Entanglement Entropy in XYZ model

Francesco Ravanini

ALMA MATER STUDIORUM UNIVERSITÀ DI BOLOGNA

INFN Istituto Nazionale di Fisica Nucleare

Collaboration with **Elisa Ercolessi** *and* **Stefano Evangelisti**

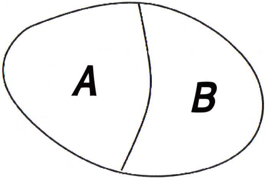
GGI Firenze, 4 sep 2008

Entanglement

- Consider a quantum system (e.g. a 1d quantum spin chain) in a pure state $|\psi\rangle$, whose density matrix is $\rho = |\psi\rangle\langle\psi|$.
- Divide the whole system into two noninteracting subsystems, A and its complement B. The Hilbert space then separates into two non-interacting parts

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$

- Suppose to do separated measures on each subsystem

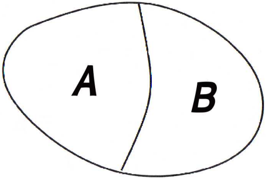


Entanglement

- Consider a quantum system (e.g. a 1d quantum spin chain) in a pure state $|\psi\rangle$, whose density matrix is $\rho = |\psi\rangle\langle\psi|$.
- Divide the whole system into two noninteracting subsystems, A and its complement B. The Hilbert space then separates into two non-interacting parts

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$

- Suppose to do separated measures on each subsystem

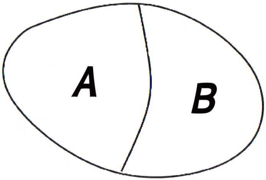


Entanglement

- Consider a quantum system (e.g. a 1d quantum spin chain) in a pure state $|\psi\rangle$, whose density matrix is $\rho = |\psi\rangle\langle\psi|$.
- Divide the whole system into two noninteracting subsystems, A and its complement B. The Hilbert space then separates into two non-interacting parts

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$

- Suppose to do separated measures on each subsystem



Separable vs. Entangled states

- States that can be written as $|\psi\rangle = |\psi_A\rangle \otimes |\psi_B\rangle$ are called separable
- Not all states are separable

$$\left.\begin{array}{ll} \text{Basis in } \mathcal{H}_\mathrm{A} & \{|i_A\rangle\} \\ \text{Basis in } \mathcal{H}_B & \{|j_B\rangle\} \end{array}\right\} \Longrightarrow \text{Basis in } \mathcal{H} \quad \{|i_A\rangle \otimes |j_B\rangle\}$$

- Generic state in $\mathcal{H}$

$$|\psi\rangle = \sum_{ij} a_{ij} |i_A\rangle \otimes |j_B\rangle$$

- Non separable states are called entangled

Separable vs. Entangled states

- States that can be written as $|\psi\rangle = |\psi_A\rangle \otimes |\psi_B\rangle$ are called separable
- Not all states are separable

$$\left.\begin{array}{ll} \text{Basis in } \mathcal{H}_\mathrm{A} & \{|i_A\rangle\} \\ \text{Basis in } \mathcal{H}_B & \{|j_B\rangle\} \end{array}\right\} \Longrightarrow \text{Basis in } \mathcal{H} \quad \{|i_A\rangle \otimes |j_B\rangle\}$$

- Generic state in $\mathcal{H}$

$$|\psi\rangle = \sum_{ij} a_{ij} |i_A\rangle \otimes |j_B\rangle$$

- Non separable states are called entangled

Separable vs. Entangled states

- States that can be written as $|\psi\rangle = |\psi_A\rangle \otimes |\psi_B\rangle$ are called separable
- Not all states are separable

$$\left.\begin{array}{ll} \text{Basis in } \mathcal{H}_\mathrm{A} & \{|i_A\rangle\} \\ \text{Basis in } \mathcal{H}_B & \{|j_B\rangle\} \end{array}\right\} \Longrightarrow \text{Basis in } \mathcal{H} \quad \{|i_A\rangle \otimes |j_B\rangle\}$$

- Generic state in $\mathcal{H}$

$$|\psi\rangle = \sum_{ij} a_{ij} |i_A\rangle \otimes |j_B\rangle$$

- Non separable states are called entangled

Separable vs. Entangled states

- States that can be written as $|\psi\rangle = |\psi_A\rangle \otimes |\psi_B\rangle$ are called separable
- Not all states are separable

$$\left.\begin{array}{ll} \text{Basis in } \mathcal{H}_\mathrm{A} & \{|i_A\rangle\} \\ \text{Basis in } \mathcal{H}_B & \{|j_B\rangle\} \end{array}\right\} \Longrightarrow \text{Basis in } \mathcal{H} \quad \{|i_A\rangle \otimes |j_B\rangle\}$$

- Generic state in $\mathcal{H}$

$$|\psi\rangle = \sum_{ij} a_{ij} |i_A\rangle \otimes |j_B\rangle$$

- Non separable states are called entangled

Observers and measures

- In subsystems A and B we have two observers

  (resp. ALICE  and BOB ) each capable of doing measures on his/her subsystem only

- Consider state $|\chi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

If Alice measures	then $\vert\chi\rangle$ collapses to	and Bob measures
$\vert\uparrow\rangle$	$\vert\uparrow\downarrow\rangle$	$\vert\downarrow\rangle$
$\vert\downarrow\rangle$	$\vert\downarrow\uparrow\rangle$	$\vert\uparrow\rangle$

- Bob measure is affected by Alice's one
- NON LOCALITY intrinsic in Quantum Mechanics?
- EPR paradox (Einstein, Podolsky, Rosen 1935) (solved by no-communication theorem)

Observers and measures

- In subsystems A and B we have two observers

  (resp. ALICE  and BOB ) each capable of doing measures on his/her subsystem only

- Consider state $|\chi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

If Alice measures	then $|\chi\rangle$ collapses to	and Bob measures
$|\uparrow\rangle$	$|\uparrow\downarrow\rangle$	$|\downarrow\rangle$
$|\downarrow\rangle$	$|\downarrow\uparrow\rangle$	$|\uparrow\rangle$

- Bob measure is affected by Alice's one
- NON LOCALITY intrinsic in Quantum Mechanics?
- EPR paradox (Einstein, Podolsky, Rosen 1935) (solved by no-communication theorem)

Observers and measures

- In subsystems A and B we have two observers

(resp. ALICE  and BOB ) each capable of doing measures on his/her subsystem only

- Consider state $|\chi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

If Alice measures	then $\|\chi\rangle$ collapses to	and Bob measures
$\|\uparrow\rangle$	$\|\uparrow\downarrow\rangle$	$\|\downarrow\rangle$
$\|\downarrow\rangle$	$\|\downarrow\uparrow\rangle$	$\|\uparrow\rangle$

- Bob measure is affected by Alice's one
- NON LOCALITY intrinsic in Quantum Mechanics?
- EPR paradox (Einstein, Podolsky, Rosen 1935) (solved by no-communication theorem)

Ensembles: pure vs. mixed states

- System with multiple subsystems (e.g. multiparticle states, or spin chains)
$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \ldots \otimes \mathcal{H}_N$$
For example, in XXX, XXZ, XYZ chains each $\mathcal{H}_i = \mathbb{C}^2$
- State $|\psi\rangle = |\alpha\rangle \otimes |\alpha\rangle \otimes \ldots \otimes |\alpha\rangle$ with all α equal is a pure ensemble
- State with more pure "1 particle" states $|\alpha_1\rangle, \ldots, |\alpha_n\rangle$ is said a mixed ensemble. Each $|\alpha_k\rangle$ appears with a percentage frequence ω_k in the ensemble ($\sum_k \omega_k = 1$)
- States $|\alpha_k\rangle$ need not be orthogonal and may exceed $\dim \mathcal{H}$ in number
- A state is pure iff all $\omega_k = 0$ but one, which is equal to 1

Ensembles: pure vs. mixed states

- System with multiple subsystems (e.g. multiparticle states, or spin chains)

$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \ldots \otimes \mathcal{H}_N$$

For example, in XXX, XXZ, XYZ chains each $\mathcal{H}_i = \mathbb{C}^2$

- State $|\psi\rangle = |\alpha\rangle \otimes |\alpha\rangle \otimes \ldots \otimes |\alpha\rangle$ with all α equal is a pure ensemble

- State with more pure "1 particle" states $|\alpha_1\rangle, \ldots, |\alpha_n\rangle$ is said a mixed ensemble. Each $|\alpha_k\rangle$ appears with a percentage frequence ω_k in the ensemble ($\sum_k \omega_k = 1$)

- States $|\alpha_k\rangle$ need not be orthogonal and may exceed $\dim \mathcal{H}$ in number

- A state is pure iff all $\omega_k = 0$ but one, which is equal to 1

Ensembles: pure vs. mixed states

- System with multiple subsystems (e.g. multiparticle states, or spin chains)

$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \ldots \otimes \mathcal{H}_N$$

  For example, in XXX, XXZ, XYZ chains each $\mathcal{H}_i = \mathbb{C}^2$

- State $|\psi\rangle = |\alpha\rangle \otimes |\alpha\rangle \otimes \ldots \otimes |\alpha\rangle$ with all α equal is a pure ensemble

- State with more pure "1 particle" states $|\alpha_1\rangle, \ldots, |\alpha_n\rangle$ is said a mixed ensemble. Each $|\alpha_k\rangle$ appears with a percentage frequence ω_k in the ensemble ($\sum_k \omega_k = 1$)

- States $|\alpha_k\rangle$ need not be orthogonal and may exceed $\dim \mathcal{H}$ in number

- A state is pure iff all $\omega_k = 0$ but one, which is equal to 1

Ensembles: pure vs. mixed states

- System with multiple subsystems (e.g. multiparticle states, or spin chains)

$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \ldots \otimes \mathcal{H}_N$$

  For example, in XXX, XXZ, XYZ chains each $\mathcal{H}_i = \mathbb{C}^2$

- State $|\psi\rangle = |\alpha\rangle \otimes |\alpha\rangle \otimes \ldots \otimes |\alpha\rangle$ with all α equal is a pure ensemble

- State with more pure "1 particle" states $|\alpha_1\rangle, \ldots, |\alpha_n\rangle$ is said a mixed ensemble. Each $|\alpha_k\rangle$ appears with a percentage frequence ω_k in the ensemble ($\sum_k \omega_k = 1$)

- States $|\alpha_k\rangle$ need not be orthogonal and may exceed $\dim \mathcal{H}$ in number
- A state is pure iff all $\omega_k = 0$ but one, which is equal to 1

Density matrix

- An observable A has expectation value

$$\langle A \rangle = \sum_i \omega_k \langle \alpha_k | A | \alpha_k \rangle$$

the average is taken over both statistical and quantum fluctuations

- Define density matrix (Von Neumann 1927)

$$\rho = \sum_k \omega_k |\alpha_k\rangle\langle\alpha_k|$$

- then $\langle A \rangle$ is expressed by

$$\langle A \rangle = \mathrm{Tr}(\rho A)$$

- Von Neumann entropy

$$\boxed{S = -\mathrm{Tr}(\rho \log \rho)}$$

Density matrix

- An observable A has expectation value

$$\langle A \rangle = \sum_i \omega_k \langle \alpha_k | A | \alpha_k \rangle$$

the average is taken over both statistical and quantum fluctuations

- Define density matrix (Von Neumann 1927)

$$\rho = \sum_k \omega_k |\alpha_k\rangle\langle\alpha_k|$$

- then $\langle A \rangle$ is expressed by

$$\langle A \rangle = \mathrm{Tr}(\rho A)$$

- Von Neumann entropy

$$\boxed{S = -\mathrm{Tr}(\rho \log \rho)}$$

Density matrix

- An observable A has expectation value

$$\langle A \rangle = \sum_i \omega_k \langle \alpha_k | A | \alpha_k \rangle$$

the average is taken over both statistical and quantum fluctuations

- Define density matrix (Von Neumann 1927)

$$\rho = \sum_k \omega_k | \alpha_k \rangle \langle \alpha_k |$$

- then $\langle A \rangle$ is expressed by

$$\langle A \rangle = \mathrm{Tr}(\rho A)$$

- Von Neumann entropy

$$\boxed{S = -\mathrm{Tr}(\rho \log \rho)}$$

Density matrix and entanglement

- A (pure) state in a quantum system is described by a ray (vector of norm 1) in its Hilbert space $\mathcal{H}$.
- However, if $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$ is composed of two subsystems, an observer (e.g. Alice) that sees only one of the subsystems (A) may have an uncomplete description of the state
- Alice defines a reduced density matrix of system A by tracing over the unobserved part B of the system: $\rho_A = \mathrm{Tr}_B \rho$
- Pure state in $\mathcal{H}$: $|\psi\rangle = \sum_{ij} a_{ij} |i_A\rangle \otimes |j_B\rangle$ has density matrix

$$\rho = |\psi\rangle\langle\psi| = \sum_{ij} |a_{ij}|^2 |i_A\rangle\langle i_A| \otimes |j_B\rangle\langle j_B|$$

- Alice takes the trace over B and sees the state as mixed

$$\rho_A = \sum_k p_k |i_A\rangle\langle i_A|$$

Density matrix and entanglement

- A (pure) state in a quantum system is described by a ray (vector of norm 1) in its Hilbert space $\mathcal{H}$.
- However, if $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$ is composed of two subsystems, an observer (e.g. Alice) that sees only one of the subsystems (A) may have an uncomplete description of the state
- Alice defines a reduced density matrix of system A by tracing over the unobserved part B of the system: $\rho_A = \mathrm{Tr}_B \rho$
- Pure state in $\mathcal{H}$: $|\psi\rangle = \sum_{ij} a_{ij} |i_A\rangle \otimes |j_B\rangle$ has density matrix

$$\rho = |\psi\rangle\langle\psi| = \sum_{ij} |a_{ij}|^2 |i_A\rangle\langle i_A| \otimes |j_B\rangle\langle j_B|$$

- Alice takes the trace over B and sees the state as mixed

$$\rho_A = \sum_k p_k |i_A\rangle\langle i_A|$$

Density matrix and entanglement

- A (pure) state in a quantum system is described by a ray (vector of norm 1) in its Hilbert space $\mathcal{H}$.
- However, if $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$ is composed of two subsystems, an observer (e.g. Alice) that sees only one of the subsystems (A) may have an uncomplete description of the state
- Alice defines a reduced density matrix of system A by tracing over the unobserved part B of the system: $\rho_A = \mathrm{Tr}_B \rho$
- Pure state in $\mathcal{H}$: $|\psi\rangle = \sum_{ij} a_{ij} |i_A\rangle \otimes |j_B\rangle$ has density matrix

$$\rho = |\psi\rangle\langle\psi| = \sum_{ij} |a_{ij}|^2 |i_A\rangle\langle i_A| \otimes |j_B\rangle\langle j_B|$$

- Alice takes the trace over B and sees the state as mixed

$$\rho_A = \sum_k p_k |i_A\rangle\langle i_A|$$

Density matrix and entanglement

- A (pure) state in a quantum system is described by a ray (vector of norm 1) in its Hilbert space $\mathcal{H}$.
- However, if $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$ is composed of two subsystems, an observer (e.g. Alice) that sees only one of the subsystems (A) may have an uncomplete description of the state
- Alice defines a reduced density matrix of system A by tracing over the unobserved part B of the system: $\rho_A = \mathrm{Tr}_B \rho$
- Pure state in $\mathcal{H}$: $|\psi\rangle = \sum_{ij} a_{ij} |i_A\rangle \otimes |j_B\rangle$ has density matrix

$$\rho = |\psi\rangle\langle\psi| = \sum_{ij} |a_{ij}|^2 |i_A\rangle\langle i_A| \otimes |j_B\rangle\langle j_B|$$

- Alice takes the trace over B and sees the state as mixed

$$\rho_A = \sum_k p_k |i_A\rangle\langle i_A|$$

Density matrix and entanglement

- A (pure) state in a quantum system is described by a ray (vector of norm 1) in its Hilbert space $\mathcal{H}$.
- However, if $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$ is composed of two subsystems, an observer (e.g. Alice) that sees only one of the subsystems (A) may have an uncomplete description of the state
- Alice defines a reduced density matrix of system A by tracing over the unobserved part B of the system: $\rho_A = \mathrm{Tr}_B \rho$
- Pure state in $\mathcal{H}$: $|\psi\rangle = \sum_{ij} a_{ij} |i_A\rangle \otimes |j_B\rangle$ has density matrix

$$\rho = |\psi\rangle\langle\psi| = \sum_{ij} |a_{ij}|^2 |i_A\rangle\langle i_A| \otimes |j_B\rangle\langle j_B|$$

- Alice takes the trace over B and sees the state as mixed

$$\rho_A = \sum_k p_k |i_A\rangle\langle i_A|$$

Entanglement entropy

- Alice sees only subsystem A, her definiton of the von Neumann entropy tracing out part B gives the entanglement entropy (EE)

$$S_A = -\mathrm{Tr}_A \rho_A \log \rho_A$$

- If $|\psi\rangle$ is the vacuum, EE is equal in both subsystems: $S_A = S_B$
- For a separable state $S_A = 0$
- S_A is maximal for a maximally entangled state: it is a measure of entanglement

Entanglement entropy

- Alice sees only subsystem A, her definiton of the von Neumann entropy tracing out part B gives the entanglement entropy (EE)

$$S_A = -\mathrm{Tr}_A \rho_A \log \rho_A$$

- If $|\psi\rangle$ is the vacuum, EE is equal in both subsystems: $S_A = S_B$
- For a separable state $S_A = 0$
- S_A is maximal for a maximally entangled state: it is a measure of entanglement

Entanglement entropy

- Alice sees only subsystem A, her definiton of the von Neumann entropy tracing out part B gives the entanglement entropy (EE)
$$S_A = -\mathrm{Tr}_A \rho_A \log \rho_A$$
- If $|\psi\rangle$ is the vacuum, EE is equal in both subsystems: $S_A = S_B$
- For a separable state $S_A = 0$
- S_A is maximal for a maximally entangled state: it is a measure of entanglement

Entanglement entropy

- Alice sees only subsystem A, her definiton of the von Neumann entropy tracing out part B gives the entanglement entropy (EE)
$$S_A = -\mathrm{Tr}_A \rho_A \log \rho_A$$
- If $|\psi\rangle$ is the vacuum, EE is equal in both subsystems: $S_A = S_B$
- For a separable state $S_A = 0$
- S_A is maximal for a maximally entangled state: it is a measure of entanglement

Lattice models

- Consider a square lattice with IRF. To each site i assign a spin σ_i and to each plaquette delimited by sites i, j, k, l Boltzmann weights

$$w(\sigma_i, \sigma_j, \sigma_k, \sigma_l) = \exp\{-\epsilon(\sigma_i, \sigma_j, \sigma_k, \sigma_l)/kT\}$$

- Total energy of the system

$$\mathcal{E} = \sum_{\square} \epsilon(\sigma_i, \sigma_j, \sigma_k, \sigma_l)$$

the sum is over all plaquettes (faces) of the lattice and i, j, k, l are the surrounding sites. The partition function is

$$\mathcal{Z} = \sum_{conf} \prod_{\square} w(\sigma_i, \sigma_j, \sigma_k, \sigma_l)$$

Lattice models

- Consider a square lattice with IRF. To each site i assign a spin σ_i and to each plaquette delimited by sites i, j, k, l Boltzmann weights

$$w(\sigma_i, \sigma_j, \sigma_k, \sigma_l) = \exp\{-\epsilon(\sigma_i, \sigma_j, \sigma_k, \sigma_l)/kT\}$$

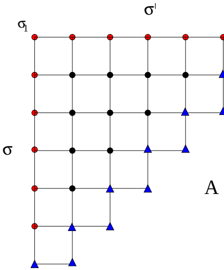
- Total energy of the system

$$\mathcal{E} = \sum_{\square} \epsilon(\sigma_i, \sigma_j, \sigma_k, \sigma_l)$$

the sum is over all plaquettes (faces) of the lattice and i, j, k, l are the surrounding sites. The partition function is

$$\mathcal{Z} = \sum_{conf} \prod_{\square} w(\sigma_i, \sigma_j, \sigma_k, \sigma_l)$$

- Consider the following quadrant of the whole lattice

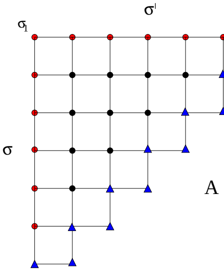


- Define the element of the Corner Transfer Matrix (CTM) as

$$A_{\bar{\sigma}\bar{\sigma}'} = \begin{cases} \sum\limits_{\bullet}\prod\limits_{\square} w(\sigma_i,\sigma_j,\sigma_k,\sigma_l) & \text{if} \quad \sigma_1 = \sigma'_1 \\ \\ = 0 & \text{if} \quad \sigma_1 \neq \sigma'_1 \end{cases}$$

where $\bar{\sigma} = (\sigma_1, ..., \sigma_m)$; $\bar{\sigma}' = (\sigma'_1, ..., \sigma'_m)$

- Consider the following quadrant of the whole lattice



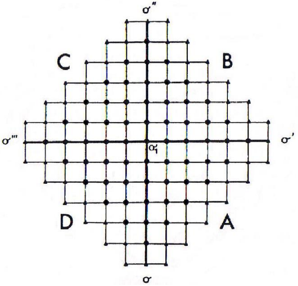
- Define the element of the Corner Transfer Matrix (CTM) as

$$A_{\bar{\sigma}\bar{\sigma}'} = \begin{cases} \sum_{\bullet} \prod_{\square} w(\sigma_i, \sigma_j, \sigma_k, \sigma_l) & \text{if} \quad \sigma_1 = \sigma_1' \\ = 0 & \text{if} \quad \sigma_1 \neq \sigma_1' \end{cases}$$

where $\bar{\sigma} = (\sigma_1, ..., \sigma_m)$; $\bar{\sigma}' = (\sigma_1', ..., \sigma_m')$

Partition function and CTM

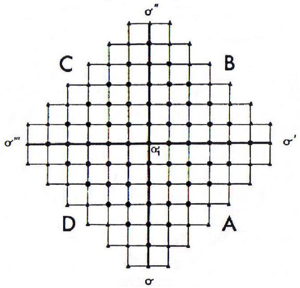
- Define $B_{\bar{\sigma}\bar{\sigma}'}$ in the same way as $A_{\bar{\sigma}\bar{\sigma}'}$ only with the last figure rotated anticlockwise by 90°. Similarly define $C_{\bar{\sigma}\bar{\sigma}'}$ and $D_{\bar{\sigma}\bar{\sigma}'}$ by rotating by 180° and 270°.
- Now we can build up the whole lattice by using the 4 CTM's



- Partition function

$$\mathcal{Z} = \sum_{\bar{\sigma},\bar{\sigma}',\bar{\sigma}'',\bar{\sigma}'''} A_{\bar{\sigma}\bar{\sigma}'} B_{\bar{\sigma}'\bar{\sigma}''} C_{\bar{\sigma}''\bar{\sigma}'''} D_{\bar{\sigma}'''\bar{\sigma}} = \mathrm{Tr}(ABCD)$$

- Define $B_{\bar{\sigma}\bar{\sigma}'}$ in the same way as $A_{\bar{\sigma}\bar{\sigma}'}$ only with the last figure rotated anticlockwise by 90°. Similarly define $C_{\bar{\sigma}\bar{\sigma}'}$ and $D_{\bar{\sigma}\bar{\sigma}'}$ by rotating by 180° and 270°.
- Now we can build up the whole lattice by using the 4 CTM's



- Partition function

$$\mathcal{Z} = \sum_{\bar{\sigma},\bar{\sigma}',\bar{\sigma}'',\bar{\sigma}'''} A_{\bar{\sigma}\bar{\sigma}'} B_{\bar{\sigma}'\bar{\sigma}''} C_{\bar{\sigma}''\bar{\sigma}'''} D_{\bar{\sigma}'''\bar{\sigma}} = \mathrm{Tr}(ABCD)$$

Density matrix and corner transfer matrix I

- Quantum spin chain with L sites, Hamiltonian H and ground state $|0\rangle$. Vacuum wave function $\langle\bar{\sigma}|0\rangle = \psi_0(\bar{\sigma})$. Density matrix $\rho = |0\rangle|\langle 0|$.
- Matrix element (assume ψ_0 real)

$$\rho(\bar{\sigma}, \bar{\sigma}') = \langle\bar{\sigma}|0\rangle\langle 0|\bar{\sigma}'\rangle = \psi_0(\bar{\sigma})\,\psi_0(\bar{\sigma}')$$

- Suppose there is a relation between this quantum chain of hamiltonian H and a classical spin lattice model of row to row transfer matrix T in the sense that $[H, T] = 0$
- Then the ground state of H is the eignestate with highest eigenvalue of T.

- Quantum spin chain with L sites, Hamiltonian H and ground state $|0\rangle$. Vacuum wave function $\langle\bar{\sigma}|0\rangle = \psi_0(\bar{\sigma})$. Density matrix $\rho = |0\rangle|\langle 0|$.
- Matrix element (assume ψ_0 real)

$$\rho(\bar{\sigma},\bar{\sigma}') = \langle\bar{\sigma}|0\rangle\langle 0|\bar{\sigma}'\rangle = \psi_0(\bar{\sigma})\,\psi_0(\bar{\sigma}')$$

- Suppose there is a relation between this quantum chain of hamiltonian H and a classical spin lattice model of row to row transfer matrix T in the sense that $[H,T]=0$
- Then the ground state of H is the eignestate with highest eigenvalue of T.

Density matrix and corner transfer matrix I

- Quantum spin chain with L sites, Hamiltonian H and ground state $|0\rangle$. Vacuum wave function $\langle\bar{\sigma}|0\rangle = \psi_0(\bar{\sigma})$. Density matrix $\rho = |0\rangle|\langle 0|$.
- Matrix element (assume ψ_0 real)

$$\rho(\bar{\sigma}, \bar{\sigma}') = \langle\bar{\sigma}|0\rangle\langle 0|\bar{\sigma}'\rangle = \psi_0(\bar{\sigma})\,\psi_0(\bar{\sigma}')$$

- Suppose there is a relation between this quantum chain of hamiltonian H and a classical spin lattice model of row to row transfer matrix T in the sense that $[H, T] = 0$
- Then the ground state of H is the eignestate with highest eigenvalue of T.

- Consider a vector $|\psi\rangle \in \mathcal{H}$ Hilbert space of H (or of T)

$$|\psi\rangle = |0\rangle + \sum_{k\neq 0} c_k |k\rangle$$

where $|k\rangle$ are the excited states of H with T eignevalues λ_k.

- Apply N times the operator T to such vector

$$T^N|\psi\rangle = \lambda_0^N \left(|0\rangle + \sum_k \left(\frac{\lambda_k}{\lambda_0}\right)^N c_k|k\rangle \right)$$

- In the limit $N \to \infty$

$$T^N|\psi\rangle \sim \lambda_0^N|0\rangle \qquad \text{or} \qquad \langle\bar{\sigma}|0\rangle \sim \lambda\langle\bar{\sigma}|T^N|\psi\rangle$$

i.e. $\psi_0(\bar{\sigma})$ is the partition function evolving the model from an initial $|\bar{\sigma}\rangle$ to a final $|0\rangle$and $\rho(\bar{\sigma}, \bar{\sigma}')$ is a product of two semi-infinite partition functions evolving the system from $\bar{\sigma}$ to $+\infty$ and from $\bar{\sigma}'$ to $-\infty$.

Density matrix and CTM

- Consider a vector $|\psi\rangle \in \mathcal{H}$ Hilbert space of H (or of T)

$$|\psi\rangle = |0\rangle + \sum_{k\neq 0} c_k |k\rangle$$

where $|k\rangle$ are the excited states of H with T eignevalues λ_k.

- Apply N times the operator T to such vector

$$T^N|\psi\rangle = \lambda_0^N \left(|0\rangle + \sum_k \left(\frac{\lambda_k}{\lambda_0}\right)^N c_k |k\rangle \right)$$

- In the limit $N \to \infty$

$$T^N|\psi\rangle \sim \lambda_0^N|0\rangle \qquad \text{or} \qquad \langle\bar\sigma|0\rangle \sim \lambda\langle\bar\sigma|T^N|\psi\rangle$$

i.e. $\psi_0(\bar\sigma)$ is the partition function evolving the model from an initial $|\bar\sigma\rangle$ to a final $|0\rangle$and $\rho(\bar\sigma,\bar\sigma')$ is a product of two semi-infinite partition functions evolving the system from $\bar\sigma$ to $+\infty$ and from $\bar\sigma'$ to $-\infty$.

Density matrix and CTM

- Consider a vector $|\psi\rangle \in \mathcal{H}$ Hilbert space of H (or of T)

$$|\psi\rangle = |0\rangle + \sum_{k \neq 0} c_k |k\rangle$$

where $|k\rangle$ are the excited states of H with T eignevalues λ_k.

- Apply N times the operator T to such vector

$$T^N|\psi\rangle = \lambda_0^N \left(|0\rangle + \sum_k \left(\frac{\lambda_k}{\lambda_0}\right)^N c_k |k\rangle \right)$$

- In the limit $N \to \infty$

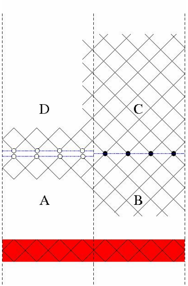
$$T^N|\psi\rangle \sim \lambda_0^N|0\rangle \qquad \text{or} \qquad \langle\bar{\sigma}|0\rangle \sim \lambda\langle\bar{\sigma}|T^N|\psi\rangle$$

i.e. $\psi_0(\bar{\sigma})$ is the partition function evolving the model from an initial $|\bar{\sigma}\rangle$ to a final $|0\rangle$and $\rho(\bar{\sigma}, \bar{\sigma}')$ is a product of two semi-infinite partition functions evolving the system from $\bar{\sigma}$ to $+\infty$ and from $\bar{\sigma}'$ to $-\infty$.

Reduced density matrix and CTM

- Now suppose to divide the spins in two subsystems A: $\bar{\sigma}_A = (\sigma_1, ..., \sigma_p)$ and B: $\bar{\sigma}_B = (\sigma_{p+1}, ..., \sigma_L)$, i.e. $\bar{\sigma} = (\bar{\sigma}_A, \bar{\sigma}_B)$
- Reduced density matrix of subsystem A (entanglement density matrix)

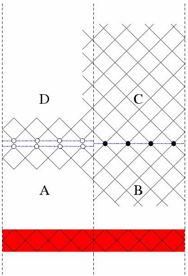
$$\rho_A(\bar{\sigma}_A, \bar{\sigma}'_A) = \sum_{\bar{\sigma}_B} \psi_0(\bar{\sigma}_A, \bar{\sigma}_B)\, \psi_0(\bar{\sigma}'_A, \bar{\sigma}_B)$$



Reduced density matrix and CTM

- Now suppose to divide the spins in two subsystems A: $\bar{\sigma}_A = (\sigma_1, ..., \sigma_p)$ and B: $\bar{\sigma}_B = (\sigma_{p+1}, ..., \sigma_L)$, i.e. $\bar{\sigma} = (\bar{\sigma}_A, \bar{\sigma}_B)$
- Reduced density matrix of subsystem A (entanglement density matrix)

$$\rho_A(\bar{\sigma}_A, \bar{\sigma}'_A) = \sum_{\bar{\sigma}_B} \psi_0(\bar{\sigma}_A, \bar{\sigma}_B)\, \psi_0(\bar{\sigma}'_A, \bar{\sigma}_B)$$



Reduced density matrix and EE

- The unnormalized reduced density matrix is

$$\hat{\rho}_A = (ABCD)_{\bar{\sigma},\bar{\sigma}'}$$

- Normalization by dividing by the trace

$$\rho_A = \frac{\hat{\rho}_A}{\mathrm{Tr}\hat{\rho}_A}$$

- Entanglement entropy

$$S_A = -\mathrm{Tr}\rho_A \log \rho_A = -\mathrm{Tr}\frac{\hat{\rho}_A \log \hat{\rho}_A}{\mathrm{Tr}\hat{\rho}_A} + \mathrm{Tr}\hat{\rho}_A$$

Reduced density matrix and EE

- The unnormalized reduced density matrix is

$$\hat{\rho}_A = (ABCD)_{\bar{\sigma},\bar{\sigma}'}$$

- Normalization by dividing by the trace

$$\rho_A = \frac{\hat{\rho}_A}{\mathrm{Tr}\hat{\rho}_A}$$

- Entanglement entropy

$$S_A = -\mathrm{Tr}\rho_A \log \rho_A = -\mathrm{Tr}\frac{\hat{\rho}_A \log \hat{\rho}_A}{\mathrm{Tr}\hat{\rho}_A} + \mathrm{Tr}\hat{\rho}_A$$

- Hamiltonian

$$
\begin{aligned}
H_{XYZ} &= -\sum_k (J_x \sigma_k^x \sigma_{k+1}^x + J_y \sigma_k^y \sigma_{k+1}^y + J_z \sigma_k^z \sigma_{k+1}^z) \\
&= -J \sum_k (\sigma_k^x \sigma_{k+1}^x + \Gamma \sigma_k^y \sigma_{k+1}^y + \Delta \sigma_k^z \sigma_{k+1}^z)
\end{aligned}
$$

- for $J_x = J_y = J_z$ (or $\Gamma = \Delta = 1$) it gives XXX chain
- for $J_x = J_y = 0$ gives Ising quantum chain
- for $J_x = J_y$ (or $\Gamma = 1$) gives XXZ chain

- it can also be seen as a particulary interesting lattice regularization of the sine-Gordon model

XYZ model

- Hamiltonian

$$
\begin{aligned}
H_{XYZ} &= -\sum_k (J_x \sigma_k^x \sigma_{k+1}^x + J_y \sigma_k^y \sigma_{k+1}^y + J_z \sigma_k^z \sigma_{k+1}^z) \\
&= -J \sum_k (\sigma_k^x \sigma_{k+1}^x + \Gamma \sigma_k^y \sigma_{k+1}^y + \Delta \sigma_k^z \sigma_{k+1}^z)
\end{aligned}
$$

- for $J_x = J_y = J_z$ (or $\Gamma = \Delta = 1$) it gives XXX chain
- for $J_x = J_y = 0$ gives Ising quantum chain
- for $J_x = J_y$ (or $\Gamma = 1$) gives XXZ chain

- it can also be seen as a particulary interesting lattice regularization of the sine-Gordon model

XYZ model

- Hamiltonian

$$\begin{aligned} H_{XYZ} &= -\sum_k (J_x \sigma_k^x \sigma_{k+1}^x + J_y \sigma_k^y \sigma_{k+1}^y + J_z \sigma_k^z \sigma_{k+1}^z) \\ &= -J \sum_k (\sigma_k^x \sigma_{k+1}^x + \Gamma \sigma_k^y \sigma_{k+1}^y + \Delta \sigma_k^z \sigma_{k+1}^z) \end{aligned}$$

- for $J_x = J_y = J_z$ (or $\Gamma = \Delta = 1$) it gives XXX chain
- for $J_x = J_y = 0$ gives Ising quantum chain
- for $J_x = J_y$ (or $\Gamma = 1$) gives XXZ chain

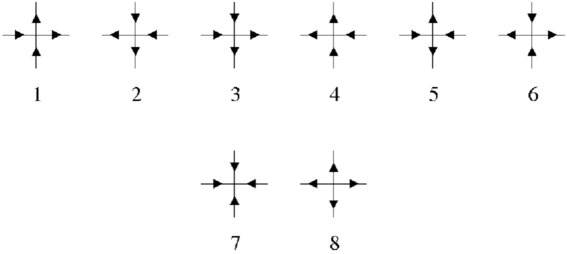
- it can also be seen as a particulary interesting lattice regularization of the sine-Gordon model

8-vertex model

- XYZ is the hamiltonian limit of 8-vertex model, with partition function

$$Z = \sum \prod_{i=1}^{8} w_i^{n_i}$$

where the 8 Boltzmann weights $w_i = e^{-\beta\epsilon_i}$ appear n_i times each on the lattice.



$$w_1 = w_2 = a,\ w_3 = w_4 = b,\ w_5 = w_6 = c,\ w_7 = w_8 = d$$

Transfer matrix of 8-vertex

- Square lattice with M rows and N columns with periodic b.c. The vertical 8-vertex variables $t_i = \uparrow, \downarrow$ and the horizontal ones $s_j = \rightarrow, \leftarrow$ live on the links.
- Denote a row of arrows $\phi_r = (t_1, t_2, ..., t_N)$ $(r = 1...M)$. Row-to-row transfer matrix

$$T(\phi, \phi') = \prod_{n=1}^{N} w \begin{pmatrix} & t'_n & \\ s_n & & s_{n+1} \\ & t_n & \end{pmatrix}$$

can be diagonalized by Bethe ansatz (Baxter)
- The partition function is

$$Z = \prod_{r=1}^{M} T(\phi_r, \phi_{r+1})$$

- This can be generalized to nontrivial b.c. by the introduction of suitable double row tranfer matrix

Transfer matrix of 8-vertex

- Square lattice with M rows and N columns with periodic b.c. The vertical 8-vertex variables $t_i = \uparrow, \downarrow$ and the horizontal ones $s_j = \rightarrow, \leftarrow$ live on the links.
- Denote a row of arrows $\phi_r = (t_1, t_2, ..., t_N)$ $(r = 1...M)$.
  Row-to-row transfer matrix

$$T(\phi, \phi') = \prod_{n=1}^{N} w \begin{pmatrix} & t'_n & \\ s_n & & s_{n+1} \\ & t_n & \end{pmatrix}$$

  can be diagonalized by Bethe ansatz (Baxter)
- The partition function is

$$Z = \prod_{r=1}^{M} T(\phi_r, \phi_{r+1})$$

- This can be generalized to nontrivial b.c. by the introduction of suitable double row tranfer matrix

Transfer matrix of 8-vertex

- Square lattice with M rows and N columns with periodic b.c. The vertical 8-vertex variables $t_i = \uparrow, \downarrow$ and the horizontal ones $s_j = \rightarrow, \leftarrow$ live on the links.
- Denote a row of arrows $\phi_r = (t_1, t_2, ..., t_N)$ $(r = 1...M)$. Row-to-row transfer matrix

$$T(\phi, \phi') = \prod_{n=1}^{N} w \begin{pmatrix} & t'_n & \\ s_n & & s_{n+1} \\ & t_n & \end{pmatrix}$$

  can be diagonalized by Bethe ansatz (Baxter)
- The partition function is

$$Z = \prod_{r=1}^{M} T(\phi_r, \phi_{r+1})$$

- This can be generalized to nontrivial b.c. by the introduction of suitable double row tranfer matrix

Transfer matrix of 8-vertex

- Square lattice with M rows and N columns with periodic b.c. The vertical 8-vertex variables $t_i = \uparrow, \downarrow$ and the horizontal ones $s_j = \rightarrow, \leftarrow$ live on the links.
- Denote a row of arrows $\phi_r = (t_1, t_2, ..., t_N)$ $(r = 1...M)$. Row-to-row transfer matrix

$$T(\phi, \phi') = \prod_{n=1}^{N} w \left(\begin{array}{ccc} & t'_n & \\ s_n & & s_{n+1} \\ & t_n & \end{array} \right)$$

can be diagonalized by Bethe ansatz (Baxter)

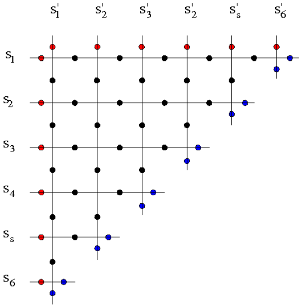
- The partition function is

$$Z = \prod_{r=1}^{M} T(\phi_r, \phi_{r+1})$$

- This can be generalized to nontrivial b.c. by the introduction of suitable double row tranfer matrix

CTM of 8-vertex

- CTM is defined with a slight modification w.r.t. the IRF models. There is no common spin on the two edges



$$A_{\bar{s},\bar{s}'} = \sum_{\bullet} \prod w_i$$

- and analogously B, C, D with 90° rotations. One can prove that $A = C$ and $B = D$.

Elliptic parametrization

- A convenient parametrization of the Boltzmann weights

$$\begin{aligned} a &= \rho\, \mathrm{snh}(\lambda - u) \\ b &= \rho\, \mathrm{snh}\, u \\ c &= \rho\, \mathrm{snh}\, \lambda \\ d &= \rho\, k\, \mathrm{snh}\lambda\, \mathrm{snh}u\, \mathrm{snh}(\lambda - u) \end{aligned}$$

- In this parametrization ($\mathrm{snh}x = -i\,\mathrm{sn}ix$, etc...)

$$\Gamma = \frac{1 - k^2 \mathrm{snh}^2\lambda}{1 + k^2 \mathrm{snh}^2\lambda} \quad , \quad \Delta = -\frac{\mathrm{cnh}\lambda\, \mathrm{dnh}\lambda}{1 + k^2 \mathrm{snh}^2\lambda}$$

- Phases:
  - ferroelettric order for $a > b + c + d$, $\Delta > 1$
  - ferroelettric order for $b > a + c + d$, $\Delta > 1$
  - disorder for $a, b, c, d < \frac{1}{2}(a + b + c + d)$, $-1 < \Delta < 1$

Elliptic parametrization

- A convenient parametrization of the Boltzmann weights

$$\begin{aligned} a &= \rho\,\mathrm{snh}(\lambda - u) \\ b &= \rho\,\mathrm{snh} u \\ c &= \rho\,\mathrm{snh}\lambda \\ d &= \rho\, k\,\mathrm{snh}\lambda\,\mathrm{snh} u\,\mathrm{snh}(\lambda - u) \end{aligned}$$

- In this parametrization ($\mathrm{snh} x = -i\,\mathrm{sn} ix$, etc...)

$$\Gamma = \frac{1 - k^2 \mathrm{snh}^2\lambda}{1 + k^2 \mathrm{snh}^2\lambda} \qquad , \qquad \Delta = -\frac{\mathrm{cnh}\lambda\,\mathrm{dnh}\lambda}{1 + k^2 \mathrm{snh}^2\lambda}$$

- Phases:
  - ferroelettric order for $a > b + c + d$, $\Delta > 1$
  - ferroelettric order for $b > a + c + d$, $\Delta > 1$
  - disorder for $a, b, c, d < \frac{1}{2}(a + b + c + d)$, $-1 < \Delta < 1$

Elliptic parametrization

- A convenient parametrization of the Boltzmann weights

$$\begin{aligned} a &= \rho\,\mathrm{snh}(\lambda - u) \\ b &= \rho\,\mathrm{snh} u \\ c &= \rho\,\mathrm{snh}\lambda \\ d &= \rho\,k\,\mathrm{snh}\lambda\,\mathrm{snh} u\,\mathrm{snh}(\lambda - u) \end{aligned}$$

- In this parametrization $(\mathrm{snh} x = -i\mathrm{sn} ix$, etc...)

$$\Gamma = \frac{1 - k^2\mathrm{snh}^2\lambda}{1 + k^2\mathrm{snh}^2\lambda} \qquad , \qquad \Delta = -\frac{\mathrm{cnh}\lambda\,\mathrm{dnh}\lambda}{1 + k^2\mathrm{snh}^2\lambda}$$

- Phases:
  - ferroelettric order for $a > b + c + d$, $\Delta > 1$
  - ferroelettric order for $b > a + c + d$, $\Delta > 1$
  - disorder for $a, b, c, d < \frac{1}{2}(a + b + c + d)$, $-1 < \Delta < 1$

Diagonalization of CTM

- In the thermodynamic limit Baxter (1977) proved the following formula for the diagonalized CTM

$$
\begin{aligned}
A_d(u) &= C_d(u) = \begin{pmatrix} 1 & 0 \\ 0 & s \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & s^2 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & s^3 \end{pmatrix} \otimes \dots \\
B_d(u) &= D_d(u) = \begin{pmatrix} 1 & 0 \\ 0 & t \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & t^2 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & t^3 \end{pmatrix} \otimes \dots
\end{aligned}
$$

where

$$
s = \exp\left(-\frac{\pi u}{2I(k)}\right) \qquad , \qquad t = \exp\left(-\frac{\pi(\lambda - u)}{2I(k)}\right)
$$

and $I(k)$ is the elliptic integral of I kind of modulus k

Reduced density matrix

- Define $x = (st)^2 = \exp\left(-\frac{\pi\lambda}{I(k)}\right)$ and use the CTM density matrix formula

$$\rho_A = ABCD = (AB)^2 = \left(\begin{array}{cc} 1 & 0 \\ 0 & x \end{array}\right) \otimes \left(\begin{array}{cc} 1 & 0 \\ 0 & x^2 \end{array}\right) \otimes \left(\begin{array}{cc} 1 & 0 \\ 0 & x^3 \end{array}\right) \otimes \ldots$$

- $\rho = e^{\epsilon \mathcal{O}}$ where $\mathcal{O}$ is a operator with integer spectrum

$$\mathcal{O} = \left(\begin{array}{cc} 0 & 0 \\ 0 & 1 \end{array}\right) \otimes \left(\begin{array}{cc} 0 & 0 \\ 0 & 2 \end{array}\right) \otimes \left(\begin{array}{cc} 0 & 0 \\ 0 & 3 \end{array}\right) \otimes \ldots$$

$\epsilon = -\frac{\pi\lambda}{I(k)}$ depends on the XYZ parameters through elliptic functions

Reduced density matrix

- Define $x = (st)^2 = \exp\left(-\frac{\pi\lambda}{I(k)}\right)$ and use the CTM density matrix formula

$$\rho_A = ABCD = (AB)^2 = \begin{pmatrix} 1 & 0 \\ 0 & x \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & x^2 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & x^3 \end{pmatrix} \otimes \ldots$$

- $\rho = e^{\epsilon \mathcal{O}}$ where $\mathcal{O}$ is a operator with integer spectrum

$$\mathcal{O} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix} \otimes \begin{pmatrix} 0 & 0 \\ 0 & 3 \end{pmatrix} \otimes \ldots$$

$\epsilon = -\frac{\pi\lambda}{I(k)}$ depends on the XYZ parameters through elliptic functions

Entanglement entropy of XYZ model

- The trace of the reduced density matrix

$$\mathcal{Z} = \mathrm{Tr}\rho_A = \prod_{j=1}^{\infty}(1+x^j) \qquad \text{and} \qquad S_A = -\epsilon\frac{\log\mathcal{Z}}{\partial\epsilon} + \log\mathcal{Z}$$

leads to the final formula

$$\boxed{S_A = \epsilon\sum_{j=1}^{\infty}\frac{j}{(1+e^{j\epsilon})} + \sum_{j=1}^{\infty}\log(1+e^{-j\epsilon})}$$

- Notice that for $\epsilon \to 0$ this can be approximated by a dilogarithmic integral

$$S_A = \int_0^{\infty} \mathrm{d}x \left(\frac{x\epsilon}{1+e^{x\epsilon}} + \log(1+e^{-x\epsilon})\right) = \frac{\pi^2}{6}\frac{1}{\epsilon}$$

Entanglement entropy of XYZ model

- The trace of the reduced density matrix

$$\mathcal{Z} = \mathrm{Tr}\rho_A = \prod_{j=1}^{\infty}(1 + x^j) \qquad \text{and} \qquad S_A = -\epsilon \frac{\log \mathcal{Z}}{\partial \epsilon} + \log \mathcal{Z}$$

leads to the final formula

$$\boxed{S_A = \epsilon \sum_{j=1}^{\infty} \frac{j}{(1 + e^{j\epsilon})} + \sum_{j=1}^{\infty} \log(1 + e^{-j\epsilon})}$$

- Notice that for $\epsilon \to 0$ this can be approximated by a dilogarithmic integral

$$S_A = \int_0^{\infty} \mathrm{d}x \, \left(\frac{x\epsilon}{1 + e^{x\epsilon}} + \log(1 + e^{-x\epsilon}) \right) = \frac{\pi^2}{6} \frac{1}{\epsilon}$$

Approaching phase transition

- In the limit $\Gamma \to 1$ and $\Delta \to -1^-$ it turns out that $\epsilon \to 0$ and

$$\lambda \approx \frac{2\sqrt{2}I(k)}{\pi}\sqrt{-1-\Delta}$$

the correlation lenght $\xi \to \infty$ approaching $c = 1$ CFT in the XXZ massless regime $\epsilon \sim \log(a/\xi)$

$$S_A = \frac{1}{6}\log\frac{\xi}{a}$$

- A more lengthy calculation can lead also to the Ising limit of XYZ where

$$S_A = \frac{1}{12}\log\frac{\xi}{a}$$

- In both cases the Calabese - Cardy (2004) formula holds

$$S_A = \frac{c}{6}\log\frac{\xi}{a}$$

Approaching phase transition

- In the limit $\Gamma \to 1$ and $\Delta \to -1^-$ it turns out that $\epsilon \to 0$ and
$$\lambda \approx \frac{2\sqrt{2}I(k)}{\pi}\sqrt{-1-\Delta}$$
the correlation lenght $\xi \to \infty$ approaching $c = 1$ CFT in the XXZ massless regime $\epsilon \sim \log(a/\xi)$
$$S_A = \frac{1}{6}\log\frac{\xi}{a}$$

- A more lengthy calculation can lead also to the Ising limit of XYZ where
$$S_A = \frac{1}{12}\log\frac{\xi}{a}$$

- In both cases the Calabese - Cardy (2004) formula holds
$$S_A = \frac{c}{6}\log\frac{\xi}{a}$$

Approaching phase transition

- In the limit $\Gamma \to 1$ and $\Delta \to -1^-$ it turns out that $\epsilon \to 0$ and

$$\lambda \approx \frac{2\sqrt{2}I(k)}{\pi}\sqrt{-1-\Delta}$$

  the correlation lenght $\xi \to \infty$ approaching $c = 1$ CFT in the XXZ massless regime $\epsilon \sim \log(a/\xi)$

$$S_A = \frac{1}{6}\log\frac{\xi}{a}$$

- A more lengthy calculation can lead also to the Ising limit of XYZ where

$$S_A = \frac{1}{12}\log\frac{\xi}{a}$$

- In both cases the Calabese - Cardy (2004) formula holds

$$S_A = \frac{c}{6}\log\frac{\xi}{a}$$

Summary

- The entanglement entropy for the XYZ spin chain (and consequently XXZ, XXX, Ising...) has been derived from the corner transfer matrix
- The result agrees with $c = 1$ and $c = 1/2$ CFT calculations of Calabrese-Cardy where applicable
- Way open to other possible integrable lattice models?
- Finite size effects? Finite temperature? Tests of off-critical corrections?
- and applications...
  - to condensed matter
  - to information theory and quantum computing
  - to quantum field theory
  - and maybe to the poor Alice...

Summary

- The entanglement entropy for the XYZ spin chain (and consequently XXZ, XXX, Ising...) has been derived from the corner transfer matrix
- The result agrees with $c = 1$ and $c = 1/2$ CFT calculations of Calabrese-Cardy where applicable
- Way open to other possible integrable lattice models?
- Finite size effects? Finite temperature? Tests of off-critical corrections?
- and applications...
    - to condensed matter
    - to information theory and quantum computing
    - to quantum field theory
    - and maybe to the poor Alice...

Summary

- The entanglement entropy for the XYZ spin chain (and consequently XXZ, XXX, Ising...) has been derived from the corner transfer matrix
- The result agrees with $c = 1$ and $c = 1/2$ CFT calculations of Calabrese-Cardy where applicable
- Way open to other possible integrable lattice models?
- Finite size effects? Finite temperature? Tests of off-critical corrections?
- and applications...
  - to condensed matter
  - to information theory and quantum computing
  - to quantum field theory
  - and maybe to the poor Alice...

Alice falls into the rabbit hole

... falling into a BLACK HOLE ...