Gauge Dynamics and Vortex Strings

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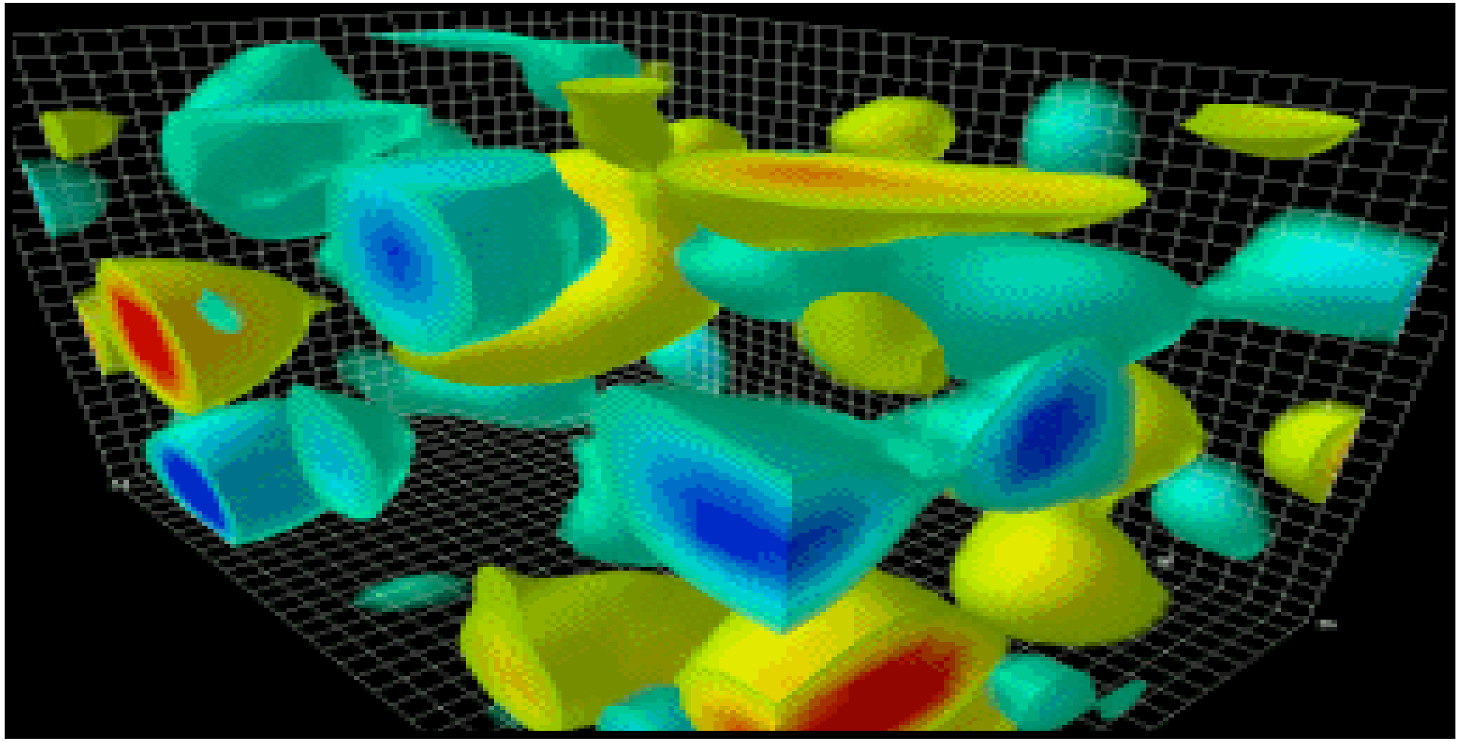
GGI, Florence, June 2008

The Take-Home Message

- For 30 years we've known that 4d non-Abelian gauge theories share certain features with 2d sigma-models
  - Asymptotic freedom
  - Confinement
  - Dynamically generated mass gap
  - Anomalies
  - Instantons
  - Theta Dependence
  - Chiral Symmetry Breaking
  - Large N limits
- In fact, there are *quantitative* links between the two. The relationship is derived through the dynamics of solitonic vortex strings.

A Cartoon of the Basic Idea

- Take a strongly coupled theory with U(N) gauge group and some fundamental scalar fields.



Graphics by Leinweber

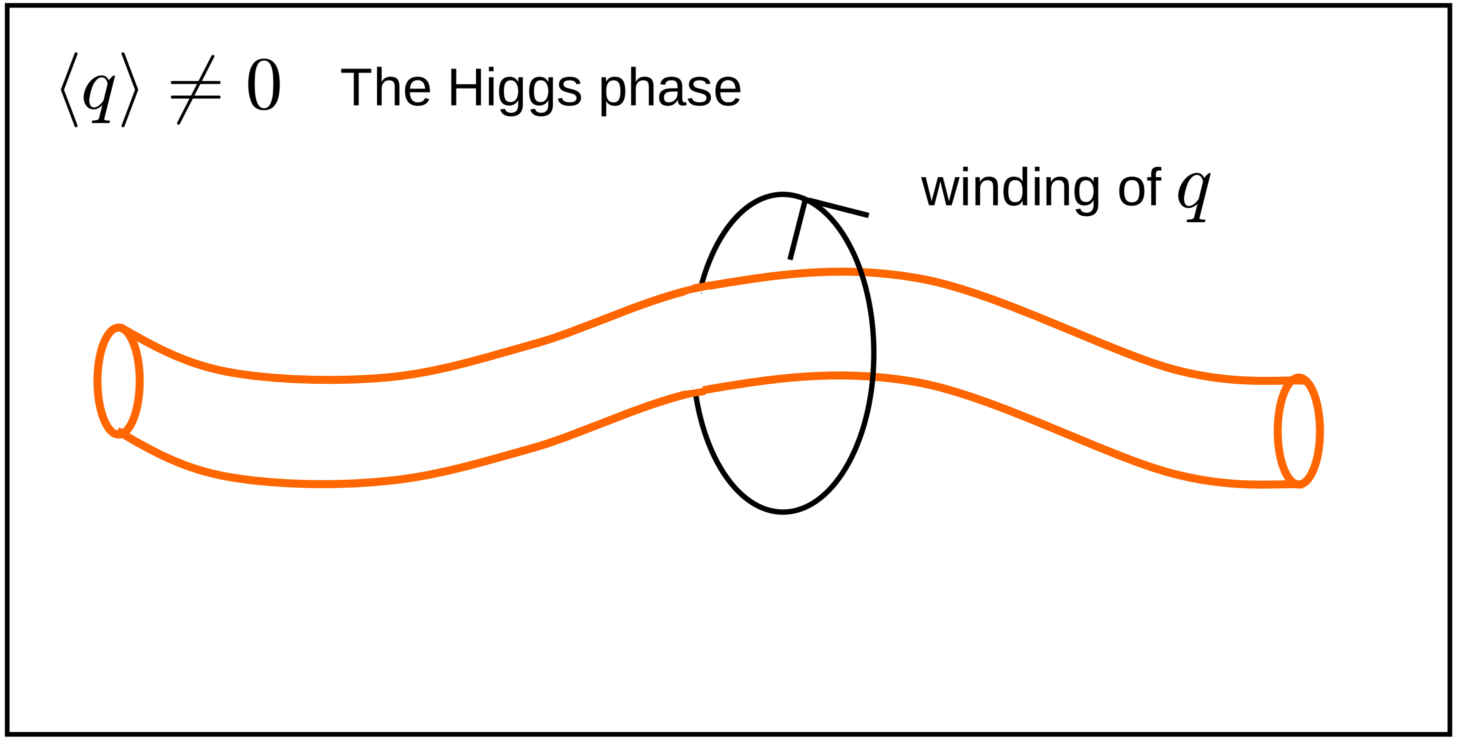
A Cartoon of the Basic Idea

- Deform the theory by inducing an expectation value for the scalar fields

$\langle q \rangle \neq 0$ The Higgs phase

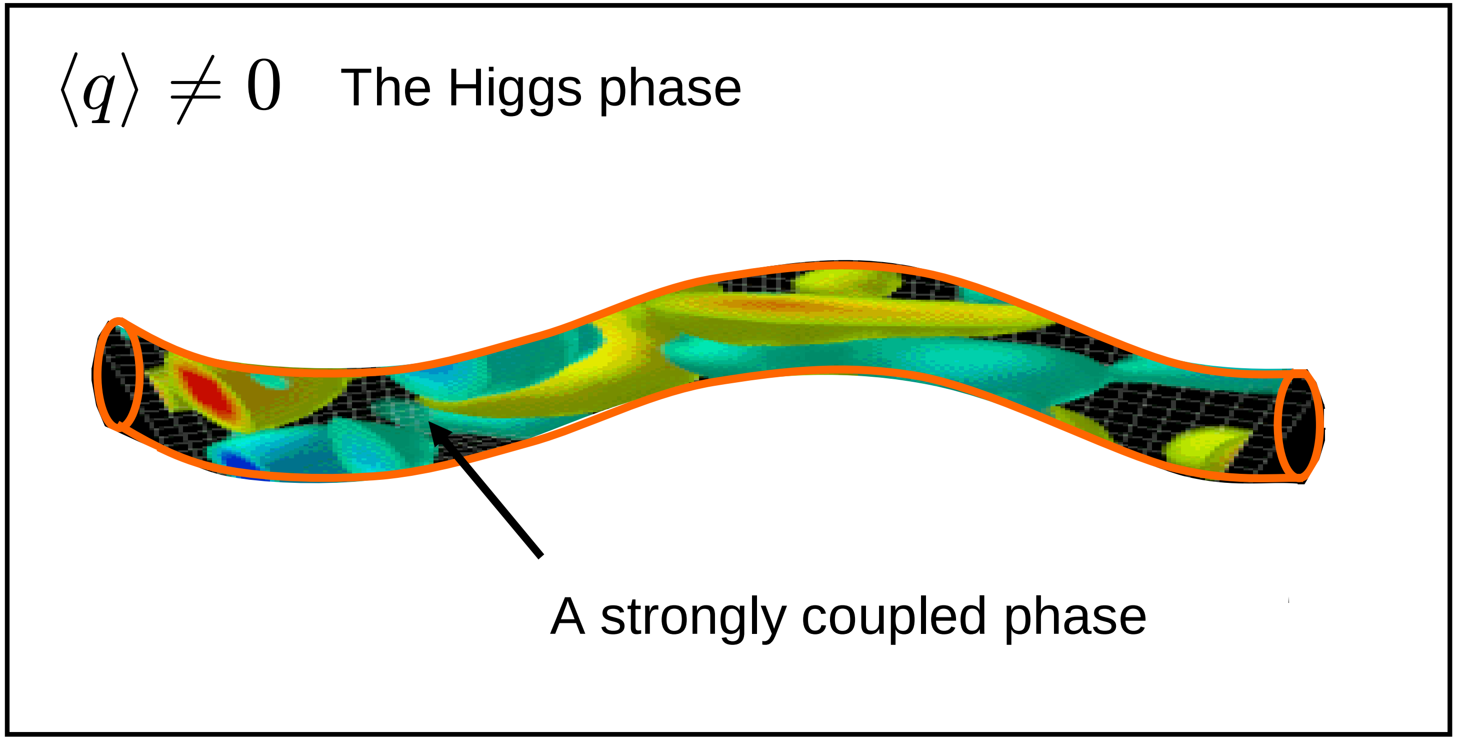
A Cartoon of the Basic Idea

- The theory now admits vortex strings, supported by the phase of the scalar winding at infinity



A Cartoon of the Basic Idea

- The interior of the vortex string is a strongly coupled system
  - The vortex string knows about the original 4d gauge theory.



The Basic Theory

Starting point: d=3+1 with U(N) gauge group and N_f =N fundamental flavours.

$$L = -\frac{1}{2e^2}\,\mathrm{Tr}\,F_{\mu\nu}F^{\mu\nu} + \sum_{i=1}^{N_f} |\mathcal{D}q_i|^2$$
$$-\frac{e^2}{2}\mathrm{Tr}\,(\sum_i q^a{}_i\, q^\dagger_{bi} - v^2\delta^a{}_b)^2$$

where a=1,…,N is the colour index, and i=1,…, N_f is the flavour index.

Classical Properties of the Theory

- Vacuum: The ground state is unique

$$q^a{}_i = v\,\delta^a{}_i$$

- Spectrum: The theory has a mass gap,with

$$m_\gamma = m_q \sim ev$$

- Symmetries: The theory lies in the “colour-flavour” locked phase

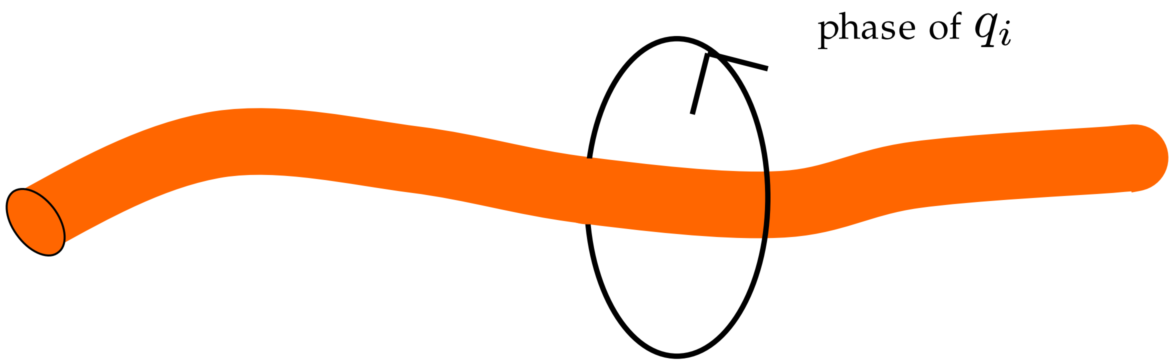
$$U(N) \times SU(N_f) \rightarrow SU(N)_{\rm diag}$$

Note that overall U(1) is broken: $\Longrightarrow$ Vortices

The Vortex

Abrikosov, '52
Nielsen and Olesen, '73

Broken U(1) gauge symmetry ⟹ Vortices



$$(B_3)^a{}_b = e^2(\textstyle\sum_i q_i^a q_{ib}^\dagger - v^2 \delta^a{}_b)$$

$$(\mathcal{D}_z q_i)^a = 0$$

$z = x_1 + ix_2$

$$T_{\text{vortex}} = 2\pi v^2$$

Orientation Modes of the Vortex

Hanany and Tong, '03
Auzzi et al. '03

Suppose we have an Abelian vortex solution $B_\star$,$q_\star$. We can trivially embed this in the non-Abelian theory.

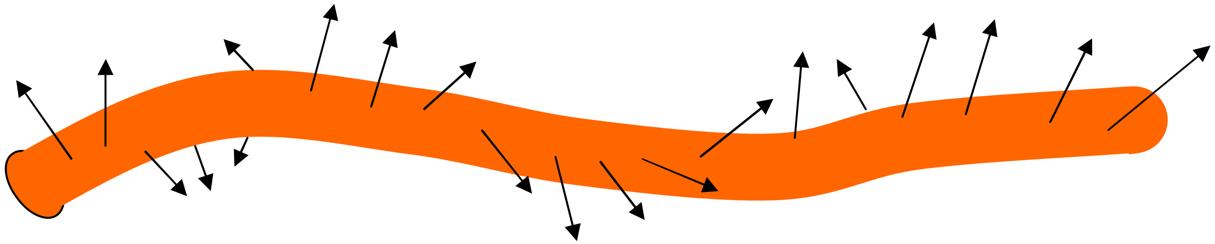
$$B = \begin{pmatrix} B_\star & & & \\ & 0 & & \\ & & \ddots & \\ & & & 0 \end{pmatrix} \qquad q = \begin{pmatrix} q_\star & & & \\ & v & & \\ & & \ddots & \\ & & & v \end{pmatrix}$$

Different embeddings $\Longrightarrow$ moduli space of vortex

$$SU(N)_{\rm diag}/SU(N-1)\times U(1) \cong \mathbf{CP}^{N-1}$$

Vortex Dynamics

The vortex may oscillate in transverse directions. But it may also vary its orientation. This is described by the 2d $\mathbf{CP}^{N-1}$ sigma model



$$L_{\rm vortex} = \sum_{i=1}^{N} |\mathcal{D}\phi_i|^2 - \lambda\left(\sum_i |\phi_i|^2 - r\right)$$

Size of $\mathbf{CP}^{N-1}$ is $\boxed{r = \frac{4\pi}{e^2}}$

Hanany and Tong, ’03
Shifman and Yung, ‘04

Vortex: 4d ⟶ 2d

4d Gauge Theory	2d Sigma Model
U(N) Gauge Theory	CP^{N-1} Sigma-Model
Add Charged Fermions	Fermion Zero Modes
Add Charged Bosons	Further Bosonic Zero Modes
Change Scalar VEVs	Induce Potentials on Target Space
Add Interactions (e.g Yukawa)	Add Interactions (e.g. 4-fermi)

Various aspects studied by several groups:
Shifman, Yung, Gorsky,
Pisa: Konishi et al.,
Tokyo: N. Sakai et. al.

Supersymmetric Vortices: Summary

<u>What the String Saw:</u>

- N=2 Supersymmetry
  - Seiberg-Witten Solution
  - Exact quantum masses of BPS states
  - No Confinement
  - Argryes-Douglas Superconformal Points
- N=1 Supersymmetry
  - Quantum deformed moduli space: $\det M - B\tilde{B} = \Lambda^{2N}$
  - Confinement

N=2 Supersymmetry

Theories with N=2 supersymmetry require the introduction of an adjoint scalar field, a.

a is a complex adjoint scalar field

$$L = -\frac{1}{2e^2}\,\mathrm{Tr}\,F_{\mu\nu}F^{\mu\nu} + \frac{1}{2e^2}\,\mathrm{Tr}|\mathcal{D}a|^2 + \sum_{i=1}^{N}|\mathcal{D}q_i|^2$$

$$-\sum_i q_i^\dagger(\phi - m_i)^2 q_i - \frac{e^2}{2}\mathrm{Tr}\left(\sum_i q^a{}_i q^\dagger_{bi} - v^2\delta^a{}_b\right)^2$$

+ fermions + …

complex masses

Vacuum: $q^a{}_i = v\,\delta^a{}_i$ $\qquad a = \mathrm{diag}(m_1, \ldots, m_N)$

Non-Abelian Vortex ⟶ Abelian Vortices

- Q: What effect do the masses have on the vortex string?
- A: Orientational modes are lifted, leaving behind *N* Abelian vortices
  - Each vortex lives in a different U(1) subgroup

$$B = \begin{pmatrix} 0 & B_\star & & \\ & \ddots & & \\ & & & 0 \end{pmatrix} \qquad q = \begin{pmatrix} v & q_\star & & \\ & \ddots & & \\ & & & v \end{pmatrix}$$

Tong '03
Shifman and Yung '04
Hanany and Tong '04

Vortex Dynamics

- The vortex dynamics has *N=(2,2)* supersymmetry
- 4d masses introduce a potential for the string orientation modes
- We introduce an auxiliary field σ on the worldsheet

$$L_{\rm vortex} = \sum_{i=1}^{N} |\mathcal{D}\phi_i|^2 - \lambda \left(\sum_i |\phi_i|^2 - r \right)$$
$$- \sum_i |\sigma - m_i|^2 |\phi_i|^2 \quad \text{+ \it fermions}$$

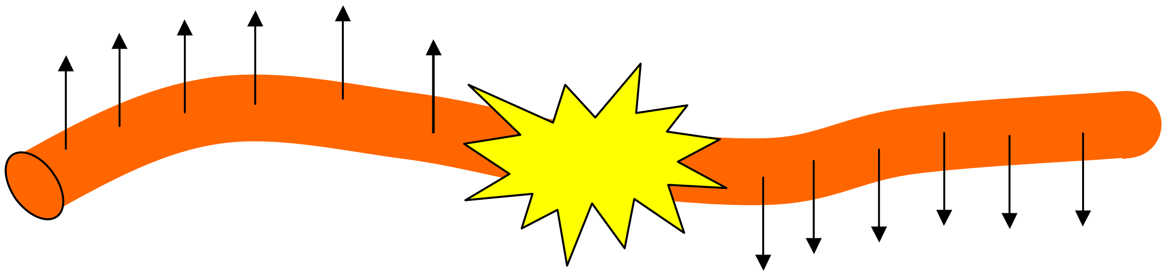
- The *N* ground states of this theory correspond to *N* vortex strings

$$\sigma = m_i \qquad |\psi_j|^2 = r\delta_{ij}$$

Beads on the String

- Isolated string vacua ⇨ kinks on the string

$B \sim \begin{pmatrix} 1 \\ 0 \end{pmatrix}$



$B \sim \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

- The kink carries magnetic flux $B \sim \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

- The mass of the bead on the string is

$$M_{\rm kink} = r|m_1 - m_2| = \frac{4\pi\,\langle a\rangle}{e^2} = M_{\rm mono}$$

- The kink is a magnetic monopole, confined by the Meissner effect

The Quantum Theory

Dorey '98,
Shifman and Yung '04
Hanany and Tong '04

The first hint that the vortex string knows about the 4d quantum theory comes from the beta functions. The relationship

$$\boxed{r = \frac{4\pi}{e^2}}$$

is preserved under RG flow of the 2d and 4d theories

$$r(\mu) = r_0 - \frac{N_c}{2\pi}\,\log\left(\frac{M_{UV}}{\mu}\right)$$

The strong coupling scale Λ of the string worldsheet is the same as that of the unbroken 4d theory.

The Quantum Spectrum

- The result $\boxed{M_{\rm kink} = M_{\rm mono}}$ also holds in the full quantum theory.

- In the regime $m \gg \Lambda$, both 2d worldsheet and the unbroken 4d theory are weakly coupled. The mass of solitons has an expansion

$$M = M_{\rm classical} + M_{\rm 1-loop} + \sum_{n=1}^{\infty} M_{\rm n-instanton}$$

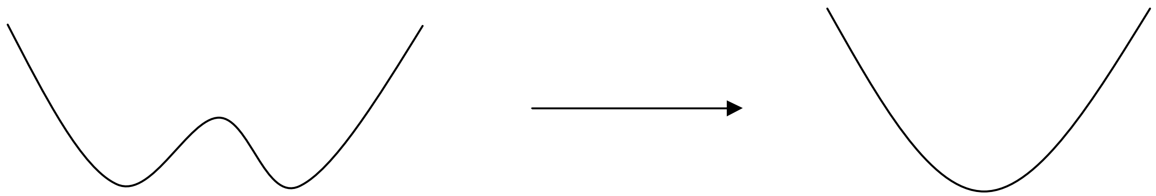
- You can compute these quantum corrections in 2d or in 4d, summing over 2d sigma-model instantons or 4d Yang-Mills instantons. They agree!

Superconformal Points

Shifman, Vainshtein, Zwicky, '06
Tong, '06

- The 4d theory has special points on its moduli space where both magnetic and electric charges become massless.
- These are "Argyres-Douglas" conformal field theories.

- Q: What happens to the vortex string at these points?
- A: The kink on the worldsheet becomes massless



$V \sim (x-a)^2(x+a)^2$ $\qquad\qquad$ $V \sim x^4$

- The worldsheet theory of the string also becomes conformal
  - The CP^N sigma model flows to the A_N minimal model
  - The dimensions of chiral primary operators in 2d and 4d agree.

Summary of N=2 Theories

- Matching of BPS spectrum in 2d and 4d
  - Monopoles, W-bosons, dyons, curves of marginal stability
- Matching of dimensions of operators at superconformal points

- The 2d superpotential gives the SW curve

- Note: the 4d N=2 theory does not confine
- The 2d sigma model with N=(2,2) supersymmetry also does not confine
  - When m=0, kinks transform in the fundamental of SU(N)

N=1 SQCD and Heterotic Vortex Strings

- When the 4d theory has *N=1* supersymmetry, the vortex worldsheet preserves *N=(0,2)* supersymmetry.
- We have dubbed them <u>heterotic vortex strings</u>

Edalati and Tong '07
Tong '07
Shifman and Yung '08

N=1 SQCD and Heterotic Vortex Strings

- <u>Two main results</u>

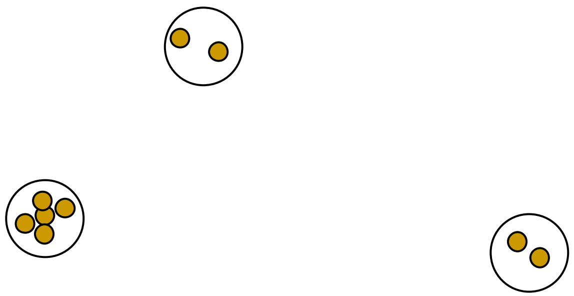
- Quantum Deformed Moduli Space: $\det M - B\tilde{B} = \Lambda^{2N}$
  - Seen as dynamical susy breaking on the worldsheet
- Confinement of spectrum

- <u>Caveat</u>
  - The theory does not have a mass gap, even when $v^2 \neq 0$
  - One zero mode of the vortex string is non-normalizable

Why Would the Spectra Agree?

Start in the strongly coupled 4d theory with $v^2 = 0$

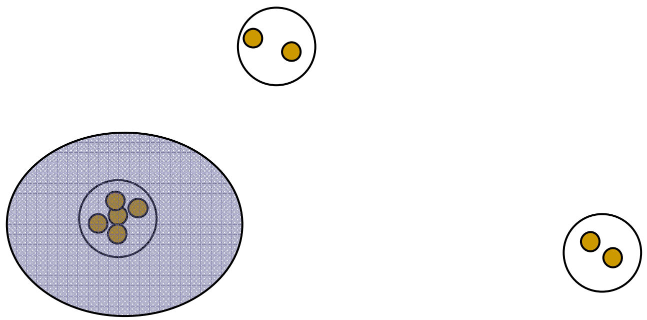
Spectrum = mesons and baryons



Why Would the Spectra Agree?

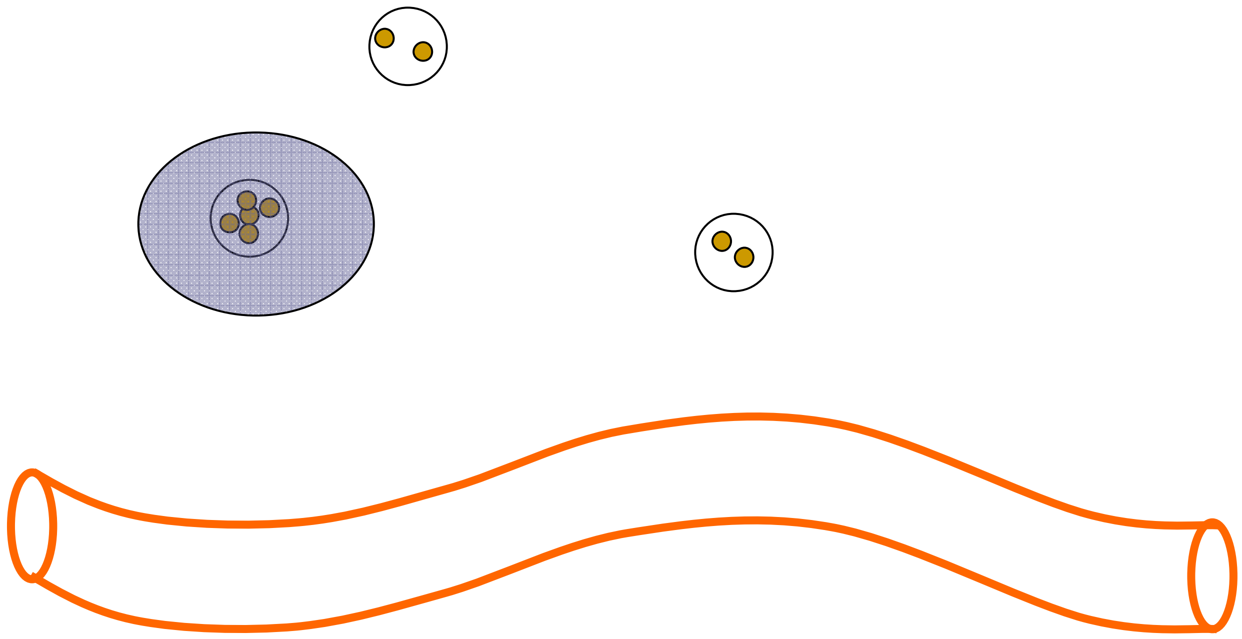
Gauge $U(1)_B$ and Higgs at scale $v \ll \Lambda$

Baryons are screened; mesons left largely unaffected.



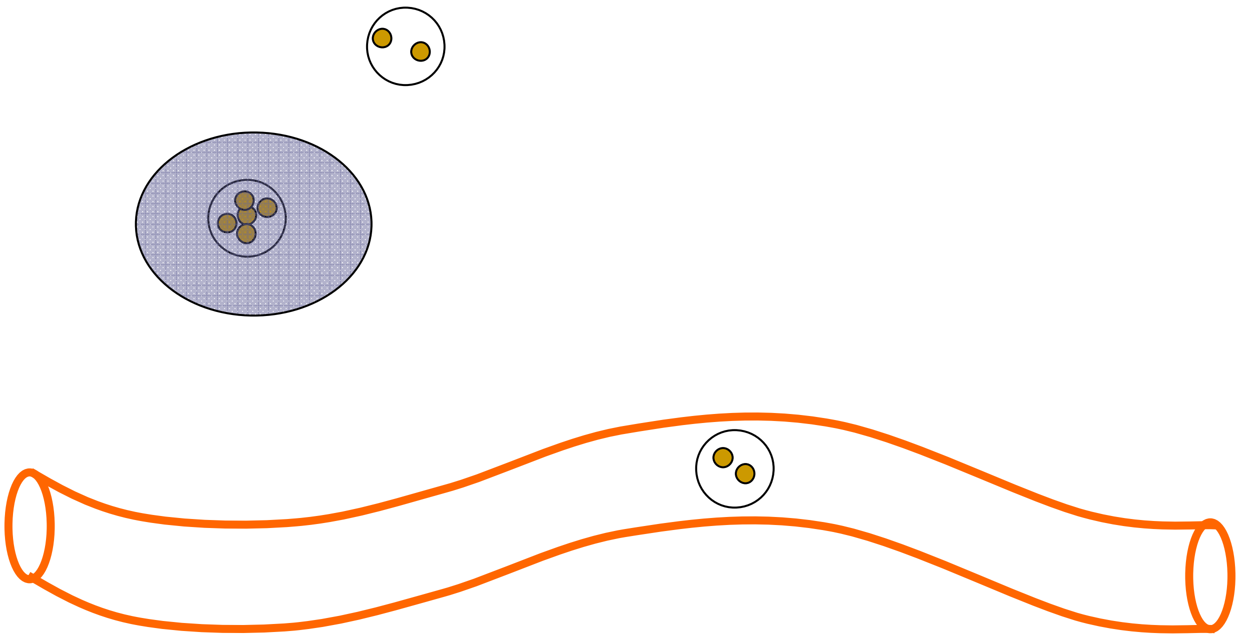
Why Would the Spectra Agree?

Introduce a vortex string. Some of the meson will form bound states with the string.



Why Would the Spectra Agree?

Now increase the ratio v/Λ. Those bound states which remain light (i.e. of order Λ) must show up as internal excitations of the 2d sigma-model.



Spectrum of the Sigma Model

d'Adda, di Vecchia, Lusher '78
Witten, '78

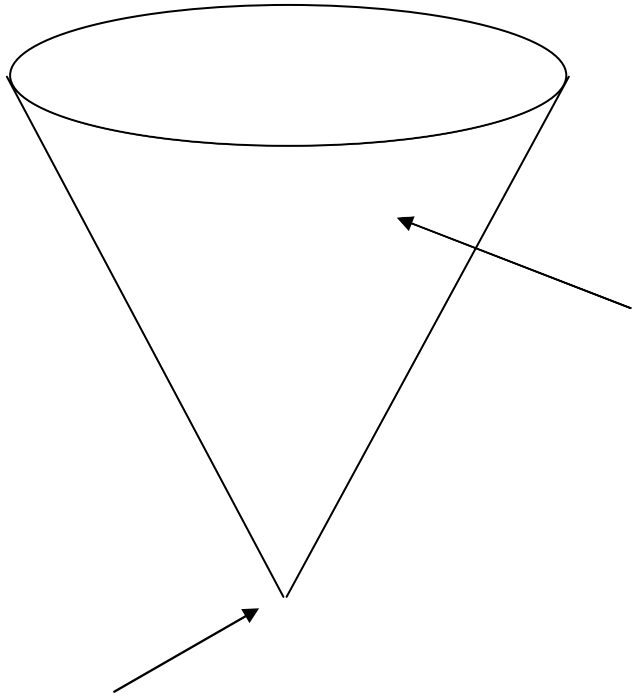
- The *N=(2,2)* sigma-model does not confine.
  - Kinks transform in the N of SU(N)
  - This matches the 4d story, where the *N=2* theory does not confine

- The *N=(0,2)* sigma-model does confine
  - The model has a chiral symmetry: $SU(N)_L \times SU(N)_R$
  - Spectrum consists of particles in singlet, adjoint and bi-fundamental

- This qualitatively matches the spectrum of the 4d theory
  - Meson Spectrum in singlet, adjoint and bi-fundamental
  - Baryon Spectrum: Slew of tensor reps under flavor symmetry…not seen

Classical Moduli Space of Vacua

$$\det M - B\tilde{B} = 0$$



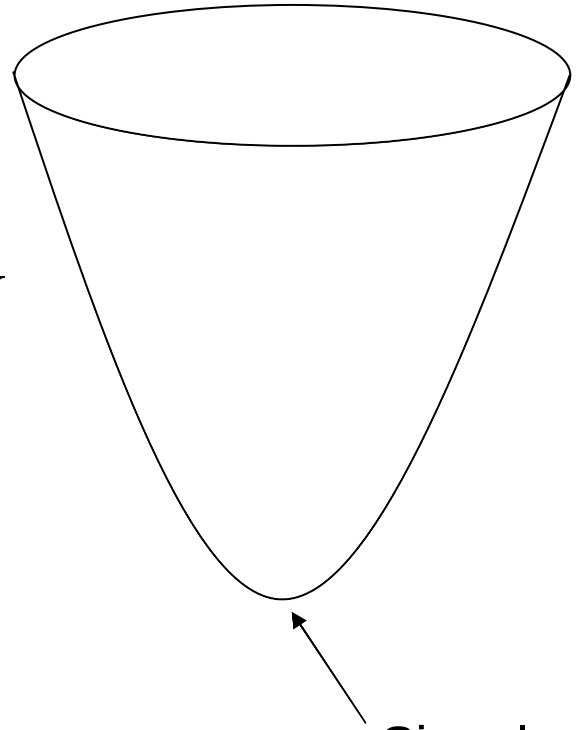
At a smooth point, we have N^2+1 massless particles

Singular point at $B = \tilde{B} = 0$ and $\mathrm{rank}(M) < N - 2$, the symmetry breaking is less than maximal $\Longrightarrow$ new massless gluons

Quantum Moduli Space

Seiberg, '94

$$\det M - B\tilde{B} = \Lambda^{2N}$$



Singularity is resolved, reflecting confinement and the fact that gluons get a mass

What Does This Mean for Vortices?

- Gauge $U(1)_B$ and introduce vev (FI parameter) $v \ll \Lambda$

- Q: When are there BPS vortices?
- A: When $\tilde{B} = 0$
  - Classically, BPS vortices exist when $\det M = 0$
  - Quantum mechanically, BPS vortices exist when $\det M = \Lambda^{2N}$

- Can we capture this 4d quantum behaviour by looking at 2d worldsheet of classical string, valid when $v \gg \Lambda$?
  - This is *N=(0,2)* $\mathbf{CP}^{N-1}$ sigma-model

  - Find dynamical susy breaking when $\det M = 0$
  - Find dynamical susy restoration when $\det M = \Lambda^{2N}$

Summary and Future Directions

- Quantitative agreement between 2d sigma models and 4d gauge dynamics
  - N=2 Gauge Theories = N=(2,2) sigma models
    - Exact agreement between BPS mass spectra
    - Agreement between superconformal theories
  - N=1 Gauge Theories = N=(0,2) sigma models
    - Baryon vevs = worldsheet supersymmetry breaking
    - qualitative agreement between spectra

<u>Still to do…</u>

- N=1 Gauge Theories with $N_f > N_c$.
  - Conformal Window? Seiberg Duality?
- Lessons for confining string in non-supersymmetric theories?