Localization properties of quarks

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Outline

1. Motivation: QCD vacuum structure and χSB
2. Puzzling results about fermion localization
3. A theoretical puzzle: the correlator of top. charge density
4. Trying to put the pieces together

See hep-lat/0611034

Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

GGI, Florence, June 2008

Motivation I: extra dimensions

- Extra dimensions not seen $\Rightarrow$ localization in *4d*

Feasible by topological defect Rubakov & Shaposhnikov, 1983

fluctuations around classical “kink” solution are localized

$\longrightarrow$ lower-dimension effective field theory

Many more: Hosotani, Randall & Sundrum, Dvali & Shifman,....

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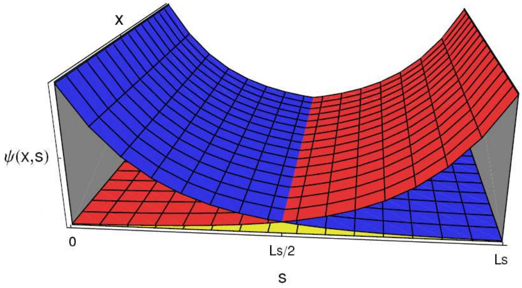
- Localization at work:

Domain-Wall fermions in lattice QCD: $5d \longrightarrow 4d$ Kaplan 1992



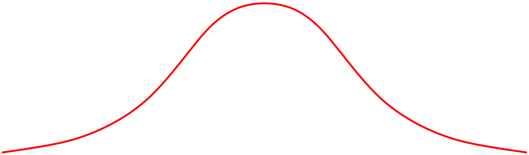
Note: $A_5 = 0$ frozen.

Motivation II: QCD vacuum structure

- "Understand" confinement $\longrightarrow$ identify relevant IR degrees of freedom
- Confinement is non-perturbative $\longrightarrow$ caused by topological excitations?

Candidates:

- instantons 't Hooft



Codimension 4: point-like topological obstruction

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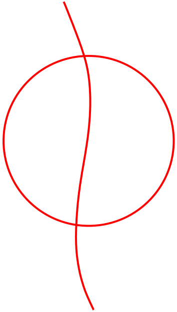
Candidates:

- Abelian monopoles 't Hooft



$A_\mu \longrightarrow$ adjoint Higgs $\longrightarrow$ BPS monopole

Codimension 3: line-like topological obstruction

Motivation II: QCD vacuum structure

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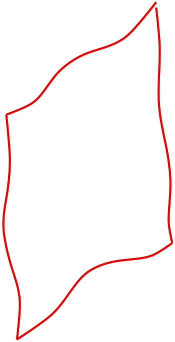
- Confinement is non-perturbative $\longrightarrow$ caused by topological excitations?

Candidates:

- center vortices Mack, 't Hooft



Codimension 2: Z_N singular transformation on sheet

Motivation II: QCD vacuum structure

- "Understand" confinement $\longrightarrow$ identify relevant IR degrees of freedom
- Confinement is non-perturbative $\longrightarrow$ caused by topological excitations?

Candidates:

instantons, Abelian monopoles, center vortices

- All objects are "thick": size $\mathcal{O}(1/\Lambda_{QCD})$
- Should also explain chiral symmetry breaking/restoration

Identify correct candidate by lattice measurements

In the past: need to filter out UV fluctuations to see structure
Smoothing/cooling/smearing to reduce action
Evolve towards action minimum, ie. classical solution $\longrightarrow$ instantons

$\boxed{\text{Can one avoid such bias?}}$

Chiral symmetry breaking/restoration

Anderson 1958: random tight-binding Hamiltonian

Random impurities, each with -localized bound e^-

-random interaction energy with crystal ions

How does conductivity depend on overlap of bound states ?

PHYSICAL REVIEW VOLUME 109, NUMBER 5 MARCH 1, 1958

Absence of Diffusion in Certain Random Lattices

P. W. ANDERSON
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received October 10, 1957)

This paper presents a simple model for such processes as spin diffusion or conduction in the "impurity band." These processes involve transport in a lattice which is in some sense random, and in them diffusion is expected to take place via quantum jumps between localized sites. In this simple model the essential randomness is introduced by requiring the energy to vary randomly from site to site. It is shown that at low enough densities no diffusion at all can take place, and the criteria for transport to occur are given.

Eigenstates of $H = \Delta + \nu$

Δ: discretized (lattice) Laplacian (hopping); ν: random potential

Localization $\equiv$ eigenmode $|\psi(r)|^2 \sim \exp(-r)$ for $r \to \infty$ with prob. 1

$\to$ no electric conductivity

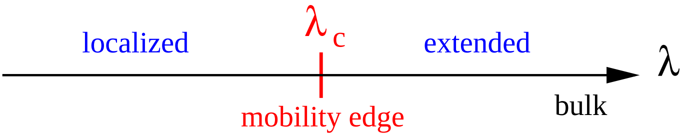
Anderson transition: $H = \Delta + \nu$

- Result: localization if - disorder sufficiently large **or**
  - energy sufficiently low

  E very large $\longrightarrow$ plane waves

  E very small $\longrightarrow$ hopping to all neighbouring sites forbidden

- Spectrum:



$$
\begin{array}{ccccc}
E & < & \lambda_c & \longrightarrow & \text{localized} \\
 & > & & & \text{extended} \\
p_{\text{Fermi}} & < & \lambda_c & \longrightarrow & \text{insulator} \\
 & > & & & \text{conductor}
\end{array}
$$

Transition driven by temperature, or by disorder ($T = 0$, quantum)

Anderson variations

1. Low dimension:
   $d = 1$: all states localized for any disorder
   $d = 2$: same Lee & Ramakrishnan, RMP 1985

2. Modify Hamiltonian: $H = \Delta + \nu$:

- randomness in hopping term Δ: qualitatively similar
- make Δ long-range: $\Delta_{ij} \propto \frac{1}{|r_{ij}|^\alpha}$ long-range

Result: transition for $\alpha = d$ ($\alpha > d \longrightarrow$ localization) Mirlin 1996

$d = 3 \longleftrightarrow$ dipole-dipole interactions

Chiral symmetry breaking à la Diakonov & Petrov (1984)

- Recall Banks-Casher: $\langle\bar{\psi}\psi\rangle = \lim_{m\to 0}\lim_{V\to\infty} -\pi\,\rho(0)$

  How to obtain density of zero-modes ?

Chiral symmetry breaking à la Diakonov & Petrov (1984)

- Instanton supports chiral Dirac zero-mode 't Hooft

Superposition of I's and A's? $\not{D}(A^I_\mu)\psi^I = 0$ but $\not{D}(\sum_{I,A} A^{I,A}_\mu)\psi^I \neq 0$

zero-modes $\longrightarrow$ displaced

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- New eigenmodes?

Write Dirac operator in basis of original I, A zero-modes ψ^I, ψ^A:

$$\not{D} = \begin{pmatrix} 0 & T_{IA} \\ T^\dagger_{IA} & 0 \end{pmatrix}$$ zero-diagonal because of chirality

Overlap $T_{ij} = \langle \psi^I_i | \psi^A_j \rangle \sim \frac{1}{|r_{ij}|^3}$ in $d = 4 \longrightarrow$ delocalization

Support of eigenmodes $\sim \bigcup I, A$

- Eigenvalues $\sim$ uniformly spread in $[-\hat{\lambda}, +\hat{\lambda}]$, $\hat{\lambda} \approx \frac{\bar{r}_I}{\bar{R}_{IA}}$

χSB: $\langle \bar{\psi}\psi \rangle \sim \frac{-1}{\bar{r}_I \bar{R}^2_{IA}}$

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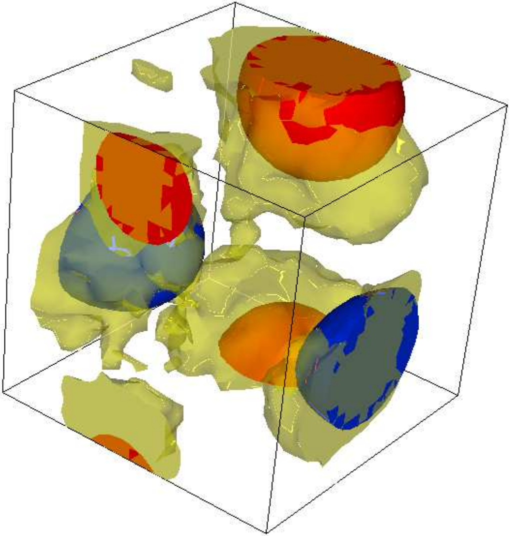
χSB: $\langle \bar{\psi}\psi \rangle \sim \frac{-1}{\bar{r}_I \bar{R}^2_{IA}}$

- Phenomenology reproduced with $\bar{r} \sim 0.3$ fm, $\bar{R}_{IA} \sim 1$ fm

Instanton liquid Shuryak, Schaefer

Dirac eigenmodes on the lattice



hep-lat/9810033 PdF et al.

lowest eigenmode of staggered $\not{D}$

no cooling

Eigenmode support $\sim$ Instanton + Antiinstanton

Comparison with Anderson

- Difference:

- Dirac eigenvalues come in pairs $\pm i\lambda$ (plus zero)

- interested in spectral properties (eg. eigenvalue repulsion) around 0
  ie. middle of spectrum $\leftrightarrow$ edge of spectrum for Anderson (bosons)

- modeled by chiral random matrix ensemble Garcia-Garcia

- Similarity:

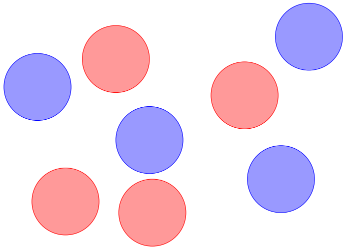
possible "depercolation" transition to localized states $\longrightarrow$ $\rho(0) = 0$

Then $\langle\bar{\psi}\psi\rangle = 0$: chiral symmetry restored from small changes in T_{IA}

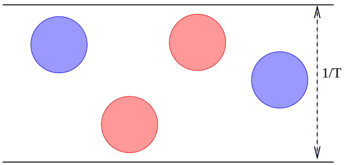
Chiral symmetry restoration at finite temperature

- Shuryak: det$\not{D}$ $\longrightarrow$ time-oriented $I - A$ molecules
  - transition in quenched theory?
  - $I - A$ molecules not seen on lattice
- Diakonov & Petrov: more subtle
  - $g(T) \searrow \Rightarrow$ instanton action $\nearrow \Rightarrow$ density of I, A decreases
  - $T_{IA} \sim \exp(-\pi R_{IA} T)$

  decreased overlap $\longrightarrow$ transition to localization



$T = 0$



$T > 0$

Vacuum structure from eigenmode $|\psi(x)|^2$?

- Diakonov-Petrov χSB scenario does not require instantons
  only chiral zero-modes
- Compatible with other topological defects:
  domain-walls, monopoles, vortices,.. Reinhardt
  [chiral zero-mode on any topological defect? Q non-integer]
- Working assumption:
  extended modes have support on $\bigcup$ topological defects

$$\Downarrow$$

deduce vacuum structure from spatial distribution of eigenmode

gauge invariant; no smoothing/cooling/smearing...

- Dirac fermions
- can compare with bosons in various representations
- Surprises; work in progress

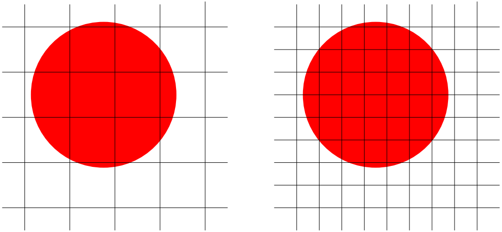
Main tool: Inverse Participation Ratio

- Definition: $IPR \equiv V \frac{\sum_x |\psi(x)|^4}{(\sum_x |\psi(x)|^2)^2}$ (ratio of moments)
- Simple cases:
  - $|\psi(x)| = 1 \ \forall x \quad \Longrightarrow \quad IPR = 1$
  - $|\psi(x)| = \delta_{x,x_0} \quad \Longrightarrow \quad IPR = V$
  - $|\psi(x)| = 1$ on fraction f of sites $\quad \Longrightarrow \quad \boxed{IPR = \frac{1}{f}}$

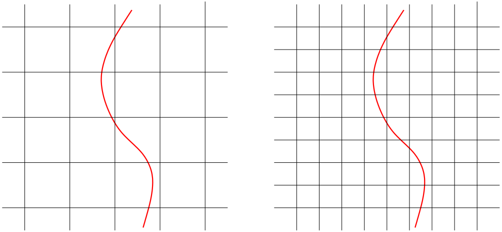
IPR: what to expect?

$IPR \ \sim \ 1/\text{fraction of occupied lattice sites}$

- any "thick" (macroscopic) object: $IPR = \text{constant}$



- "thin" (singular) instantons, monopoles, vortices: $IPR \longrightarrow \infty$ as $a \longrightarrow 0$



occupied sites $\propto a^{-1}$

total sites $\propto a^{-2}$

occupied fraction $f = a^{+1}$

$IPR \sim \frac{1}{f} \propto a^{-1}$

Strategy

- Case of interest:

$|\psi(x)| = 1$ on manifold of dim. d, "volume" $\mathcal{V}_d$

Fraction of lattice sites $f = \frac{\mathcal{V}_d/a^d}{V/a^4}$ (a lattice spacing, V 4-volume)

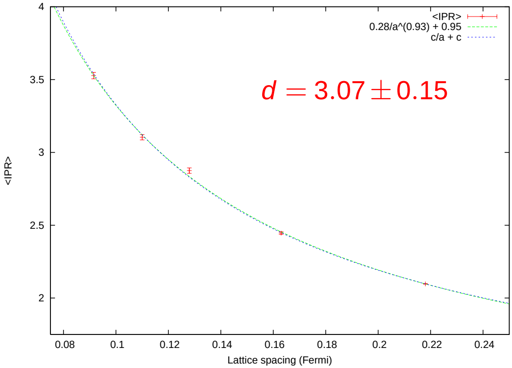
$$\boxed{IPR \propto a^{d-4}}$$

$\longrightarrow$ determine d by scaling of IPR versus a

$d = 0, 1, 2 \implies$ "thin" instantons, monopoles, vortices

IPR measurement I

- $SU(3)$, quenched, Symanzik gauge, Asqtad $\not{D}$ (no exact zero-modes)
- $IPR \longrightarrow$ constant as $V \longrightarrow \infty$ (?)
- a dependence: IPR diverges as $a \longrightarrow 0$ (Note scale of IPR)



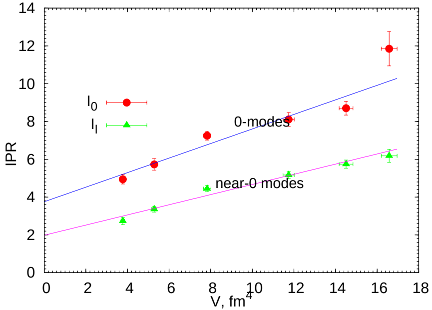
Hetrick et al. (MILC) hep-lat/0410024 + 0510025

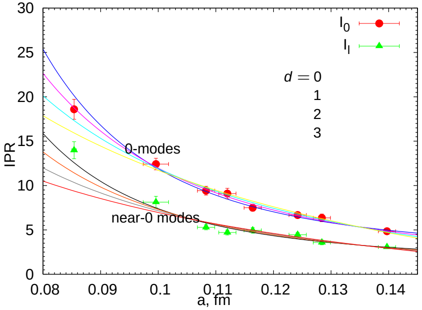
$d = 3 \longrightarrow$ eigenmodes localized on domain-walls of thickness $\ll 0.1$ fm

branes on the lattice!

IPR measurement II

- $SU(2)$, quenched, Wilson gauge, overlap $\not{D}$ ($\rightarrow$ exact zero-modes)
- $IPR = b_0 + b_1 V$, ie. eigenmodes are localized on finite nb. of sites
- $IPR = c_0 + c_1\ a^{d-4}$ with $d = 0$, ie. eigenmodes support is point-like





Polikarpov, Zakharov et al., hep-lat/0505016 + 0510098

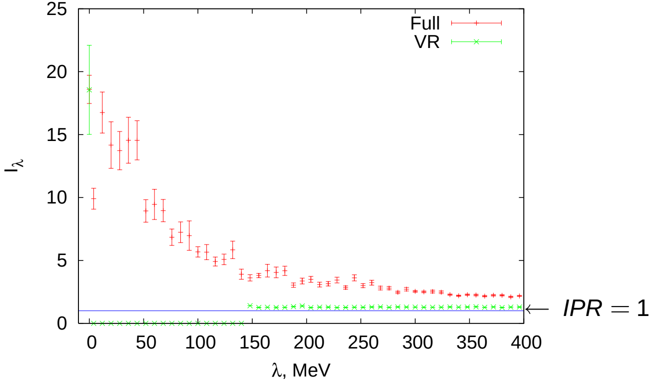
Note scale of IPR $\gg$ $SU(3)$

Speculative interpretation (Zakharov)

- $\langle\bar{\psi}\psi\rangle$ and eigenvalues behave 'normally' as $a \longrightarrow 0$ while $IPR \longrightarrow \infty$

  evidence of fine-tuning energy vs entropy
- confinement is caused by $d = 2$ “thin” center vortex sheets
- topological density at point-like sheet intersections: $\varepsilon_{ijkl} F_{ij} F_{kl}$
- Circumstancial evidence:

  Removing center vortices destroys confinement and χSB



My conservative interpretation

- $d = 3$ ($SU(3)$) vs $d = 0$ ($SU(2)$): lack of universality at short distance?

Defects at scale a may become dense depending on details of lattice action

dislocations (size a instantons) in $SU(2)$ Pugh & Teper '89

Energy vs entropy:

- energetic suppression: $\exp(-\frac{4}{g_0^2}S^*)$
- entropic enhancement: nb. of positions $V_{\rm phys.}/a^4 \sim \exp(+\frac{\beta_1}{2\beta_0}\frac{1}{g_0^2})$

Result:

- Entropy wins for $SU(2)$ with Wilson action
- Energy wins (dislocations suppressed) for $SU(3)$ with Symanzik action

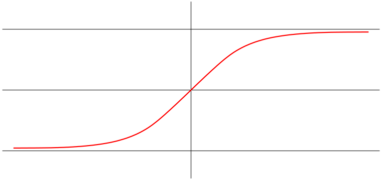
Problem cured by adding irrelevant terms in lattice action

$\Rightarrow$ forget $d = 0$ result. Can one understand $d = 3$ "branes"?

Why $d \neq 4$?

$d = 4$ is the dimension of macroscopic, classical objects, BUT:

- A kink does not look smooth as $a \longrightarrow 0$



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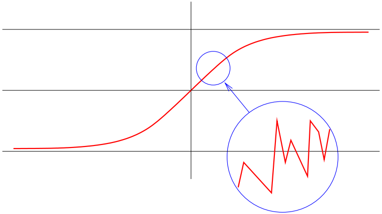

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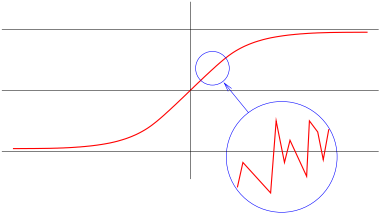
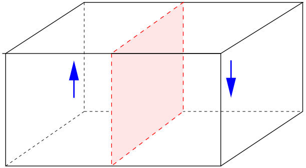


- Interface in $3d$ Ising model:

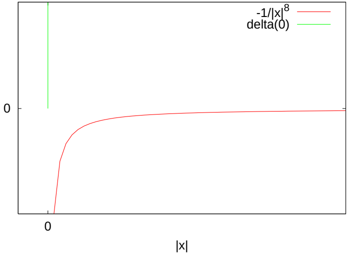


genus diverges as $a \to 0$ Caselle, Gliozzi, Vinti '93

Quantum fields are rough $\longrightarrow$ $d < 4$. Why $d = 3$?

Correlator of topological charge density

- Continuation Minkowski $\leftrightarrow$ Euclidean $\Rightarrow \langle q(0)q(x \neq 0)\rangle_{\rm Eucl.} < 0$
  (reflection positivity, or $q \sim \vec{E}\cdot\vec{B} \;\longrightarrow\; i\,\vec{E}\cdot\vec{B}$) Seiler & Stamatescu
- But $\langle \int d^4x\, q(0)q(x)\rangle = \chi_{\rm top} \sim (190{\rm MeV})^4 \;\Rightarrow$ contact term
- $q(x)$ has canonical dim. 4 $\;\longrightarrow\; \int d^4x\, 1/|x|^8$ UV-divergent
  Divergence cancelled by contact term $\;\longrightarrow$ "fine tuning"



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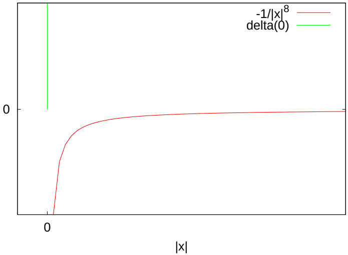

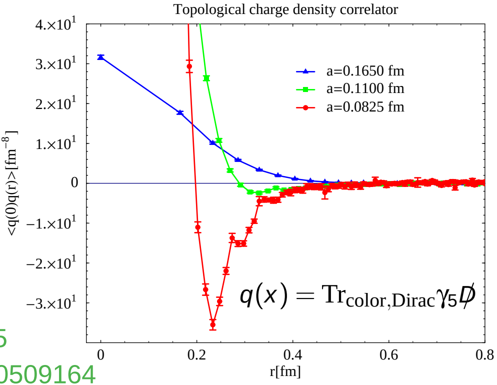

Horvath et al., hep-lat/0504005

Also Schierholz et al., hep-lat/0509164

Antiferromagnetic structure for $q(x)$?

- Kentucky group (Horvath et al.):

sign($q(x)$) forms 2 space-filling **$3d$ structures** (transverse size $\mathcal{O}(a)$)

- Reproduced by effective anti-ferromag. model Boyko & Gubarev

Koma, Ilgenfritz, Schierholz et al., hep-lat/0509164

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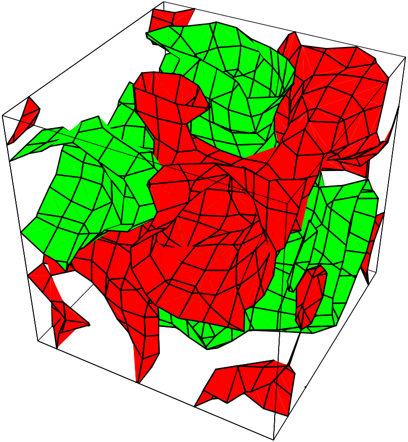

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Roma, Ilgenfritz, Schierholz et al., hep-lat/0509164

- Same for $(1+1)d$ CP^3 model Thacker et al., hep-lat/0509066

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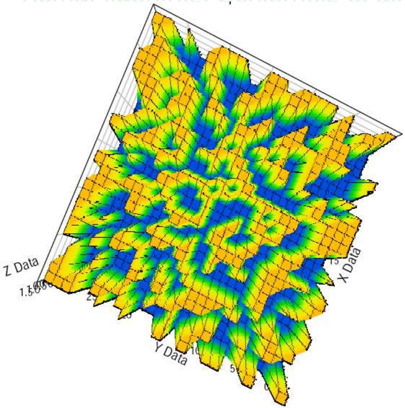

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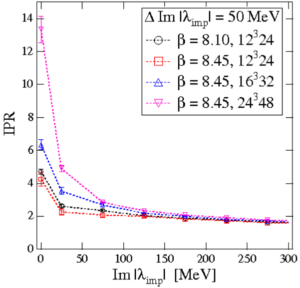
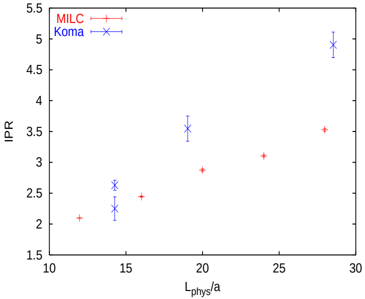
Trying to make sense of it all...

- To the rescue: Koma, Ilgenfritz, Schierholz et al., hep-lat/0509164

$SU(3)$, Lüscher-Weisz gauge (no disloc.), overlap $\not{D}$ (exact zero-modes)





- IPR numerically similar to MILC (for non-zero modes)
- scaling consistent with $d = 3$

Unifying interpretation:

- evidence for localization of quarks on $d = 3$ **domain-walls**:

consistent with observed topological charge domains

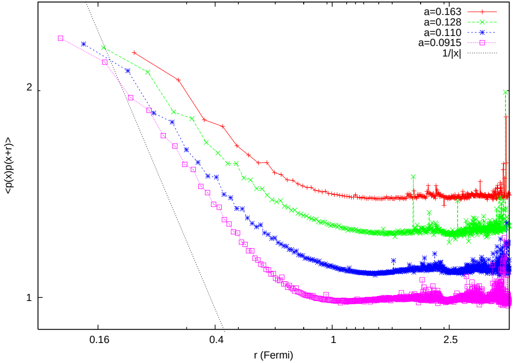
- update: multi-fractal Ilgenfritz et al., 0705.0018

Spatial correlator of Dirac eigenmode

- Check spatial structure of $3d$ support

IF eigenmode $|\psi(x)| = 1$ on $3d$ fractal, 0 elsewhere

then $\langle|\psi(0)||\psi(x)|\rangle \sim 1/|x|$



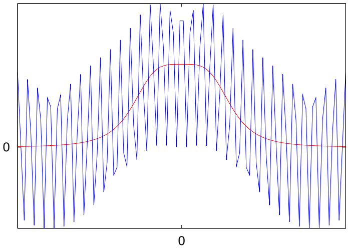
Hetrick et al. (MILC), hep-lat/0510025

- not inconsistent?

- fractal structure stops at $|x| \sim 1/\Lambda_{QCD}$

Conclusions

- Quantum fields not smooth: classical lumps $\leftrightarrow$ quantum descendents
- Wild goose chase? learn nothing about "structure" [at scale $1/\Lambda_{QCD}$] by looking at UV distances
- UV? Theoretical argument + some numerical evidence $\longrightarrow$ sandwich alternating $3d$ layers of diverging $\pm$ topological charge density
  Bizarre but allowed?
- not inconsistent with instanton, monopole, vortices at scale $1/\Lambda_{QCD}$



Vacuum structure depends on scale