Static Properties and Form Factors of Baryons in Holographic QCD

Shigeki Sugimoto (IPMU, Univ of Tokyo)

(New Institute in Kashiwa, Japan)

Today!

Based on

arXiv:0806.3122 K.Hashimoto, T.Sakai and S.S.

hep-th/0701280 H.Hata, T.Sakai, S.S. and S.Yamato

Closely related works:

arXiv:0803.0180 H.Hata, M.Murata and S.Yamato

hep-th/0701276, arXiv:0705.2632, arXiv:0710.4615, … D.K.Hong, M.Rho, H.U.Yee and P.Yi

Non-Perturbative Methods in Strongly Coupled Gauge Theories @ The Galileo Galilei Institute for Theoretical Physics, Florence

1 Introduction

- **Meson effective theory (traditional approach)**

effective action consistent with chiral sym, hidden local sym.

$$
\begin{aligned}
S_{4\text{dim}} = \int d^4x \Bigg[& \frac{f_\pi^2}{4}\,\mathrm{tr}[D_\mu U^\dagger D^\mu U] \\
& + L_1 \left(\mathrm{tr}[D_\mu U^\dagger D U]\right)^2 + L_2\,\mathrm{tr}[D_\mu U^\dagger D_\nu U]\mathrm{tr}[D^\mu U^\dagger D^\nu U] \\
& + L_3\,\mathrm{tr}[D_\mu U^\dagger D^\mu U D^\nu U^\dagger D^\nu U] \\
& - iL_9\,\mathrm{tr}[F^L_{\mu\nu} D^\mu U^\dagger D^\nu U^\dagger + F^R_{\mu\nu} D^\mu U^\dagger D^\nu U] + L_{10}\,\mathrm{tr}[U^\dagger F^L_{\mu\nu} U F^{R\,\mu\nu}] \\
& + \frac{1}{2}\mathrm{tr} F^v_{\mu\nu} F^{v\,\mu\nu} + m_\rho^2\,\mathrm{tr}[(v_\mu - g^{-1}\beta_\mu)^2] \Bigg] \\
& - \frac{N_c}{24\pi^2}\int_{4\text{dim}} \Bigg[\mathrm{Tr}[(A_R dA_R + dA_R A_R + A_R^3)(U^{-1}A_L U + U^{-1}dU) - \text{p.c.}] \\
& \quad + \mathrm{Tr}[dA\_dU^{-1}A_L U - \text{p.c.}] + \mathrm{Tr}[A_R(dU^{-1}U)^3 - \text{p.c.}] \\
& \quad + \frac{1}{2}\mathrm{Tr}[(A_R dU^{-1}U)^2 - \text{p.c.}] + \mathrm{Tr}[U A_R U^{-1} A_L dU dU^{-1} - \text{p.c}] \\
& \quad - \mathrm{Tr}[A_R dU^{-1} U A_R U^{-1} A_L U - \text{p.c.}] + \frac{1}{2}\mathrm{Tr}[(A_R U^{-1} A_L U)^2] \\
& \quad + C_1\,\mathrm{tr}[\alpha_L^3\alpha_R - \alpha_R^3\alpha_L] + C_2\,\mathrm{tr}[\alpha_L\alpha_R\alpha_L\alpha_R] \\
& \quad + C_3\,\mathrm{tr}[F^v \alpha_L\alpha_R\alpha_L\alpha_R] \\
& \quad + C_4\,\mathrm{tr}[F^L(\alpha_L\alpha_R - \alpha_R\alpha_L) - F^R(\alpha_R\alpha_L - \alpha_L\alpha_R)] \Bigg] \\
& - \frac{N_c}{240\pi^2}\int_{5\text{dim}} \mathrm{Tr}(g d g^{-1})^5 \\
& + (\text{much more})
\end{aligned}
$$

pion

ρ meson

WZW term

- A lot of terms
- A lot of parameters

$$
\begin{aligned}
D_\mu U &= \partial_\mu U - iA^L_\mu U + iU A^R_\mu \\
U &= \xi_L^\dagger \xi_R \\
\beta_\mu &= \frac{1}{2i}(\partial_\mu \xi_R \cdot \xi_R^\dagger + \partial_\mu \xi_L \cdot \xi_L^\dagger) \\
D_\mu \xi_L &= \partial_\mu \xi_L - igv_\mu \xi_L + i\xi_L A^L_\mu \\
D_\mu \xi_R &= \partial_\mu \xi_R - igv_\mu \xi_R + i\xi_R A^R_\mu \\
\alpha_{L\mu} &= \frac{1}{i} D_\mu \xi_L \cdot \xi_L^\dagger, \quad \alpha_{R\mu} = \frac{1}{i} D_\mu \xi_R \cdot \xi_R^\dagger
\end{aligned}
$$

"Top down approach" of holographic QCD

1. Find a D-brane configuration that realizes QCD
2. Use the Gauge/String duality
3. Some approximation

[Sakai-S.S. 2004]

Wait for the explanation

Meson effective theory

5 dim U(N_f) YM–CS theory in a curved space-time

$$S_{\text{5dim}} \simeq S_{\text{YM}} + S_{\text{CS}}$$

$$k(z) = 1 + z^2$$

CS5-form

$$S_{\text{YM}} = \kappa \int d^4x dz \, \text{Tr} \left(\frac{1}{2} h(z) F_{\mu\nu}^2 + k(z) F_{\mu z}^2 \right) \qquad S_{\text{CS}} = \frac{N_c}{24\pi^2} \int_5 \omega_5(A)$$

$$\kappa = \frac{\lambda N_c}{216\pi^3} \equiv a\lambda N_c \qquad h(z) = (1 + z^2)^{-1/3}$$

$(M_{\text{KK}} = 1 \text{ unit})$

- Just one line
- Just 2 parameters

M_{KK} ~ cut off scale

λ ~ bare coupling

5 dim YM-CS theory = 4 dim meson theory

$$A_\mu(x^\mu, z) = \sum_{n\geq 1} B_\mu^{(n)}(x^\mu)\psi_n(z)$$

$$A_z(x^\mu, z) = \sum_{n\geq 0} \varphi^{(n)}(x^\mu)\phi_n(z)$$

complete sets

Chosen to diagonalize kinetic & mass terms of $B_\mu^{(n)}, \varphi^{(n)}$

$\varphi^{(0)} \sim$ pion $\quad B_\mu^{(1)} \sim \rho$ meson $\quad B_\mu^{(2)} \sim a_1$ meson $\quad \cdots$

$$\boxed{S_{5\text{dim}}(A) \;=\; S_{4\text{dim}}(\pi, \rho, a_1, \rho', a_1', \cdots)}$$

- Reproduces old phenomenological models
  - Vector meson dominance [Gell-Mann-Zachariasen 1961, Sakurai 1969]
  - Gell-Mann Sharp Wagner model [Gell-Mann -Sharp-Wagner 1962]
  - Hidden local symmetry [Bando-Kugo-Uehara-Yamawaki-Yanagida 1985]
- Masses and couplings roughly agree with experiments.

Quantitative tests

[Sakai-S.S. 2004, 2005]

(Our model vs Experiment)

Meson mass

mass	ρ	a_1	ρ'	(a'_1)	ρ''
exp.(MeV)	776	1230	1465	(1640)	1720
our model	[776]	1189	1607	2023	2435
ratio	[1]	1.03	0.911	(0.811)	0.706

input ($M_{\text{KK}} \simeq 949$ MeV)

coupling

coupling		fitting m_ρ and f_π	experiment
f_π	$1.13 \cdot \kappa^{1/2} M_{\text{KK}}$	[92.4 MeV]	92.4 MeV
L_1	$0.0785 \cdot \kappa$	0.584×10^{-3}	$(0.1 \sim 0.7) \times 10^{-3}$
L_2	$0.157 \cdot \kappa$	1.17×10^{-3}	$(1.1 \sim 1.7) \times 10^{-3}$
L_3	$-0.471 \cdot \kappa$	-3.51×10^{-3}	$-(2.4 \sim 4.6) \times 10^{-3}$
L_9	$1.17 \cdot \kappa$	8.74×10^{-3}	$(6.2 \sim 7.6) \times 10^{-3}$
L_{10}	$-1.17 \cdot \kappa$	-8.74×10^{-3}	$-(4.8 \sim 6.3) \times 10^{-3}$
$g_{\rho\pi\pi}$	$0.415 \cdot \kappa^{-1/2}$	4.81	5.99
g_ρ	$2.11 \cdot \kappa^{1/2} M_{\text{KK}}^2$	0.164 GeV^2	0.121 GeV^2
$g_{a_1\rho\pi}$	$0.421 \cdot \kappa^{-1/2} M_{\text{KK}}$	4.63 GeV	$2.8 \sim 4.2$ GeV

What about baryons?

- In 1961, Skyrme proposed

  Baryons are solitons (Skyrmion) in a pion effective theory.

- In 1983, Adkins-Nappi-Witten (ANW)

  succeeded to calculate the static properties
  (mean square radii, magnetic moment, axial coupling, etc.)
  by quantizing the collective modes of the Skyrmion.

  ➡ Roughly agree with the experimental data!

Q. Can we apply the idea of ANW to our system?

Plan

1. Introduction
2. Brief summary of the model
3. Baryons as instantons
4. Quantization
5. Currents
6. Exploration
7. Conclusion

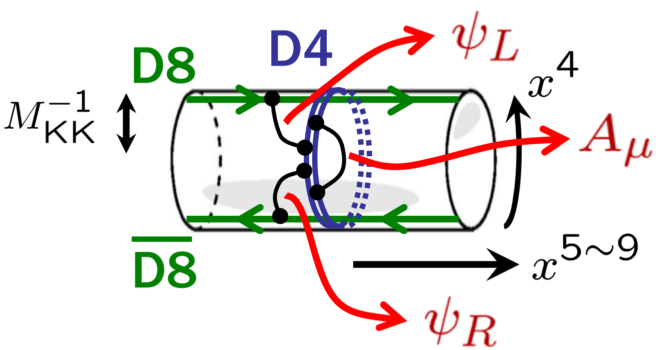
2 Brief summary of the model [Sakai-S.S. 2004]

Type IIA string theory
in Witten's D4 background
+ N_f Probe D8-branes
(assuming $N_c \gg N_f$)

dual ⟷ at low energy

4 dim QCD with
N_f massless quarks

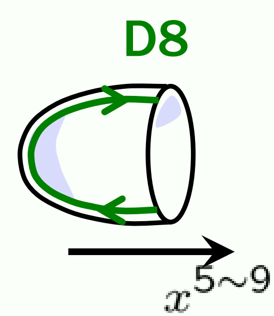
N_c $\underbrace{\phantom{D8-\overline{D8}}}$ N_f **pairs**
D4-D8-$\overline{\text{D8}}$ system
on ~~SUSY~~ S^1



QCD with N_f massless quarks
(at low energy)

dual

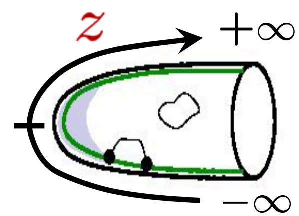
String theory
in the D4 background
+ N_f probe D8-branes
(assuming $N_c \gg N_f$)



The effective theory on the D8-branes

N_f D8-branes extended along $(x^\mu, z) \times S^4 \subset \mathbf{R}^{1,3} \times \mathbf{R}^2 \times S^4$

← Low energy



9 dim $U(N_f)$ gauge theory

← Reducing S^4 (Here we only consider *SO(5)* invariant states)

5 dim $U(N_f)$ YM-CS theory

$A_\mu(x^\nu, z), A_z(x^\nu, z)$ $\mu, \nu = 0 \sim 3$

5 dim gauge field

$$S_{\text{5dim}} \simeq S_{\text{YM}} + S_{\text{CS}}$$

$k(z) = 1 + z^2$

CS5-form

$$S_{\text{YM}} = \kappa \int d^4x dz \, \text{Tr} \left(\frac{1}{2} h(z) F_{\mu\nu}^2 + k(z) F_{\mu z}^2 \right) \qquad S_{\text{CS}} = \frac{N_c}{24\pi^2} \int_5 \omega_5(A)$$

$h(z) = (1 + z^2)^{-1/3}$

$$\kappa = \frac{\lambda N_c}{216\pi^3} \equiv a \lambda N_c$$

$(M_{\text{KK}} = 1 \text{ unit})$

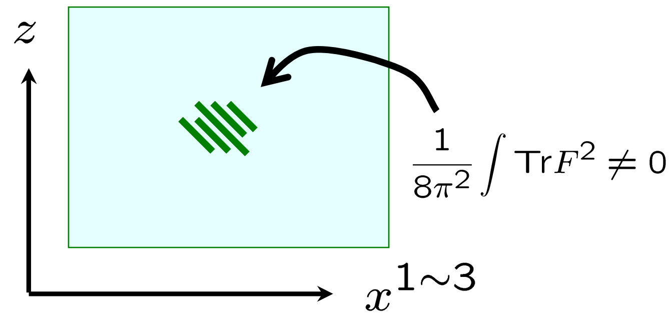
[See also, Son-Stephanov 2003]

③ Baryons as instantons

- Consider an instanton config. in $x^M = (\vec{x}, z) \in \mathbf{R}^4$ ($M = 1,2,3,z$)

  ➡ behaves as a point-like particle

  ➡ Interpreted as a baryon



- In fact,

$$\boxed{\underbrace{N_B}_{\text{baryon \#}} = \underbrace{\frac{1}{8\pi^2}\int \mathrm{tr} F \wedge F}_{\text{Instanton \#}}}$$

- <u>Comment</u>

D4 wrapped on *S*^{4} $\simeq$ instanton on D8 $\simeq$ Skyrmion

[Witten, Gross-Ooguri 1998] [Atiyah-Manton 1989] [Skyrme 1961]

Realization of Atiyah-Manton:

$$\underbrace{U(x^\mu)}_{\text{Skyrmion}} \equiv P\exp\left\{-\int_{-\infty}^{\infty} dz \underbrace{A_z(x^\mu, z)}_{\text{Instanton}}\right\}$$

Classical solution (We consentrate on the $N_f = 2$ case.)

- The instanton solution for the Yang-Mills action

$$S_{\mathrm{YM}} = \kappa \int d^4x dz \, \mathrm{Tr} \left(\frac{1}{2} h(z) F_{\mu\nu}^2 + k(z) F_{\mu z}^2 \right)$$

shrinks to zero size !

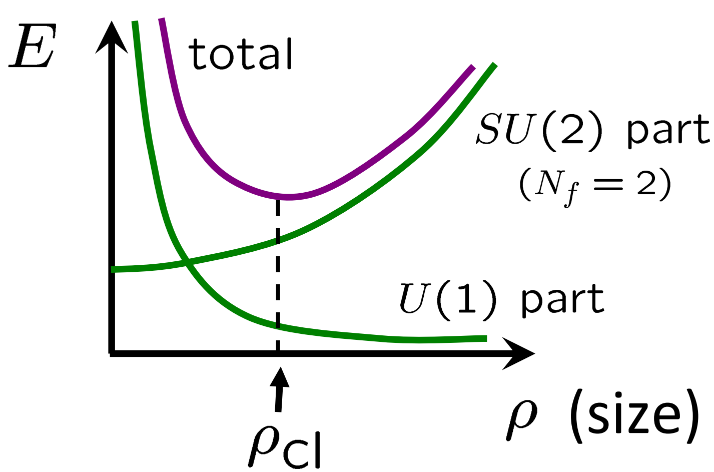
- The Chern-Simons term makes it larger

$U(1)$ part

$$S_{\mathrm{CS}} = \frac{N_c}{24\pi^2} \int_5 \omega_5(A) = \frac{N_c}{16\pi^2} \int d^4x dz \, A_0^{U(1)} \underbrace{\epsilon^{ijk} \mathrm{Tr} F_{ij} F_{kz}}_{} + \cdots$$

Non-zero for instanton

➡ source of the U(1) charge

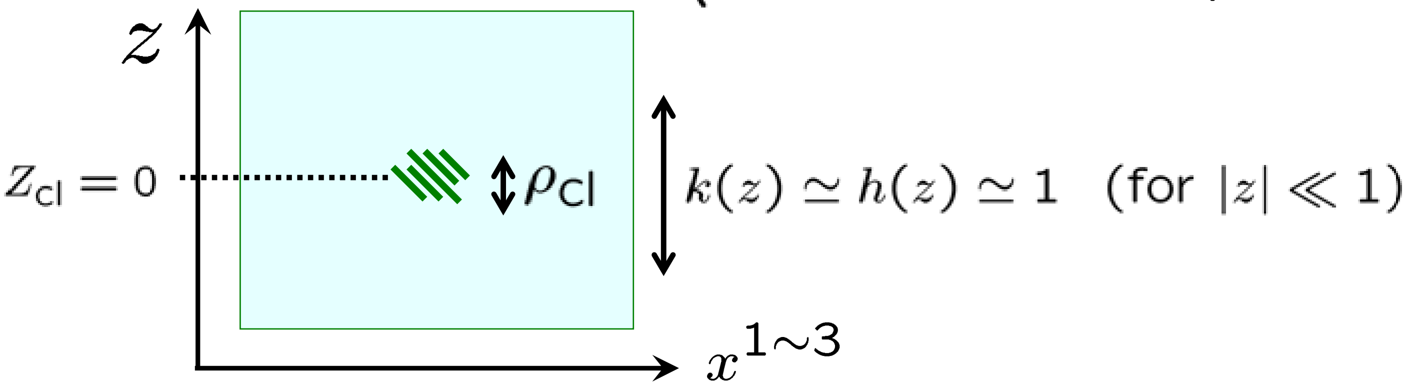


➡ Stabilized at $\rho_{\mathrm{cl}}^2 = \frac{N_c}{8\pi^2\kappa} \sqrt{\frac{6}{5}}$

[Hong-Rho-Yee-Yi 2007]
[Hata-Sakai-S.S.-Yamato 2007]

- Note that $\rho_{\text{cl}} \sim \mathcal{O}(\lambda^{-1/2})$

If λ is large enough, the 5 dim space-time can be approximated by the flat space-time. (The effect of the non-trivial z-dependence is taken into account perturbatively.)



➡ The leading order classical solution is the BPST instanton with $\rho = \rho_{\text{cl}}$ and $Z = Z_{\text{cl}} = 0$

$$A_M^{\text{cl}} = -i \frac{\xi^2}{\xi^2 + \rho^2} g \partial_M g^{-1} \qquad g = \frac{(z - Z) - i(\vec{x} - \vec{X}) \cdot \vec{\tau}}{\xi}$$

$$\xi = \sqrt{(\vec{x} - \vec{X})^2 + (z - Z)^2}$$

ρ : **size** $\quad (\vec{X}, Z)$: **position of the instanton**

4 Quantization

[Hata-Sakai-S.S.-Yamato 2007]

- Consider a slowly moving (rotating) baryon configuration.
  Use the moduli space approximation method :

Instanton moduli $\mathcal{M} \ni (X^\alpha) \rightarrow (X^\alpha(t))$ $(\alpha = 1, 2, \cdots, \dim \mathcal{M})$

time

$$\boxed{A_M(t,x) \sim A_M^{\mathrm{cl}}(x; X^\alpha(t))}$$

$S_{5\mathrm{dim}}$ $\rightarrow$ **Quantum Mechanics for** $X^\alpha(t)$

- For SU(2) one instanton,

$$\mathcal{M} \simeq \{(\underbrace{\vec{X}, Z}_{\text{position}}, \rho)\} \times SU(2)/\mathbf{Z}_2 \qquad \mathbf{Z}_2 : \mathbf{a} \rightarrow -\mathbf{a}$$

size

$\cup\!\!\!\!\!\!\!\;\,\mathbf{a}$ ← *SU(2)* orientation

$$\rightarrow \quad L_{\mathrm{QM}} = \frac{G_{\alpha\beta}}{2} \dot{X}^\alpha \dot{X}^\beta - U(X^\alpha) \qquad U(X^\alpha) = 8\pi^2\kappa\left(1 + \left(\frac{\rho^2}{6} + \frac{3^6\pi^2}{5\,\lambda^2\rho^2} + \frac{Z^2}{3}\right) + \cdots\right)$$

<u>**Note**</u> $(\vec{X}, \mathbf{a})$: genuine moduli (the same as in the Skyrme model)

(ρ, Z) : new degrees of freedom, added since they are light compared with the other massive modes.

- Solving the Schrodinger equation for this Quantum mechanics, we obtain the baryon states

→ Generalization of Adkins-Nappi-Witten including vector mesons and ρ , Z modes

We can construct baryon states for

$$n, p, \Delta(1232), N(1440), N(1530), \cdots$$

<u>Example</u> Nucleon wave function:

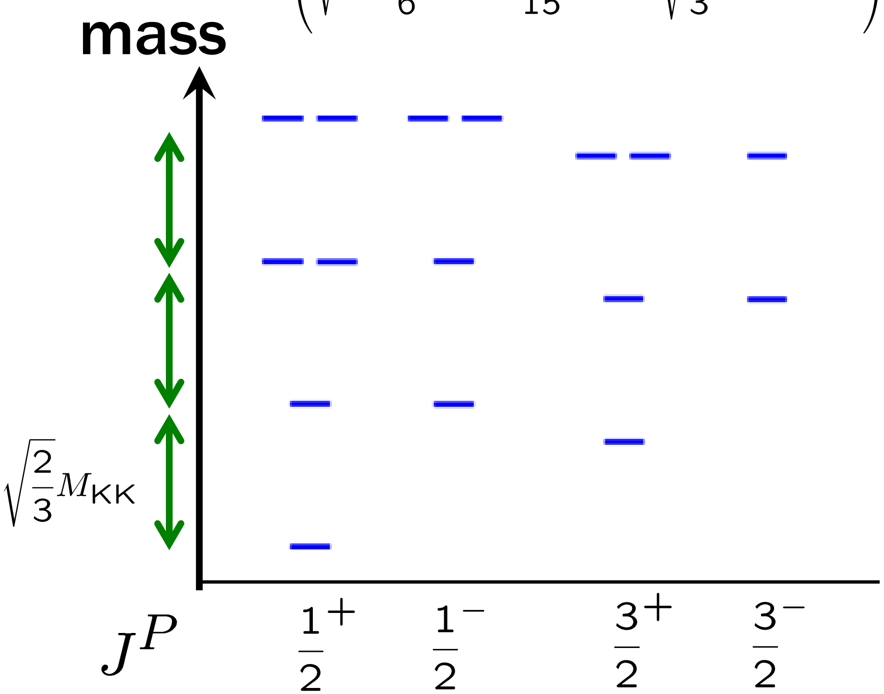
$$\psi(\vec{X}, \mathbf{a}, \rho, Z) \propto e^{i\vec{p}\cdot\vec{X}} R(\rho)\psi_Z(Z)T(\mathbf{a})$$

$$\left(\begin{array}{ll} R(\rho) = \rho^{\tilde{l}} e^{-A\rho^2} & \tilde{l} = -1 + 2\sqrt{1 + N_c^2/5} \\ \psi_Z(\rho) = e^{-AZ^2} & A = \frac{8\pi^2\kappa}{\sqrt{6}} \\ T(\mathbf{a}) = a_1 + ia_2 \ \ \text{for} \ \ |p\uparrow\rangle & \text{etc.} \end{array} \right)$$

Baryon spectrum

Theory

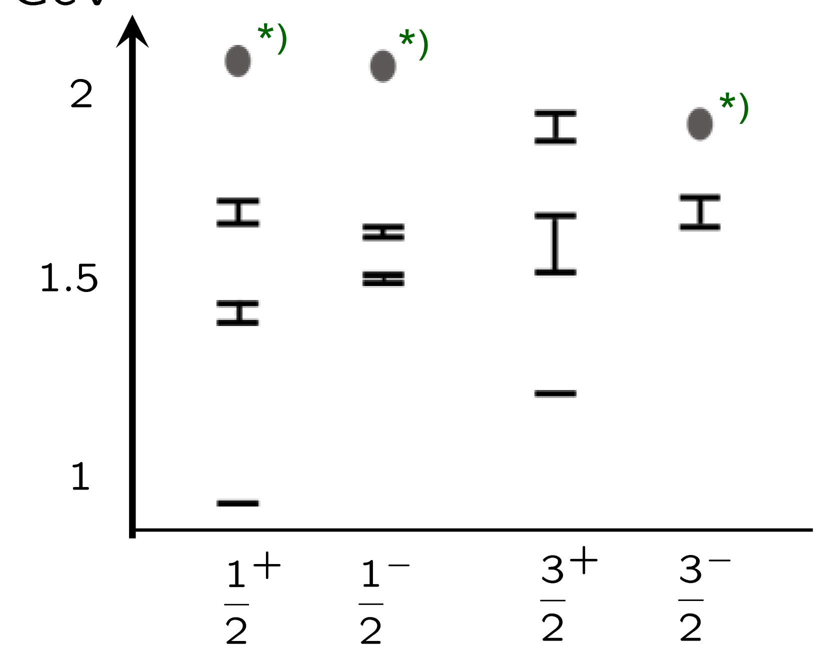
$$M \simeq M_0 + \left(\sqrt{\frac{(l+1)^2}{6}+\frac{2}{15}N_c^2}+\sqrt{\frac{2}{3}}(n_\rho+n_z)\right) M_{\mathrm{KK}}$$



Experiment

($I = J$ states from PDG)

*) Evidence for existence is poor



Note:

- We only consider the mass difference, since $\mathcal{O}(N_c^0)$ term in M_0 is not known.
- $M_{\mathrm{KK}} \simeq 949$ MeV (fixed by ρ -meson mass) is a bit too large. It looks better if M_{KK} were around 500 MeV .

5 Currents

[Hashimoto-Sakai-S.S.2008]

[See also, Hata-Murata-Yamato 2008]

- Chiral symmetry

$$U(N_f)_L \times U(N_f)_R \xrightarrow{\text{gauge}} (A_{L\mu}(x), A_{R\mu}(x))$$

- Interpreted as

$$A_{L\mu}(x) = \lim_{z\to+\infty} A_\mu(x,z) \qquad A_{R\mu}(x) = \lim_{z\to-\infty} A_\mu(x,z)$$

$$\Rightarrow \quad S_{5\,\mathrm{dim}}\Big|_{\mathcal{O}(A_L,A_R)} = -\int d^4x \left(A^a_{L\mu} J^{a\mu}_L + A^a_{R\mu} J^{a\mu}_R\right)$$

with

$$\boxed{J_{L\mu} = -\kappa\,(k(z)F_{\mu z})\Big|_{z=+\infty} \qquad J_{R\mu} = +\kappa\,(k(z)F_{\mu z})\Big|_{z=-\infty}}$$

- vector and axial vector currents

$$J^\mu_V \equiv J^\mu_L + J^\mu_R = -\kappa\Big[k(z)F^{\mu z}\Big]^{z=+\infty}_{z=-\infty}$$

$$J^\mu_A \equiv J^\mu_L - J^\mu_R = -\kappa\Big[\psi_0(z)k(z)F^{\mu z}\Big]^{z=+\infty}_{z=-\infty} \quad (\psi_0(\pm\infty) = \pm 1)$$

How to calculate

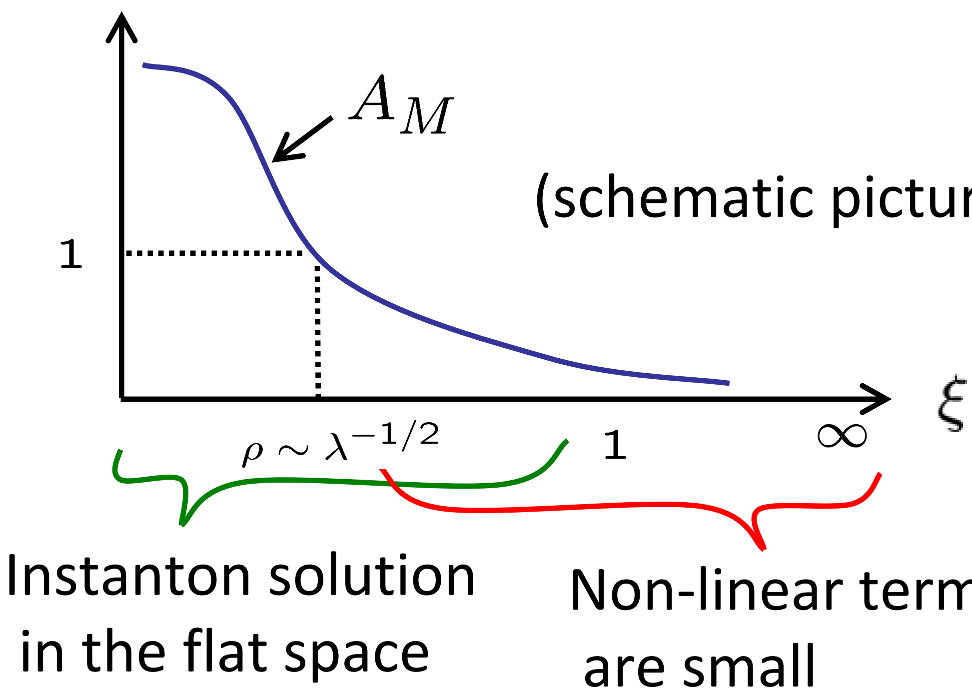
- We need to know how $F_{\mu z}(x,z)$ behaves at $z \to \pm\infty$

  ➡ We cannot use the solution in the flat space.

- The EOM are complicated non-linear equations.

  ➡ difficult to solve exactly.

- We use the following trick to calculate the currents.



Approximated by linearized EOM

Now we can solve!

6 Exploration

[Hashimoto-Sakai-S.S.2008]

[See also, Hong-Rho-Yee-Yi 2007,
Hata-Murata-Yamato 2008]

Now we are ready to calculate various physical quantities

But, don’t trust too much !

- λ may not be large enough.
- Higher derivative terms may contribute.
- $N_c = 3$ is not large enough.
- The model deviates from real QCD at high energy $\sim M_{\rm KK}$
- We use $M_{\rm KK} \simeq 949$ MeV (value consistent with ρ meson mass)
  But we know this is too large to fit the baryon mass differences.

Baryon number current

$$J_B^\mu = -\frac{2}{N_c}\kappa\left[k(z)F_{U(1)}^{\mu z}\right]_{z=-\infty}^{z=+\infty}$$

U(1) part of the U(2) gauge field

$\Downarrow$

$$J_B^0 \simeq \left[k(z)\partial_z G\right]_{z=-\infty}^{z=+\infty} \qquad J_B^i \simeq -\frac{J^j}{16\pi^2\kappa}\epsilon^{ijk}\partial_k J_B^0 + \cdots$$

G : Green's function $\quad (h(z)\partial_i^2 + \partial_z k(z)\partial)G = \delta^3(\vec{x}-\vec{X})\delta(z-Z)$

J^j : Spin operator $\quad J^j = -i4\pi^2\kappa\rho^2\,\mathrm{tr}(\tau^j \mathrm{a}^{-1}\dot{\mathrm{a}})$

Note: $\quad k(z) \sim z^2, \quad \partial_z G \sim 1/z^2 \quad$ at $\quad z \to \pm\infty$

$\Rightarrow$ J_B^μ is non-zero, finite.

Isoscalar mean square radius

$$\langle r^2 \rangle_{I=0} = \int d^3x \, r^2 \, J_B^0 \simeq (0.742 \ \mathrm{fm})^2$$

Numerical estimate using $M_{\mathrm{KK}} \simeq 949 \ \mathrm{MeV}$
(fixed by ρ -meson mass)

$$\left(\mathrm{cf.} \ \langle r^2 \rangle^{1/2}_{I=0} \Big|_{\mathrm{exp}} = 0.806 \ \mathrm{fm} \, , \ \langle r^2 \rangle^{1/2}_{I=0} \Big|_{\mathrm{ANW}} = 0.59 \ \mathrm{fm} \right)$$

Isoscalar magnetic moment

$$J_B^i \simeq -\frac{J^j}{16\pi^2\kappa}\epsilon^{ijk}\partial_k J_B^0 + \cdots$$

$$\mu^i_{I=0} = \frac{1}{2}\epsilon^{ijk}\int d^3x\, x^j J_B^k \simeq \frac{J^i}{16\pi^2\kappa}$$

For a spin up proton state $|p\uparrow\rangle$

Isoscalar g-factor

$$\langle p\uparrow|\,\mu^i_{I=0}\,|p\uparrow\rangle = \frac{\delta^{i3}}{32\pi^2\kappa} \equiv \frac{g_{I=0}}{4M_N}\delta^{i3}$$

Nucleon mass

($M_N \simeq 940$ MeV)

$$\Rightarrow \quad g_{I=0} = \frac{M_N}{8\pi^2\kappa\, M_{\mathrm{KK}}} \simeq 1.68$$

$M_{\mathrm{KK}} \simeq 949$ MeV, $\kappa \simeq 0.00745$

(fixed by m_ρ) (fixed by f_π)

$$\left(\text{cf. } g_{I=0}\big|_{\mathrm{exp}} \simeq 1.76,\ \ g_{I=0}\big|_{\mathrm{ANW}} = 1.11\right)$$

Summary of the results

	our result	exp.	ANW
$\langle r^2\rangle_{I=0}^{1/2}$	0.74 fm	0.81 fm	0.59 fm
$\langle r^2\rangle_{I=1}^{1/2}$	0.74 fm	0.94 fm	∞ ※
$\langle r^2\rangle_{A}^{1/2}$	0.54 fm	0.67 fm	—
$g_{I=0}$	1.7	1.8	1.1
$g_{I=1}$	7.0	9.4	6.4
g_A	0.73	1.3	0.61

※ pion loop contribution is log divergent in the chiral limit.
Our calculation corresponds to the tree level in ChPT.

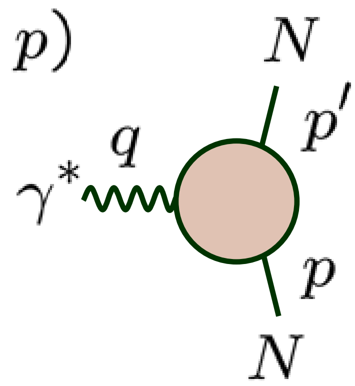
- We can also evaluate these for excited baryons such as $\Delta(1232), N(1440), N(1535), \cdots$

Form factors

Dirac form factor — Pauli form factor

$$\langle N, \vec{p}'|J^{\mu}_{\rm em}(0)|N, \vec{p}\rangle = \overline{u}(p', s')\left[\gamma^{\mu} F_1(q^2) + \frac{i}{2m_N}\sigma^{\mu\nu} q_{\nu} F_2(q^2)\right] u(p, s)$$

$$(q = p' - p)$$



Breit frame: $\vec{p}' = -\vec{p} = \vec{q}/2$

$$\langle N, \vec{q}/2|J^{0}_{\rm em}(0)|N, -\vec{q}/2\rangle = G_E(\vec{q}^2)\, \chi^{\dagger}_{s'}\chi_s$$

$$\langle N, \vec{q}/2|J^{i}_{\rm em}(0)|N, -\vec{q}/2\rangle = \frac{i}{2m_N} G_M(\vec{q}^2)\, \chi^{\dagger}_{s'}(\vec{q} \times \vec{\sigma})\chi_s$$

Sachs form factor

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4m_N^2} F_2(q^2)$$ Electric form factor

$$G_M(q^2) = F_1(q^2) + F_2(q^2)$$ Magnetic form factor

Dipole behavior

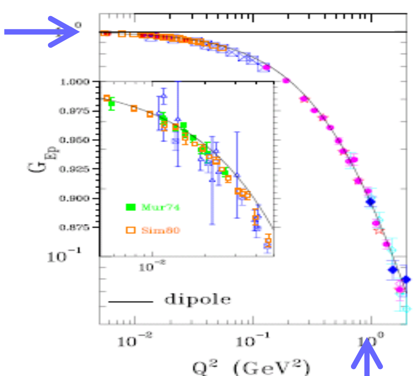
Experimental data suggest

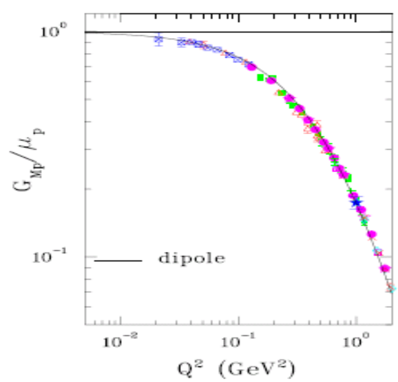
dipole $(\Lambda \simeq 0.71\ \mathrm{GeV}^2)$

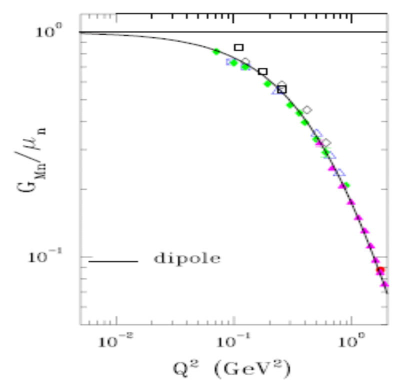
$$G_E^p(Q^2) \simeq \frac{1}{\mu_p} G_M^p(Q^2) \simeq \frac{1}{\mu_n} G_M^n(Q^2) \simeq \frac{1}{(1+\frac{Q^2}{\Lambda^2})^2} \qquad G_E^n(Q^2) \simeq 0$$

G_E^p $\qquad$ G_M^p/μ_p $\qquad$ G_M^n/μ_n

1







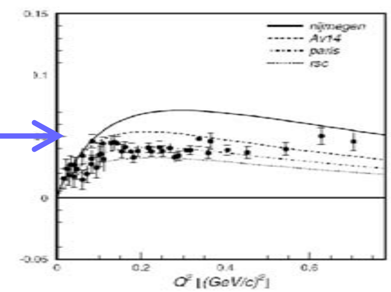
1 GeV^2

—— : dipole

dots : data

G_E^n

0.05

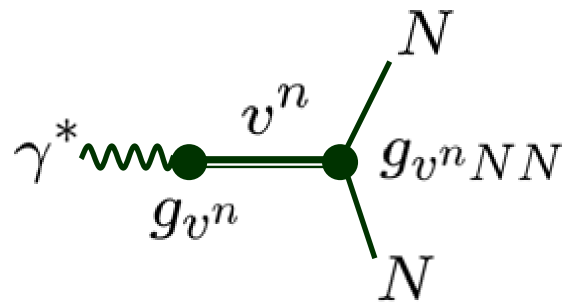


Our result

$$G_E^p(Q^2) = \frac{1}{\mu_p} G_M^p(Q^2) = \frac{1}{\mu_n} G_M^n(Q^2) = \sum_{n \geq 1} \frac{g_{v^n} g_{v^n NN}}{Q^2 + m_n^2} \qquad G_E^n(Q^2) = 0$$

with $\quad g_{v^n} = -2\kappa (k(z)\partial_z \psi_{2n-1})\Big|_{z=+\infty}$

$$g_{v^n NN} = \langle \psi_{2n-1}(Z) \rangle$$



Vector meson dominance

- Can this be compatible with dipole?

$$G_E^p(Q^2) \simeq 1 - 2.38Q^2 + 4.02(Q^2)^2 - 6.20(Q^2)^3 + 9.35(Q^2)^4 - 14.0(Q^2)^5 + \cdots$$

$$\frac{1}{(1 + Q^2/\Lambda^2)^2} \simeq 1 - 2.38Q^2 + 4.24(Q^2)^2 - 6.71(Q^2)^3 + 9.97(Q^2)^4 - 14.2(Q^2)^5 + \cdots$$

with $\quad \Lambda^2 = 0.758 \text{ GeV}^2$ $\qquad (M_{\mathrm{KK}} = 1 \text{ unit})$

5 Conclusion

- We proposed a new method to analyze static properties of baryons.
- Our model automatically includes the contributions from various massive vector and axial-vector mesons.
- Compared with the similar analysis in the Skyrme model (ANW), the agreement with the experimental values are improved in most of the cases.
- But, we should keep in mind that our analysis is still very crude and there are a lot of ambiguities remain unsolved.

Back up slides

- Isovector current

$$J_V^{a\,\mu} = -\kappa\left[k(z)F_{SU(2)}^{a\,\mu z}\right]_{z=-\infty}^{z=+\infty}$$

SU(2) part of the U(2) gauge field

$$\Downarrow$$

$$J_V^{a\,0} \simeq I^a J_B^0 + \cdots \qquad J_V^{a\,i} \simeq 2\pi^2\kappa\rho^2 \mathrm{tr}(\tau^a \mathbf{a} \tau^j \mathbf{a}^{-1})\,\epsilon^{ijk}\partial_k J_B^0 + \cdots$$

(I^a : Isospin operator $I^a = -i4\pi^2\kappa\rho^2\,\mathrm{tr}(\tau^a \mathbf{a}\,\dot{\mathbf{a}}^{-1})$)

- We can easily check that

$$Q_V^a = \int d^3x\, J_V^{a0} = I^a$$: iso-spin operator

- The ele-mag current is given by

$$J_{\mathrm{em}}^{\mu} = J_V^{3\mu} + J_B^{\mu}/2 \longleftarrow Q_{\mathrm{em}} = I^3 + Q_B/2$$

- Isovector magnetic moment

$$\mu^i_{I=1} = \epsilon^{ijk}\int d^3x\, x^j J_V^{3,k} \simeq -4\pi^2\kappa\rho^2 \mathrm{tr}(\mathbf{a}\tau^i\mathbf{a}^{-1}\tau^3)$$

$$J_V^{a\,i} \simeq 2\pi^2\kappa\rho^2 \mathrm{tr}(\tau^a\mathbf{a}\tau^j\mathbf{a}^{-1})\,\epsilon^{ijk}\partial_k J_B^0 + \cdots$$

For a spin up proton state $|p\uparrow\rangle$

$$\langle p\uparrow|\,\mu^i_{I=1}\,|p\uparrow\rangle = \frac{8\pi^2\kappa}{3}\langle\rho^2\rangle\delta^{i3} \equiv \frac{g_{I=1}}{4M_N}\delta^{i3}$$

- If we approximate $\langle\rho^2\rangle$ by its classical value $\rho^2_{\mathrm{cl}} = \frac{N_c}{8\pi^2\kappa}\sqrt{\frac{6}{5}}$

➡ $g_{I=1} \simeq \frac{M_N}{M_{\mathrm{KK}}}\cdot\frac{N_c}{3}\cdot 4\sqrt{\frac{6}{5}} \simeq 4.34$ $\left(\text{cf. } g_{I=1}\big|_{\mathrm{exp}} \simeq 9.4,\ g_{I=0}\big|_{\mathrm{ANW}} = 6.38\right)$

- If we evaluate $\langle\rho^2\rangle$ by using the nucleon wavefunction,

$$\langle\rho^2\rangle = \rho^2_{\mathrm{cl}}\left(\sqrt{1+\frac{5}{N_c^2}}+\frac{\sqrt{5}}{2N_c}\right) \simeq 1.62\,\rho^2_{\mathrm{cl}}$$ ➡ $g_{I=1} \simeq 7.03$

magnetic moment

The magnetic moments for proton and neutron (in the unit of Bohr magneton $\mu_N = \dfrac{1}{2M_N}$) are

$$\mu_p = \frac{1}{4}(g_{I=0} + g_{I=1}) \simeq 2.18 \qquad \mu_n = \frac{1}{4}(g_{I=0} - g_{I=1}) \simeq -1.34$$

$$g_{I=0} \simeq 1.68, \quad g_{I=1} \simeq 7.03$$

$$\left(\begin{array}{lll} \text{cf.} & \mu_p|_{\text{exp}} \simeq 2.79\,, & \mu_n|_{\text{exp}} \simeq -1.91\,, \\ & \mu_p|_{\text{ANW}} \simeq 1.87\,, & \mu_n|_{\text{ANW}} \simeq -1.31\,, \end{array} \right)$$

※ Since $g_{I=0} = \mathcal{O}(N_c^0)$ and $g_{I=1} = \mathcal{O}(N_c^2)$

these values may not be meaningful.

Axial coupling

The axial coupling g_A is defined by

$$\int d^3x \, \langle J_A^{a\,i} \rangle = \frac{g_A}{3} \langle \underset{\text{spin}}{\sigma^i} \underset{\text{isospin}}{\tau^a} \rangle$$

$\langle \cdots \rangle$: expectation value w.r.t. a nucleon state

$$\boxed{J_A^{a\,i} \simeq -2\pi^2 \kappa \rho^2 \, \mathrm{tr}(\mathbf{a}\tau^j \mathbf{a}^{-1}\tau^a) \left[\psi_0(z) k(z) (\partial_i \partial_j - \delta_{ij}\partial_k^2) H \right]_{z=-\infty}^{z=+\infty} + \cdots}$$

(H : Green's function

$$(k(z)\partial_i^2 + k(z)\partial_z h(z)^{-1}\partial k(z))H = \delta^3(\vec{x} - \vec{X})\delta(z - Z)$$)

formula

$$\int d^3x \, \partial_i^2 \left(-\frac{1}{4\pi} \frac{1}{r} \right) = 1$$

$$\langle \mathrm{tr}(\mathbf{a}\tau^j \mathbf{a}^{-1}\tau^a) \rangle = -\frac{2}{3} \langle \sigma^i \tau^a \rangle$$

$\Rightarrow$

$$\boxed{g_A = \frac{16\pi\kappa}{3} \left\langle \frac{\rho^2}{k(Z)} \right\rangle}$$

$$\boxed{g_A = \frac{16\pi\kappa}{3}\left\langle \frac{\rho^2}{k(Z)} \right\rangle}$$

- If we approximate $\left\langle \frac{\rho^2}{k(Z)} \right\rangle$ by its classical value $\rho_{\rm cl}^2 = \frac{N_c}{8\pi^2\kappa}\sqrt{\frac{6}{5}}$

$$\Rightarrow \quad g_A \simeq \frac{2N_c}{3\pi}\sqrt{\frac{6}{5}} \underset{\substack{\uparrow \\ N_c = 3}}{\simeq} 0.697 \qquad \left(\text{cf. } g_A\big|_{\rm exp} \simeq 1.27,\ g_A\big|_{\rm ANW} = 0.61\right)$$

- If we evaluate $\left\langle \frac{\rho^2}{k(Z)} \right\rangle$ by using the nucleon wavefunction,

$$\langle\rho^2\rangle \simeq 1.62\,\rho_{\rm cl}^2 \qquad \left\langle \frac{1}{k(Z)} \right\rangle \simeq 0.639 \quad \Rightarrow \quad g_A \simeq 0.722$$

- <u>Note</u>: It is possible to show that
  the Goldberger-Treiman relation is satisfied.

$$\boxed{g_A = \frac{f_\pi g_{\pi NN}}{M_N}}$$

Summary table

	our model	Skyrmion[14]	experiment
$\langle r^2 \rangle_{I=0}^{1/2}$	0.742 fm	0.59 fm	0.806 fm
$\langle r^2 \rangle_{\mathrm{M},I=0}^{1/2}$	0.742 fm	0.92 fm	0.814 fm
$\langle r^2 \rangle_{\mathrm{E,p}}$	$(0.742\ \mathrm{fm})^2$	∞	$(0.875\ \mathrm{fm})^2$
$\langle r^2 \rangle_{\mathrm{E,n}}$	0	$-\infty$	$-0.116\ \mathrm{fm}^2$
$\langle r^2 \rangle_{\mathrm{M,p}}$	$(0.742\ \mathrm{fm})^2$	∞	$(0.855\ \mathrm{fm})^2$
$\langle r^2 \rangle_{\mathrm{M,n}}$	$(0.742\ \mathrm{fm})^2$	∞	$(0.873\ \mathrm{fm})^2$
$\langle r^2 \rangle_{A}^{1/2}$	0.537 fm	–	0.674 fm
μ_p	2.18	1.87	2.79
μ_n	-1.34	-1.31	-1.91
$\left\lvert \frac{\mu_p}{\mu_n} \right\rvert$	1.63	1.43	1.46
g_A	0.73	0.61	1.27
$g_{\pi NN}$	7.46	8.9	13.2
$g_{\rho NN}$	5.80	–	$4.2 \sim 6.5$