

# Magnetic monopoles in high temperature QCD

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GGI workshop, Florence

# Outline

- 1 Magnetic monopoles in lattice QCD
- 2 Results
  - Monopole-(anti)monopole correlation function
  - Monopole density
- 3 Open problems
  - The gauge dependence problem
  - The Gribov ambiguity

# Motivation

Abelian **magnetic monopoles** are candidates for explaining color confinement within the dual superconducting model of the QCD vacuum (confinement is induced by the breaking of a magnetic  $U(1)$  symmetry via monopole condensation).

The magnetic component is supposed to be relevant (Chernodub & Zakharov '06, Liao & Shuryak '06) in explaining the physical properties of the Quark Gluon Plasma phase (above the transition).

It has been identified (Chernodub & Zakharov '06) with abelian magnetic monopoles “evaporating” from the condensate at  $T > T_c$ .

# The Abelian Projection

How can we get abelian monopoles from a non abelian theory such as QCD?

- First we fix a gauge that leaves a  $U(1)$  residual symmetry: in the Maximal Abelian Gauge we maximize

$$F_{\text{MAG}} = \sum_{\mu, x} \text{Re tr} \left[ U_{\mu}(x) \sigma_3 U_{\mu}^{\dagger}(x) \sigma_3 \right]$$

- Then we take the diagonal part of the links (Abelian Projection)

Possible dependence of the abelian observables on the gauge fixed prior the projection!!!

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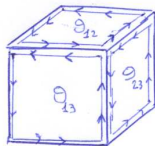
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# De Grand-Toussaint



De Grand elementary cube (in 3D)

On abelian projected configurations  
monopole currents are defined as

$$m_\mu = \frac{1}{2\pi} \varepsilon_{\mu\nu\rho\sigma} \hat{\partial}_\nu \bar{\theta}_{\rho\sigma}$$

where  $\bar{\theta}_{\rho\sigma}$  is the compactified  
part of the abelian plaquette  
phase (De Grand & Toussaint '80).

- Quantization of charge
- Closure of monopole currents:  $\hat{\partial}_\mu m_\mu = 0$



# The thermal monopole density

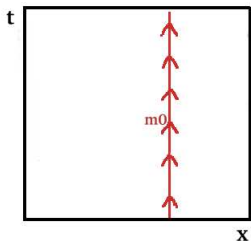
At  $T < T_c$  magnetic currents are virtual;

At  $T > T_c$  currents and monopoles become real (magnetic currents percolate in temporal direction).

Real particle = wrapped trajectory on the compact  $t$  direction  
 (Chernodub & Zakharov '07).

$$\rho = \frac{\sum_{\vec{x}} |N_{wrap}(m_0(\vec{x}, t))|}{V_s}$$

$m_0(\vec{x}, t)$  = magnetic trajectory  
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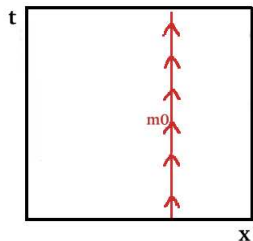
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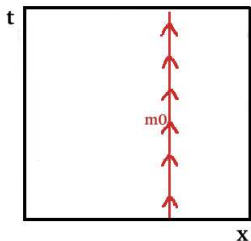
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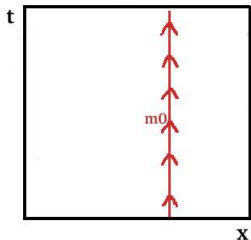
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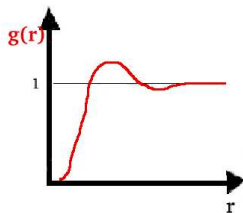
# The monopole-(anti)monopole correlation function

$$g(r) = \frac{\langle \rho(0)\rho(r) \rangle}{\langle \rho \rangle \langle \rho \rangle} \text{ (monopole-monopole)}$$

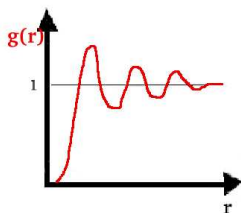
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$$g(r) = 1$$

$g(r)$ -free gas



$g(r)$ -liquid



$g(r)$ -solid

- $g(r) = 1 \Rightarrow$  no interaction
- If the interaction potential  $V(r)$  is weak we can extract it through  $g(r) = \exp(-V(r)/T)$ .

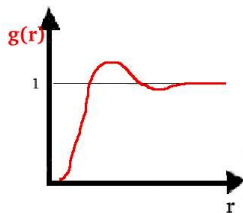
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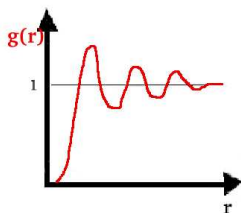
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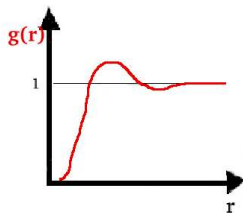
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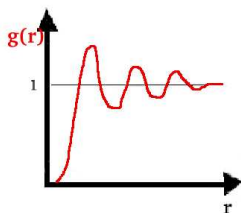
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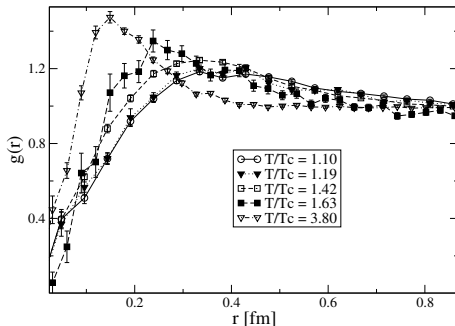
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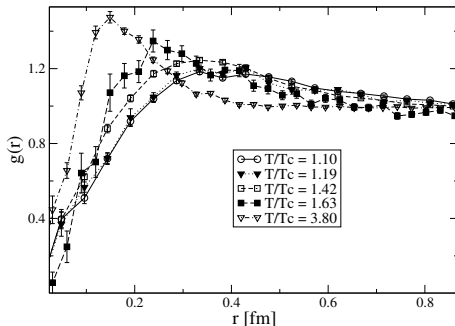
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- Fit with screened Coulomb  $V(r) = \alpha_M e^{-r/\lambda}/r$ ,  $\lambda \sim 0.2$  fm;
- Liquid-like structure!  
Stronger  $\alpha_M$  coupling at high  $T$  (Liao & Shuryak '07);
- Agreement with MD simulation of std. EM plasma (Liao & Shuryak '07)

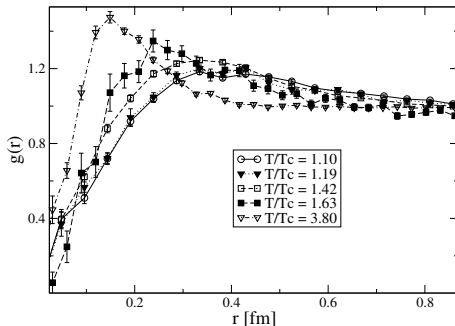


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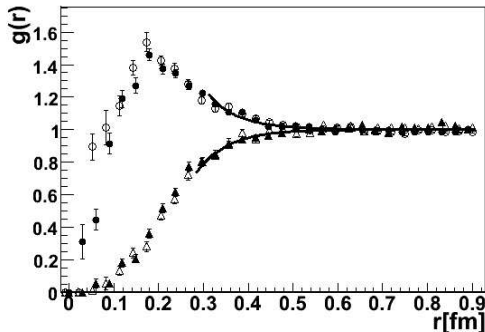
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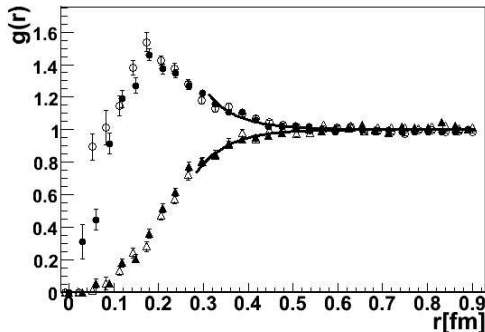
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Monopole-monopole (triangles) Vs. Monopole-antimonopole (circles) at different  $\beta$ 's

- Monopoles repel monopoles and attract antimonopoles;
- The scaling is good.

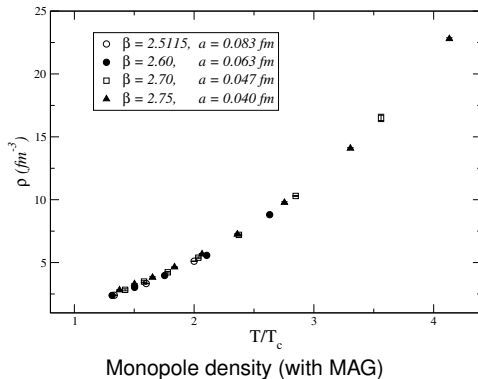
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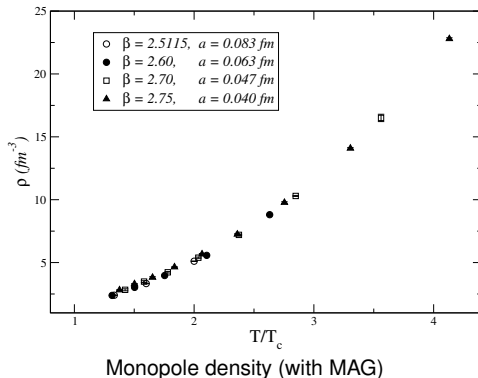


- $\rho \neq \frac{\zeta(3)}{\pi^2} T^3$  (free particles)  $\Rightarrow$  interactions are important!!!

Nice fit with  $\rho \sim T^3 / (\log(T/\Lambda_{\text{eff}}))^\alpha$  with  $\alpha \sim 2 - 3$

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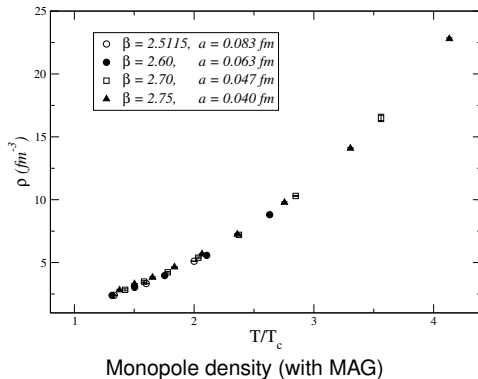


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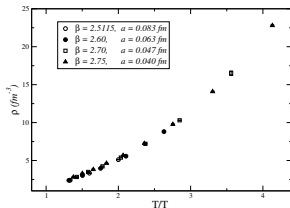
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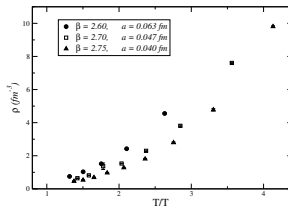
- In the Landau gauge, defined by maximizing  $F_L = \sum_{\mu, x} \text{Re tr } U_{\mu}(x)$  before the Abelian projection, the monopole density is compatible with zero.



# The Gribov ambiguity



Monopole density (MAG)



Monopole density (Landau preconditioning + MAG)

**Within the same MAG gauge** we start the gauge fixing iterative algorithm from a Landau gauged configuration: the density is now different and the scaling is lost. We are on a different local maximum of  $F_{\text{MAG}}$ .

A similar behavior was observed for vortices in center dominance studies (Bornyakov et al. '96, Kovacs & Tomboulis '99, Greensite et al. '01)

# Summary

- We measured the density of thermal monopoles in the deconfined phase. An interacting behavior is observed, as  $\rho \approx T^3$  (precisely  $\rho \sim T^3 / (\log(T/\Lambda_{eff}))^\alpha$  with  $\alpha \sim 2$ , even 3 at high T)
- We observed the monopole-(anti)monopole correlation function. A liquid-like behavior is observed.
- A very good physical scaling is observed for monopoles obtained with the standard Maximal Abelian Gauge;
- Physical properties, like contribution to QGP yet to be studied (see [Chernodub et al. PosLAT07](#)).

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