Topological objects contribute to thermodynamics of gluon plasma

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- Magnetic component in Yang-Mills theory at $T = 0$.
  - Models of color confinement.
  - Strings and monopoles at $T = 0$
- Magnetic component at $T > 0$:
  - Condensate-liquid-gas transition?
  - Existence of real(?) strings/monopoles at $T > T_c$?
  - Strings/monopoles and equation of state (lattice)

► K.Ishiguro, A.Nakamura, T.Sekido, T.Suzuki, V.I.Zakharov, M.N.Ch., Proceedings of Science (LATTICE 2007) 174 [arXiv:0710.2547]

► V.I.Zakharov, M.N.Ch., Phys.Rev.Lett. 98 (2007) 082002

Phase structure of pure gluons

Nontrivial dynamics of QCD is determined by the gluon sector:

[Talk by Frithjof Karsch $\approx$ 27 hours ago]

Phase structure of $SU(N)$ gluodynamics, $N = 2, 3$

- $T < T_c$: confinement of color
- $T > T_c$: deconfinement of color

In the deconfinement phase:

- $T \sim [T_c \ldots 2 - 5T_c]$: plasma, strongly interacting gluons
- higher T : predictions scale towards perturbation theory
- $T \gg T_c$: perturbative electric gluons
  plus logarithmically decaying
  non-perturbative magnetic sector

Properties of gluon plasma are unexpected:

- Similar to ideal(!) liquid

review in, e.g., [Shuryak, hep-ph/0608177]

- Shear viscosity of plasma is low , $\eta/s \approx 0.1 \ldots 0.4$

* interpretation of RHIC experiment

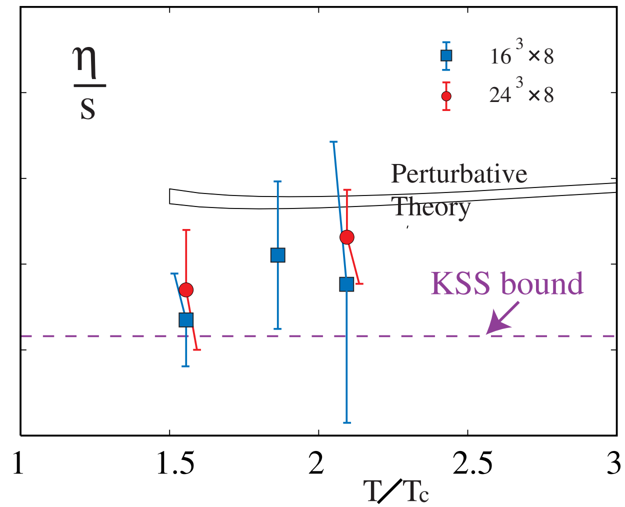
[Teaney, 03]

* simulations of quenched QCD

[= SU(3) lattice gauge theory]

[A.Nakamura, S.Sakai, 05]

[H.Meyer, '07]



- Stefan-Boltzmann law seems to be reached at $T \to \infty$

$$\varepsilon_{\mathrm{free}} = 3P_{\mathrm{free}} = N_{d.f.}\, C_{\mathrm{SB}}\, T^4 \,, \qquad C_{\mathrm{SB}} = \frac{\pi^2}{30}$$

$$N_{d.f.} = 2(N_c^2 - 1) \quad \text{degrees of freedom}$$

numerical simulations [Karsch et al, NPB'96]

[Bringoltz, Teper, PLB'05; Gliozzi, '07]

- Many features may be described by
  - perturbation theory
  - large-N_c supersymmetric Yang–Mills theory

    a review can be found in [Klebanov, hep-ph/0509087]
  - quasiparticle models (work also around $T \approx T_c$)

    [Rischke et al' 90, Peshier et al' 96; Levai, Heinz' 98]

$T < T_c$ (confinement phase)

Widely discussed mechanisms of color confinement:

- **Dual superconductor picture**

  ['t Hooft, Mandelstam, Nambu, '74-'76]

  * Based on existence of special gluonic configurations, called "magnetic monopoles"
  * Monopoles are classified with respect to the Cartan subgroup $[U(1)]^{N-1}$ of the $SU(N)$ gauge group

- **Center vortex mechanism**

  [Del Debbio, Faber, Greensite, Olejnik, '97]

  * a realization of spaghetti (Copenhagen) vacuum
  * Center strings are classified with respect to the center $\mathbb{Z}_N$ of the $SU(N)$ gauge group

Popular models of confinement of quarks

- Condensation of magnetic monopoles
  Abelian Dominance [T.Suzuki, I. Yotsuyanagi '00]
  Monopole Dominance [T.Suzuki, H.Shiba '00]
- Percolation of magnetic strings
  Center/Vortex Dominance
  [L. Del Debbio, M. Faber, J. Greensite, S. Olejnik '97]
- These approaches are related:
  - The percolation is related to the condensation (presence of the IR component in the density)
  - Monopoles are related to strings.

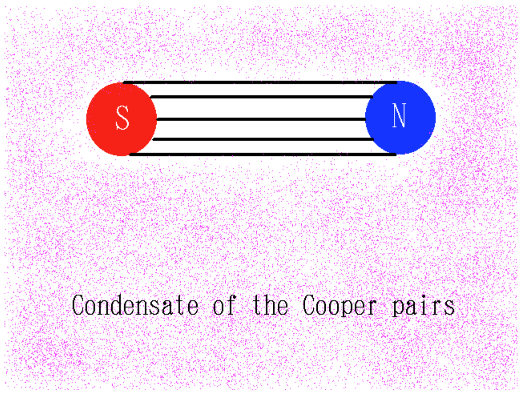
numerical fact [Ambjorn, Giedt & Greensite '00]

required analytically [Zakharov '05]

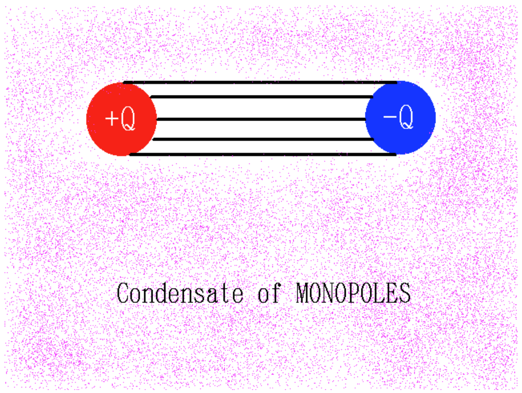
compact gauge models [Feldmann, Ilgenfritz, Schiller & Ch. '05]

$T < T_c$ (Dual Superconductor)

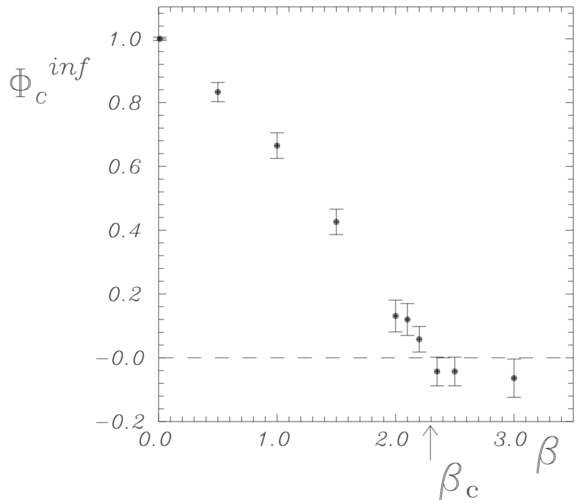
* condensation of monopoles $\longrightarrow$ dual Meissner effect
* dual Meissner effect $\longrightarrow$ chromoelectric string formation
* chromoelectric string = (dual) analogue of Abrikosov string
* quarks are sources of chromoelectric flux $\longrightarrow$ confinement



Abrikosov string in superconductor



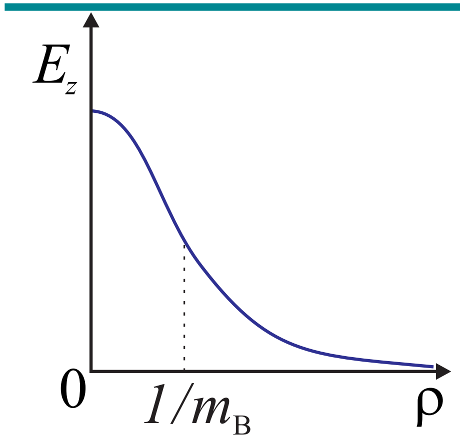
chromoelectric string in QCD vacuum

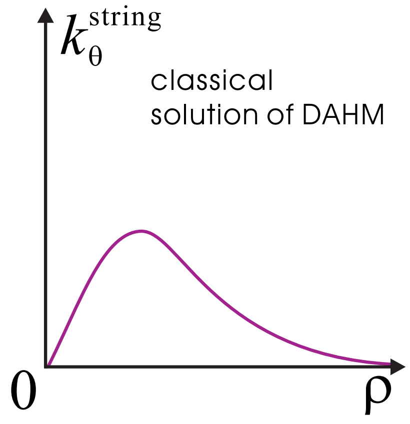


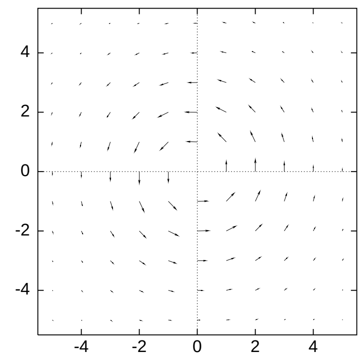
monopole condensate (numerical results)

[Di Giacomo & Paffuti'97]
monopole condensate from [Polikarpov, Veselov & Ch. '97]

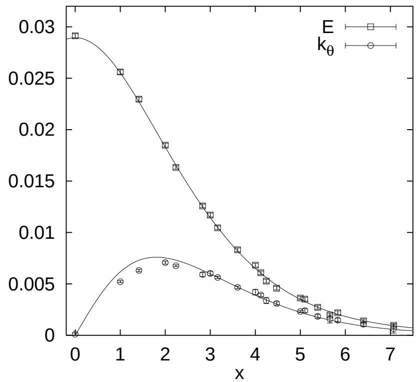
$T < T_c$ (Dual Superconductor and QCD string)

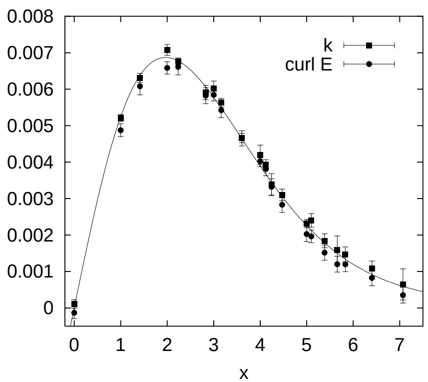


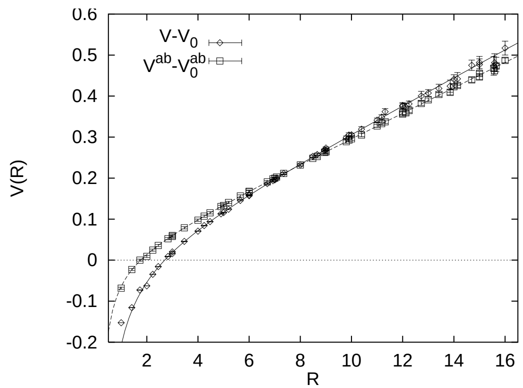




electric field (theor.) monopole current (theor.) 2D-monopole curl (num.!)



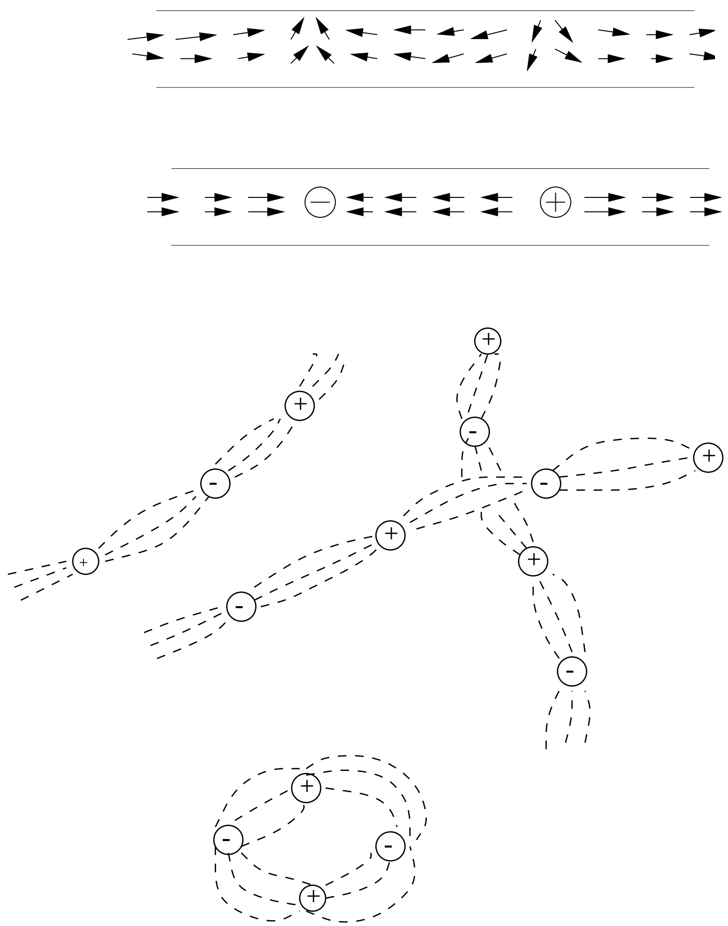




electric field, magn. current London (Ampere) equation quark-antiquark potential

[Bali, Schlichter, Schilling ’98; Bali, Bornyakov, Müller-Preussker, Schilling ’96]

Center/Monopole mechanisms are linked



- non-oriented half-flux of magnetic field

- monopoles are at points at which the flux alternates
- vortices are chains of monopoles

  [Ambjorn, Giedt, Greensite, '00]

- a similar string–monopole structure appears also in SUSY models

  [Hanany, Tong, '03; Auzzi et al '03]

  and in non-SUSY theories

  [Feldmann, Ilgenfritz, Schiller & Ch. '05]

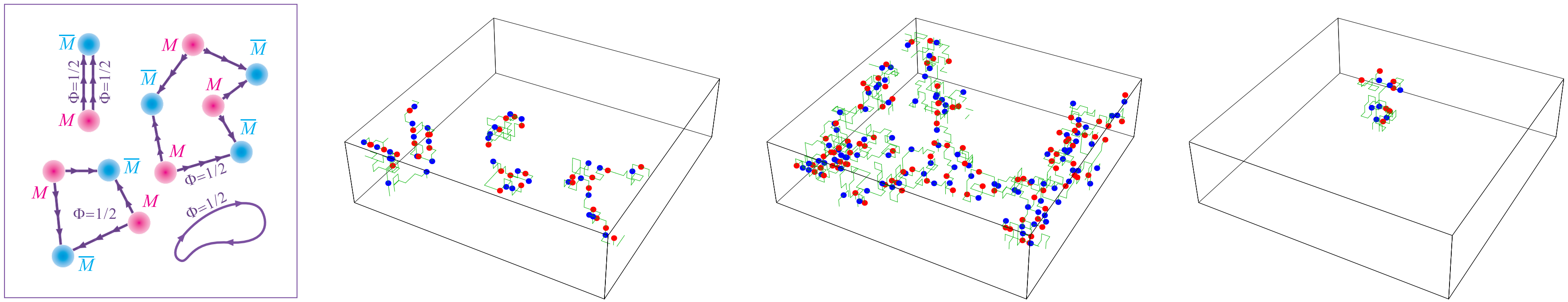
  [Gorsky, Shifman, Yung, '04 ... '07]

- required analytically [Zakharov '05]

Monopoles vs. vortices

▶ The vortices may organize the monopoles into dipole-like and chain-like structures, which are also present in compact Abelian models with doubly charged matter fields

▶ Examples of monopoles-vortex configurations:



[results of numerical simulations are taken from Feldmann, Ilgenfritz, Schiller & M.Ch. ’05]

▶ Observation of monopoles in the vortex chains: monopole is a point defect, where the flux of the vortex alternates.

Confinement ($T < T_c$) and plasma ($T > T_c$)

- The monopoles are condensed at $T < T_c$

... and not condensed at $T > T_c$

- The magnetic strings are percolating at $T < T_c$

... and not percolating at $T > T_c$

- What happens with topological defects at finite temperature $T > 0$?

- SUGGESTION: Degrees of freedom condensed at $T = 0$ form a light component of the thermal plasma at $T > 0$.

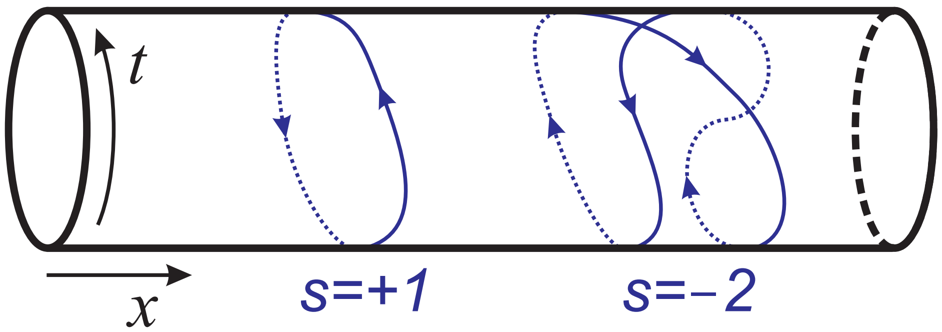
- The magnetic monopoles and the magnetic vortices become real (thermal) particles at $T > 0$

[Zakharov, M.N.Ch., '07]

[Liao and Shuryak, '07]

Use lattice simulations?

- Lattice simulations provide us with ensembles of magnetic defects.
- Which defect is real and which is virtual?



- s: the wrapping number with respect to the compact T–direction.

- Properties of thermal particles are encoded in the wrapped trajectories, $s \neq 0$, and the virtual particles are non-wrapped, $s = 0$.

[Zakharov, M.N.Ch., '07]

Example of a free scalar particle

- How to get thermal component of the density

$$\rho^{\text{th}}(T) = \int \frac{\text{d}^3 p}{(2\pi)^3} f_T(p)$$

from the trajectories of the particles?

- The propagator of the scalar particle is:

$$G(x-y) \propto \sum_{P_{x,y}} e^{-S_{\text{cl}}[P_{x,y}]}$$

is the sum over all trajectories $P_{x,y}$ connecting points x and y.

- The propagator in momentum space,

$$\mathcal{G}_s(\mathbf{p}) = \int \text{d}^3\mathbf{x}\, e^{-i(\mathbf{p},\mathbf{x})} G(\mathbf{x}, t = s/T)\,.$$

where s is the wrapping number of trajectories in the T−direction.

Example of a free scalar particle

- Then the vacuum ($s = 0$) part of propagator is divergent:

$$\mathcal{G}^{\text{vac}} \equiv \mathcal{G}_0 = \frac{4\,\Lambda_{\text{UV}}^2}{\omega_{\mathbf{p}}}$$

- ... while the ratio

$$f_T(\omega_{\mathbf{p}}) = \frac{1}{2}\frac{\mathcal{G}^{\text{wr}}(\mathbf{p})}{\mathcal{G}^{\text{vac}}(\mathbf{p})}, \quad \mathcal{G}^{\text{wr}} \equiv \sum_{s\neq 0} \mathcal{G}_s$$

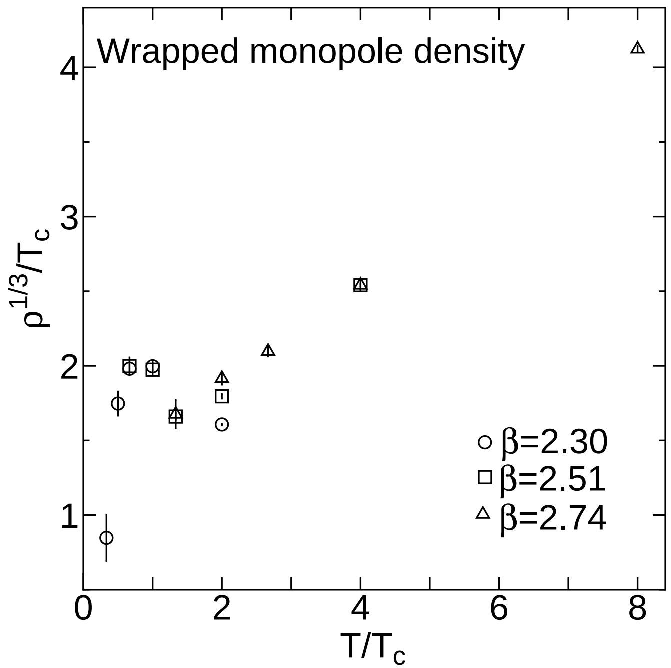
is finite as it gives the thermal distribution of the free particles

$$f_T = \frac{1}{e^{\omega_{\mathbf{p}}/T} - 1}, \qquad \omega_{\mathbf{p}} = (\mathbf{p}^2 + m_{\text{phys}}^2)^{1/2}$$

- CONCLUSION: Wrapped trajectories in the Euclidean space correspond to real particles in Minkowski space.

Density of thermal particles

- The average number of wrappings s in a time slice of volume V_{3d} is directly related to the density of real particles



$$\rho^{\text{th}}(T) = n_{\text{wr}} = \frac{\langle |s| \rangle}{V_{3d}}$$

[V.I.Zakharov, M.N.Ch., '07]

Density of thermal monopoles vs. T

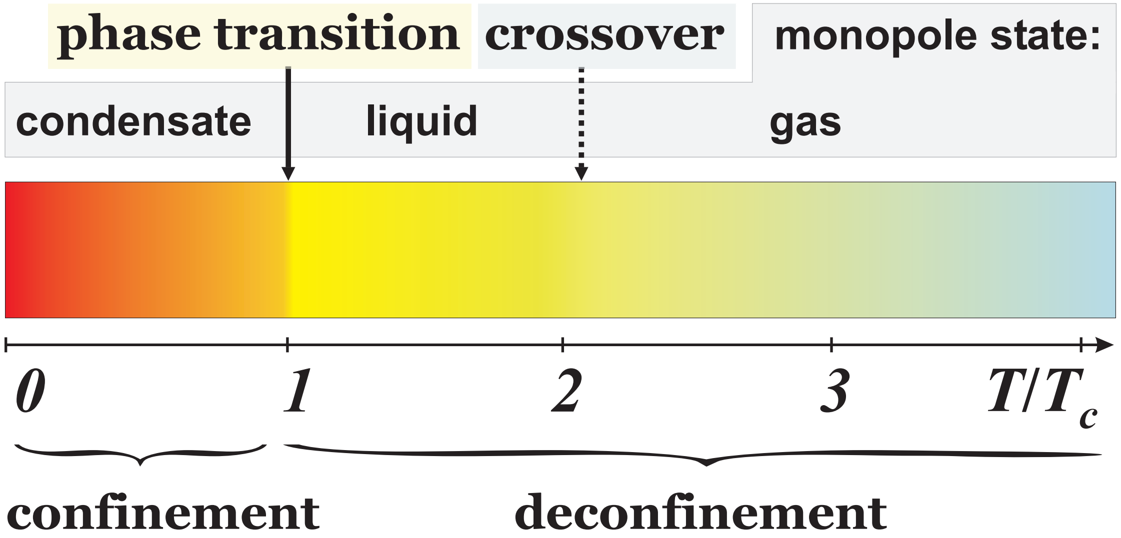
[Bornyakov, Mitrjushkin, Müller-Preussker '92]

[T.Suzuki, S.Ejiri, '95]

[T.Ejiri, '96]

First reliable lattice calculation:

[A.D'Alessandro, M.D'Elia, '07]



- At $T = T_c$: The condensed cluster $\to$ wrapped trajectories
- Wrapped trajectories correspond to real (thermal) particles:

$$\begin{array}{c} \text{condensate + virtual} \\ (T < 0) \end{array} \Rightarrow \begin{array}{c} \text{condensate + thermal + virtual} \\ (0 < T < T_c) \end{array} \Rightarrow \begin{array}{c} \text{thermal + virtual} \\ (T > T_c) \end{array}$$

- Analogy with superfluid Helium-4 [Zakharov & M.Ch'07]

In He-4 at $T = 1K \approx 0.5 T_c$ only 7% of particles are in the condensate! The rest (93%) is thermal!

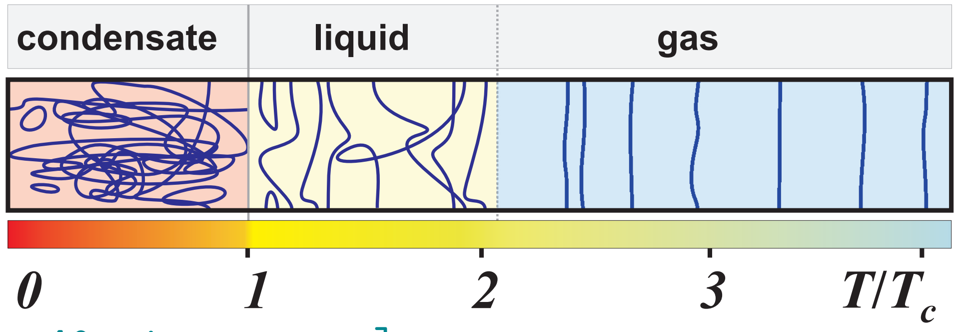
Liquid state of monopoles at finite T

- Monopole correlations $\langle \rho_\pm(x)\rho_\pm(y)\rangle$
- Calculation in lattice Yang-Mills:

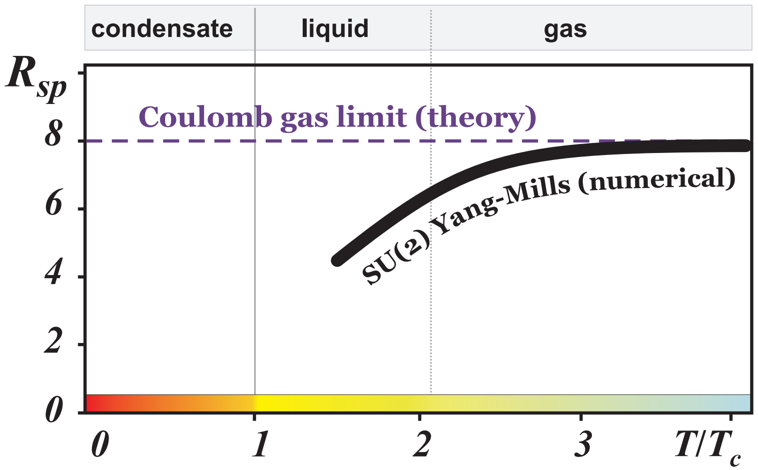
[A.D'Alessandro & M.D'Elia, '07]

- Liquid state interpretation:

[Liao, Shuryak '08] + [talk by Shuryak ≈40 minutes ago]



▶ A gas parameter for the monopole gas in Yang-Mills theory:



▶ The static monopoles contribute to spatial string tension $\sigma_{\rm sp}$.

▶ If the monopoles form a gas, then

$$R_{\rm sp} = \frac{\sigma_{\rm sp}(T)}{\lambda_D(T)\rho(T)} = 8 \text{ [theory]}$$

▶ We find: $R_{\rm sp} \approx 8$ at $T \gtrsim 2.5 T_c$

[Ishiguro, Suzuki, M.Ch '03]

- Free Energy (T is temperature and V is *spatial* volume)

$$F = -T \log \mathcal{Z}(T,V)$$

- Pressure

$$p = \frac{T}{V}\frac{\partial \log Z(T,V)}{\partial \log V} = -\frac{F}{V} = \frac{T}{V}\log \mathcal{Z}(T,V)$$

- Energy density

$$\varepsilon = \frac{T}{V}\frac{\partial \log Z(T,V)}{\partial \log T}$$

- Entropy density

$$s(T) = \frac{\varepsilon + p}{T} = \frac{\partial\, p(T)}{\partial T}$$

Thermodynamics: Trace Anomaly

- Trace anomaly of the energy–momentum tensor $T_{\mu\nu}$

$$\theta(T) = \langle T^{\mu}_{\mu} \rangle \equiv \varepsilon - 3p = T^5 \frac{\partial}{\partial T} \frac{p(T)}{T^4}$$

- Pressure via trace anomaly

$$p(T) = T^4 \int^{T} \frac{\mathrm{d}\, T_1}{T_1} \frac{\theta(T_1)}{T_1^4}$$

- Energy density via trace anomaly

$$\varepsilon(T) = 3\, T^4 \int^{T} \frac{\mathrm{d}\, T_1}{T_1} \frac{\theta(T_1)}{T_1^4} + \theta(T)$$

- Trace anomaly is a key quantity

Trace Anomaly for pure gluons

- Partition Function

$$\mathcal{Z}(T,V) = \int DU\ \mathrm{exp}\{-\beta \sum_P S_P[U]\}\,, \qquad S_P[U] = (1 - \frac{1}{2}\mathrm{Tr}\, U_P)$$

- Trace Anomaly

$$\theta(T) = T^5 \frac{\partial}{\partial T} \frac{\log \mathcal{Z}(T,V)}{T^3 V}$$

- Asymmetric $N_s^3 N_t$ lattice:

$$T = 1/(N_t a)\,, \qquad V = (N_s a)^3$$

- Trace anomaly on the lattice

$$\frac{\theta(T)}{T^4} = 6\, N_t^4 \left(\frac{\partial \beta(a)}{\partial \log a} \right) \cdot \boxed{(\langle S_P \rangle_T - \langle S_P \rangle_0)}$$

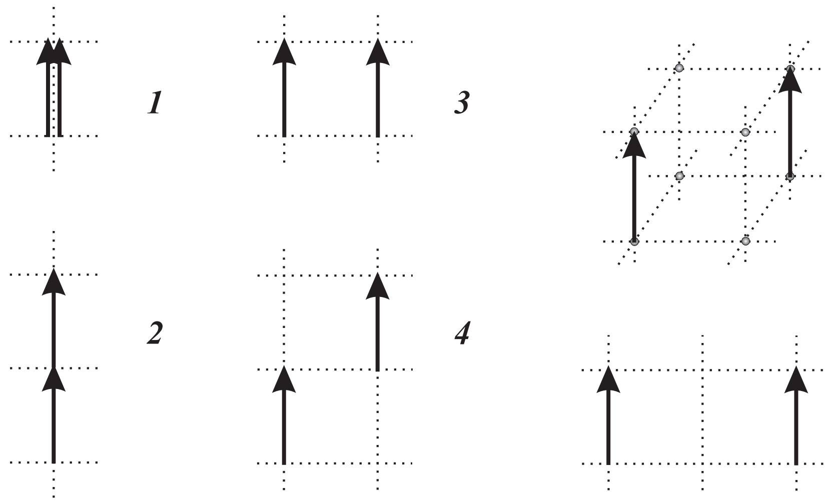
Trace Anomaly from monopoles

- Fix Maximal Abelian gauge $D_\mu^{\text{diag}} A_\mu^{\text{off}} = 0$
- Define particular singular gluon objects (monopoles) $k_\mu = \partial_\nu \tilde{F}_{\mu\nu}^{\text{diag}}$
- Determine the monopole action by inverse Monte Carlo algorithm

[Shiba, Suzuki '95]

- Partition function and the Trace of energy–momentum tensor:

$$\mathcal{Z} = \mathcal{Z}^{\text{mon}} \mathcal{Z}^{\text{rest}}, \qquad \theta = \theta^{\text{mon}} + \theta^{\text{rest}}$$



- Monopole partition function:

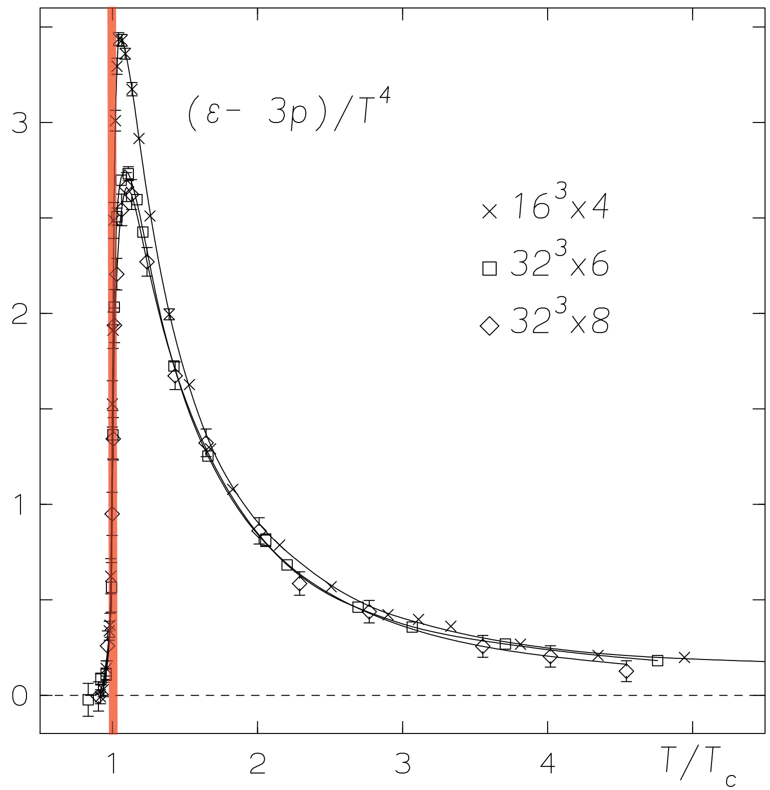
$$\mathcal{Z}^{\text{mon}} = \sum_{\text{monopoles},k} \exp\left\{-\sum_x \sum_{i=1}^{n} f_i(\beta) S_i^{\text{mon}}(k)\right\}$$

- Contribution of monopoles into the trace anomaly:

$$\theta^{\text{mon}} = N_t^4 \left(a\frac{\partial\beta}{\partial a}\right) \sum_i \left(\frac{\partial f_i(\beta)}{\partial\beta}\right) \left[\langle S_i^{\text{mon}}\rangle_T - \langle S_i^{\text{mon}}\rangle_0\right]$$

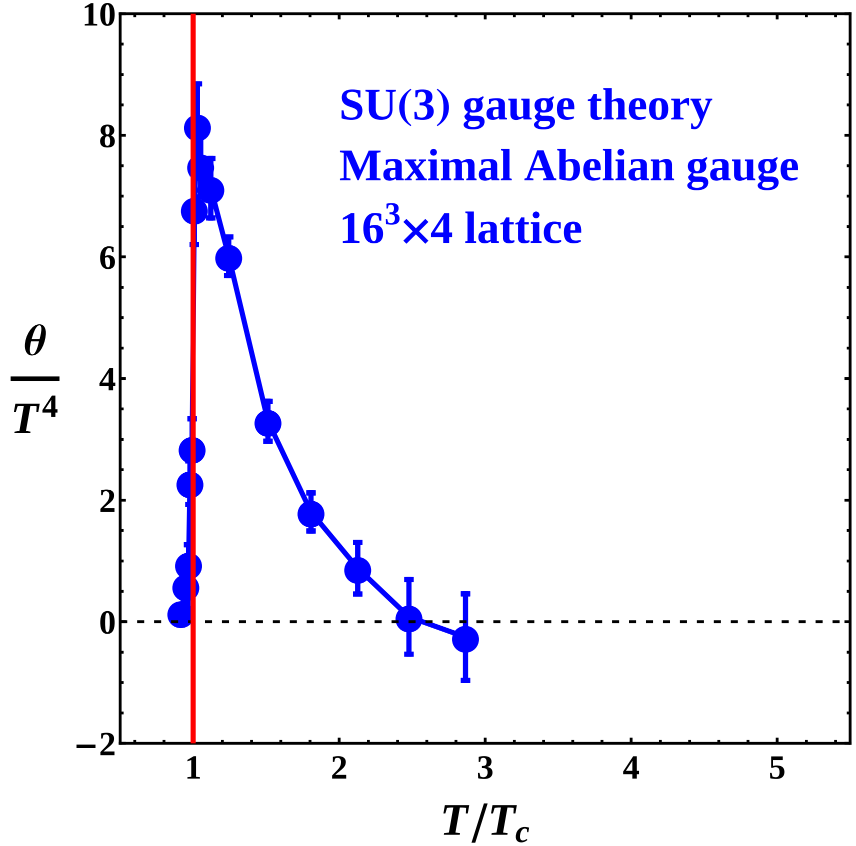
▶ Trace anomaly of energy momentum tensor

pure SU(3) glue



from [G.Boyd, J.Engels, F.Karsch, E.Laermann, C.Legeland, M.Lütgemeier, B.Petersson, '96]

contribution from monopoles



Yes, there is a contribution

large deviation is due to non-local action
(we need larger volumes - in preparation)

Trace Anomaly from vortices

- Fix Maximal Center gauge $\min_\Omega \left(\mathrm{Tr}\, U_{x\mu}^{(\Omega)}\right)^2$
- Define singular string-like gluon objects (vortices)

  with the worldsheet current $\sigma_P = \prod_{l \in \partial P} Z_l$
- Separate all plaquettes into two sets:

  i) $\sigma_P = -1$ (belong to the vortices)

  ii) $\sigma_P = +1$ (outside the vortices)
- Action splits trivially:

$$\sum_P S_P = \sum_{P \in \mathrm{vort}} S_P + \sum_{P \neq \mathrm{vort}} S_P$$

- Trace anomaly splits as well:

$$\theta = \theta^{\mathrm{vort}} + \theta^{\mathrm{outside}}$$

- Trace anomaly of energy momentum tensor

pure SU(2) glue

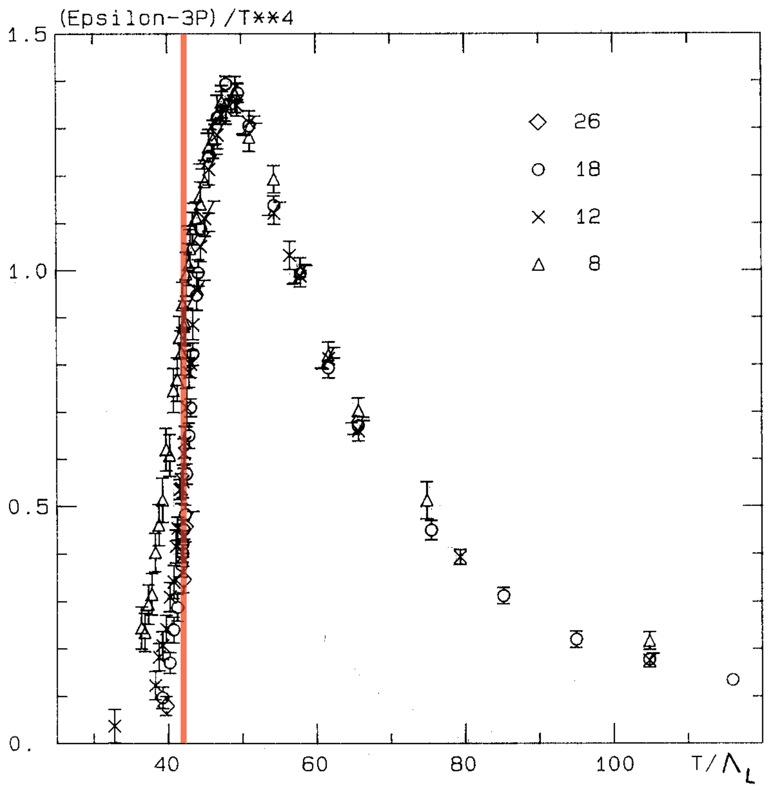
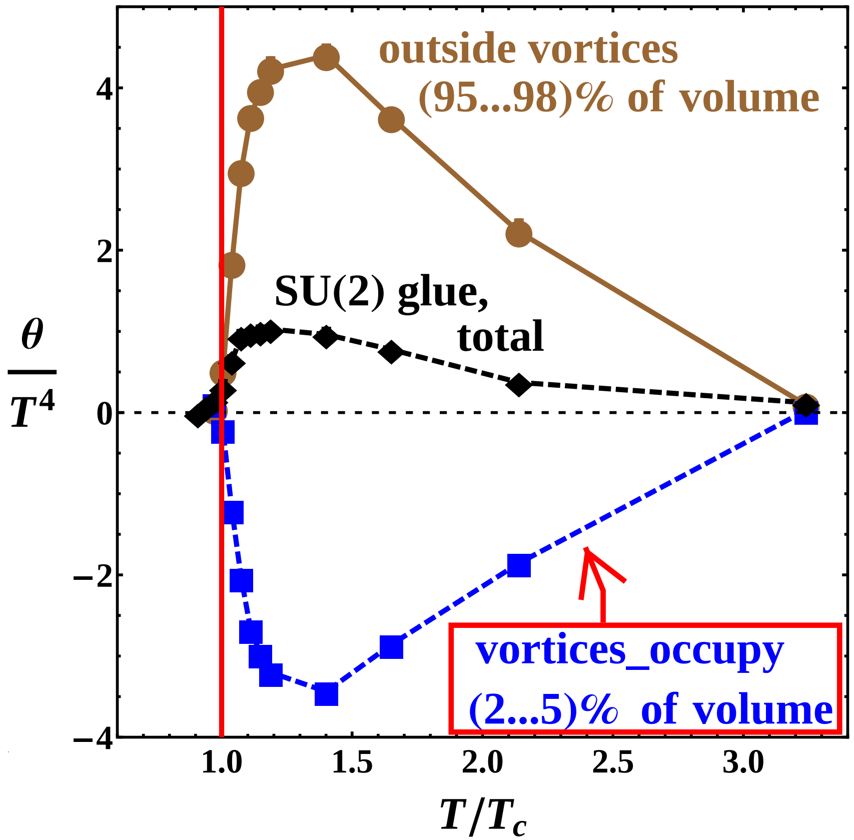


FIG. 3

from [J.Engels, J.Fingberg, K.Redlich, H.Satz, and M.Weber, '88]

contribution from vortices



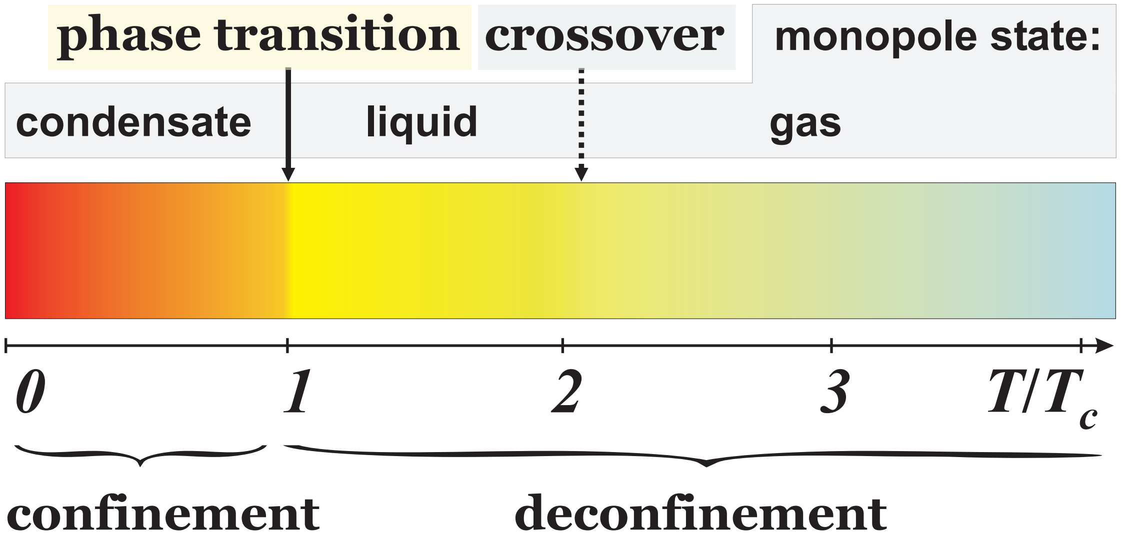
Yes, there is a contribution

Negative sign: [Gorsky, Zakharov '07]

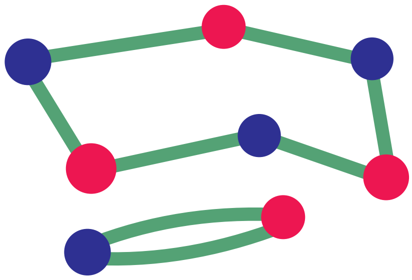
Discussion/Conclusion

▶ String-like and monopole-like magnetic gluonic configurations must be present as thermal excitations in the YM plasma.

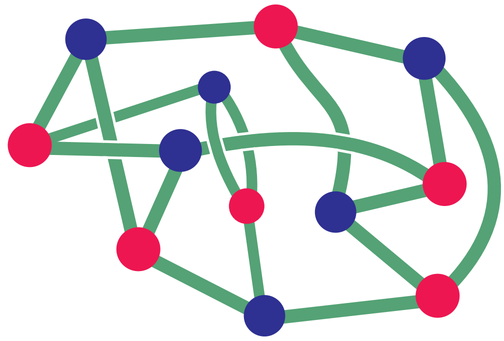
▶ Evolution of the magnetic component of the YM vacuum:



▶ Strong contribution of magnetic component to the trace anomaly, and, consequently, to the equation of state of the Yang-Mills plasma.

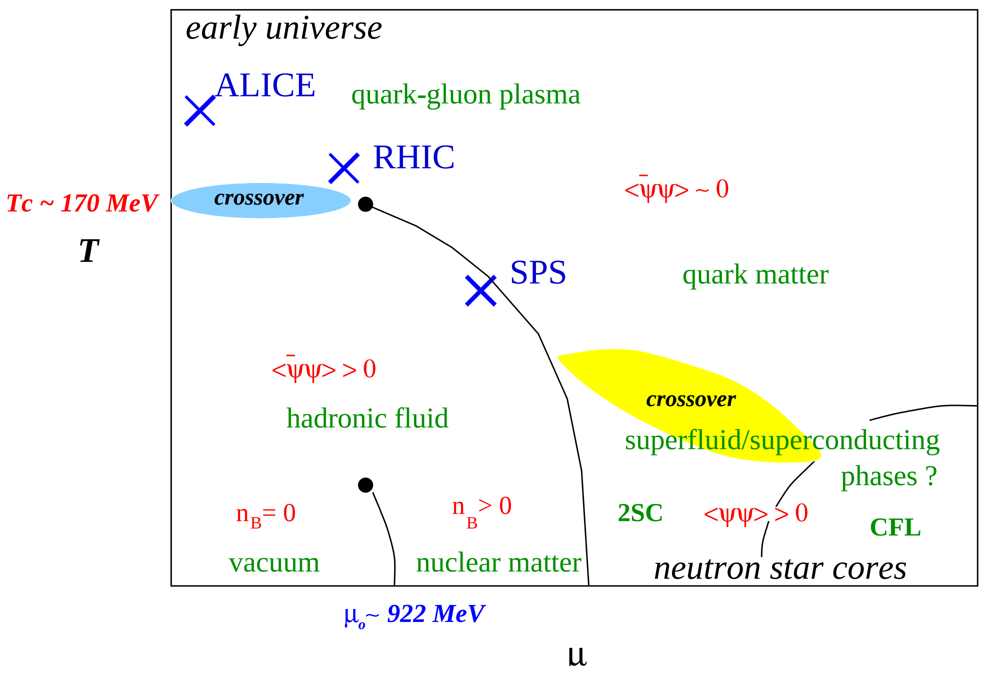


SU(2) chains



SU(3) nets

Phase structure of QCD



Various phases:

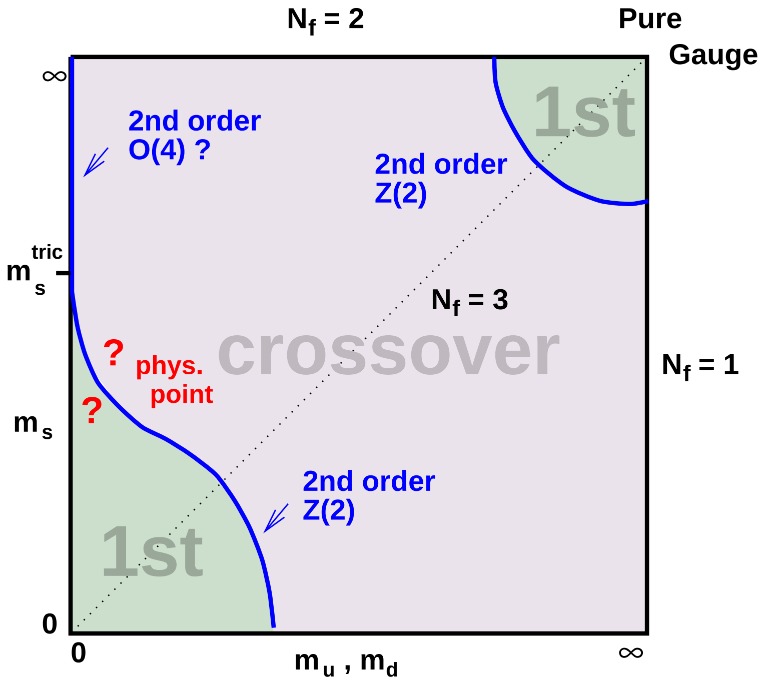
- quark-gluon phase
- hadron phase
- superconducting phases

Low-μ structure confirmed in lattice simulations [Fodor & Katz (2002),...]

Reviews: arXiv:0711.0661, arXiv:0711.0656, arXiv:0711.0336

May be observable in heavy-ion collision experiments!

Order of the $\mu = 0$ transition



Pure gauge:

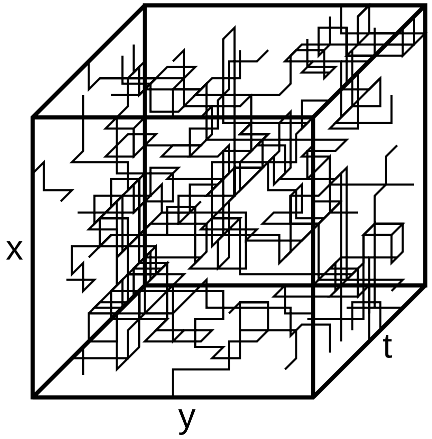
- $N_f = 0$ or $m_{u,d,s} = \infty$
- weak 1st order phase transition
- order parameter: Polyakov loop

Two light quarks:

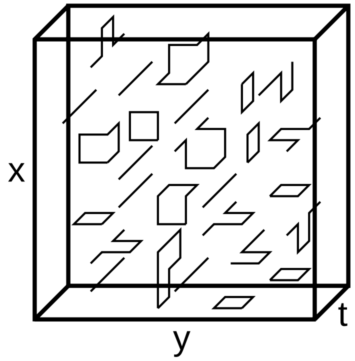
- $N_f = 2$, $m_{u,d} = 0$, $m_s = \infty$
- 2nd order phase transition
- order parameter: chiral condensate

In pure gauge case $T_c \approx 265(1)$ MeV [from lattice simulations]

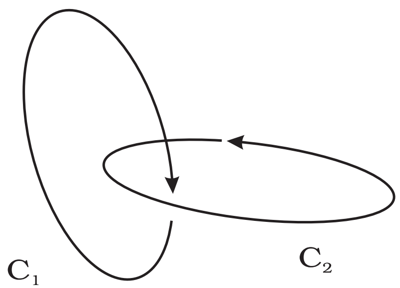
Center vortex mechanism

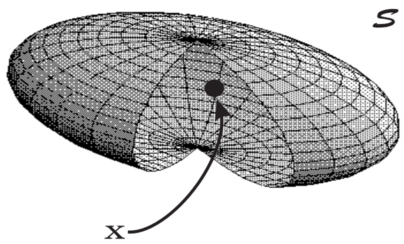


confined phase



deconfined phase





$T < T_c$ Vortices are percolating

$=$ Vortices are condensed

$=$ Center disorder

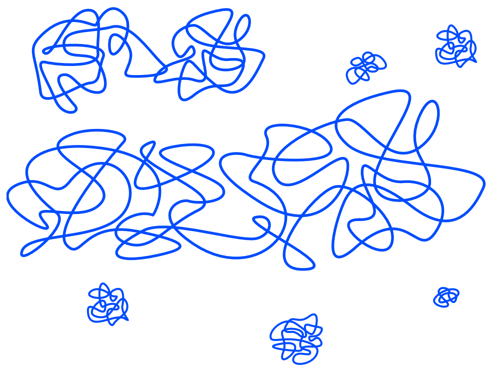
$T \geqslant T_c$ No percolation

- Interaction with quarks: Aharonov–Bohm effect "magnetic (center) flux links with particle (quark) trajectories"

Percolation transition [Engelhardt, Langfeld, Reinhardt, Tennert '99]

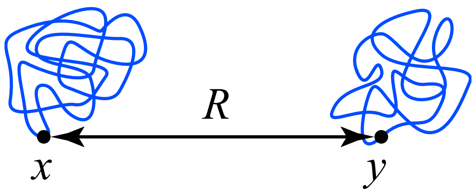
Aharonov-Bohm mechanism [Polikarpov, Veselov, Zubkov, Ch. '98]

Percolation vs. condensation

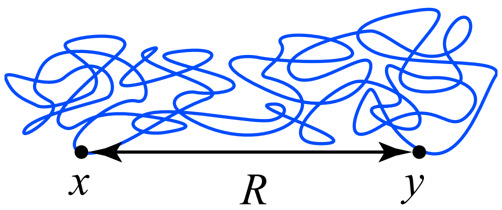


General structure of ensembles: many small (UV) clusters plus one big (IR) cluster

$$\rho = \rho_{\mathrm{UV}} + \rho_{\mathrm{IR}}$$



Percolation: probability to find two points x and y separated by the distance R and connected by any trajectory C



$$P(R) = \frac{\sum_{x,y}\sum_C \delta_C(x)\delta_C(y)\delta(|x-y|-R)}{\sum_{x,y}\sum_C \delta(|x-y|-R)})$$

Condensation: $P(R) \simeq P_\infty + P_0 \exp\{-\mu R\}$ with $P_\infty > 0$

Dimensional Reduction at $T \gg T_c$

- Non-perturbative magnetodynamics: 3D YM with the coupling

$$g^2_{3d}(T) = g^2_{4d}(T) \cdot T \sim T/\log T$$

- All dimensional quantities are expressed in terms of $g^2_{3d}(T)$ only.
- The monopole density is

$$\rho(T) = C_\rho \, g^6_{3d}(T) \propto \left(\frac{T}{\log T/\Lambda_{QCD}} \right)^3 \qquad T \gg T_c$$

found also in [Giovannangeli, Korthals Altes, '05]

- Reproduced by T-dependent chemical potential

$$\mu \sim 3T \log \log T/\Lambda$$