Holographic Mesons

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Motivation + Aim

To generalize the AdS/CFT correspondence such that it describes realistic field theories

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Holographic description of

- fields in the fundamental representation of the gauge group (quarks)
- chiral symmetry breaking and meson spectra
- finite temperature field theories (+ finite density)

Outline

1. Adding flavour to AdS/CFT

2. Chiral symmetry breaking

3. Mesons

4. Finite temperature and finite density

Generalizations of the AdS/CFT correspondence

$\mathcal{N} = 4$ $SU(N)$ theory:

- $N \to \infty$
- Supersymmetry
- Conformal symmetry
- All fields in the adjoint representation of the gauge group

QCD:

- $N = 3$
- No supersymmetry
- Confinement
- Quarks in fundamental representation of the gauge group

Desirable extensions of AdS/CFT:

- Break SUSY and conformal symmetry $\Leftrightarrow$ Deformation of AdS space
- Add quarks in fundamental representation of gauge group
- Relax $N \to \infty$ limit ($1/N$ corrections)
  $\Leftrightarrow$ String theory instead of supergravity

Models of generalized AdS/CFT with flavour

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D3/D7 model:

Embed $D7$ brane probes in (deformed versions of) $AdS_5 \times S^5$

$U(1)$ axial (chiral) symmetry

In UV, theory returns to $d = 4\, \mathcal{N} = 2$ theory

($\mathcal{N} = 4$ + fundamental hypermultiplet)

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Sakai-Sugimoto model

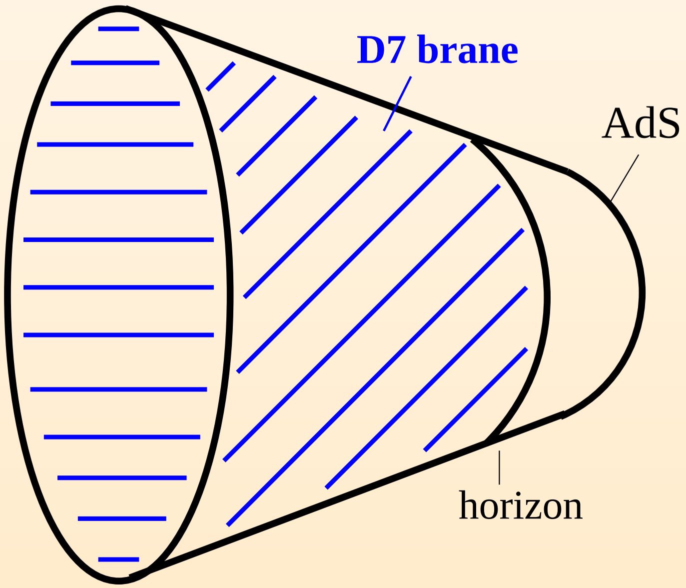
$D8$ and $\overline{D8}$ in $D4$ background with compactified space direction

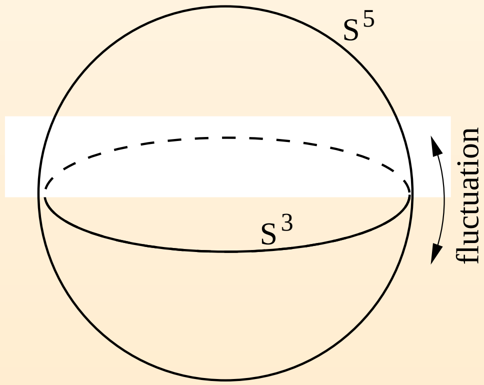
$SU(N_f) \times SU(N_f)$ chiral symmetry

UV theory five-dimensional

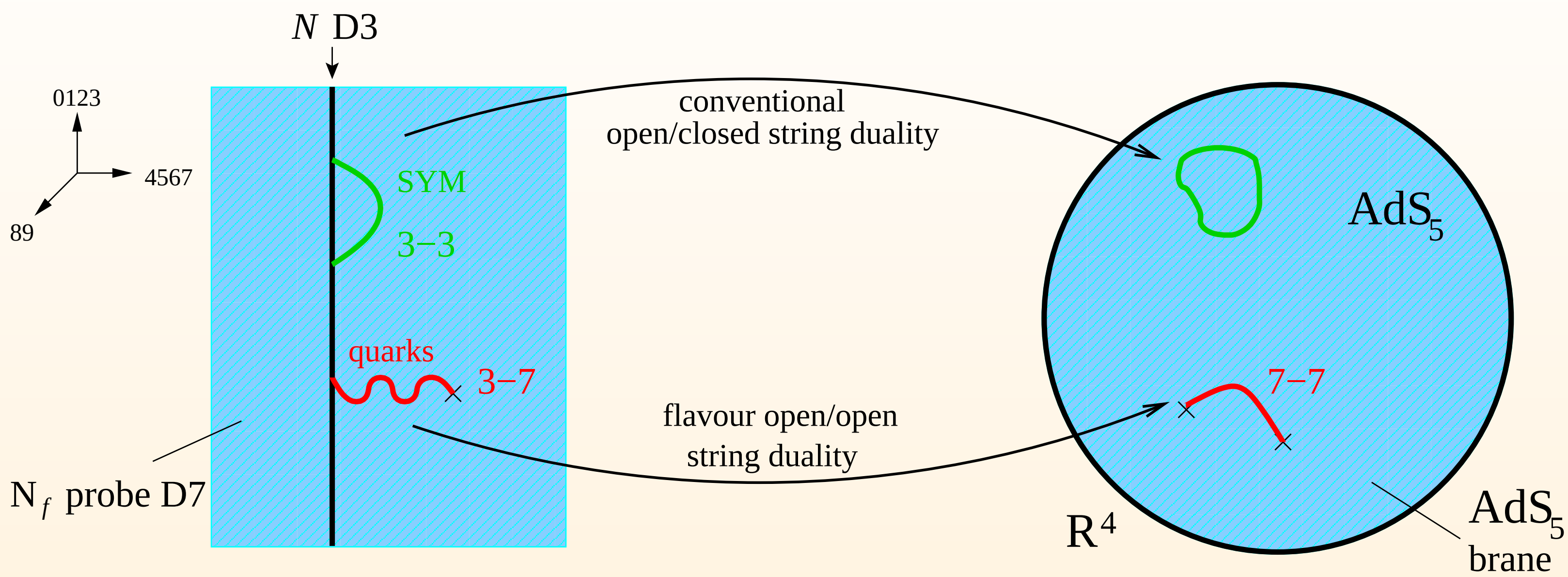
D7 brane probe:

	0	1	2	3	4	5	6	7	8	9
D3	X	X	X	X						
D7	X	X	X	X	X	X	X	X		





Quarks (fundamental fields) from brane probes



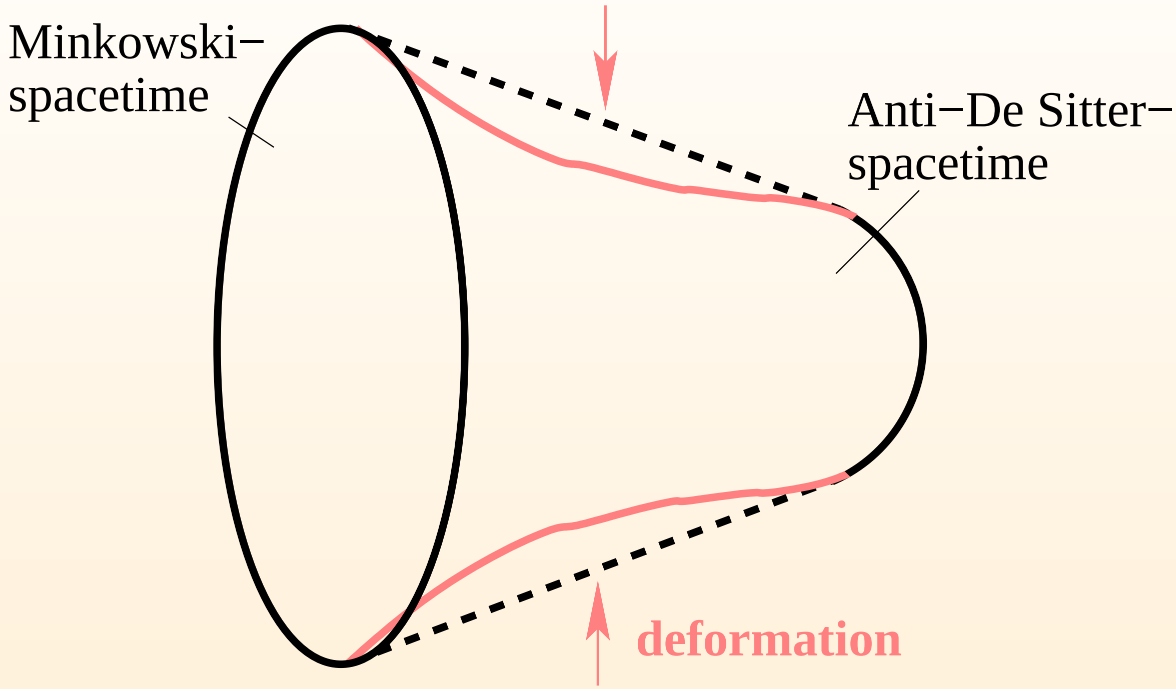
$N \to \infty$ (standard Maldacena limit), N_f small (probe approximation)

duality acts twice:

$\mathcal{N} = 4$ SU(N) Super Yang-Mills theory coupled to $\mathcal{N} = 2$ fundamental hypermultiplet	$\longleftrightarrow$	IIB supergravity on $AdS_5 \times S^5$ + Probe brane DBI on $AdS_5 \times S^3$

Karch, Katz 2002

Deformations of AdS space



Fifth Dimension $\Leftrightarrow$ Energy scale

Renormalization group flow from supergravity

$\Rightarrow$ 'holographic' Renormalization Group flow

SUSY broken by deformation of S^5

Combine the deformation of the supergravity metric

with the addition of brane probes:

Dual gravity description of chiral symmetry breaking and Goldstone bosons

J. Babington, J. E., N. Evans, Z. Guralnik and I. Kirsch, hep-th/0306018

D7 brane probe in deformed backgrounds

D7 brane probe in gravity backgrounds dual to confining gauge theories without supersymmetry.

Example:

Constable-Myers background (particular deformation of $AdS_5 \times S^5$ metric)

- non-constant dilaton
- non-constant S^5 radius
- naked singularity in IR
- dual field theory confining

- The deformation introduces a new scale into the metric.
- In UV limit, geometry returns to $AdS_5 \times S^5$ with D7 probe wrapping $AdS_5 \times S^3$.

General strategy

1. start from Dirac-Born-Infeld action for a D7-brane embedded in deformed background

2. derive equations of motion for transverse scalars (w_5, w_6)

3. solve equations of motion numerically using shooting techniques

   solution determines embedding of D7-brane (e.g. $w_5 = 0, w_6 = w_6(\rho)$)

4. meson spectrum:

   consider fluctuations δw_5, δw_6 around a background solution obtained in 3.

   solve equations of motion linearized in $\delta w_5, \delta w_6$

UV asymptotic behaviour of solutions to equation of motion:

$$w_6 \propto m\, e^{-r} + c\, e^{-3r}$$

Identification of the coefficients as in the standard AdS/CFT correspondence:

m quark mass, $c = \langle \bar{q} q \rangle$ quark condensate

Here:

$m \neq 0$: explicit breaking of $U(1)_A$ symmetry

$c \neq 0$: spontaneous breaking of $U(1)_A$ symmetry

The Constable-Myers deformation

$\mathcal{N} = 4$ super Yang-Mills theory deformed by VEV for $tr\, F^{\mu\nu} F_{\mu\nu}$
(R-singlet operator with $D = 4$) $\quad \rightarrow$ non-supersymmetric QCD-like field theory

The Constable-Myers background is given by the metric

$$ds^2 = H^{-1/2} \left(\frac{w^4 + b^4}{w^4 - b^4} \right)^{\delta/4} dx_4^2 + H^{1/2} \left(\frac{w^4 + b^4}{w^4 - b^4} \right)^{(2-\delta)/4} \frac{w^4 - b^4}{w^4} \sum_{i=1}^{6} dw_i^2,$$

where

$$H = \left(\frac{w^4 + b^4}{w^4 - b^4} \right)^{\delta} - 1 \qquad (\Delta^2 + \delta^2 = 10)$$

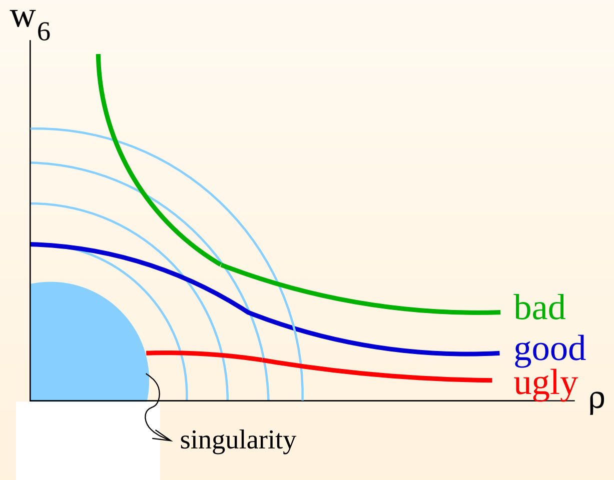
and the dilaton and four-form

$$e^{2\phi} = e^{2\phi_0} \left(\frac{w^4 + b^4}{w^4 - b^4} \right)^{\Delta}, \qquad C_{(4)} = -\frac{1}{4} H^{-1} dt \wedge dx \wedge dy \wedge dz$$

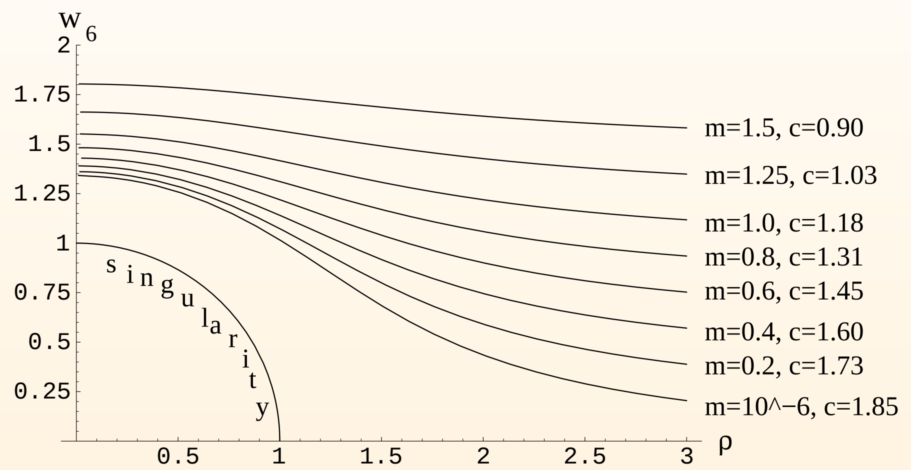
This background has a singularity at $w = b$

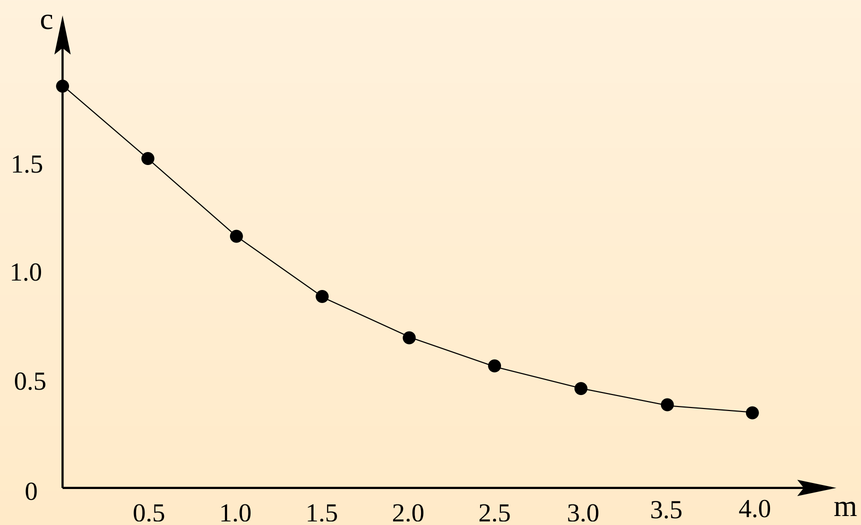
Chiral symmetry breaking

Solution of equation of motion for probe brane



Numerical Result:





Result:

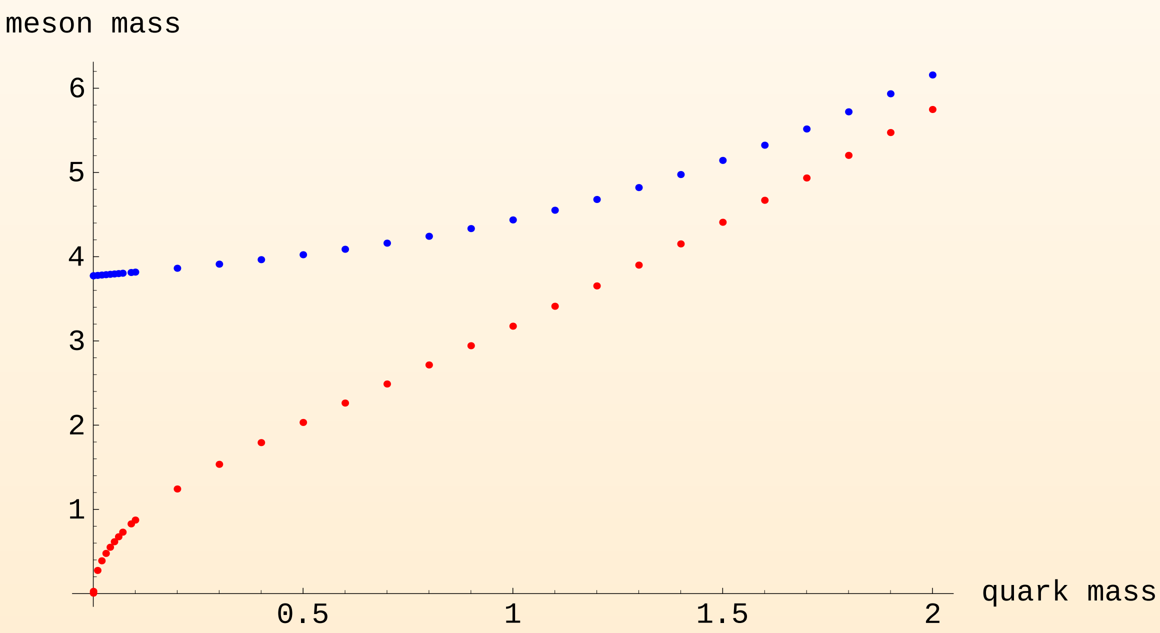
Screening effect: Regular solutions do not reach the singularity

Spontaneous breaking of $U(1)_A$ symmetry: For $m \to 0$ we have $c \equiv \langle \bar{\psi}\psi \rangle \neq 0$

Meson spectrum

From fluctuations of the probe brane

Ansatz: $\delta w_i(x,\rho) = f_i(\rho) sin(k \cdot x)\,, \qquad M^2 = -k^2$



Goldstone boson (η')

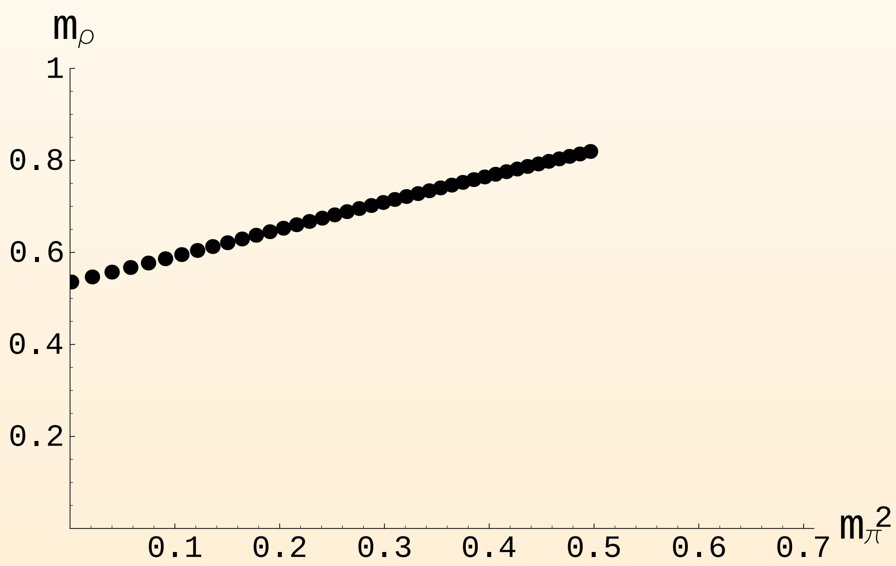
Gell-Mann-Oakes-Renner relation: $M_{Meson} \propto \sqrt{m_{Quark}}$

Comparison with lattice gauge theory

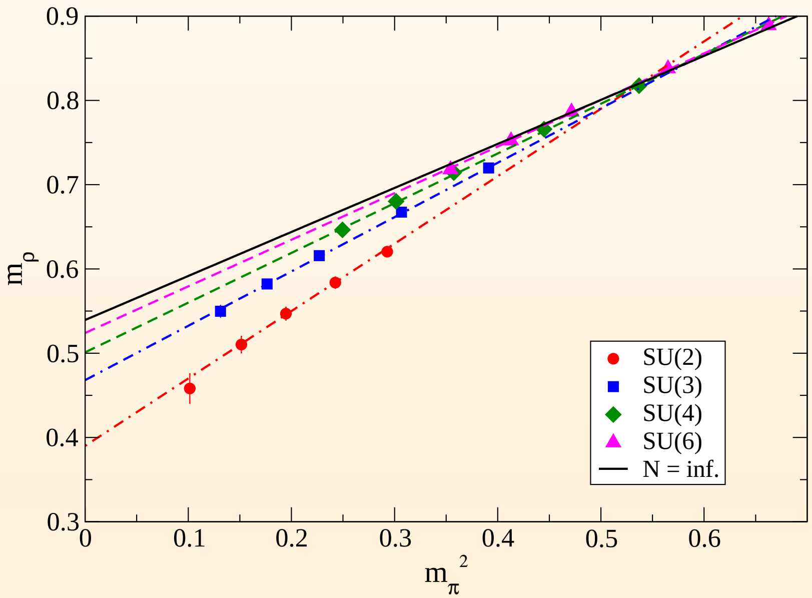
J.E., Evans, Kirsch, Threlfall 0711.4467, EPJA

m_ρ vs. ${m_\pi}^2$

(Lattice: Lucini, Del Debbio, Patella, Pica 0712.3036)



Slope: 0.57
Normalized to scale in metric



Slope: 0.52
Normalized to lattice spacing

(Similar results by Bali and Bursa)

Flavour in the AdS Black Hole geometry

Consider $\mathcal{N} = 4\ SU(N)$ SYM at finite temperature (Witten, 1998)

Dual string theory background: Euclidean AdS-Schwarzschild solution

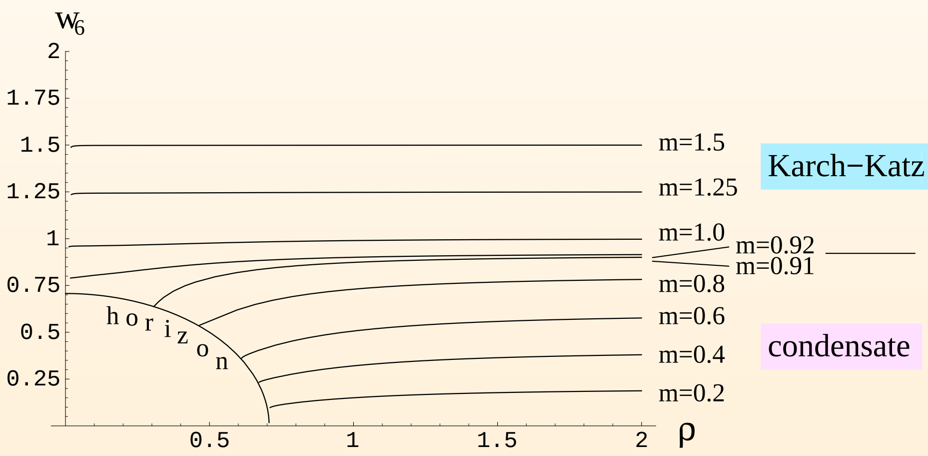
$$ds^2 = \left(w^2 + \frac{b^4}{4w^2}\right) d\vec{x}^2 + \frac{(4w^4 - b^4)^2}{4w^2(4w^4 + b^4)} d\tau^2 + \frac{1}{w^2} \sum_{i=1}^{6} dw_i^2$$

with radial coordinate $w^2 = \rho^2 + w_5^2 + w_6^2$

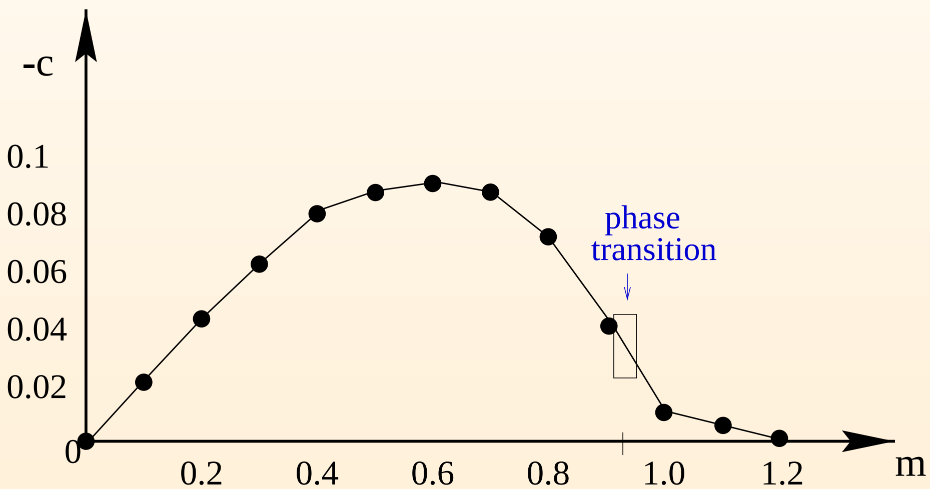
b deformation parameter, τ periodic (period $\pi b = T^{-1}$)
horizon: S^1 collapses at $w = \frac{1}{2} b$

Condensate in field theory at finite temperature

D7 brane embedding in black hole background



Condensate c versus quark mass m (c, m normalized to T)

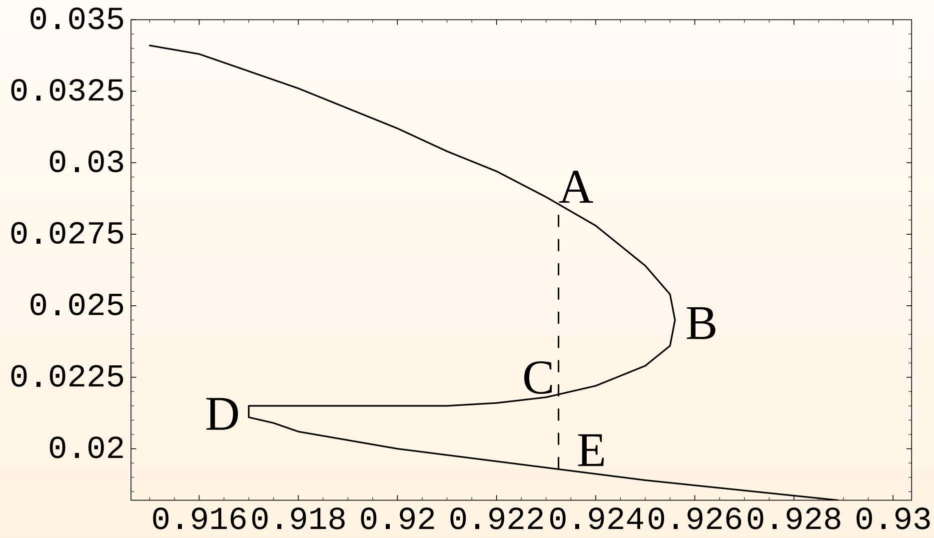


Phase transition at $m_c \approx 0.92$

No condensate for $m = 0$
(no spontaneous chiral symmetry breaking)

BEEGK 0306018

Phase transition



First order phase transition in type II B AdS black hole background

Ingo Kirsch, PhD thesis 2004

(Related work by Mateos, Myers et al)

Dusling, J.E., Kaminski, Rust, Teaney, Young in progress

Diffusion and momentum broadening of heavy mesons

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Diffusion and momentum broadening of heavy mesons

Perturbative effective field theory results: Manohar et al; Peskin

$$\mathcal{L} = +\phi_v^\dagger iv \cdot \partial\phi_v + \frac{c_E}{N^2}\phi_v^\dagger \mathcal{O}_E \phi_v + \frac{c_B}{N^2}\phi_v^\dagger \mathcal{O}_B \phi_v$$

ϕ_v: heavy scalar meson with velocity v^μ (use rest frame $v^\mu = (1,0,0,0)$)

$\mathcal{O}_E = E^A \cdot E^A, \quad \mathcal{O}_B = B^A \cdot B^A$

Non-relativistic polarizabilities: $c_E = \frac{28\pi}{3} {a_0}^3$, $c_B = 0$

Bohr radius: $a_0 = (m_q \frac{N}{2} \alpha_s)^{-1}$

In-medium mass shift: $\delta M = -\langle \mathcal{L}_{int} \rangle = T\,(\pi T a_0)^3\,\frac{14}{45}$

Kinetics of heavy meson in medium

κ: Drag coefficient describing momentum broadening in Langevin theory

Microscopically, with dipole force $\vec{F} = -\frac{1}{2}\vec{\nabla}(E^a \cdot E^a)$:

$$\kappa = \frac{1}{3}\frac{c_E^2}{N^4}\int \frac{d^3q}{(2\pi)^3} q^2 \left[-\frac{2T}{\omega}\mathrm{Im}\, G_R^{E^2E^2}(\omega, q)\right]$$

From perturbative calculation

$$\kappa_{QCD} \simeq \frac{T^3}{N^2}(\pi T a_0)^6\, 130$$

To compare with strong coupling calculation consider

$$\frac{\kappa}{\delta M^2} \simeq \frac{\pi T}{N^2}\, 426$$

AdS/CFT calculation

$\mathcal{N} = 4$ SYM:

$$\mathcal{L}_{eff} = \phi_v(x,t) i v \cdot \partial \phi_v(x,t) + \frac{c_T}{N^2} \phi_v^\dagger(x,t) T^{\mu\nu} v_\mu v_\nu \phi_v(x,t) + \frac{c_F}{N^2} \phi_v(x,t)^\dagger (tr F^2) \phi_v(x,t)$$

Perturbative result for $\mathcal{N} = 4$ SYM:

$$\frac{\kappa}{\delta M^2} \simeq \frac{\pi T}{N^2} 37$$

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Strong coupling calculation from gauge/gravity duality

Polarization coefficients to be determined from mass shifts

$$\delta M_T = \frac{c_T}{N^2}\langle T^{00}\rangle\,, \qquad \delta M_F = \frac{c_F}{N^2}\langle trF^2\rangle$$

(Meson mass: Lowest mode $M = \frac{m_q}{\sqrt{\lambda}} 2\sqrt{2}$ in $AdS_5 \times S^5$)

AdS/CFT calculation

To obtain the polarizabilities, we calculate

δM_T from linear response to switching on black hole background

δM_F from linear response to switching on dilaton flow background

Dilaton background of Liu, Tseytlin 1999:

$$e^{\phi} = g_s(1 + \frac{q^4}{r^4})\,, \qquad q^4 = \frac{2\pi^2 R^8}{N^2}\langle trF^2\rangle$$

δM is obtained analytically
by expanding new eigenfunctions in basis of solutions of the unperturbed case

$$-\partial_\rho \rho^3 \partial_\rho \phi(\rho) = \bar{M}^2 \frac{\rho^3}{(\rho+1)^2}\phi(\rho) \, + \, \Delta(\rho)\phi(\rho)$$

Drag coefficient

$$\kappa = \lim_{\omega \to 0} \int \frac{d^3 q}{(2\pi)^3} \frac{q^2}{3} \left[\left(\frac{c_F}{N^2} \right)^2 \frac{-2T}{\omega} \mathrm{Im}\, G_R^{F^2 F^2}(\omega, q) + \left(\frac{c_T}{N^2} \right)^2 \frac{-2T}{\omega} \mathrm{Im}\, G_R^{TT}(\omega, q) \right]$$

Green functions calculated
from propagation through AdS black hole background

Putting everything together:

$$\kappa = \frac{T^3}{N^2} \left(\frac{2\pi T}{M} \right)^6 \left(\left(\frac{8}{5\pi} \right)^2 67.258 + \left(\frac{12}{5\pi} \right)^2 355.1 \right)$$

$$= \frac{T^3}{N^2} \left(\frac{2\pi T}{M} \right)^6 224.7$$

Temperature, scale and N dependence agree with perturbative result

AdS/CFT calculation - result

This gives

$$\frac{\kappa}{(\delta M)^2} = \frac{\pi T}{N^2} 8.37$$

Result five times smaller than perturbative $\mathcal{N} = 4$ SYM result!

Conclusions

- Holographic description of chiral symmetry breaking by a quark condensate

  Light mesons as Goldstone bosons

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- Meson diffusion in $\mathcal{N} = 4$ plasma

  $\kappa/(\delta M)^2$ smaller in strongly coupled case