Anomalous neutrino-photon interactions in the Standard Model

June 5, 2008
Based on

- Baryon-Number-Induced Chern-Simons Couplings of Vector and Axial-Vector Mesons in Holographic QCD, PRL 99,14 (2007); arXiv:0704.1604 w/ Sophia Domokos

- Anomaly mediated neutrino-photon interactions at finite baryon density; arXiv:0708.1281 w/ Chris Hill and Richard Hill.


- Work in progress w/ Chris Hill and Richard Hill
Outline

- Motivation from AdS/QCD
- Low-Energy QCD
- WZW term and its Standard Model gauging
  \[ f_1 \rightarrow \rho + \gamma \] as a check of the formalism
- Anomalous neutrino-photon interactions and neutrino-nucleon scattering
- Comparison to the MiniBoone excess
- Other possible applications
Motivation from AdS/QCD:

In QCD the $SU(N_f)_L \times SU(N_f)_R$ flavor symmetry has anomalies:

The AdS dual involves terms which are gauge invariant in bulk, but vary on the bndy in the same way that QCD would if coupled to fictitious flavor gauge fields.
\[ S_{CS} = \frac{N_c}{24\pi^2} \int \omega_5(A_L) - \omega_5(A_R) \]

\[ d\omega_5 = Tr F^3 \]

\[ \delta\omega_5 = d\omega_4^1 \]

This can also be understood in the S-S model where such couplings arise from a term on the D8-branes

\[ S_{anom} = \int_{\Sigma_p} C \wedge \text{ch}(F) = \frac{1}{6(2\pi)^3} \int_{\Sigma_9} C_3 \wedge Tr F^3 \]

which in the D4-background with gives the same couplings.

\[ \int_{S^4} G_4 = 2\pi N_c \]
Restrict to lightest modes:

\[
\pi^a(x, z) = \pi^a(x) \psi_\pi(z)
\]
\[
V_\mu^a(x, z) = g_5 \rho_\mu^a(x) \psi_\rho(z)
\]
\[
A_\mu^a(x, z) = g_5 \alpha_\mu^a(x) \psi_\alpha(z)
\]

Then the 5-d Chern-Simons term gives rise to a variety of 4-d couplings involving the epsilon tensor, including cubic couplings between vector and axial-vector mesons such as

\[
S_{\omega-\rho-a_1} \sim g_{\omega\rho a_1} \int d^4x \epsilon^{\mu\nu\lambda\rho} \omega_\mu a_\nu \partial_\rho \rho_\lambda + \cdots
\]

and similar terms for the isoscalar \( f_1 \).
The coefficients of these “pseudo-Chern-Simons” terms are predicted by AdS/QCD. They have some possibly interesting consequences in QCD, but in general they are not well constrained by experiment.

Could there be such couplings involving SM gauge fields? In AdS the $\rho, a_1$ (or $\omega, f_1$) are gauge fields coupling to vector and axial-vector currents. They are roughly analogous to the $\gamma, Z^0$ in the Standard Model.
We claim that in a well defined sense the SM does contain such couplings, for example it contains a coupling

\[ \frac{N_c}{48\pi^2} \frac{e g_\omega g_2}{\cos \theta_W} \epsilon_{\mu \nu \rho \sigma} \omega^\mu Z^\nu F^{\rho \sigma} \]
This interaction gives a new source of photon-neutrino couplings in the presence of nuclear matter which should affect, among other things,

Neutrino Scattering  Energy transfer in neutron stars and supernovae
These couplings can be derived as part of the low-energy description of QCD.

QCD at low-energies is described by the interactions of the lightest particles, the pions (and kaons) treated as NGB. These interact with other light fields, the photon, electron, muon and neutrinos through renormalizable interactions as well as through non-renormalizable terms of higher dimension.

At somewhat higher mass (700-1200 MeV) one encounters vector and axial-vector mesons: $\rho, \omega, a_1, f_1$. These couple to the isospin and baryon currents and their axial counterparts. They act in some ways like gauge fields (Yang-Mills! and VMD)
As mentioned earlier, the $U(N_f)_L \times U(N_f)_R$ symmetry is anomalous. If we try to gauge it by coupling to gauge fields $A_L, A_R$, the effective action is not gauge invariant, but instead varies by

$$\delta S_{q}^{\text{eff}} = \frac{N_c}{24\pi^2} \int \left[ -\epsilon_L dA_L dA_L + \epsilon_R dA_R dA_R + \cdots \right]$$
On general grounds this structure must be reflected in the low-energy description of QCD. The way this works is a familiar story. The pion action is a function of

\[ U = e^{2i\pi a T^a / f_\pi}. \]

The kinetic terms can be made gauge invariant under \( U(N_f)_L \times U(N_f)_R \) simply by replacing ordinary derivatives by covariant derivatives:

\[ \mathcal{L}_K = \frac{f_\pi^2}{4} \text{Tr}(D_\mu U^\dagger D^\mu U) \]
To correctly reflect the anomaly structure, and to break a “fake” natural parity symmetry, $U \rightarrow U^\dagger$ there must be another term in the action. The missing term is the “WZW” term:

$$\Gamma_{WZW} = -\frac{iN_c}{240\pi^2} \int_{M_5} \text{Tr}[(dUU^\dagger)^5]$$

$\Gamma_{WZW}$ is proportional to the area of the image of spacetime in the $SU(N_f)$ group manifold. It breaks the fake parity symmetry.
Wonderful things happen when $\Gamma_{WZW}(U)$ is generalized to $\Gamma_{WZW}(U, A_L, A_R)$, including gauge fields for $U(N_f)_L \times U(N_f)_R$:

- There is an anomalous gauge variation which matches that of the quarks and tells us that without other fields we cannot gauge the full $U(N_f)_L \times U(N_f)_R$.

- For an anomaly free subgroup e.g. $U(1)_{EM}$ the coupling of photons to the pion gives the correct rate for the anomaly-driven decay $\pi^0 \rightarrow \gamma + \gamma$. 
In the real world there are additional effects which should be included into this description:

- There are charged and neutral current weak interactions. We must gauge $SU(2)_L \times U(1)_Y$ and the anomalies cancel between quarks and leptons.

- There are environments with background baryon and isospin densities which lead to background values of the vector mesons of QCD. These backgrounds must not destroy anomaly cancellation.
We start with $\Gamma_{WZW}(U, A_L, A_R)$ with $A_L, A_R$ gauging $SU(2)_L \times U(1)_Y$ and then add a background of QCD vector and axial-vector mesons $\rho, \omega, a_1, f_1$.

This gives us $\Gamma_{WZW}(U, A_L, B_L, A_R, B_R)$ where

for two flavors,

$$B_L + B_R = \begin{pmatrix} \rho^0 + \omega & \sqrt{2}\rho^+ \\ \sqrt{2}\rho^- & -\rho^0 + \omega \end{pmatrix}$$

$$B_L - B_R = \begin{pmatrix} a_1^0 + f_1 & \sqrt{2}a_1^+ \\ \sqrt{2}a_1^- & -a_1^0 + f_1 \end{pmatrix}$$
This generates many new interaction terms. We will focus on the “pseudo-Chern-Simons” terms of the form

\[ S_{pCS} = \int A_1 \wedge A_2 \wedge dA_3 = \int d^4 x \, \epsilon^{\mu \nu \lambda \rho} A_{1 \mu} A_{2 \nu} \partial_\lambda A_{3 \rho} \]

with the \( A_i \) vector fields.

These are related to true Chern-Simons terms in three dimensions (by putting in background values for some \( A \)) or in five-dimensions (by dimensional reduction).
This leads to a large number of pCS terms:

\[
\Gamma_{pCS} = C \int dZZ \left[ \frac{sW}{cW} \rho^0 \left( \frac{3}{2cW} - 3 \right) \omega - \frac{1}{2cW} f \right] + dAZ \left[ -\frac{sW}{cW} \rho^0 - \frac{3sW}{cW} \omega \right] + dZ \left[ W^- \rho^+ + W^+ \rho^- \right] \frac{s^2W}{cW}
\]

\[
+ dA \left[ W^- \rho^+ + W^+ \rho^- \right] \left( -sW \right) + \left( DW^+ W^- + DW^- W^+ \right) \left[ -\frac{3}{2} \omega - \frac{1}{2} f \right],
\]

\[
+ C \int d\rho^0 \left[ -\frac{3}{2cW} \omega - \frac{s^2W}{cW} \rho^0 + \left( -\frac{3}{2cW} + 3cW \right) f \right] + d\omega \left[ -\frac{3}{2cW} \rho^0 + \left( \frac{3}{2cW} + 3cW \right) \rho^0 - \frac{s^2W}{cW} f \right]
\]

\[
+ d\rho^0 \left[ \frac{s^2W}{cW} \rho^0 \left( \frac{3}{2cW} - 3cW \right) \omega - \frac{1}{2cW} f \right] + d\rho^0 \left[ \left( \frac{3}{2cW} - 3cW \right) \rho^0 + \frac{s^2W}{cW} \rho^0 - \frac{1}{2cW} \rho^0 a^0 \right]
\]

\[
+ dA \left( sW \rho^0 a^0 + 3sW \rho^0 f + 3sW \omega a^0 + sW \omega f \right) + dZ \left( -\frac{s^2W}{cW} (\rho^+ a^- + \rho^- a^+) \right)
\]

\[
+ dA \left( sW ((\rho^+ a^- + \rho^- a^+) \right)
\]

\[
+ \frac{3}{2} \left[ W^+ D\rho^- + W^- D\rho^+ \right] f + \frac{3}{2} \left[ W^-(\rho^- - a^-) + W^-(\rho^+ - a^+) \right] d\omega
\]

\[
+ \frac{1}{2} \left[ W^+ D\rho^- + W^- D\rho^+ \right] \left( -3\omega - f \right) + \frac{1}{2} \left[ W^+(\rho^+ - a^+) + W^-(-3\rho^- - a^-) \right] df,
\]

\[
+ C \int 2 \left( \rho^- f + \omega a^- \right) D\rho^+ + (\omega a^+ + \rho^+ f) D\rho^- + (\omega a^0 + \rho^0 f) D\rho^0 + (\rho^+ a^- + \rho^- a^+ + \omega f + \rho^0 a^0) df.
\]

\[
+ C \int \left\{ W^+ W^- \left[ 3cW Z \right] \omega + W^+ W^- \left[ (cW + \frac{1}{2cW}) Z \right] f \right\},
\]

\[
+ C \int \left\{ W^+ W^- \left[ \frac{3}{2} (\rho^0 + a^0) \omega - \frac{1}{2} (\rho^0 - a^0) f \right\]
\]

\[
+ W^+ Z \left[ \frac{3cW}{2} \rho^- f - \frac{3cW}{2} \rho^- \omega - \frac{cW}{2} a^- f + \frac{3cW}{2} \omega a^- - \frac{1}{cW} \rho^- f \right]
\]

\[
+ W^- Z \left[ - \frac{3cW}{2} \rho^+ f + \frac{3cW}{2} \rho^+ \omega + \frac{cW}{2} a^+ f - \frac{3cW}{2} \omega a^+ + \frac{1}{cW} \rho^+ f \right]
\]

\[
+ C \int \left\{ W^+ \left[ \rho^- \omega f - \rho^- \omega f \right] - \rho^- \omega f + a^0 \omega f + \rho^0 a^0 - \omega a^0 \right\}
\]
What do we do with such couplings?

- Integrate out the massive $W^\pm, Z$ to get couplings for light fields (e.g. $\gamma, \nu, \bar{\nu}$) in the presence of background fields (e.g. baryon number).

- Treat the QCD mesons as fundamental fields in the spirit of Vector Meson Dominance.

The first is more clearly justified, the second involves an approximation which is not under good control, but often works reasonably well, and receives some justification from AdS/QCD.
The decay $f_1 \rightarrow \rho + \gamma$ provides a useful sanity check of this analysis. It is observed with a 5% branching ratio $\Gamma(f_1 \rightarrow \rho + \gamma) = 1.32\text{MeV}$.

Our coupling leads to

$$\Gamma = \frac{3\alpha}{256\pi^4} \frac{E_\gamma^2}{m_\rho^2} g_\rho^2 g_f^2 \left(1 + \frac{m_\rho^2}{m_f^2}\right)$$

where $E_\gamma = (m_f^2 - m_\rho^2)/2m_f$ is the photon energy in the $f_1$ rest frame. Agreement with the measured rate requires $g_\rho g_f \sim 50$, a not unreasonable value.
The helicity structure of the amplitude provides additional information. Computing the ratio of decays in which the $\rho$, in its rest frame, is longitudinally or transversely polarized gives

$$\frac{\Gamma(\text{long})}{\Gamma(\text{trans})} = \frac{m_{f}^{2}}{m_{\rho}^{2}} \sim 2.8$$

This disagrees with an earlier quark model calculation of Babcock&Rosner. The experimental situation is a bit confused. The primary PDG reference (Coffman et. al.) has conflicting statements. A 1995 experiment by Amelin et.al. gives $\Gamma(\text{long})/\Gamma(\text{trans}) = 3.9 \pm 0.9(\text{stat}) \pm 1.0(\text{syst})$
In any event, the analysis using pCS terms is in better agreement with data than previous calculations, and makes further predictions for other decays which may be measured in the future ($f_1 \rightarrow \omega + \gamma$, $a_1 \rightarrow \omega + \gamma$).
I now want to focus on the term

$$\frac{N_c}{48\pi^2} \frac{e g_\omega g_2}{\cos \theta_W} \epsilon_{\mu\nu\rho\sigma} \omega^\mu Z^\nu F^{\rho\sigma}$$

which gives rise to the “321 widget” which links the strong, weak, and EM interactions:
Note that this in not invariant under the “baryon gauge transformation” \( \delta \omega = d \epsilon_B \).

This is a reflection of the fact that the baryon current is anomalous in the Standard Model, a result which has played an important role in scenarios for weak scale baryogenesis.
How can we test this anomalous interaction? We need a context where other interactions (e.g. E&M) do not swamp the effect. To treat it as a term in the low-energy effective action we should replace the $Z$ by the low-energy part of the currents it couples to. This suggests we look at processes involving neutrinos. We also need a source that couples to the $\omega$, that is, an object with baryon number. Thus we might expect to see the effects of this interaction in the scattering of neutrinos off of nuclei.
This is quite analogous to the Primakoff effect which probes the anomalous $\gamma - \gamma - \pi^0$ interaction by using nuclei as a source of electric charge. We now use the nucleus as a source of baryon charge, and look for a photon in the final state:
Since this process involves $Z^0$ exchange, the rate will be very small at low-energies. The approximations used to derive this interaction break down at an energy scale of $4\pi f_\pi \sim 1 \text{ GeV}$ and above this energy the rate will be reduced by form-factor effects. Thus we might hope to see this effect in scattering of neutrinos off nuclei with $100 \text{ MeV} \leq E_\nu \leq 1000 \text{ MeV}$

Luckily there are both current (MiniBoone) and future (T2K) experiments that may be sensitive to this effect that operate in this energy regime.
The MiniBooNE experiment looks for $\nu_\mu \rightarrow \nu_e$ oscillations followed by charged current scattering to produce final state electrons which are detected through their Cerenkov radiation.
MiniBoone sees an excess at low energies:

\[ 300 < E_\nu < 475 \text{MeV} \]

\[ 96 \pm 17 \pm 20 \text{events} \]

excess \[ 3.7 \sigma \]

(MiniBoone presentation at Lepton-Photon '07)
MiniBooNE distinguishes electrons from muons, but cannot discriminate between final state photons and electrons:

- **Electron**
  - Short track, no multiple scattering
  - Electrons: short track, multi. scat., brems.

- **Muon**
  - Long track, slows down

- **Two Photons**
  - Neutral pions: 2 electron-like tracks

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Cerenkov Light...

- **Ring**
  - Sharp Ring
  - Fuzzy Ring

- **Sharp Outer Ring with Fuzzy Inner Region**

- **Two Fuzzy Rings**

Thanks to Fermilab for graphics.
Thus the $Z - \omega - \gamma$ vertex gives a background to the charged current events $(\nu_e + N \rightarrow e^- + N')$ MiniBooNE is looking for:
Competing Processes:

Rho exchange suppressed by

\[ \frac{g_\rho}{g_\omega} \sim \frac{1}{3} \]

Pion exchange suppressed by

\[ 1 - 4 \sin^2 \theta_W \ll 1 \]

Brehmstrahlung suppressed by

\[ \frac{1}{M_N} \]
The most naive estimate ignores recoil, form factors, nuclear physics effects (Fermi motion, Pauli blocking), and replaces the neutrino beam by a mono-energetic beam at the peak energy of 700 MeV. This gives

\[ \sigma \simeq \frac{1}{480\pi^6} G_F^2 \alpha \frac{g^4}{m^4_\omega} E^6_\nu \]

Which for every \(2 \times 10^5\) CCQE events gives

\[ \sim 140 \left(\frac{g_\omega}{10}\right)^4 \]

events from the anomaly-induced neutrino-photon interaction.
We are working on a more detailed comparison. Including some of the simpler effects leads to reasonable fits to data. Including nuclear recoil and a simple choice of form factor, but using a mono-energetic beam and scattering off of nucleons rather than nuclei gives, up to normalization.
Note that MiniBoone plots the number of events vs. the reconstructed neutrino energy, assuming a two body final state (electron + nucleon).

The previous graphs plots the number of events vs. the (visible) photon energy, which is shared roughly equally with the final state neutrino in a three body final state (neutrino + nucleon + photon). A neutrino beam with a distribution of neutrino energies peaked at 700 MeV is shifted to a distribution of events with final state photons with the photon energy peaked at ~ 350 MeV.
Other Applications:

Neutron Star Cooling, Supernova dynamics?

Detection of coherent neutrino scattering off of nuclei.

New contributions to atomic parity violating effects (e.g. nuclear anapole moments).
Conclusions

There are a set of SM couplings which involve both the SM gauge fields and the vector fields of QCD which are distinguished by their violation of “natural parity” and their relation to anomalies.

The couplings give a reasonable prediction for certain vector meson decays and may account for the MiniBoone excess at low-energies.

These couplings should have a variety of other applications in astrophysics and nuclear physics/QCD.
Extra Slides/Toy Model
A Toy Model with pCS

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We gauge $U(1)_L \times U(1)_R$ with generators

$$Q_L = B_L - L_L$$
$$Q_R = B_R - L_R$$
The action is

\[
S = \int d^4 x \left( \bar{q}_L (i \partial - A_L) q_L + \bar{q}_R (i \partial + A_R) q_R 
+ \bar{\ell}_L (i \partial - A_L) \ell_L + \bar{\ell}_R (i \partial - A_R) \ell_R 
- \frac{1}{4} (F_L)^2 - \frac{1}{4} (F_R)^2 \right)
\]

The quark sector by itself has triangle anomalies
We add a quark mass term and integrate out the quarks.

We add in a background field coupling to the (anomalous) vector baryon current.

But this anomaly is cancelled by the leptons so that we can consistently gauge $U(1)_L \times U(1)_R$

We now generalize this model in two ways:

- We add a quark mass term and integrate out the quarks.
- We add in a background field coupling to the (anomalous) vector baryon current.
To add a mass term consistent with gauge invariance we add a Higgs field with a non-zero vev $\Phi = ve^{i\phi/f}$ coupled to the quarks leading to

$$m_q e^{i\phi/f} \bar{q}_L q_R + h.c. \quad \phi/f \sim \pi^0/f_\pi \quad \text{in QCD}$$

The $U(1)_L \times U(1)_R$ acts as

$$\delta_L q_L = i\epsilon_L q_L, \quad \delta_L A_L = d\epsilon_L, \quad \delta_L \phi/f = \epsilon_L$$

$$\delta_R q_R = i\epsilon_R q_R, \quad \delta_R A_R = d\epsilon_R, \quad \delta_R \phi/f = -\epsilon_R$$
Now consider the low-energy theory after integrating out the massive quarks. This is a function of $\phi, A_L, A_R$ and since the anomalies must still cancel, it must have an anomalous variation equal to that of $S_{q}^{\text{eff}}$ (D’Hoker and Farhi). This “WZW” contribution is

$$
\Gamma_{WZW} = -\frac{1}{24\pi^2} \int \left( A_L A_R dA_L + A_L A_R dA_R 
+ \frac{\phi}{f} [dA_L dA_L + dA_R dA_R + dA_L dA_R] \right)
$$
We now add a new ingredient: a background field coupled to the baryon current (like the omega meson in QCD).

\[ S_q \rightarrow \int d^4x \, \bar{q}_L(i\partial/ - A_L/ - \phi)q_L + \bar{q}_R(i\partial/ - A_R/ - \phi)q_R \]

This leads to new terms in the variation of the WZW term proportional to omega:

\[ \delta \Gamma_{WZW} = -\frac{1}{24\pi^2} \int \epsilon_L (2dA_Ld\omega + d\omega d\omega) \]

\[ -\epsilon_R (2dA_Rd\omega + d\omega d\omega) \]
Since omega doesn't couple to leptons, the anomaly no longer cancels. But, as one might expect, this can be fixed by adding a local counterterm whose variation cancels the omega dependent terms in $\delta \Gamma_{WZW}$:

$$\Gamma_c = \frac{1}{24\pi^2} \int \left( 2\omega A_R dA_R - \omega A_R d\omega - (R \leftrightarrow L) \right)$$

The effective action then has the anomaly related terms

$$\Gamma_{eff} = \Gamma_{WZW}(\phi, A_L + \omega, A_R + \omega) + \Gamma_c$$

$$= \Gamma_{WZW}(\phi, A_L, A_R) + \Gamma_{pCS}$$

Variation cancelled by leptons
where in terms of \( A_L = Z + A, \ A_R = A \)

\[
\Gamma_{pCS} = \frac{1}{8\pi^2} \int (\omega [2dA + dZ] + \omega d\omega) \left( Z - \frac{d\phi}{f} \right)
\]

This term correctly generates the anomaly in the baryon current by varying \( \delta \omega = d\epsilon_B \)