Phase Transitions in Hot and Dense QCD at Large N

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I. Introduction

- At large $T \gg \Lambda_{QCD}$ the system in the deconfined phase
- At small $T \ll \Lambda_{QCD}$ the system in the confined (hadronic) phase
- At large $\mu \gg \Lambda_{QCD}$ the system in the deconfined phase
- At small $\mu \ll \Lambda_{QCD}$ the system in the confined (hadronic) phase

It is clear:
something drastic must be happening on the way when temperature (chemical potential) varies

Question we want to address:
what are the most important vacuum configurations which are responsible for the transitions when $\mu(T)$ *varies ?*

2- Basic technique and methods:

- Main object: Large N QCD $(N_f \ll N)$
- Main technique-1: dual representation
- Main technique-2: holographic description
- Crucial element: Θ -parameter
- Basic trick: light η'-field as a probe of topological charges of the constituents

The basic **Conjecture**:

The Θ parameter suddenly changes its behavior precisely at the same point T_c where the phase transition happens

$$E_{vac} = N^2 \min_k \, h\left(\frac{\theta + 2\pi k}{N}\right), \qquad T < T_c$$

$$E_{vac} \sim \cos\theta, \qquad T > T_c$$

3. Support for the ***CONJECTURE*** from the holographic model of QCD

- The large N QCD is known to have a holographic description;
- Confined / deconfined phases in the holographic description can be studied in the standard way by analyzing the Polyakov's loop;
- Transition from confined to deconfined phase corresponds to the transition *from one background metric to another* at temperature T_c ;
- The Θ behavior has been also studied in both phases with the result: the confinement- deconfinement *phase transition takes place precisely at T_c where Θ dependence drastically changes.*
- $$\chi(T) \sim \frac{\partial^2 E_{vac}}{\partial \theta^2} \sim 1, \qquad T < T_c$$
  $$\chi(T) \sim \frac{\partial^2 E_{vac}}{\partial \theta^2} \sim 0, \qquad T > T_c$$

4. Support for the ***CONJECTURE*** from the lattices:

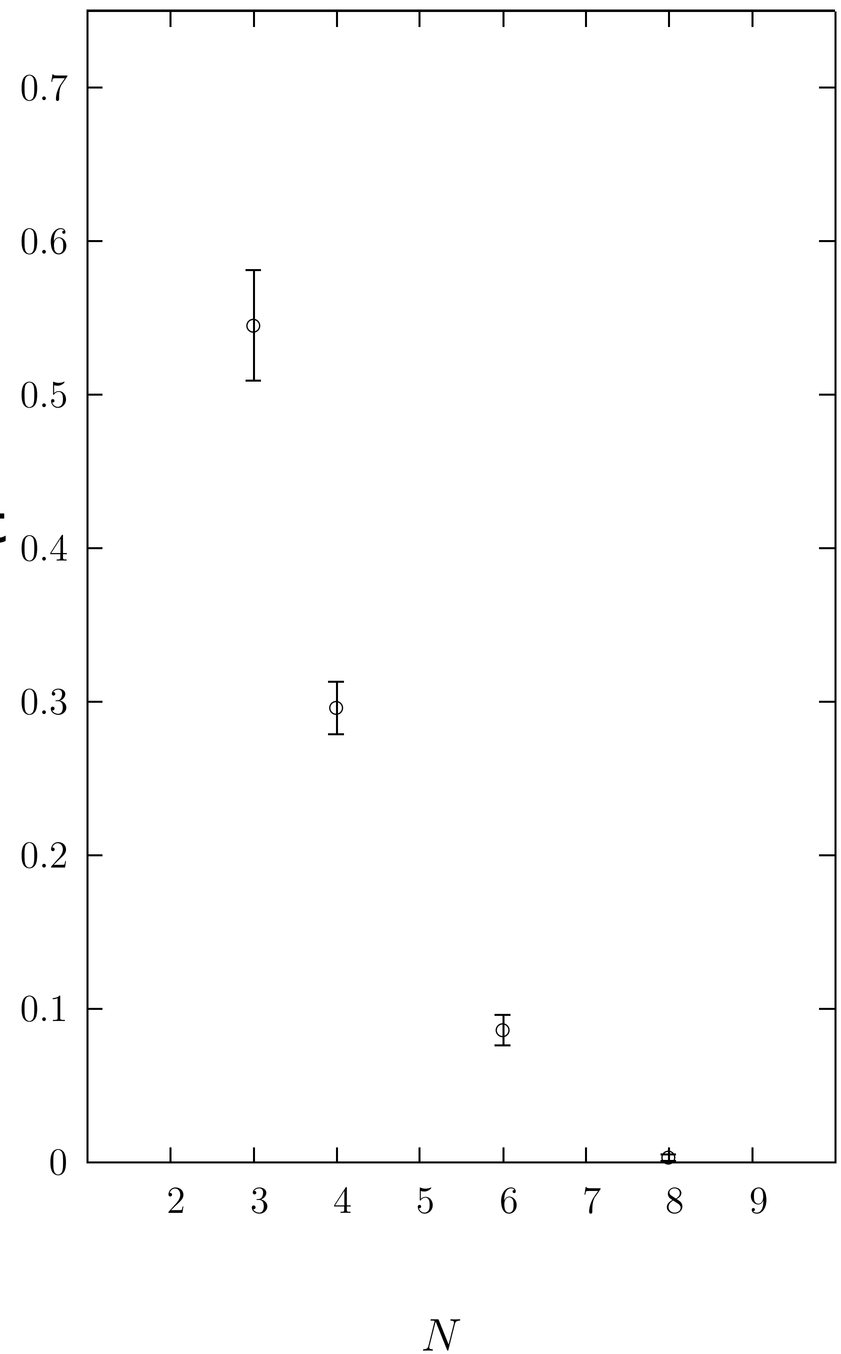
the ratio

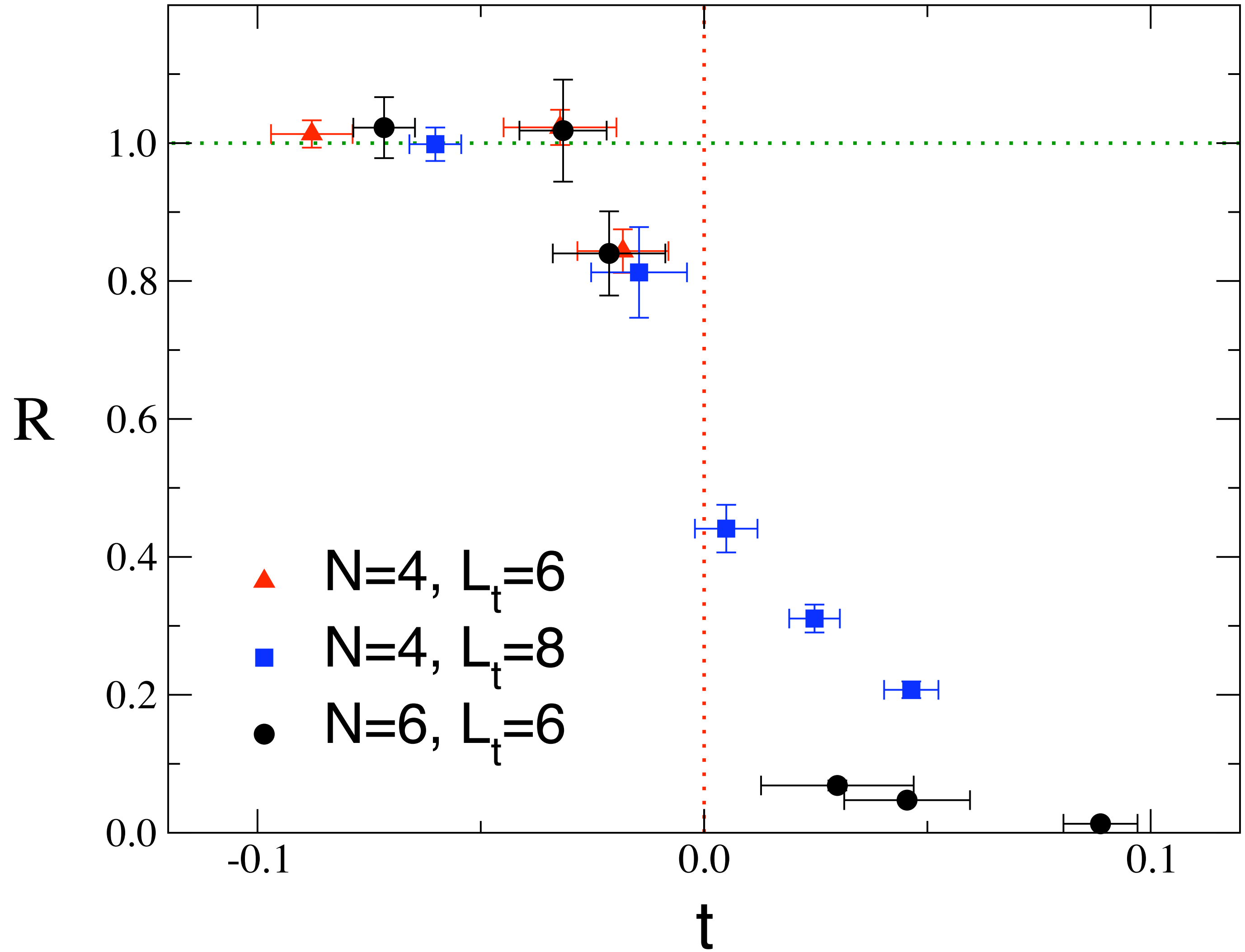
$$R \equiv \chi(T = T_c + \epsilon)/\chi(T = T_c - \epsilon)$$

in deconfined and confined phases at

$$T \simeq T_c$$

B. Lucini, M. Teper, U. Wenger, 2004





Support for the ***CONJECTURE*** from the lattices:

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the ratio $R(T) \equiv \chi(T)/\chi(T=0)$ as a function of reduced temperature $t = T/T_c - 1$ for N=4, 6, L.Del Debbio, et al.2004

5. Deconfined Phase, $T > T_c$

- According to the *Conjecture*, one can study the confinement -deconfinement phase transition by analyzing the θ dependence rather than Polyakov's loop.
- The θ dependence for $T > T_c$ is determined by instantons.
- Instanton expansion converges at $T > T_c$

$$V_{\text{inst}}(\theta) \sim e^{-\gamma N} \cos\theta, \qquad \gamma = \left[\frac{11}{3} \ln\left(\frac{\pi T}{\Lambda_{QCD}}\right) - 1.86\right],$$

- Critical temperature is determined by the condition

$$\gamma = \left[\frac{11}{3} \ln\left(\frac{\pi T_c}{\Lambda_{QCD}}\right) - 1.86\right] = 0 \quad \Rightarrow \quad T_c(N = \infty) \simeq 0.53\Lambda_{QCD},$$

Deconfined phase--continue

- For any positive $\gamma > 0$ the instanton density is parametrically small and calculations are under complete theoretical control even in close vicinity of T_c

$$V \sim \cos\theta \cdot e^{-\alpha N\left(\frac{T-T_c}{T_c}\right)}, \quad 1 \gg \left(\frac{T-T_c}{T_c}\right) \gg 1/N.$$

- Topological susceptibility $\chi(T > T_c) \sim e^{-N} = 0$ obviously vanishes in agreement with results from holographic QCD

- One can compute $T_c(\mu)$ for small chemical potential.

$$T_c(\mu) = T_c(\mu = 0)\left[1 - \frac{3N_f\mu^2}{2(2N+N_f)\pi^2 T_c^2}\right], \quad \mu \ll T_c$$

- As expected, there is no dependence on μ at large N, in agreement with the lattice results: Fodor, et al, 2004; Fodor, Katz, Schmidt, 2007.

- The Θ dependence may only experience drastic changes in the vicinity where the instanton expansion breaks down, which explains our conjecture on connection between the two parameters.

6. Coulomb Gas Representation (CGR)

- We introduce η' field as a probe to investigate the topological charges of the constituents in both phases. It appears in unique combination $(\varphi - \theta)$, $\eta' \sim f_{\eta'}\varphi$ in both phases.

- The partition function for light (almost massless) η' field in deconfined phase is given by

$$\int \mathcal{D}\varphi \; e^{-\frac{1}{2T} f_{\eta'}^2 \int d^3x (\vec{\nabla}\varphi)^2} \, e^{\lambda \int d^3x \cos(\varphi(x) - \theta)}, \qquad \lambda = \Lambda_{QCD}^3 \cdot e^{-\gamma N}$$

- Mapping between sine-Gordon theory and its CGR is well known

$$Z = \sum_{M_\pm = 0}^{\infty} \frac{(\lambda/2)^M}{M_+! M_-!} \int d^3x_1 \ldots \int d^3x_M \; e^{-i\theta \sum_{a=0}^{M} Q_a} \, e^{-\frac{T}{f_{\eta'}^2} \sum_{a>b=0}^{M} Q_a G(x_a - x_b) Q_b} \, .$$

$$G(x_a - x_b) = \frac{1}{4\pi |\vec{x}_a - \vec{x}_b|} \, .$$

7. Coulomb gas representation. Physical interpretation.

- The charges were introduced in a formal way.
- Physical interpretation of charges: they are topological charges as follows from identification

$$e^{i\theta \sum Q_a}$$

- The following hierarchy of scales exists

$$\begin{array}{ccccc} (\text{size},\ \rho) & \ll & (\text{distance},\ \bar{r}) & \ll & (\text{Debye},\ r_D) \\ \frac{1}{T} & \ll & \frac{1}{\Lambda_{QCD}\sqrt[3]{a}} & \ll & \frac{1}{\Lambda_{QCD}\sqrt{a}} \end{array} \qquad a \equiv e^{-\gamma N} \ll 1$$

- Typical size of the instantons $\rho \sim T^{-1}$

- The average distance between the instantons $\bar{r} \sim \lambda^{-1/3} \sim \Lambda_{QCD}^{-1} a^{-1/3}$

- Charge Q_a is identified with an integer topological charge localized at point x_a. This by definition corresponds to a small instanton at x_a

- $\lambda \sim a \ll 1$ is the fugacity of the instanton gas in deconfined phase.

- The instanton-anti-instanton interaction at large distances is the same as instanton-instanton. They both are Coulomb-like interactions (in contrast with semiclassical picture).

- The η' mass emerges as a result of Debye screening

- The η' was defined as the phase of the det(..) which does not vanish even if chiral symmetry is unbroken. In holographic model the chiral symmetry is broken in deconfined phase in a small window above T_c

The basic **Question**:

We identified the point T_c with the place where Θ behavior drastically changes.

It implies that some topological configurations (which couple to Θ)must be responsible for these drastic changes. In deconfined phase they are nice dilute instantons. What happens to them at

$$T < T_c \ ?$$

8.Confined phase. Speculations.

- We want to speculate here on the fate of instantons when we cross the phase transition line from above

- The instanton expansion is not justified. We do not attempt to use semiclassical ideas in this region

- We argue that the instantons do not disappear from the system, but rather dissociate into the instanton quarks, the quantum objects with fractional topological charges 1/N.

- Instanton quarks carry the magnetic charges along with topological charges.

- the η' field will play a crucial role in identification of topological charges 1/N of the constituents.

9. Instanton quarks: few historical remarks.

- Instanton quarks originally appeared in 2d models. namely, using the resummation of exact n-instanton solution in 2d CP^{N-1} models, the original problem was mapped into 2d system of pseudo -particles with fractional 1/N topological charges, *Fateev et al, 79; Berg, Luscher, 79.*

- The picture leads to elegant explanation of the confinement.

- Similar calculations in 4d is proven to difficult to carry out, *Belavin et al, 79.*

10. Confined phase. Lagrangian for η'

- We want to use the same trick (tested in weak coupling regime) with η' as a probe of the topological charges of constituents.
- Effective lagrangian has the form

$$L_\varphi = \frac{1}{2} f_{\eta'}^2 (\partial_\mu \varphi)^2 + E_{vac} \cos\left(\frac{\varphi - \theta}{N}\right)$$

- It follows from the following (2k)-th correlators (Veneziano,79)

$$\frac{\partial^{2k} E_{vac}(\theta)}{\partial\, \theta^{2k}}|_{\theta=0} \sim \int \prod_{i=1}^{2k} dx_i \langle Q(x_1)...Q(x_{2k})\rangle \sim (\frac{i}{N})^{2k}, \qquad \text{where} \;\; Q \equiv \frac{g^2}{32\pi^2} G_{\mu\nu}\widetilde{G}_{\mu\nu}.$$

- There are few additional arguments supporting the SG structure
- It satisfies U(1) anomalous WI and in large N limit leads to the standard expression $E_{vac}(\theta/N)^2 \sim 1$

11. Coulomb Gas Representation (CGR). Confined Phase

- We want to use the trick to present the effective η' lagrangian in the dual form (CGR). The η' is a unique field which explicitly measures the topological charges of constituents.
- Repeating all previous steps we arrive at CGR,

$$Z = \sum_{M_\pm=0}^{\infty} \frac{(\frac{E_{vac}}{2})^M}{M_+!M_-!} \int d^4x_1 \dots \int d^4x_M \cdot e^{-i\theta \sum_{(a=0,Q_a=\pm 1/N)}^{M} Q_a} \cdot e^{-\frac{1}{f_{\eta'}^2} \sum_{(a>b=0,Q_a=\pm 1/N)} Q_a G(x_a-x_b) Q_b},$$

$$G(x_a - x_b) = \frac{1}{4\pi^2(x_a - x_b)^2}.$$

- The fundamental difference in comparison with deconfined case: while the total charge is integer, the individual charges are fractional 1/N.

- This is a direct consequence of θ/N dependence of the underlying theory.

- Due to 2π periodicity only the configurations with total integer topological charges contribute. therefore the number of particles with charges 1/N in each configuration must be proportional to N.

- as a result, the moduli space in CGR is 4Nk where k- integer. This number is precisely the number of zero modes in k-instanton background.

- This is the basis for identification of charges from CGR with instanton quarks suspected long ago.

- For gauge group G the number of integration is 4kC_2(G) where C_2(G) is the quadratic Casimir. This is precisely the number of zero-modes in k- instanton background for the gauge group G.

- We recover the moduli space which we identify with strongly interacting instantons in confined phase of QCD

- role of the fugacity for this ensemble plays $E_{vac} \sim N^2$

- average distance between constituents $\bar{r} \sim N^{-1/2}$

- The Debye screening length is large $r_D \sim m_{\eta'}^{-1} \sim \sqrt{N}$

- Density of instantons is ~N (instanton quarks $\sim N^2$). It is consistent with observation from holographic QCD: finite number of instantons will disappear from the system.

12. The relation to other studies.

- *Pierre van Baal et al:* there seems to be a close relation with periodic instantons at nonzero temperature.

- *M. Unsal and L. Yaffe, hep-th/0803.0344.* A specific deformation of gluodynamics supports a weak coupling analysis in confined phase. Objects which resemble the instanton quarks (with action and topological charge $S = 8\pi^2/(g^2 N),\ Q = 1/N$) are found.

- *D. Diakonov and V. Petrov, hep-th/0704.3181.* Fractionally charged objects 1/N appear in semiclassical analysis.

- *E. Shuryak, hep-ph/061113; M.Chernodub, V. Zakharov, hep-ph/0611228* Instanton quarks in narrow deconfined window $0 < (T - T_c) < 1/N$ behave like wrapped monopoles. They form well-defined small instanton at larger temperature $(T - T_c) \geq 1/N$.

- *A.Gorsky, V. Zakharov, hep-th/0707.1284.* magnetic strings connected by wrapped monopoles which are related to the instanton quarks, see above.

13. Propaganda

- We presented the arguments that the sharp changes in θ happen at the point of the phase transition T_c (support from holographic QCD and lattices)

- At sufficiently large $(T - T_c) \gg 1/N$ the instanton is small and well defined object.

- We speculated that in confined region the same instantons dissociate into N-instanton quarks.

- One can test these ideas by studying a narrow window $0 \le (T - T_c) \le 1/N$ in deconfined phase where instanton quarks (wrapped monopoles) start to form the instanton with zero monopole charge.