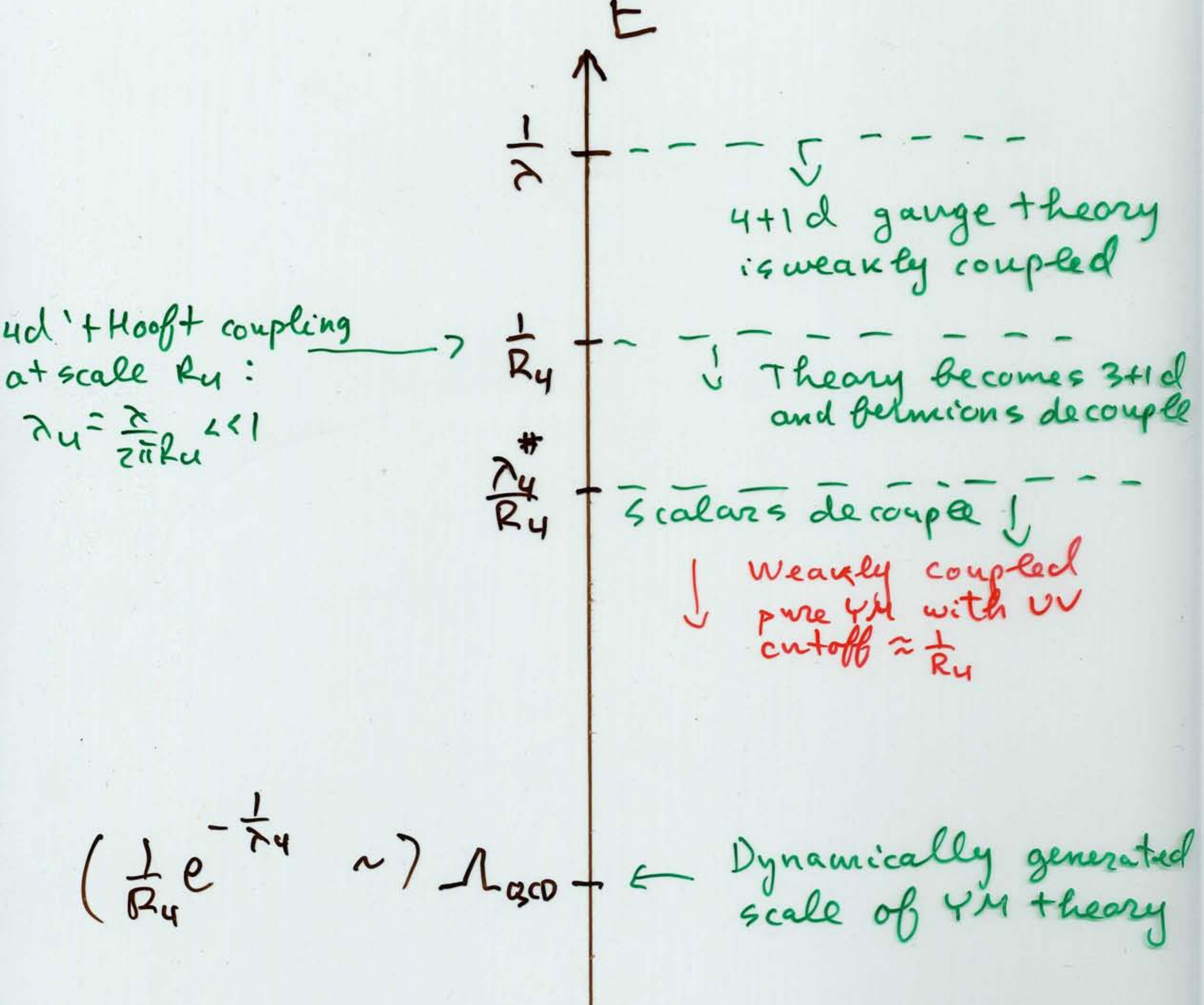
$$\psi(x^4 + 2\pi R_4) = -\psi(x^4)$$

One might think that to get 3+1 d YM, we should take R_4 to be small, but in fact the opposite is true:

pure YM is obtained when R_4 is large,

$$R_4 \gg \lambda$$

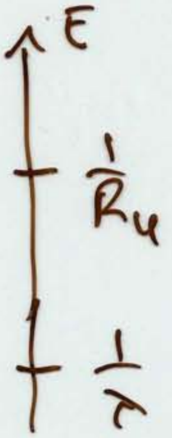
To understand why, consider the different scales in this limit:



The key point is at the energy of interest, Λ_{QCD}, dynamics of branes is just that of pure (large N_c) YM theory, with all the extra d.o.f. decoupled.

A nice feature of the brane construction is that it embeds YM theory in a larger set of theories, with a tunable parameter (λ_4), so we can ask what happens as we change this parameter.

Consider, in particular, the limit $\lambda \gg R_4$. Here, the hierarchy of scales is inverted



so the 5d gauge theory is strongly coupled at the scale R_4, and the YM description is not useful.

The good description in that regime is in terms of supergravity in the near-horizon geometry of D4-branes.

Metric is:

$$ds^2 = \left(\frac{U}{R}\right)^{\frac{3}{2}} \left[\overbrace{dx^\mu dx_\mu}^{(0123)} + f(U)(dx^4)^2\right] +$$
$$+ \left(\frac{U}{R}\right)^{-\frac{3}{2}} \left[\underbrace{\frac{dU^2}{f(U)} + U^2 d\Omega_4^2}_{(56789)}\right]$$

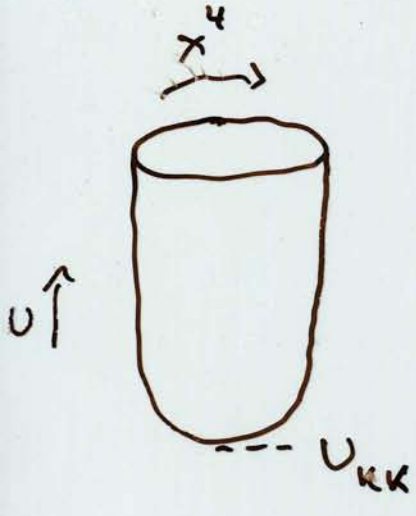
where

$$\alpha' = 1$$

$$R^3 = \pi \lambda$$

$$f(U) = 1 - \left(\frac{U_{KK}}{U}\right)^3$$

$$U_{KK} = \frac{4\pi}{9} \frac{\lambda}{R_4^2}$$



In this solution, $U \geq U_{KK}$. This is related to confinement (force between static quarks is linear). One can compute spectrum of glueballs, by studying sugra fluctuations about the solution. Can also compute deconfinement temperature,

$$T_d \sim \frac{1}{R_4}$$

This energy scale is $\gg \frac{1}{\lambda}$, so indeed 4+1d gauge theory is strongly coupled at T_d.

Also, fermions and scalars on D4-branes do not decouple.

So, the picture we arrive at is this:

$R_4 >> \lambda \Rightarrow$ large N_c YM

$R_4 << \lambda \Rightarrow$ can calculate using gravity.

The hope is that there is no phase transition as a function of $\frac{\lambda}{R_4}$.

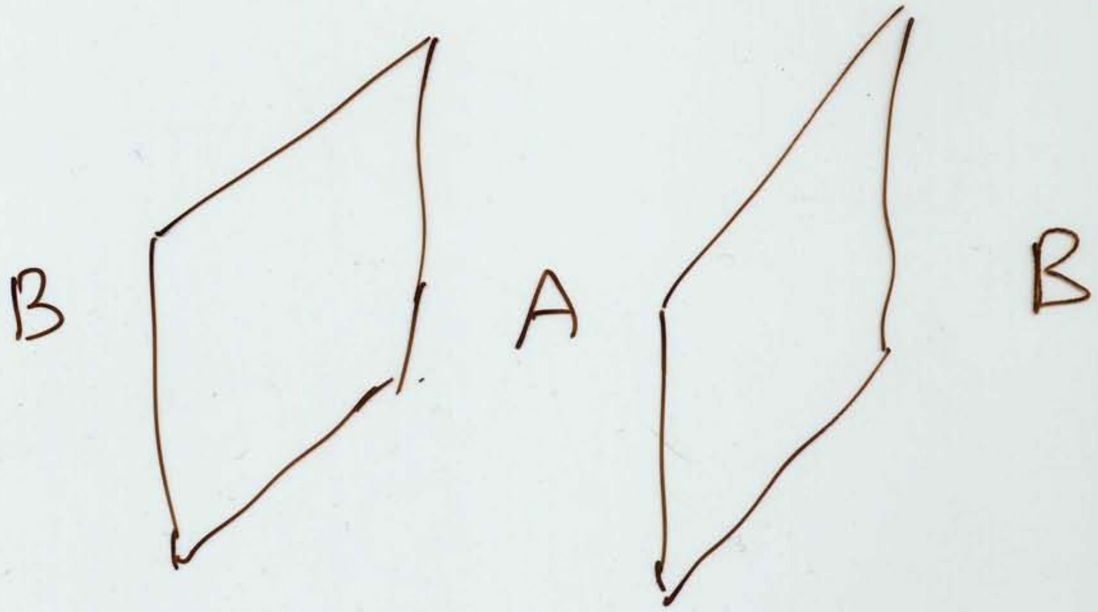
Detailed features of spectrum and correlation fnctns of gravity and YM need not agree - indeed, they are known to be generically quite different.

However, one can use gravity as a qualitative guide to features such as the phase structure as a function of temperature, chemical potential, etc.

One such quantity is entanglement entropy, to which we turn next.

Entanglement entropy as a probe of confinement

Divide R^3 into two complementary regions:



A = "inside"

B = "outside"

Define the density matrix

$$\rho_A = \mathrm{Tr}_B\, |0\rangle\langle 0|$$

By tracing over all d.o.f. in "outside" region B.

The entanglement entropy of A, B is defined as:

$$(S_B =) S_A = - \mathrm{Tr}_A\, \rho_A \ln \rho_A$$

In general, S_A is UV divergent, but one can show that $\frac{\partial}{\partial \ell} S_A$ is finite. Thus, below we will consider S_A up to an ℓ-independent (infinite) constant.

Q: How does $S_A(\ell)$ behave in a confining theory, such as large N_c YM?

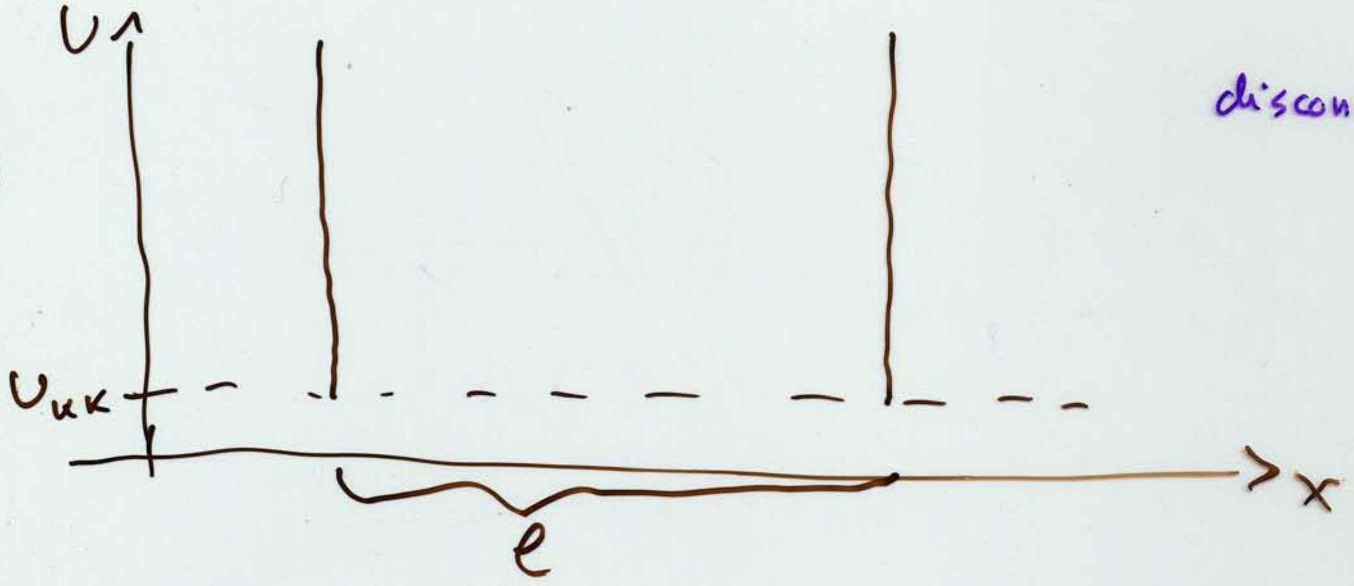
In gauge theory, it is non-trivial to calculate $S_A(\ell)$ from 1st principles, But in gravity regime this is feasible. Ryu & Takayanagi proposed a formula for S_A for any theory with a gravity dual:

$$S_A = \frac{1}{4G_N} \int_{\mathcal{M}} d^8\sigma \, e^{-2\Phi} \sqrt{G^{(8)}_{ind}}$$

where $\mathcal{M}$ is a spatial manifold, which minimizes S_A over all bulk surfaces that approach the boundary of A (or B) as $U \to \infty$ (i.e. near the boundary of the bulk spacetime).

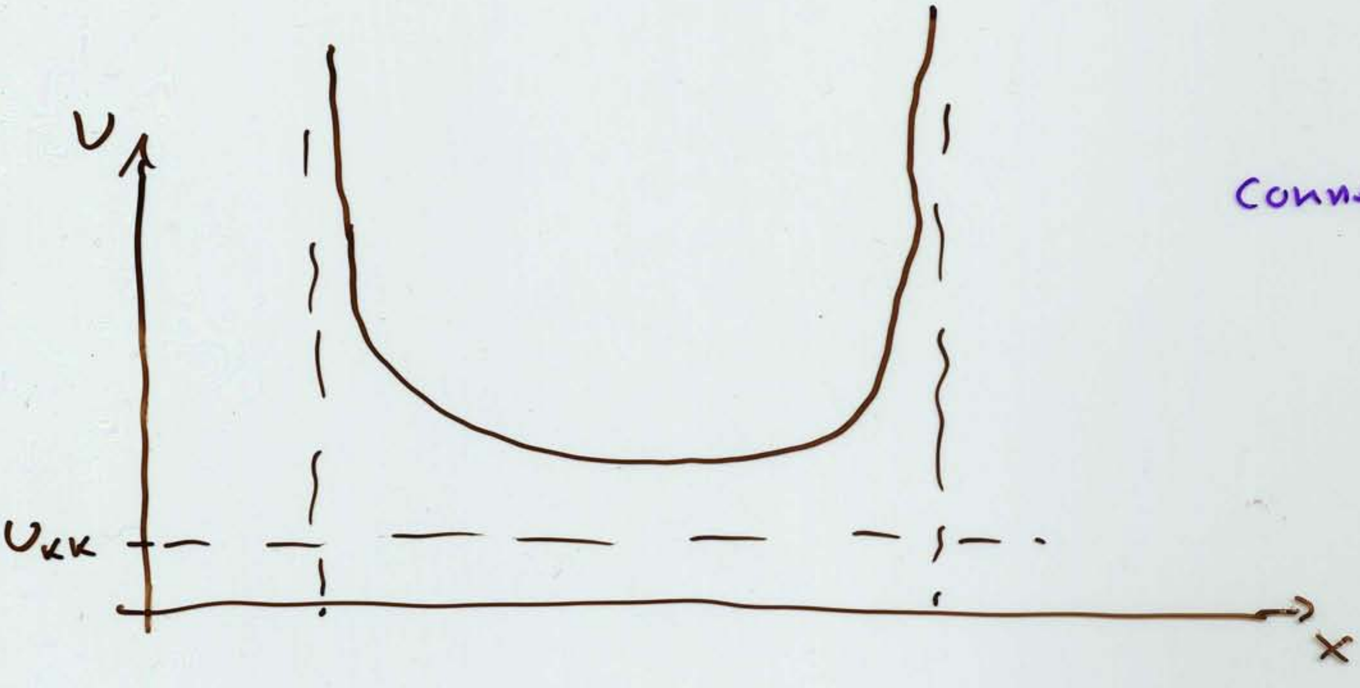
In our case, there are 2 candidate surfaces with the right b.c.'s:

(1)



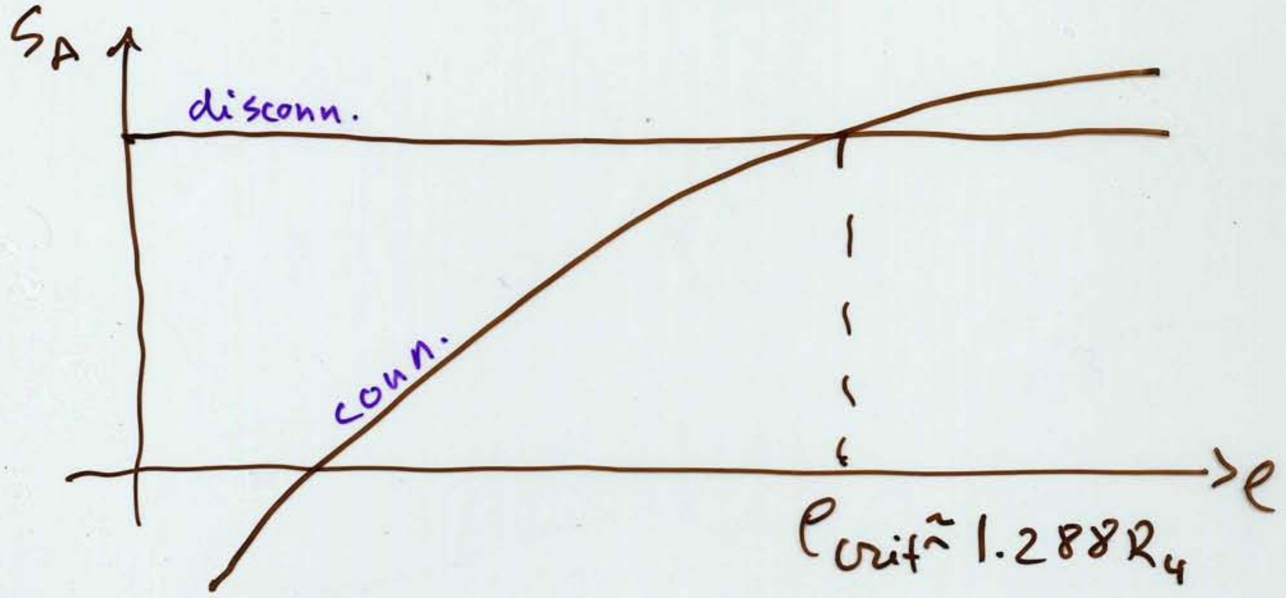
disconnected

(2)



connected

Calculating S_A for them using the Ryu-Takayanagi prescription one gets:



* For $\ell > \ell_{crit}$, disconnected solution has smaller S_A and entanglement entropy is ℓ-independent. Can set it to zero by a choice of constant.
* For $\ell < \ell_{crit}$, connected solution dominates. S_A depends non-trivially on ℓ.

* All the above results concern terms that go like N_c^2 in entanglement entropy. Thus, we find:

$$S_A(\ell) \sim \begin{cases} N_c^2 & \ell < \ell_{crit} \\ N_c^0 & \ell > \ell_{crit} \end{cases}$$

At $\ell = \ell_{crit}$ we have a 1st order phase transition (the surface that dominates the entropy changes discontinuously). Very reminiscent of thermodynamics ($\ell \leftrightarrow \beta$).

At large N_c, one can argue for such a phase transition directly in YM theory. Assume that to leading order in $\frac{1}{N_c}$ the theory reduces to a free field theory of glueballs, whose density of states exhibits Hagedorn growth:

$$\rho(m) \sim m^{\alpha} e^{\beta_H m}$$

with $\beta_H \sim \frac{1}{\Lambda_{QCD}}$ and α a constant.

To calculate $S_A(\ell)$, need to calculate contribution of a single free field, and sum over glueballs.

For a single free field of mass $m \gg \frac{1}{\ell}$ one finds:

$$S_A^{(\text{free})}(\ell) \sim e^{-2m\ell}$$

Thus,

$$S_A(\ell) \sim \int^{\infty} dm \, m^{\beta} e^{(\beta_H - 2\ell)m}$$

* For $\ell > \frac{1}{2}\beta_H$, integral converges $\Rightarrow$

$$S_A(\ell) \sim N_c^0$$

* For $\ell < \frac{1}{2}\beta_H$, integral diverges $\Rightarrow$

$$S_A(\ell) \sim N_c^2$$

In analogy to thermodynamic behavior.

We conclude that large N_c confining field theories have a phase transition in $S_A(\ell)$ at a critical value of ℓ. One would expect such a transition at finite N_c as well.

Would be interesting to investigate this on the lattice. Some preliminary results:

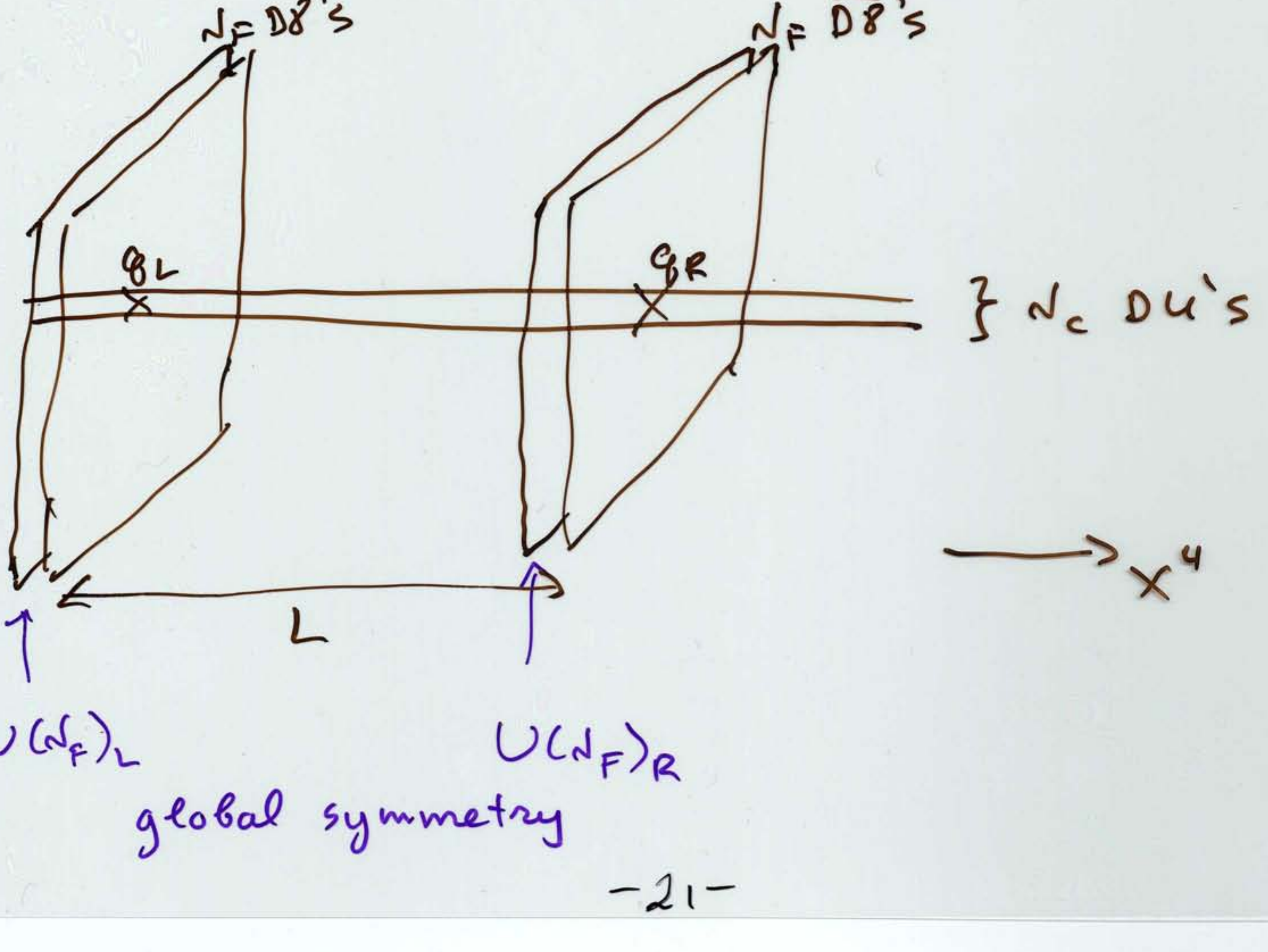
A. Velytsky, 0801.4111

P. Buividovich, M. Polikarpov, 0802.4247.

Adding quarks

So far we discussed holographic dual of pure YM using branes. To add quarks, one can proceed as follows (Sakai & Sugimoto).

Add D8 and $\overline{D8}$ branes, which intersect D4-branes along $R^{3,1}$:

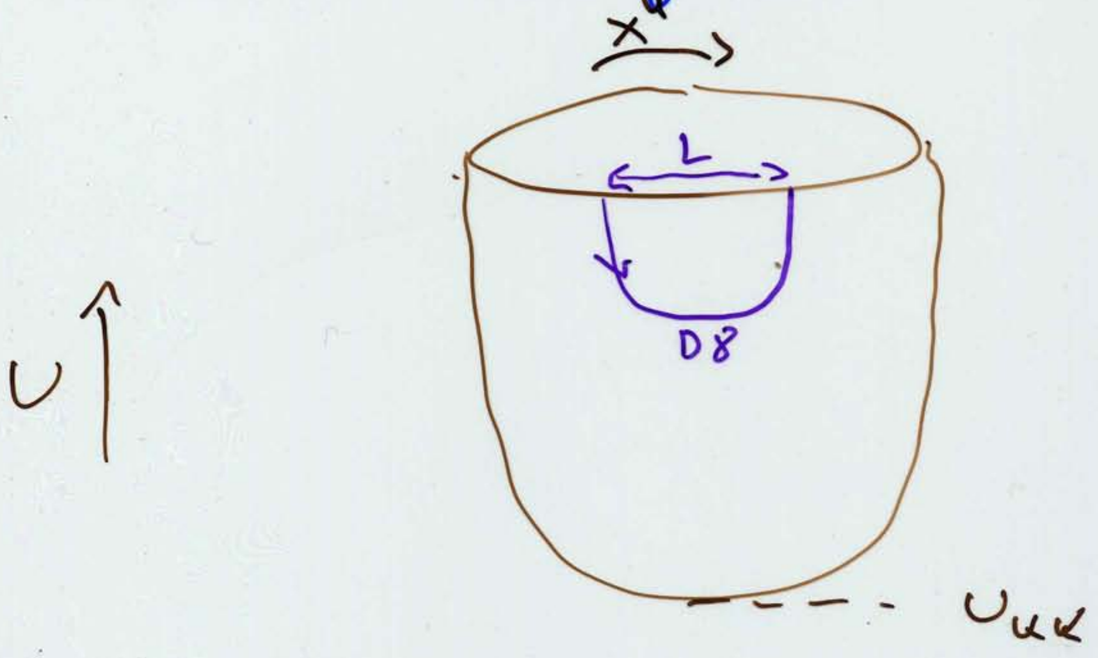


* Now, in addition to $SU(N_c)$ gauge field, we have massless fermions in the fundamental rep of $SU(N_c)$, q_L, q_R, coming from 4-8, 4-$\bar{8}$ intersections.

* The brane configuration preserves $U(N_F)_L \times U(N_F)_R$ chiral symmetry.

* The dynamics depends on $\frac{\lambda}{R_4}$, as before, but now also on $\frac{\lambda}{L}$. To understand this dependence, consider the gravity regime $\lambda \gg R_4, L$.

* Can replace D4-branes by their geometry.
* D8-branes are governed by DBI action in this geometry. Minimal energy solution has form:

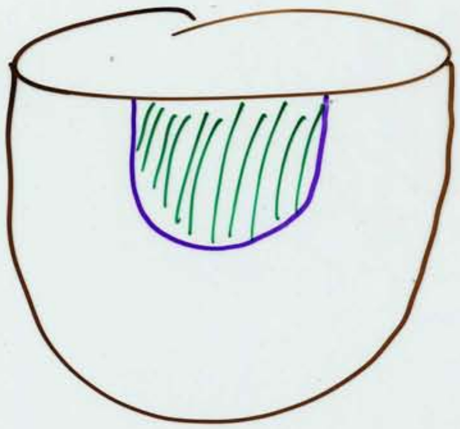


Note that D8 and $\overline{D8}$-branes are connected, and form one stack of N_f curved D8-branes. Hence, chiral symmetry is broken:

$$U(N_F)_L \times U(N_F)_R \to U(N_F)_{diag}$$

Order parameter for this breaking is an 8-$\bar{8}$ string stretched between the D8 & $\overline{D8}$-branes. It transforms as $(N_F, \bar{N}_F)$ under $U(N_F)_L \times U(N_F)_R$ and if it has a v.e.v., chiral symmetry is broken.

This v.e.v., $\langle OW \rangle$ is given by a disk instanton calculation:



$$\langle OW \rangle \sim e^{-S_{string}}$$

S_{string} = action for a Euclidean string worldsheet with the right b.c.'s at $U=\infty$.

This calculation can be done exactly. One finds:

$$\langle OW \rangle \sim \begin{cases} e^{c_1 \frac{\lambda}{L}} & L \ll R_4 \\ e^{\frac{\lambda}{18\pi R_4}} & L = \pi R_4 \end{cases}$$

with a known interpolation between the two limits. Note that $\langle OW \rangle$ is large in the gravity regime.

Comments

* One may be tempted to identify the stretched string OW with $q_L^\dagger \cdot q_R$, but this is not quite right, since the latter is not gauge invariant. Rather, one has

$$OW = q_L^\dagger \underbrace{e^{\int dx^\mu (iA_\mu + \vec{n}\cdot\vec{\Phi})}}_{\text{open Wilson line}} q_R$$

For $L, R_u \gg \lambda$ (in QCD regime), can ignore Wilson line and $OW = q_L^\dagger \cdot q_R$. In gravity regime Wilson line is important, and in fact gives the bulk of the contribution to $\langle OW \rangle$.

* As is clear from shape of D8-branes and dependence of $\langle OW \rangle$ on R_u, L, the brane configurations exhibit a decoupling between chiral symmetry breaking and confinement. The latter is determined by R_u ($\to U_{KK}$). The former by L.

E.g. even when $R_u \to \infty$ so $U_{KK} \to 0$ and no confinement, $\langle OW \rangle \neq 0$ and is large for small L.

From QCD pt of view, this decoupling is due to direct interactions between fermions, e.g. $q_L^+ \cdot q_R \, q_R^+ \cdot q_L$. Gravity analysis shows that when they are strong enough, they can induce χSB even without confinement.

This leads to a physical realization of the NJL model in string theory.

Very recently (0805.4627) D. Sinclair showed that adding 4-Fermi interactions on the lattice indeed separates the scales of χSB & Confinement, as predicted by gravity analysis.

* Can use above ideas to add mass for the quarks in the SS model.

Will leave for next time ..