Intersecting branes from supergravity

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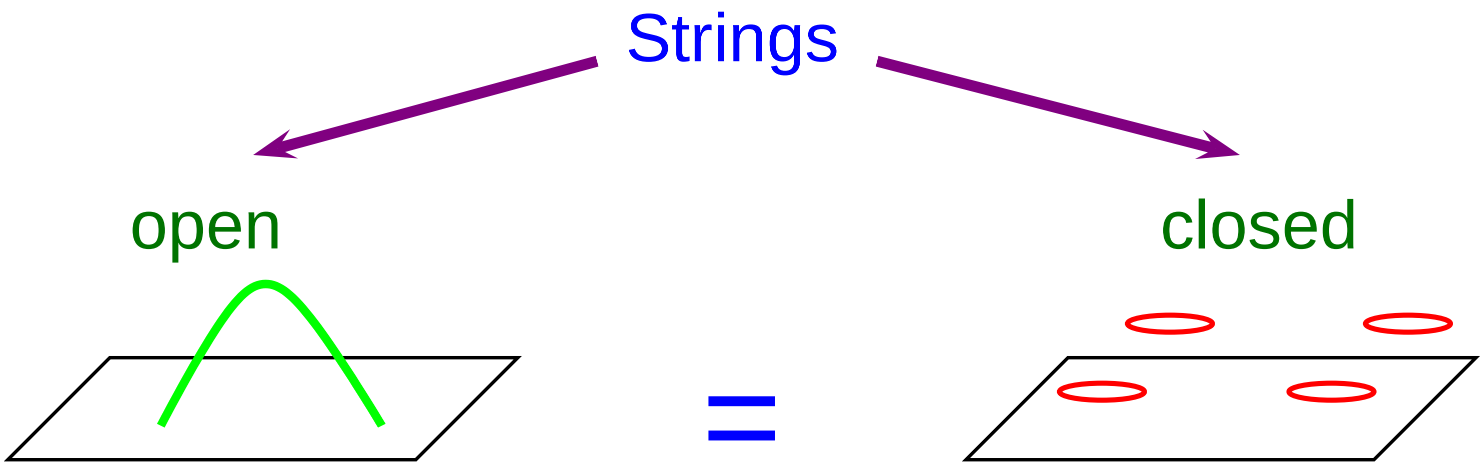
Branes in string theory



Dai–Leigh–Polchinski ’89 Polchinski ’95 Horowitz–Strominger ’91

- D–brane
- effective action: DBI
- flat branes
- intersections
  - shapes from dynamics

- black brane (geometry)
- supergravity
- large symmetry
- special solutions
  - ???

The uses of branes

- Applications to gauge theory
  - low energy dynamics: super–Yang–Mills
  - intersecting branes: colors and flavors

    Hanany–Witten ’96; Witten ’97
  - geometric picture for Seiberg duality

The uses of branes

- Applications to gauge theory
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- Applications to quantum gravity
  - information paradox and states of the black holes

$$\begin{array}{ccc} \text{black hole} & \longrightarrow & \text{thermal state} \\ & & \downarrow \\ \text{"microstate"} & \stackrel{?}{\longleftarrow} & \text{member of the ensemble} \end{array}$$

Strominger, Vafa ’96

The uses of branes

- Applications to gauge theory
  - low energy dynamics: super–Yang–Mills
  - intersecting branes: colors and flavors

    Hanany–Witten '96; Witten '97
  - geometric picture for Seiberg duality
- Applications to quantum gravity
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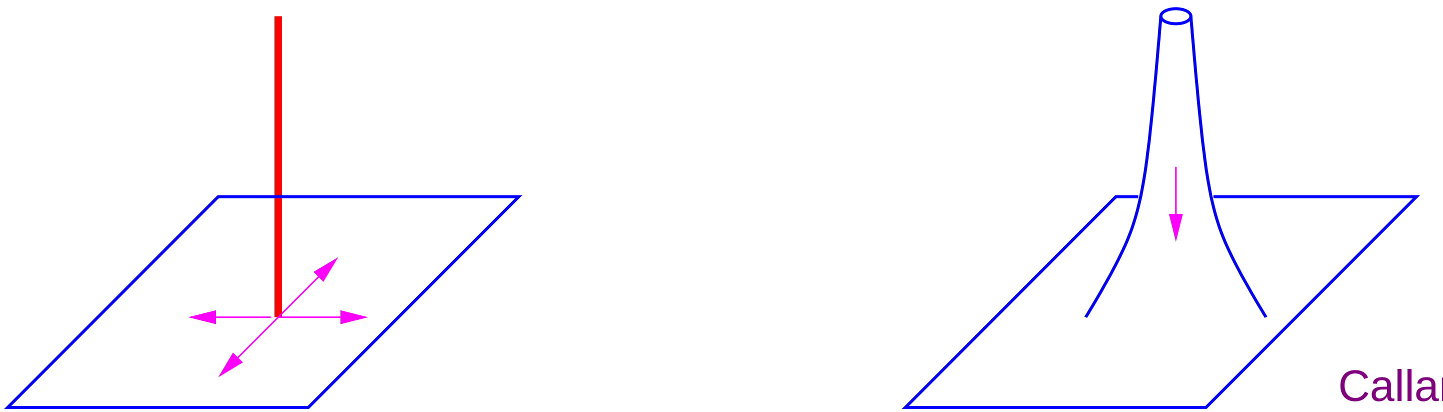
    Strominger–Vafa '96
  - attempts to build cosmological models KKLT '03
- Gauge/gravity duality
  - field theory = string theory
  - strong/weak coupling complementarity Maldacena '97

Outline

- Motivation
  - quantum gravity, gauge dynamics
  - understanding shapes on the gravity side
- Probe approximation
- Solutions in supergravity
  - technique for constructing the geometries
  - local description and consistency conditions
  - "DBI/SUGRA correspondence"
- Applications to AdS/CFT
  - Wilson lines at strong coupling
  - brane webs and dual picture for local states
- Open questions

BIons in flat space

- Strings ending on a brane:



Callan–Maldacena '97

- Dirac–Born–Infeld: nonlinear electrodynamics

$$S_{DBI} = -T \int d^{p+1}\xi \sqrt{-\det(G + 2\pi\alpha' F)}$$

  - induced metric → electric field determines shape

$$F_{ti} = \frac{1}{2\pi\alpha'}\nabla_i X, \qquad \nabla^2 X = 0$$

BIons in curved space

- Spike preserves 8 supercharges

	1	2	3	4	5	6	7	8	9
$D3$	•	•	•						
$F1$				•					
$D5$					•	•	•	•	•

- Coupling to the RR fluxes:

$$S_p = S_{DBI} + T_p \sum \int e^{2\pi\alpha' F} \wedge P[C_q]$$

- D3 brane probe in D3 geometry: harmonic profile
- D3 probe in D5 geometry (or vice versa):

$$-(1 + (\nabla X)^2)\partial_X H + H\nabla^2 X + 2\nabla H \nabla X = 0$$

Constructing the gravity solution

- Expected symmetries
  - eight supercharges
  - solution is static
  - all fields of IIB SUGRA are excited (ex. axion)
- Strategy:
  - find special solution with accidental symmetries
  - guess the generalization, check the result
  - match the number of degrees of freedom with DBI
- Special geometry
  - $SO(3) \times SO(5) \times U(1)_t$ isometries
  - complete solution of eqns for Killing spinors

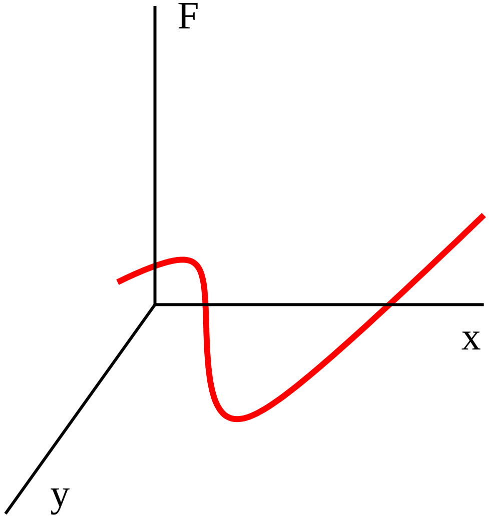
Structure of the geometry

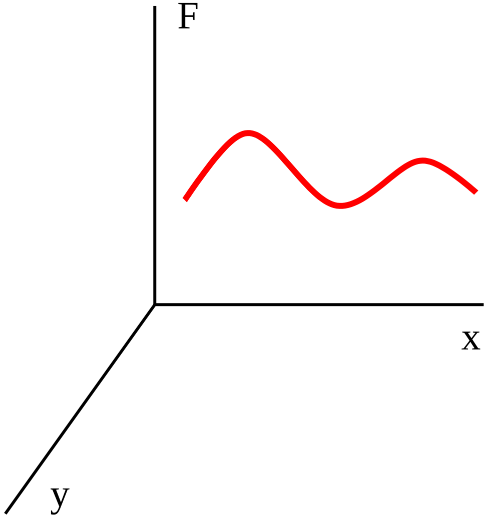
- Local description
  - two functions of 9 variables $(w, \vec{x}_3, \vec{y}_5)$:

$$\partial_w e^{-2\phi} + \Delta_{\mathbf{x}} F|_{y,w} = 0, \quad \partial_F e^{-2G-2\phi} + (\Delta_{\mathbf{y}} w)|_{x,F} = 0$$

$$e^{-2G} = \partial_w F$$

  - metric and fluxes are algebraic in F and e^{ϕ}
- Boundary conditions for D3 branes





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  - finite dilaton, $w = \tilde{w}[\vec{y}, F - f(\vec{x})]$

$$\left(\partial_{\mathbf{x}} - \frac{\partial_{\mathbf{x}} w}{\partial_F w}\partial_F\right)\frac{\partial_{\mathbf{x}} w}{\partial_F w} = 0 \;\rightarrow\; \Delta_{\mathbf{x}} f = 0$$

- Harmonic profiles for D branes
- Positions of branes $\rightarrow$ unique geometry

Summary of the solution

- Metric

$$ds^2 = e^H \left[-e^{3\phi/2}dt^2 + e^{-\phi/2}d\mathbf{x}_3^2\right] + e^{-H-\phi/2}d\mathbf{y}_5^2$$
$$+e^{-H+3\phi/2}(\partial_w F dw + \partial_{\mathbf{y}} F d\mathbf{y})^2, \qquad e^{2H} = \partial_w F$$

- Fluxes

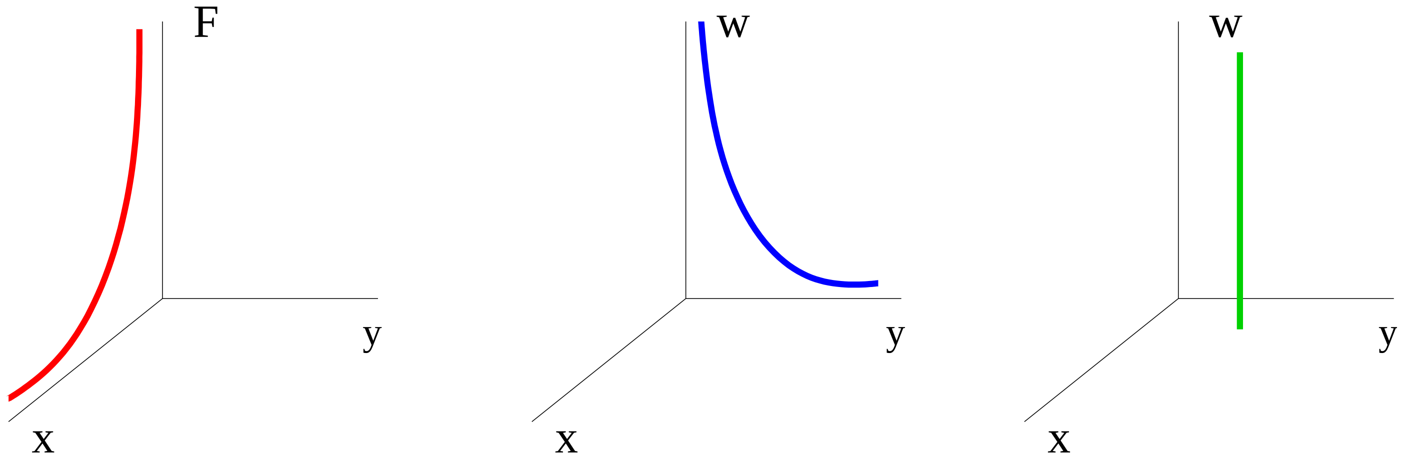
$$F_5 = -\frac{1}{4\cdot 4!} d\left[e^{-2H}\varepsilon_{ijklm}\partial^{y_m} F dy^{ijkl}\right] + dual,$$
$$H_3 = d\left[e^{2\phi}(\partial_w F dw + \partial_{\mathbf{y}} F d\mathbf{y})\right] dt, \quad F_3 = \frac{1}{2} d(\varepsilon_{ijk}\partial^k F dx^{ij}).$$

- Coupled PDEs for F, e^{ϕ}
- 1/4–BPS: two projectors for the spinor

$$\varepsilon = \exp\left[\frac{1}{4}(H + \frac{3\phi}{2})\right]\varepsilon_0 : \quad \Gamma_w\Gamma_{45678}\varepsilon_0 = -i\varepsilon_0, \quad \Gamma_w\Gamma_{123}\varepsilon_0^* = i\varepsilon_0$$

Boundary conditions

- D–branes must follow harmonic profiles
  - D3 brane: $w = \tilde{w}[\vec{y}, F - f(\vec{x})]$
  - D5 brane: $F = \tilde{F}[\vec{x}, w - g(\vec{y})]$
  - fund. string: $\vec{x} = \vec{y} =$const



- Vicinity of the brane: Poisson eqn for $\tilde{w}$, $\tilde{F}$ or $e^{-2\phi}$
- Unique solution in perturbation theory

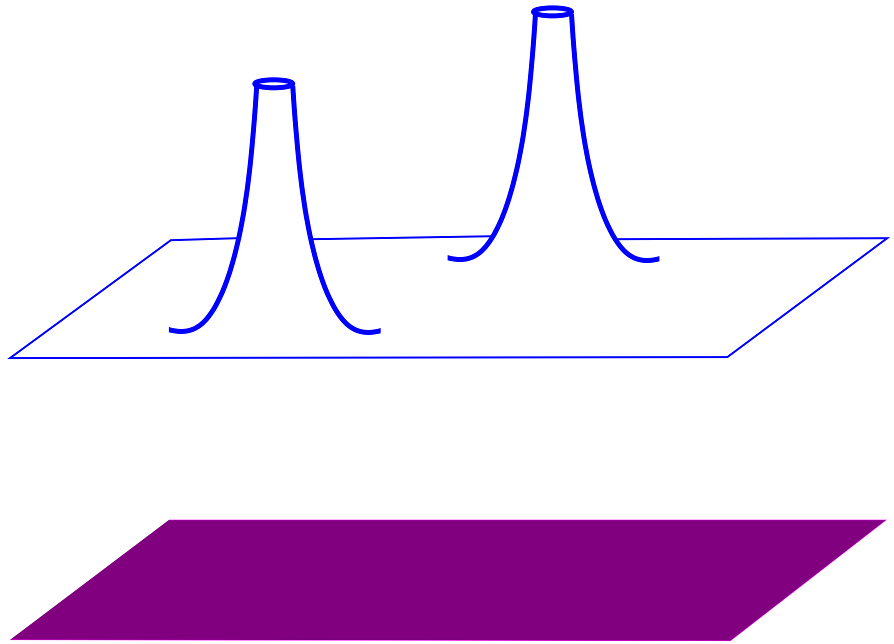
Supergravity vs DBI

- Profiles of D3 branes from SUGRA:

$$\Delta_{\mathbf{x}} f = 0$$

- D3 probe in D3 geometry:

$$\Delta_{\mathbf{x}} X = 0$$



Supergravity vs DBI

- Profiles of D3 branes from SUGRA:

$$\Delta_{\mathbf{x}} f = 0$$

- D3 probe in D3 geometry:

$$\Delta_{\mathbf{x}} X = 0$$

- D3 probe in D5 geometry

$$-(1 + (\nabla X)^2)\partial_X H + H\nabla^2 X + 2\nabla H \nabla X = 0$$

$$H = H_5(z, \vec{x})|_{z=X(\vec{x})}\,, \quad (\partial_z^2 + \Delta_x) H_5 = 0$$

Supergravity vs DBI

- Profiles of D3 branes from SUGRA:

$$\Delta_{\mathbf{x}} f|_{y,F} = 0$$

- D3 probe in D3 geometry:

$$\Delta_{\mathbf{x}} X|_{y,F} = 0$$

- D3 probe in D5 geometry

$$-(1 + (\nabla X)^2)\partial_X H + H\nabla^2 X + 2\nabla H \nabla X|_{y,w} = 0$$

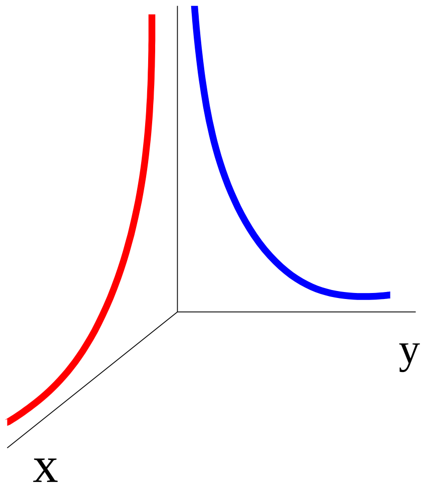
$$H = H_5(z, \vec{x})|_{z=X(\vec{x})}\,, \quad (\partial_z^2 + \Delta_x)H_5 = 0, \quad w_0 = X(\vec{x})$$

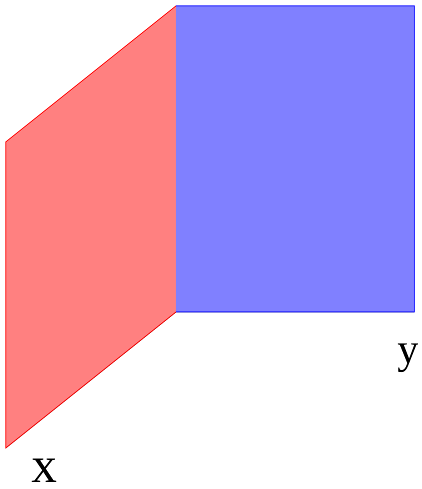
  - translation to the appropriate variable

$$\partial_w F = e^{2G}, \quad F_0(\vec{x}) = \int^{X(\vec{x})} H_5(z, \vec{x}) dz : \quad \Delta_x F_0|_{y,F} = 0$$

Explicit solutions

- Smeared D branes: harmonic functions





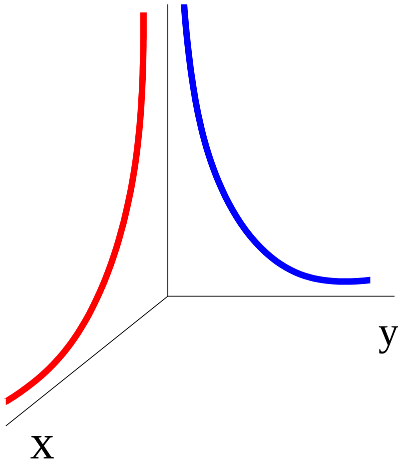
$$q(x), \quad p(y), \quad q\Delta_y(q^{-1}e^{-2\phi}) + p\Delta_x(q^{-1}e^{-2\phi}) = 0$$

Explicit solutions

- Smeared D branes: harmonic functions

$$q(x), \quad p(y), \quad q\Delta_y(q^{-1}e^{-2\phi}) + p\Delta_x(q^{-1}e^{-2\phi}) = 0$$

- Limit of flat branes



  - D3 brane: $\partial_x = 0$: dual of NCYM$_{3+1}$
  - D5 brane: $\partial_y = 0$: dual of NCYM$_{5+1}$

Hashimoto–Itzhaki, Maldacena–Russo '99

Explicit solutions

- Smeared D branes: harmonic functions

$$q(x), \quad p(y), \quad q\Delta_y(q^{-1}e^{-2\phi}) + p\Delta_x(q^{-1}e^{-2\phi}) = 0$$

- Limit of flat branes: non–commutative theories
  - D3 brane: $\partial_x = 0$: dual of NCYM$_{3+1}$
  - D5 brane: $\partial_y = 0$: dual of NCYM$_{5+1}$

Hashimoto–Itzhaki, Maldacena–Russo '99

- Near–horizon limits
  - 1/4–BPS states in AdS$_5\times$S^5/linear dilaton geometry
  - special solutions: 1/2–BPS states in AdS$_5\times$S^5
    - Wilson lines in $\mathcal{N} = 4$ SYM

Wilson lines and AdS/CFT

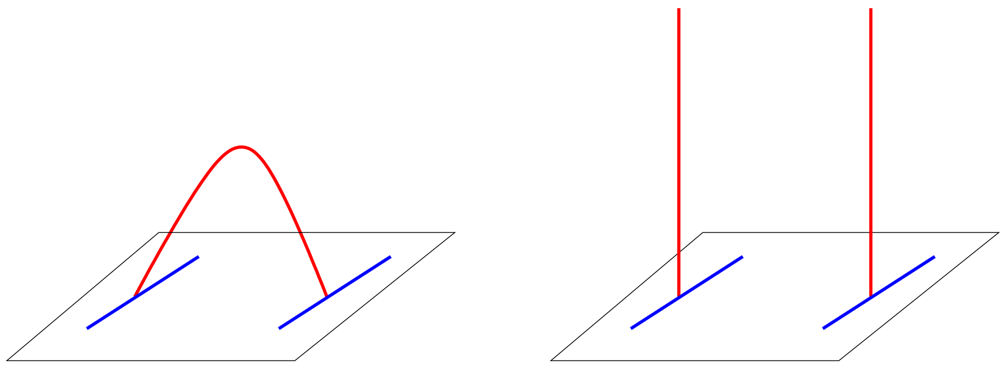
- Wilson line in field theory

$$W(\mathcal{C}) = \frac{1}{d_R} \mathrm{Tr}_R P e^{i \int_{\mathcal{C}} A}$$

- Dual description
  - fund. rep: string ending on a contour

Rey, Yee; Maldacena '98

$$\langle W(\mathcal{C}) \rangle = e^{-(S - S_0)}$$



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- Wilson line in field theory

$$W(\mathcal{C}) = \frac{1}{d_R} \mathrm{Tr}_R P e^{i \int_{\mathcal{C}} A}$$

- Dual description
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    Rey, Yee; Maldacena '98
  - reps with $\Delta \sim N$: D3 brane with flux
    Drukker, Fiol '05
  - heavy states ($\Delta \sim N^2$): gravitational backreaction

Wilson lines and AdS/CFT

- Supersymmetric Wilson line in field theory

$$W = \frac{1}{d_R}\mathrm{Tr}Pe^{i\int(A_\mu \dot{x}^\mu + i\Phi|\dot{x}|)dt}$$

- Dual description
  - fund. rep: string ending on a contour

    Rey, Yee; Maldacena ’98
  - reps with $\Delta \sim N$: D3 brane with flux

    Drukker, Fiol ’05
  - heavy states ($\Delta \sim N^2$): gravitational backreaction
- Structure of the geometries
  - symmetry: $SO(2,1) \times SO(3) \times SO(5)$
  - solutions in terms of harmonic $\Phi(x,y)$
  - regularity at $y=0$: $\partial_y \Phi = \pm \frac{\pi}{4}$

    OL ’06

1/2–BPS states

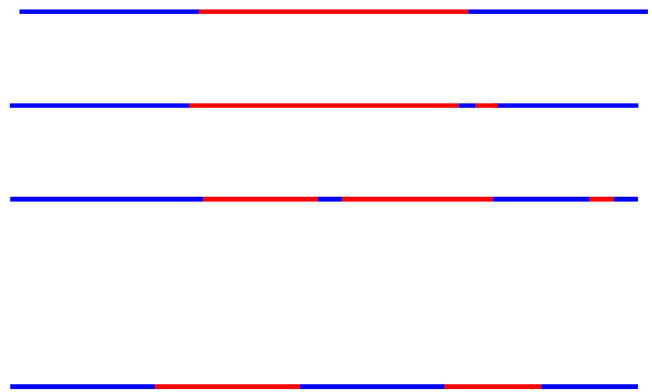
- Wilson lines: Young tableaux, branes & geometry

$\Delta = 0$

$\Delta \sim 1$

$\Delta \sim N$

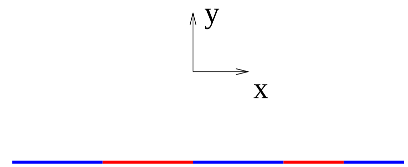
$\Delta \sim N^2$



$AdS_5 \times S^5$

string

branes



- Boundary condition $\rightarrow \Phi(x, y) \rightarrow$ geometry
- Relation to the 1/4–BPS F1–D3–D5 system
  - extra (super)symmetries
  - no sources
  - fluxes are supported by non–trivial topology

1/4–BPS intersections

- Intersections in IIB string theory
  - harmonic rule

$$(D5_{12345}, D5_{16789}, P_1)\ (D3_{123}, D7_{1456789}, P_1)\ (D1_1, P_1)$$

$$(D5_{12345}, D1_1, KK_{2345}) \qquad (D3_{123}, D7_{1234567}, KK_{4567})$$

  - "nontrivial" intersections

$$(D3_{123}, D5_{56789}, F1_4)\ (D3_{123}, D3_{145}, KK_{2345})\ (D3_{123}, D5_{12456}, \mathrm{NS5}_{12789})$$

$$(D5_{12345}, D7_{1234678}, \mathrm{NS5}_{12349}) \qquad (D7_{1234567}, F1_8, D1_9)$$

  - dualization of F1-D3-D5 system
- Intersections in M theory
  - U dualities: localized M2/M5/M5 intersections
  - agreement with probe analysis (PST action)
  - enhanced (super)symmetry near the branes

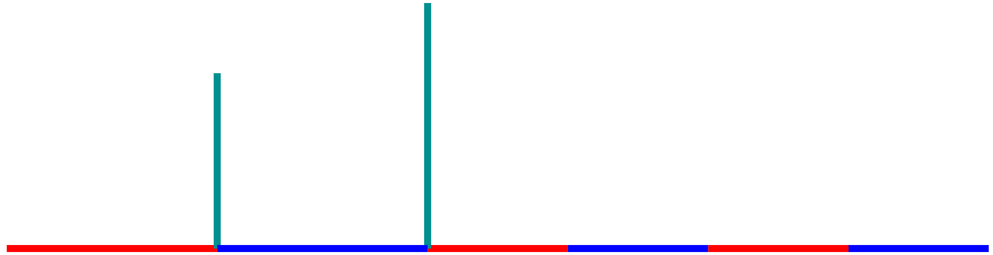
1/2–BPS geometries in M theory

- Properties of the geometries
  - dual description: defects in the CFTs
  - bosonic symmetries: $SO(2,2) \times SO(4)^2$
  - solution in terms of harmonic function $\Phi(x,y)$ and q
  - regularity at $y = 0$: two allowed values of $\partial_y \Phi$
- Examples of boundary conditions

OL '07

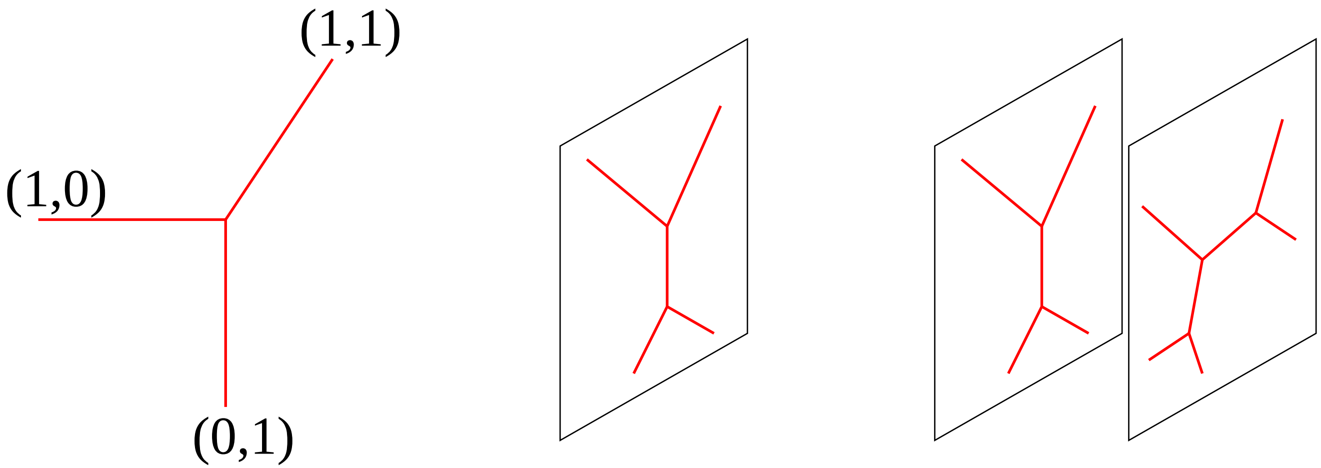
  - $AdS_7 \times S^4$ branch: $q = 1$

  - $AdS_4 \times S^7$ branch: $q = -\frac{1}{2}$



Brane webs

- String webs
  - probes: straight lines, orientation and dilaton
  - gravity solution: consistent boundary conditions



Brane webs

- String webs
  - probes: straight lines, orientation and dilaton
  - gravity solution: consistent boundary conditions
- Webs of membranes and Dp branes
  - probes: holomorphic profiles
  - gravity: restrictions on the brane locations
  - existence and uniqueness of the solution
- Near–horizon limits
  - 1/4–BPS geometries in $AdS_p \times S^q$
  - regular metrics: droplets in 2D Kahler space
  - degenerate limits and giant gravitons

1/4–BPS bubbling solutions

- Field theory
  - two adjoint scalars $X = \Phi_1 + i\Phi_2$, $Y = \Phi_3 + i\Phi_4$.
  - generic state: combination of Tr$(X^m Y^n)$

Berenstein '05

- String theory:
  - giant gravitons: holomorphic profiles — Mikhailov '00
  - large R charges: smooth geometries
- Structure of SUGRA solutions
  - $SO(4) \times U(1) \times U(1)_t$ symmetries
  - 2D Kahler space, $y = R_3 R_1$
  - regular metrics: droplets in Kahler space
  - interesting topological structure

Structure of the geometry

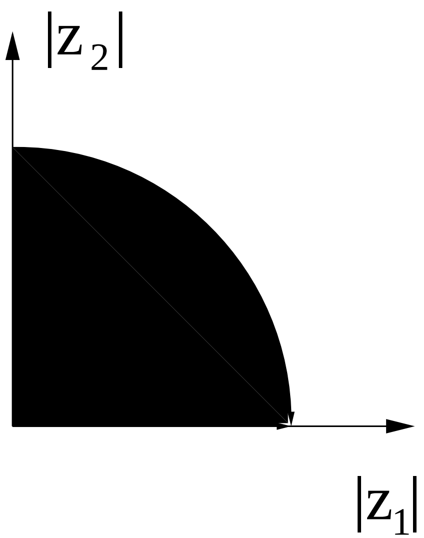
- Local description
  - Monge–Ampere type equation in $4+1$ dimensions

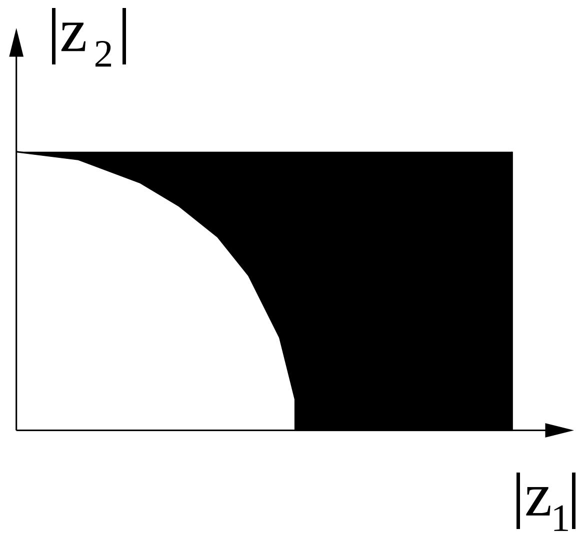
$$\text{det} h_{a\bar{b}} = -\frac{y^3}{8}\partial_y \left[y^{-1} \exp\{ y^{-1} \partial_y K \} \right]$$

  - Kahler potential in 4D $\rightarrow$ geometry in 10D

Structure of the geometry

- Local description
  - Monge–Ampere type equation in $4+1$ dimensions
  - Kahler potential in 4D $\rightarrow$ geometry in 10D
- Boundary conditions
  - generic point at $y=0$: $Z = \pm\frac{1}{2}$
  - asymptotic behavior at infinity





Structure of the geometry

- Local description
  - Monge–Ampere type equation in $4+1$ dimensions
  - Kahler potential in 4D $\to$ geometry in 10D
- Boundary conditions
  - generic point at $y=0$: $Z=\pm\frac{1}{2}$
  - asymptotic behavior at infinity
  - regularity at the domain walls: shapes of the droples are restricted

$$\partial_a\bar{\partial}_b v+\lambda\partial_a v\bar{\partial}_b v=g\partial_a w\bar{\partial}_b\bar{w}+O(v)$$

- Degenerate droplets $\to$ holomorphic curves

Boundary conditions

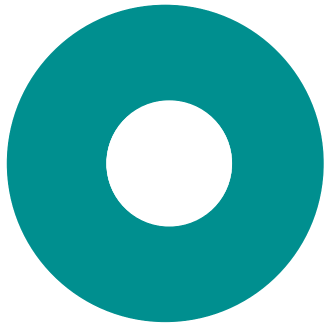
- 1/2–BPS states
  - field theory: matrix model for $X = \Phi_1 + i\Phi_2$
  - eigenvalues: free fermions in harmonic potential

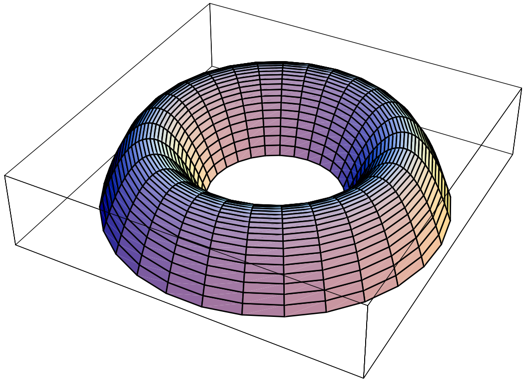
    Corley-Jevicki-Ramgoolam '02; Berenstein '04
  - gravitational picture: boundary conditions in a plane
  - map into the phase space of oscillators

    Lin-OL-Maldacena '04

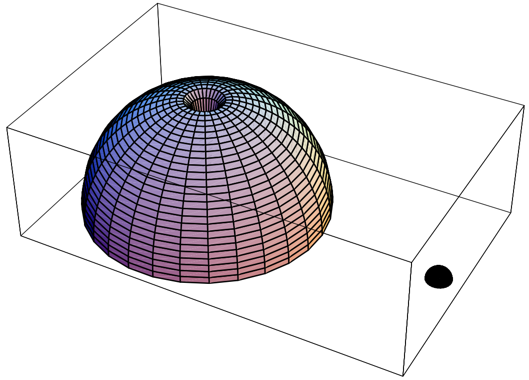
Boundary conditions

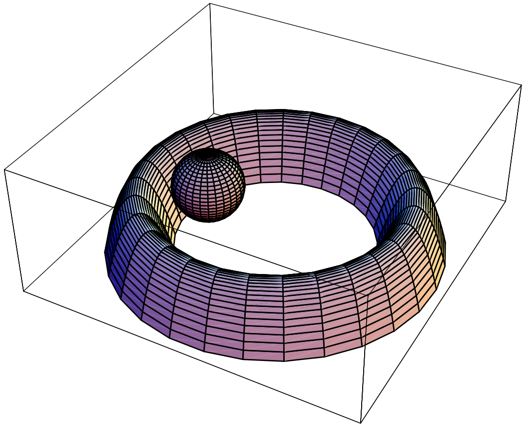
- Relation to 1/2–BPS case





- Generic boundary conditions





Outlook

- Shapes of branes are determined dynamically
  - open strings: solutions of DBI
  - closed strings: consistency of SUGRA
- Explicit example: 1/4–BPS D3/D5/F1 system
  - complete gravity solution, only $U(1)$ isometry
  - perfect agreement between DBI and SUGRA
- Brane webs and holomorphic surfaces
- Near–horizon limits
  - dual description of Wilson lines
  - local $1/4$–BPS states at strong coupling
- Open questions
  - relation to matrix model
  - extension to cases with lower SUSY