Amplitudes in N=8 supergravity and Wilson Loops

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Motivations

- Understand why scattering amplitudes (in maximally supersymmetric theories) are simple
    - geometry in Twistor Space (Witten)
    - recursive structures in the perturbative S-matrix of gauge theories
- Simplicity hidden by Feynman diagrams
    - diagrams not not separately gauge invariant
    - unphysical singularities
- Unitarity-based & twistor-inspired methods
    - gauge-invariant, on-shell data at each intermediate step of calculation
    - also in non-supersymmetric theories

- Amplitudes in N=4 super Yang-Mills are even simpler (and more mysterious...)

  - All one-loop amplitudes expressed in terms of box functions (Bern, Dixon, Dunbar, Kosower)
  - Iterative structures in N=4 splitting amplitudes and planar MHV amplitudes (Anastasiou, Bern, Dixon, Kosower; Bern, Dixon, Smirnov)
    - Splitting amplitudes: universal quantities, govern collinear limits
    - MHV: gluon helicities are a permutation of $--++....+$
    - planar: leading in $1/N$

- Intriguing relation to Wilson loops
  (Drummond, Korchemsky, Sokatchev + Henn; Brandhuber, Heslop, GT)

- Dual conformal symmetry

  - integral functions in planar amplitudes (Drummond, Henn, Smirnov, Sokatchev)

  - Wilson loops (Drummond, Henn, Korchemsky, Sokatchev)

- Maximal transcendentality

- We will consider N=8 supergravity
  - maximally supersymmetric
  - nonplanar
- Our goals:
  - look for iterative relations in N=8 supergravity MHV amplitudes
  - No multi-particle poles
    - MHV in N=4/N=8; all-plus in non-supersymmetric YM/Gravity
  - relate Wilson loops to amplitudes
  - idea: find more similarities between the two maximally supersymmetric theories

- Common features N=4/N=8:
  - Absence of triangle and bubble subgraphs in amplitudes ("no-triangle hypothesis") (Bern, Dixon, Perelstein, Rozowsky; Bern, Bjerrum-Bohr, Dunbar; Bjerrum-Bohr, Dunbar, Ita; Bjerrum-Bohr, Dunbar, Ita, Perkins, Risager; Bjerrum-Bohr, Vanhove)
  - N=8 conjectured to be perturbatively finite (Bjerrum-Bohr, Dunbar, Ita, Perkins, Risager; Chalmers; Bern, Dixon, Roiban; Green, Russo, Vanhove; Bern, Carrasco, Dixon, Johansson, Kosower, Roiban)
- Gauge theory/gravity:
  - KLT relations (Kawai, Lewellen, Tye)
  - UV behaviour under complex shifts (Bedford, Brandhuber, Spence, GT; Cachazo, Svrcek; Benincasa, Boucher-Veronneau, Cachazo; Arkany-Hamed, Kaplan)

In the rest of the talk:

- MHV amplitudes in N=4 SYM
  - iterative structures in the perturbative expansion
  - one-loop n-point amplitudes and Wilson loops

    (Brandhuber, Heslop, GT)

Gregory Korchemsky's talk this afternoon

- 4-point MHV amplitude in N=8 Supergravity

  (Brandhuber, Heslop, Nasti, Spence, GT)

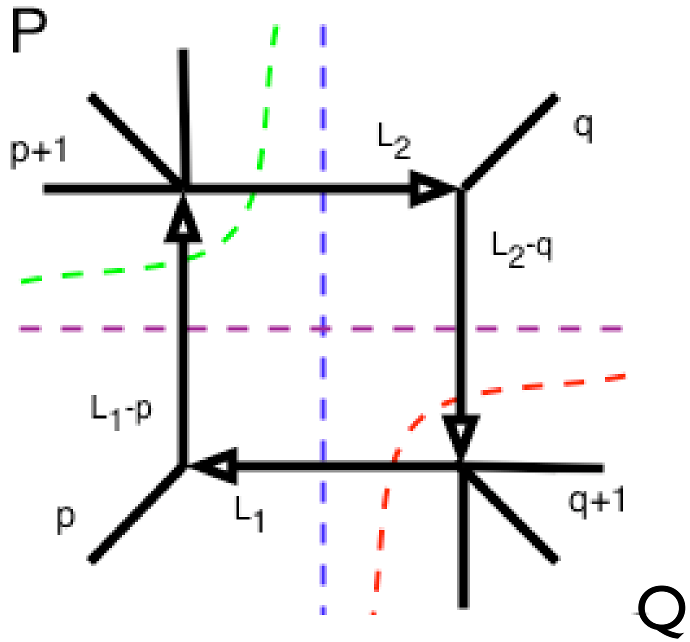
  - iterative structures
  - Wilson loops

N=4 Yang-Mills

Simplest one-loop amplitude

- n-point MHV amplitude in N=4 SYM at one loop:

$$\mathcal{A}_{\text{MHV}}^{1-\text{loop}} = \mathcal{A}_{\text{MHV}}^{\text{tree}} \sum$$



- Colour-ordered partial amplitude, leading term in $1/N$
- Sum of two-mass easy box functions, all with coefficient 1

Diagrammatic interpretation

- Computed in 1994 by Bern, Dixon, Dunbar and Kosower using unitarity

- Rederived in 2004 with loop MHV diagrams...

  (Brandhuber, Spence, GT)

- ...and, more recently, with a weakly-coupled Wilson loop calculation, with the Alday-Maldacena polygonal contour (Brandhuber, Heslop, GT)

Surprising regularities at higher loops

- n-point MHV amplitude in N=4 SYM

  - $\mathcal{A}_{n,\mathrm{MHV}} = \mathcal{A}^{\mathrm{tree}}_{n,\mathrm{MHV}} \mathcal{M}_n$

$$\mathcal{M}_n := 1 + \sum_{L=1}^{\infty} a^L \mathcal{M}_n^{(L)} \stackrel{?}{=} \exp\left[\sum_{L=1}^{\infty} a^L \Big(f^{(L)}(\epsilon) \mathcal{M}_n^{(1)}(L\epsilon) + C^{(L)} + \mathcal{O}(\epsilon) \Big)\right]$$

(Bern, Dixon, Smirnov) $a \sim g^2 N/(8\pi^2)$

  - $\mathcal{M}_n^{(1)}(\epsilon)$ is the all-orders in ϵ one-loop amplitude, $D = 4 - 2\epsilon$
  - $f^{(L)}(\epsilon) = f_0^{(L)} + \epsilon f_1^{(L)} + \epsilon^2 f_2^{(L)}$

anomalous dimension of twist-two operators at large spin, $\gamma_K^{(L)}/4$

- Higher-loop amplitudes expressed in terms of lower loop amplitudes

First few terms of BDS conjecture:

(take Log of the Ansatz)

$$\mathcal{M}_n^{(2)} = \frac{1}{2}\left(\mathcal{M}_n^{(1)}(\epsilon)\right)^2 + f^{(2)}(\epsilon)\,\mathcal{M}_n^{(1)}(2\epsilon) + \mathcal{O}(\epsilon)$$

$$\mathcal{M}_n^{(3)} = -\frac{1}{3}\left(\mathcal{M}_n^{(1)}(\epsilon)\right)^3 + \mathcal{M}_n^{(2)}(\epsilon)\mathcal{M}_n^{(1)}(\epsilon) + f^{(3)}(\epsilon)\,\mathcal{M}_n^{(1)}(3\epsilon) + \mathcal{O}(\epsilon)$$

and so on...

- Signature of two-loop iteration:

$$\mathcal{M}_n^{(2)} - \frac{1}{2}\left(\mathcal{M}_n^{(1)}(\epsilon)\right)^2 = f^{(2)}(\epsilon)\,\mathcal{M}_n^{(1)}(2\epsilon) + \mathcal{O}(\epsilon)$$

One-loop amplitude

  - Requires knowledge of lower-loop amplitude to higher orders in ϵ
  - Go up by one loop only

IR behaviour of Yang-Mills amplitudes

- Motivates BDS Ansatz (Anastasiou, Bern, Dixon, Kosower)

- Universal resummation of IR divergences

$$\mathcal{A}|_{\mathrm{IR}} = \prod_{i=1}^{n} \mathcal{A}_{\mathrm{div}}(s_{i,i+1})$$

(for colour-ordered amplitudes)

$$\mathcal{A}_{\mathrm{div}}(s) = \exp\left[-\frac{1}{8\varepsilon^2} \sum_{L=1}^{\infty} a^L \left(\frac{-s}{\mu^2}\right)^{-L\varepsilon} \frac{\gamma_K^{(L)}}{L^2} - \frac{1}{4\varepsilon} \sum_{L=1}^{\infty} a^L \left(\frac{-s}{\mu^2}\right)^{-L\varepsilon} \frac{g^{(L)}}{L} \right]$$

(Catani; Magnea, Sterman; Sterman, Tejeda-Yeomans)

- BDS: exponentiation of finite parts
  - Exponentiated finite remainders approach constants (independent of kinematics and # of particles)

Checks of BDS conjecture

- Two and three loops at four points (Anastasiou, Bern, Dixon, Kosower; Bern, Dixon, Smirnov). Confirmed result for three-loop cusp anomalous dimension obtained assuming maximal transcendentality (Kotikov, Lipatov, Onishcenko, Velizhanin)

- Two loops at five points (Bern, Czakon, Kosower, Roiban, Smirnov)

- Problems begin at six points (Bern, Dixon, Kosower, Roiban, Spradlin, Vergu, Volovich)

- Exponent requires an additional finite remainder

N=8 Supergravity

N=8 supergravity MHV amplitudes

- At four points $\mathcal{A}_{4,\mathrm{MHV}}^{\mathcal{N}=8} = \mathcal{A}_{4,\mathrm{MHV}}^{\mathrm{tree}} \mathcal{M}_4^{\mathcal{N}=8}$
  - tree-level amplitude factors out as in N=4 thanks to supersymmetric Ward identities
- Write $\mathcal{M}_4^{\mathcal{N}=8} = 1 + \sum_{L=1}^{\infty} \mathcal{M}_4^{(L)} = \exp\left[\sum_{L=1}^{\infty} m_4^{(L)}\right]$

$$m_4^{(1)} = \mathcal{M}_4^{(1)}\,, \quad m_4^{(2)} = \mathcal{M}_4^{(2)} - \frac{1}{2}\left(\mathcal{M}_4^{(1)}\right)^2$$

- Goal: compute the quantity $\mathcal{M}_n^{(2)} - \frac{1}{2}\left(\mathcal{M}_n^{(1)}(\epsilon)\right)^2$

In YM: $\mathcal{M}_{\mathrm{YM}}^{(2)} - \frac{1}{2}\left(\mathcal{M}_{\mathrm{YM}}^{(1)}(\epsilon)\right)^2 = f^{(2)}(\epsilon)\, \mathcal{M}_{\mathrm{YM}}^{(1)}(2\epsilon) + \mathcal{O}(\epsilon)$

One- and two-loop MHV amplitude

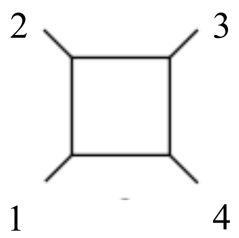
- One loop:

$$\mathcal{M}_4^{(1)} = -i\, s\, t\, u \left(\frac{\kappa}{2}\right)^2 \left[\mathcal{I}_4^{(1)}(s,t) + \mathcal{I}_4^{(1)}(s,u) + \mathcal{I}_4^{(1)}(u,t)\right]$$

(Green, Schwarz, Brink; Dunbar, Norridge)

$$\mathcal{I}_4^{(1)}(s,t) := \int \frac{d^D l}{(2\pi)^D} \frac{1}{l^2(l-p_1)^2(l-p_1-p_2)^2(l+p_4)^2}$$

zero-mass box



- No colour ordering for gravity
  - sum over permutations (1234), (1342), (1423)

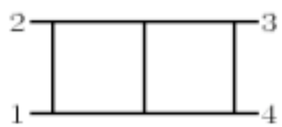
- # Two loops:

$$\mathcal{M}_4^{(2)} = \left(\frac{\kappa}{2}\right)^4 stu \left[s^2\, \mathcal{I}_4^{(2),\mathrm{P}}(s,t) + s^2\, \mathcal{I}_4^{(2),\mathrm{P}}(s,u) + s^2\, \mathcal{I}_4^{(2),\mathrm{NP}}(s,t) + s^2\, \mathcal{I}_4^{(2),\mathrm{NP}}(s,u) \;+\; \mathrm{cyclic}\right]$$

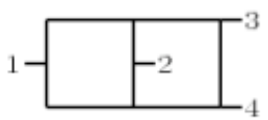
(Bern, Dunbar, Dixon, Perelstein, Rozowsky)

- $\mathcal{I}_4^{(2),\mathrm{P}}, \mathcal{I}_4^{(2),\mathrm{NP}}$ are the planar and non-planar boxes

$$\mathcal{I}_4^{(2),\mathrm{P}}(s,t) = \int \frac{d^D l}{(2\pi)^D}\, \frac{d^D k}{(2\pi)^D}\, \frac{1}{l^2\,(l-p_1)^2\,(l-p_1-p_2)^2\,(l+k)^2 k^2\,(k-p_4)^2\,(k-p_3-p_4)^2}$$



$$\mathcal{I}_4^{(2),\mathrm{NP}}(s,t) = \int \frac{d^D l}{(2\pi)^D}\, \frac{d^D k}{(2\pi)^D}\, \frac{1}{l^2\,(l-p_2)^2\,(l+k)^2\,(l+k+p_1)^2\, k^2\,(k-p_3)^2\,(k-p_3-p_4)^2}$$



$$s := (p_1+p_2)^2,\; t := (p_2+p_3)^2,\; u := (p_1+p_3)^2$$

- Laurent expansion explicitly evaluated by Smirnov and Tausk
- use it to study possible iterations

Iterative structure

- Main result: $\mathcal{M}_n^{(2)} - \frac{1}{2}\left(\mathcal{M}_n^{(1)}(\epsilon)\right)^2 = \text{finite} + \mathcal{O}(\epsilon)$

- Finite remainder has uniform transcendentality

  - $\pi, \log$ have transcendentality $1;\ \zeta_n, \mathrm{Li}_n$ have transcendentality n ...

  - Soft anomalous dimensions in N=4 obtained as leading transcendentality contribution of QCD result (Kotikov, Lipatov, Onishcenko, Velizhanin)

  - Planar one- and two-loop box are transcendental. Specific combination of two-loop nonplanar box functions is transcendental

  - another property in common with N=4 SYM ? higher loops ?

- Remainder is “simpler” compared to full $\mathcal{M}_4^{(2)}$

$$\mathcal{M}_4^{(2)} - \frac{1}{2}(\mathcal{M}_4^{(1)})^2 \;= - \left(\frac{\kappa}{8\pi}\right)^4 \Big[u^2\big[k(y) + k(1/y)\big] \,+\, s^2\big[k(1-y) + k(1/(1-y)\big]$$

$$+\, t^2\big[k(y/(y-1)) + k(1-1/y)\big]\Big] + \mathcal{O}(\epsilon)$$

where

$$\begin{aligned} k(y) \quad &:= \quad \frac{L^4}{6} + \frac{\pi^2 L^2}{2} - 4S_{1,2}(y)L + \frac{1}{6}\log^4(1-y) + 4\;S_{2,2}(y) - \frac{19\pi^4}{90} \\ &+i \quad \left(-\frac{2}{3}\pi\log^3(1-y) - \frac{4}{3}\pi^3\log(1-y) - 4L\pi\;\mathrm{Li}_2(y) + 4\pi\mathrm{Li}_3(y) - 4\pi\zeta(3)\right) \end{aligned}$$

$$y = -s/t\;, \quad L := \log(s/t)$$

IR behaviour of (super)gravity amplitudes

- Exponentiation of one-loop divergences (Weinberg)
  - Similar to QED
  - Soft and collinear amplitudes unrenormalised

    (Bern, Dunbar, Dixon, Perelstein, Rozowsky)
- No colour ordering: $\mathcal{M}|_{\mathrm{IR}} = \prod_{i<j} \mathcal{M}_{\mathrm{div}}(s_{ij})$
- 4 pts, one loop, $\mathcal{M}^{(1)}\Big|_{\mathrm{IR}} = c_\Gamma \left(\frac{\kappa}{2}\right)^2 \frac{2}{\epsilon} \Big[s \log(-s) + t \log(-t) + u \log(-u) \Big]$
  - ϵ^{-1} IR divergence softer than in YM

- Our result is in agreement with the expected IR singularities
    - Cancellation of leading and subleading singularities in the difference $\mathcal{M}_4^{(2)} - \frac{1}{2}(\mathcal{M}_4^{(1)})^2$

Steven Weinberg, Phys. Rev. 140: B516 (1965)

6. Remark

It was crucial in the above that the infrared divergences arise only from diagrams in which the soft real or virtual photon or graviton is attached to an external line, with "external line" *not* including the soft real photons or gravitons themselves. In electrodynamics this is true because photons are electrically neutral. In gravitation theory it is justified because the effective coupling constant for emission of a very soft graviton from a graviton (or photon) line with energy E is proportional to E, and the vanishing of this factor prevents simultaneous infrared divergences from a graviton and the line to which it is attached.

But these remarks do not apply to theories involving charged massless particles. In such theories (including the Yang-Mills theory) a soft photon emitted from an external line can itself emit a pair of soft charged massless particles, which themselves emit soft photons, and so on, building up a cascade of soft massless particles each of which contributes an infrared divergence. The elimination of such complicated interlocking infrared divergences would certainly be a Herculean task, and might even not be possible.

We may be thankful that the zero charge of soft photons and the zero gravitational mass of soft gravitons saves the real world from this mess. Perhaps it would not be too much to suggest that it is the infrared divergences that prohibit the existence of Yang-Mills quanta or other charged massless particles. See Sec. III for further remarks in this direction.

Beyond four points

- One-loop amplitude no longer proportional to the tree-level amplitude
- Requires more thinking!

Wilson Loops

Gregory Korchemsky's talk this afternoon

Amplitudes in N=4 and Wilson Loops

(Drummond, Korchemsky, Sokatchev; Brandhuber, Heslop, GT; Drummond, Henn, Korchemsky, Sokatchev)

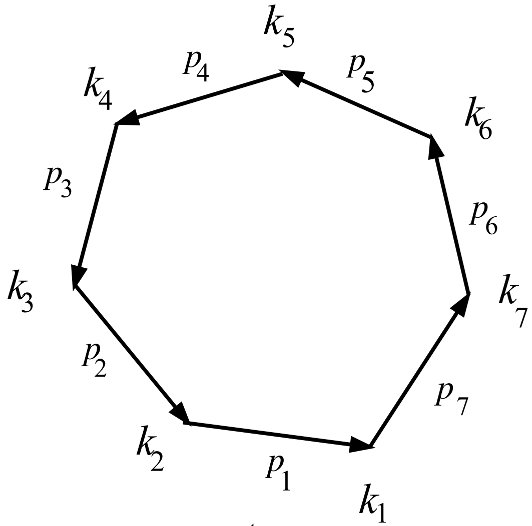
- MHV amplitudes in N=4 super Yang-Mills appear in a completely different calculation:

$$< W[C] >$$

- Contour C is determined by the momenta of the scattered particles
- Strong coupling calculation of Alday and Maldacena

- The contour of the Wilson loop:

  - this contour corresponds to a seven-point amplitude
  - colour ordering $\mathrm{Tr}(T^1 T^2 \cdots T^7)$
  - at strong coupling, boundary of worldsheet tends to boundary of dual AdS space as IR cutoff is removed



$p_i = k_i - k_{i+1}$ lightlike momenta

k's are T-dual (region) momenta

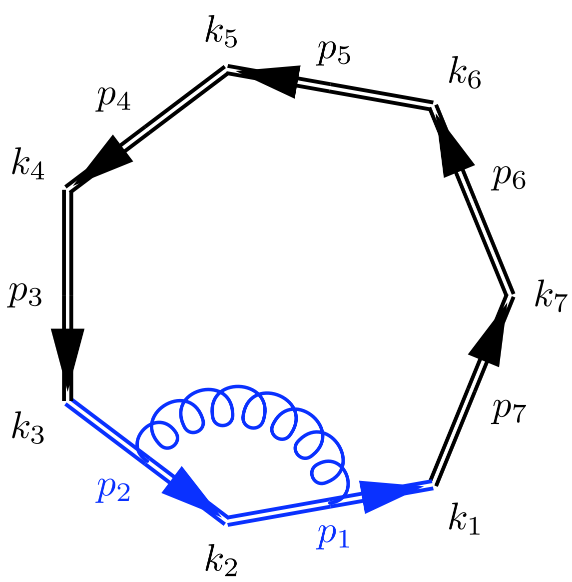
  - momentum conservation $\sum_{i=1}^{n} p_i = 0$ ➡ closed contour
  - dual conformal symmetry acts on the T-dual momenta

- Result: $< W[C] >$ is the n-point MHV amplitude in N=4 SYM (modulo tree-level prefactor)
  - Completely unexpected! Eikonal approximation usually only reproduces IR behaviour; we also get finite parts
- Conjecture: (Log) $< W[C] >$ = (Log) $\mathcal{M}$ persists at higher loops
  - Recently checked at two loops by Drummond, Henn, Korchemsky, Sokatchev for the four-, five-, and six-point case
  - Discussed earlier today by Gregory Korchemsky

$< W[C] >$ at one loop, n points

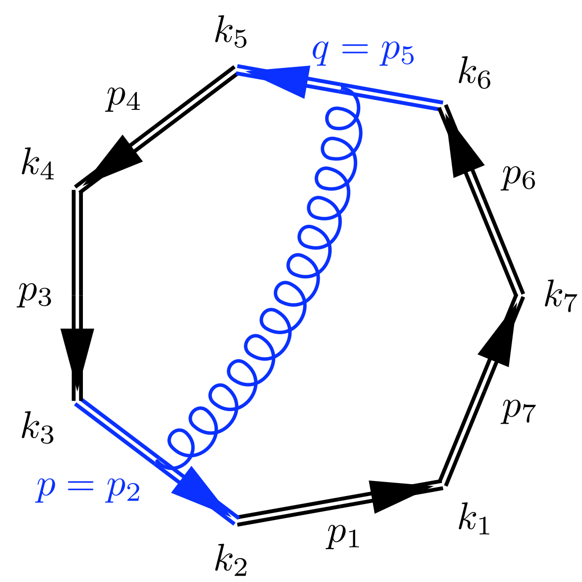
(Brandhuber, Heslop, GT)

- Calculation done (almost) instantly.
  Two classes of diagrams:



Gluon stretched between two segments meeting at a cusp

A. Infrared divergent



Gluon stretched between two non-adjacent segments

B. Infrared finite

- Clean separation between infrared-divergent and infrared-finite terms

  - Important advantage, as ε can be set to zero in the finite parts from the start

- From diagrams in class A :

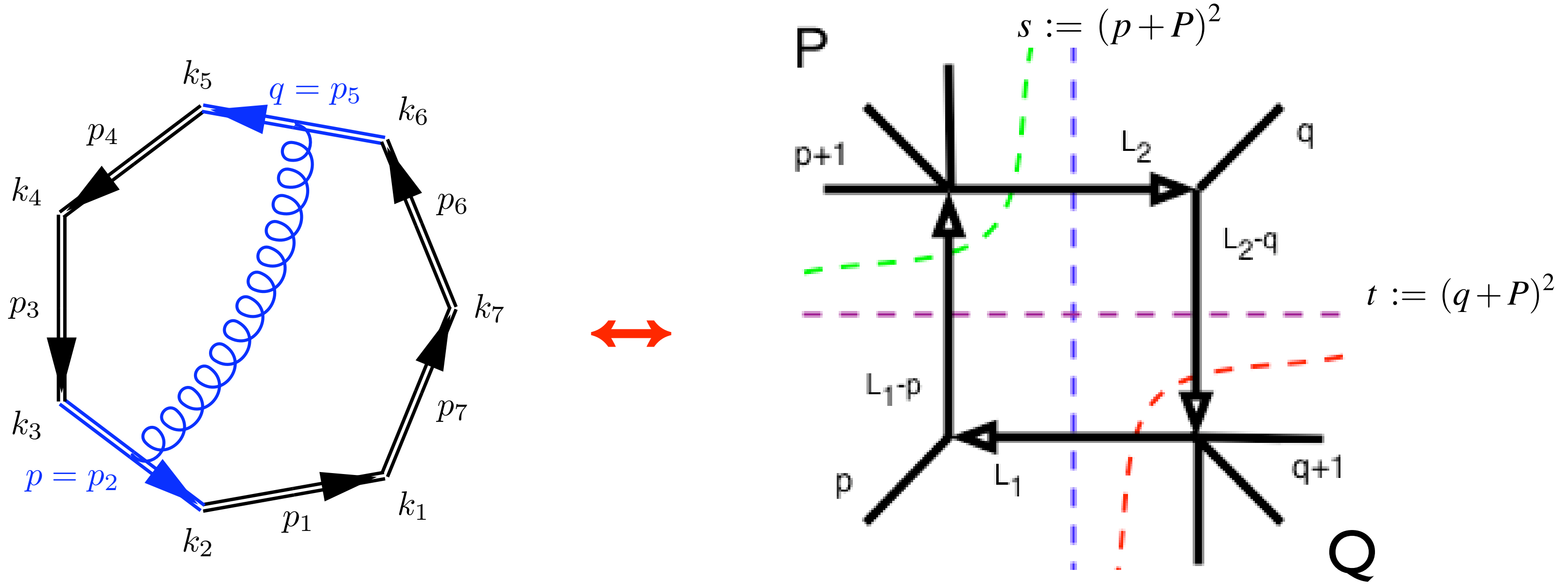
$$\mathcal{M}_n^{(1)}\Big|_{IR} = -\frac{1}{\varepsilon^2}\sum_{i=1}^{n}\left(\frac{-s_{i,i+1}}{\mu^2}\right)^{-\varepsilon}$$

  - $s_{i,i+1} = (p_i + p_{i+1})^2$ is the invariant formed with the momenta meeting at the cusp

- Diagram in class B, with gluon stretched between p and q gives a result proportional to

$$\mathcal{F}_\varepsilon(s,t,P,Q) = \int_0^1 d\tau_p d\tau_q \, \frac{P^2+Q^2-s-t}{\left[-\left(P^2+(s-P^2)\tau_p+(t-P^2)\tau_q+(-s-t+P^2+Q^2)\tau_p\tau_q\right)\right]^{1+\varepsilon}}$$

- Explicit evaluation shows that this is the finite part of a 2-mass easy box function
  - Two-dimensional representation of a four-dimensional integral function



- In the example: $p = p_2 \quad q = p_5$

  $P = p_3 + p_4\,, \quad Q = p_6 + p_7 + p_1$

- One-to-one correspondence between Wilson loop diagrams and finite parts of 2-mass easy box functions

- Explains why each box function appears with coefficient equal to 1 in the expression of the one-loop N=4 MHV amplitude

Gravity Wilson Loops

(Brandhuber, Heslop, Nasti, Spence, GT)

- Requirements for candidate Wilson loop:
  - invariance under coordinate transformations
  - contour dictated by particle momenta
  - has the same symmetries as the scattering amplitude

- Obvious choice: $\langle \mathrm{Tr}\, \mathcal{U}(C) \rangle$ where

$$\mathcal{U}^{\alpha}_{\beta}(C) \; := \; \mathcal{P} \exp\left[i\kappa \oint_C dy^{\mu} \Gamma^{\alpha}_{\mu\beta}(y) \right]$$

  - Γ is the Christoffel connection
  - invariant under coordinate transformations
  - already studied in perturbation theory by Modanese
- Result has nothing to do with amplitude !

$$\kappa^2 \oint_C dx^{\mu} dy^{\nu} \, \langle \Gamma^{\alpha}_{\mu\beta}(x) \Gamma^{\beta}_{\nu\alpha}(y) \rangle \; \sim \; \kappa^2 \oint_C \; dx_{\mu} dy^{\mu} \delta^{(D)}(x-y)$$

  - divergent expression, reminiscent of the loop equation...

- Try again
  - work in linearised approximation $g_{\mu\nu}(x) = \eta_{\mu\nu} + \kappa h_{\mu\nu}(x)$

$$W[C] \ := \ \left\langle \exp\left[i\kappa \oint_C d\tau \ h_{\mu\nu}(x(\tau))\dot{x}^\mu(\tau)\dot{x}^\nu(\tau) \right] \right\rangle$$

  - Same expression used in gravity eikonal approximation
    (Kabat & Ortiz; Fabbrichesi, Pettorino, Veneziano, Vilkovisky)
- For cusped contours, gauge invariance violated at cusps
  - Exponent can be rewritten as $\int d^D x \, \mathcal{T}^{\mu\nu}(x) h_{\mu\nu}(x)$
    where $\mathcal{T}^{\mu\nu}(x) \ := \ \int d\tau \ \dot{x}^\mu(\tau)\dot{x}^\nu(\tau) \, \delta^{(D)}(x - x(\tau))$

energy-momentum tensor of free particle

- Try anyway
  - in order to have correct symmetries, we consider

$$W \ := \ W[C_{1234}]\, W[C_{1243}]\, W[C_{1324}]$$

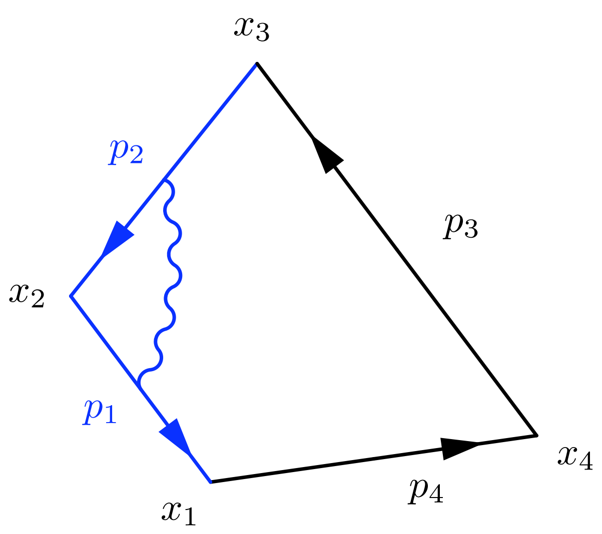
  - C_{ijkl} is a contour obtained by joining p_i, p_j, p_k, p_l in this order
  - At one loop, $W^{(1)} = W^{(1)}[C_{1234}] + W^{(1)}[C_{1243}] + W^{(1)}[C_{1324}]$

Results

- Tree-level prefactor missing (as in YM)
- Relative normalisation between IR singular and finite parts incorrect by a factor of - 2
  - 2 from overcounting cusp contributions in W; minus sign more difficult to explain
- Result gauge dependent (but very close to correct one...)

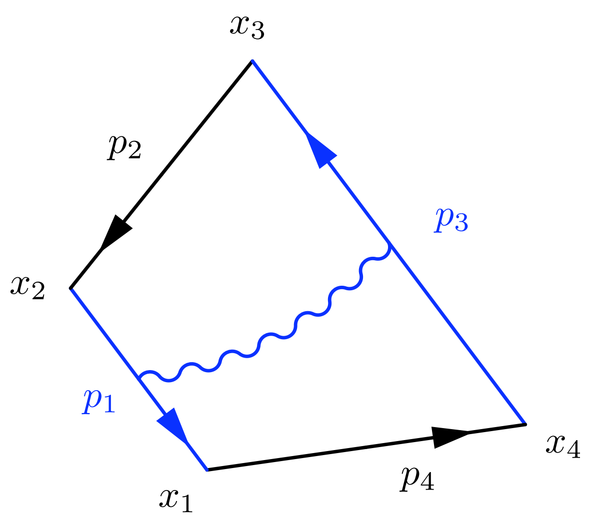
$< W >$ at one loop

- Diagrammatics identical to YM case.
  2 classes of diagrams:



Graviton stretched between two edges meeting at a cusp

A. Infrared divergent



Graviton stretched between two non-adjacent edges

B. Infrared finite

- From diagrams in class A (after summing over permutations):

$$\kappa^2 \frac{c(\epsilon)}{\epsilon^2}\left[(-s)^{1-\epsilon}+(-t)^{1-\epsilon}+(-u)^{1-\epsilon}\right]$$

  - leading divergence cancels due to $s + t + u = 0$

  - subleading term proportional to expected $1/\epsilon$ term:

$$\mathcal{M}^{(1)}\Big|_{\mathrm{IR}} = c_\Gamma \left(\frac{\kappa}{2}\right)^2 \frac{2}{\epsilon}\left[s\,\log(-s)+t\,\log(-t)+u\,\log(-u)\right]$$

- From diagrams in class B:

$$\kappa^2\, c(\epsilon)\, \frac{u}{2}\, \frac{1}{4} \left[\log^2\left(\frac{s}{t}\right) + \pi^2 \right]$$

  - finite part of zero-mass box function
  - sum over all permutations reproduces finite part of amplitude

$$\mathcal{M}_4^{(1)} \;=\; -i\, s\, t\, u \left(\frac{\kappa}{2}\right)^2 \left[\mathcal{I}_4^{(1)}(s,t) \;+ \mathcal{I}_4^{(1)}(s,u) \;+ \mathcal{I}_4^{(1)}(u,t)\right]$$

"Conformal" gauge

(Brandhuber, Heslop, Nasti, Spence, GT)

- Defined as the gauge where cusp diagrams vanish
  - used in Yang-Mills, where Wilson loop is gauge invariant
- In this gauge, we obtain the correct N=8 supergravity amplitude
- Special case of a de Donder gauge fixing:

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu h_{\nu\rho})^2 + (\partial_\nu h^\nu{}_\mu)^2 + \frac{1}{2}(\partial_\mu h^\lambda{}_\lambda)^2 + h^\lambda{}_\lambda \partial_\mu \partial_\nu h^{\mu\nu}$$

Free Lagrangian of linearised gravity

$$\mathcal{L}^{(\mathrm{gf})} = \frac{\alpha}{2}\left(\partial_\nu h^\nu_\mu - \frac{1}{2}\partial_\mu h^\alpha_\alpha\right)^2$$

- α-gauge fixing
- $\alpha = -2$ usual de Donder gauge
- $\alpha = -\frac{2\epsilon}{1+\epsilon}$ conformal gauge

- Graviton propagator in configuration space:

$$\Delta^{\rm conf}_{\mu\nu,\mu'\nu'}(x) \sim \frac{\epsilon+1}{\epsilon}\left[\frac{1}{(-x^2)^{1+\epsilon}}\left(\eta_{\mu'(\mu}\eta_{\nu)\nu'} + \frac{\epsilon}{2(\epsilon+1)^2}\eta_{\mu\nu}\eta_{\mu'\nu'}\right) + 2\frac{1}{(-x^2)^{2+\epsilon}}x_{(\mu}\eta_{\nu)(\nu'}x_{\mu')}\right]$$

- Cfr. gluon propagator in configuration space:

$$\Delta^{\rm conf}_{\mu\nu}(x) \;\sim\; \frac{\epsilon+1}{\epsilon}\frac{1}{(-x^2+i\varepsilon)^{1+\epsilon}}\left[\eta_{\mu\nu} \;-\; 2\frac{x_\mu x_\nu}{x^2}\right]$$

$$J_{\mu\nu}(x) := \eta_{\mu\nu} \;-\; 2\frac{x_\mu x_\nu}{x^2}$$ Inversion tensor

Summary

- Not quite same iterative structure as in N=4
  - uniform transcendentality of the result
  - finite remainder, relatively simple expression
- Wilson loop almost reproduces amplitude
  - Gauge-dependent expression
  - Result closely related to correct answer
  - Conformal gauge
  - Can we do better ?