GGI Workshop **on Strong Coupling**

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- Heterotic strings from N=1 SUSY field theory;
- Worldsheet theory: N=(0,2) extension of 2D CP(N-1) sigma model.

With A. Yung

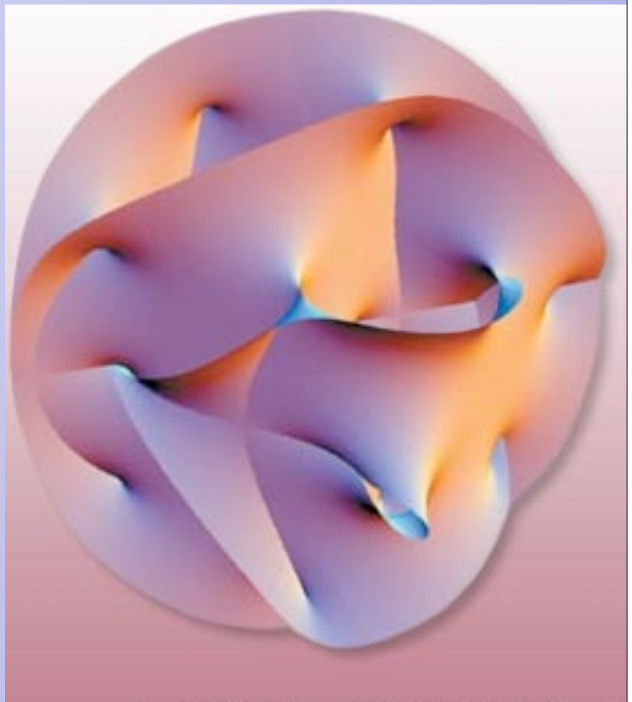


Illustration: A.J. Hanson, Indiana University

✱ If string/brane theory had not evolved from dual resonance model, it would have emerged as BPS soliton theory in various 4D SUSY Yang-Mills models

Correspondence dictionary

Yang-Mills string	Flux tube
Brane	BPS domain wall
Stack of branes	Composite wall
String-brane junction	Flux tube-wall junction

Today's topic: Heterotic strings

Non-Abelian flux tubes in N=2 super-Yang-Mills

- U(N) gauge group, N flavors

gluons + 2 gluinos + adjoint scalars

For U(2): 2 (s)quark flavors

q^{iA} & $\tilde{q}_{iA}$ (i=1,2, A=1,2)

color flavor

+ U(1) Fayet-Iliopoulos term $\longrightarrow$ "BPS-ness"

$$\mathcal{W} = \sum_{A=1}^{2} \left(\tilde{q}_A \mathcal{A} q^A + \tilde{q}_A \mathcal{A}^a \tau^a q^A \right)$$

Vacuum:

$$q^{kA} = \sqrt{\xi}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

FI

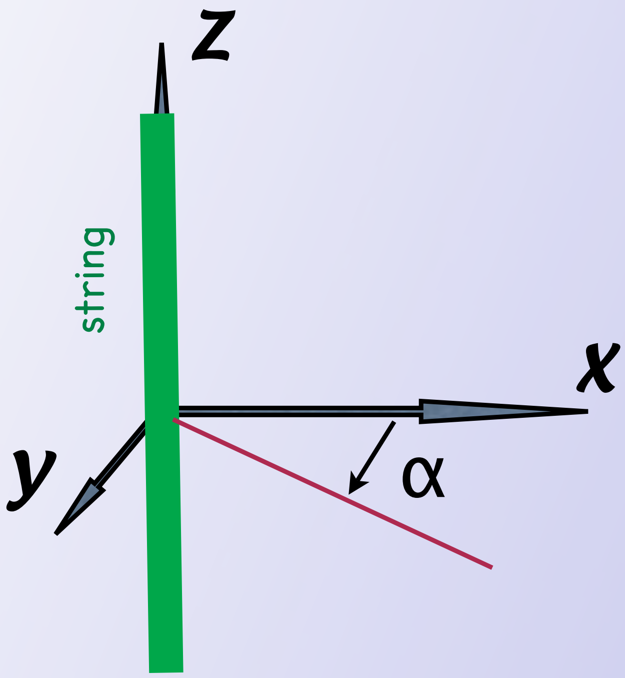
$$\tilde{q} \equiv 0\,, \qquad a^a = a = 0$$

- Bulk theory is fully Higgsed
- Color+Flavor SU(2) survives!

$$\longrightarrow \quad U_c \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} U_F^\dagger = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Non-Abelian flux tubes

$\pi_1(U(1)\times SU(2))$ nontrivial due to Z_2 center of $SU(2)$



x_0 ← string center in perp. plane

ANO

$$\sqrt{\xi}\, e^{i\alpha} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$T = 4\pi\xi$$

Non-Abelian

$$\sqrt{\xi} \begin{pmatrix} e^{i\alpha} & 0 \\ 0 & 1 \end{pmatrix}$$

$T_{U(1)} \pm T^3_{SU(2)}$

$$T = 2\pi\xi$$

$\boxed{\text{SU(2)/U(1) orientational moduli emerge}}$

8 supercharges in the bulk → **1/2 BPS flux tube** →

4 supercharges on world sheet →

MODULI:

(super)Translational

x_{0i} (i=1,2)& ζ_R, $\zeta_R^\dagger$, ζ_L, $\zeta_L^\dagger$ ⇐**decouple**

SU(2)/U(1) moduli

2 orientational + 4 superorientational → CP(1) model with N= (2,2) SUSY

$$\mathcal{L} = \int d^4\theta K\,, \qquad K = \frac{1}{g_0^2}\ln\left(1+\bar{\Phi}\Phi\right)$$

✾ Non-Abelian strings have:

✾ Dynamical IR scale Λ generated on worldsheet;

$\Lambda \ll m_W \ll \sqrt{\xi}$

String thickness $\sim 1/m_W$

String tension $\sim \xi$

✾ Lüscher coefficient different

From N=2 to N=1 in the bulk → Eliminating adjoint fields

$$\mathcal{W} = \frac{1}{2}\mu\mathcal{A}^2$$

If $\mu >> m_W$ adjoint fields are GONE!

The classical string solution remains the same, and so does the number of moduli;

N=2 is broken down to N=1 in the bulk;

Expect 4 supercharges in the bulk & 2 supercharges on the worldsheet;

BUT: CP(1) with N=1 is automatically elevated to N=2 ✔✔✔

Accidental SUSY enhancement?

supertranslational

Edalati-Tong suggestion:

assume x_0 and ζ_L decouple BUT ζ_R gets mixed with ψ from CP(1)

superorintational

Then the N(0,2) generalization of bosonic CP(1) IS possible!

Our task was to derive it directly from the bulk; explicit expressions for <u>all</u> fermion zero modes needed

$$L_{heterotic} = \zeta_R^\dagger\, i\partial_L\, \zeta_R + \left[\gamma\zeta_R R\left(i\partial_L\phi^\dagger\right)\psi_R + H.c.\right] - g_0^2|\gamma|^2\left(\zeta_R^\dagger\zeta_R\right)\left(R\,\psi_L^\dagger\psi_L\right)$$

$$\boxed{\begin{aligned} &= G\left\{\partial_\mu\phi^\dagger\,\partial^\mu\phi + \frac{i}{2}\left(\psi_L^\dagger\overleftrightarrow{\partial_R}\psi_L + \psi_R^\dagger\overleftrightarrow{\partial_L}\psi_R\right)\right. \\ &\left. -\frac{i}{\chi}\left[\psi_L^\dagger\psi_L\left(\phi^\dagger\overleftrightarrow{\partial_R}\phi\right) + \psi_R^\dagger\psi_R\left(\phi^\dagger\overleftrightarrow{\partial_L}\phi\right)\right] - \frac{2\left(1-g_0^2|\gamma|^2\right)}{\chi^2}\,\psi_L^\dagger\psi_L\,\psi_R^\dagger\psi_R\right\}\end{aligned}}$$

$g_0^2|\gamma|^2 = \alpha^2/(1+\alpha^2)$, $\alpha = 2\sqrt{2}\,\mu/m_W$ if $\mu \ll m_W$

$= 1$ if $\mu \gg m_W$

saturation

At small γ /small μ

$$\gamma\zeta_R R\left(i\partial_L\phi^\dagger\right)\psi_R$$

$$\Delta J_{sc,R}=\gamma\langle R\psi_R^\dagger\psi_L\rangle\boxed{\zeta_R^\dagger}$$

Goldstino

$$\mathcal{E}_{vac}=|\gamma|^2\left|\langle R\psi_R^\dagger\psi_L\rangle\right|^2$$

SUSY vacua are LIFTED!

SUSY is broken, and so is Z_{2N}

$Z_{2N}\rightarrow Z_2$,

as in conventional N=(2,2) CP(N-1)

Large-N solution for all μ's ➠ ➠ ➠

At $m_W \ll \mu < \infty$:

$$\mathcal{E}_{vac} = \Lambda^2$$

Goldstino is a mixture of ζ_R and $R\psi_R^\dagger \partial_L \phi_R$

The chiral condensate $\langle R\psi_R^\dagger \psi_L \rangle$ becomes small → $Z_{2N} \to Z_2$ breaking is weak!

$\boxed{\text{What happens in the limit } \mu \to \infty \text{ ?}}$

Bulk theory becomes gapless in IR → derivation of heterotic CP(N-1) fails (at $\mu \sim \xi/\Lambda$).

N vacua coalesce? Conformal? Perhaps, M model?........

Conclusions:

- Heterotic non-Abelian flux tube in N=1 SUSY YM constructed;
- Heterotic generalization of CP(N-1) on worldsheet derived from the bulk;
- Heterotic CP(N-1) solved at large N; patterns of SUSY breaking and Z_{2N} breaking established.