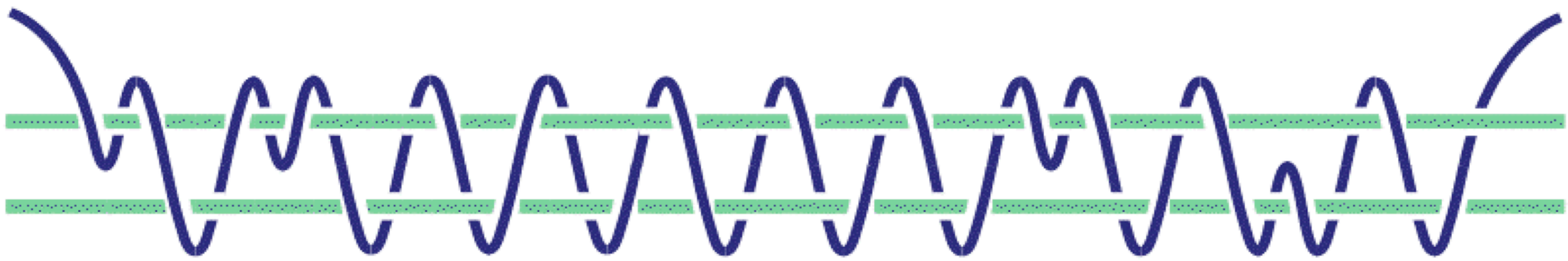
Topological Quantum Registers



Kareljan Schoutens, Universiteit van Amsterdam

Galileo Galilei Institute — 5 Sept. 2008

What’s going on?

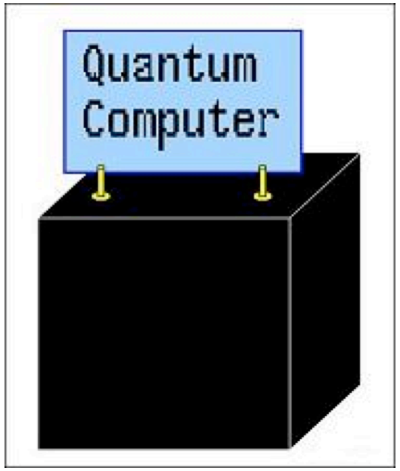
Two major goals

- establishing experimentally the existence in nature of the existence of `non-abelian anyons’ in topological phases of matter and the associated phenomenon of `non-abelian statistics’

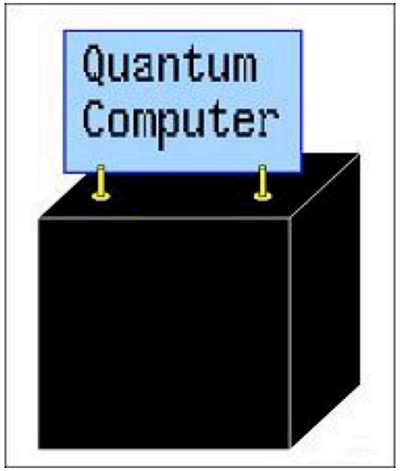
- using the above to develop what is called a `topological quantum computer’, where information is stored in quantum knots and a `topological shield’ protects against decoherence

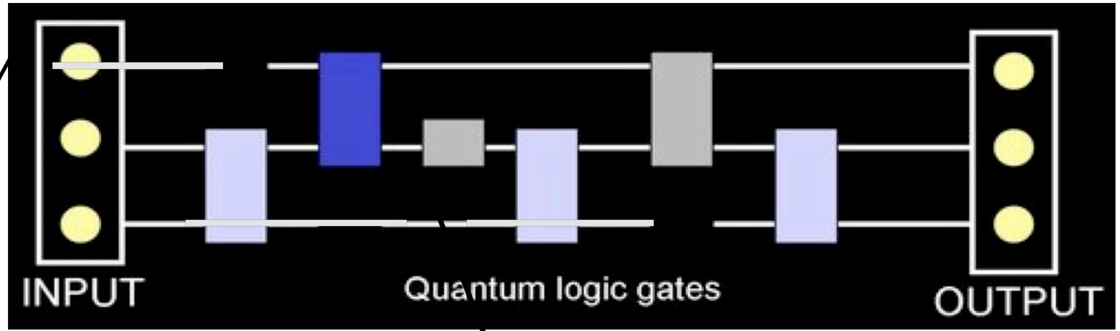
S. das Sarma, M. Freedman, C. Nayak, S.H. Simon and A. Stern, arXiv.0707.1889, to appear in Rev.Mod.Phys.

Quantum computation



Quantum computation





qubit:
quantum system
with two states
|0> and |1>

quantum register:
collection of qubits

quantum logical gate:
operation on 1 or 2 qubits

Quantum computation

Quantum software

For certain computations quantum algorithms can outperform classical algorithms by a landslide

- prime example is Peter Shor's algorithm for factorisation into prime factors (exit RSA)

- possible major application: computations in quantum many-body physics [quantum chemistry, quantum engineering, materials ...]

Quantum computer hardware

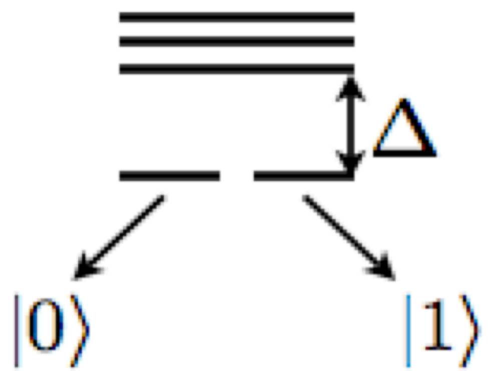
Local qubits

Information stored in local quantum degree of freedom, such as spin, (charge,) flux, etc.

Challenge to protect such qubits from decoherence due to noise and coupling to environment.

Quantum computer hardware

Topological qubits



- information stored non-locally in many body states of a suitable quantum matter system
- qubit states realized as a **quantum knots**
- topological order provides shield that protects against decoherence

↓

fault-tolerant quantum register

Quantum knots and quantum information

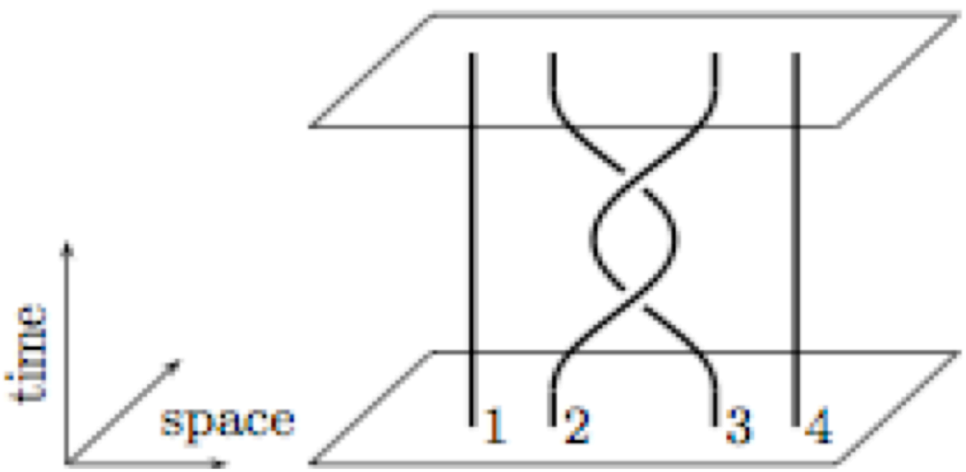
Quantum knots

- idea: find suitable quantum system such that wavefunctions form `quantum representation' of classical knots
- this will imply topological protection against decoherence

Quantum knots and quantum information

quantum knots

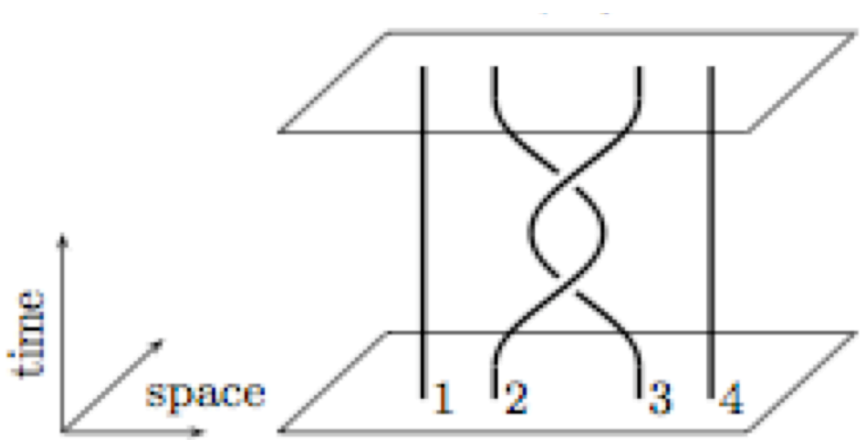
- quantum knots and braids realized by braiding word-lines in 2+1 dimensional space-time of particles existing in 2 spatial dimensions



Quantum knots and quantum information

precise statement

- time-evolution of many-particle wavefunction given by representation of braid-group for particles in two spatial dimensions

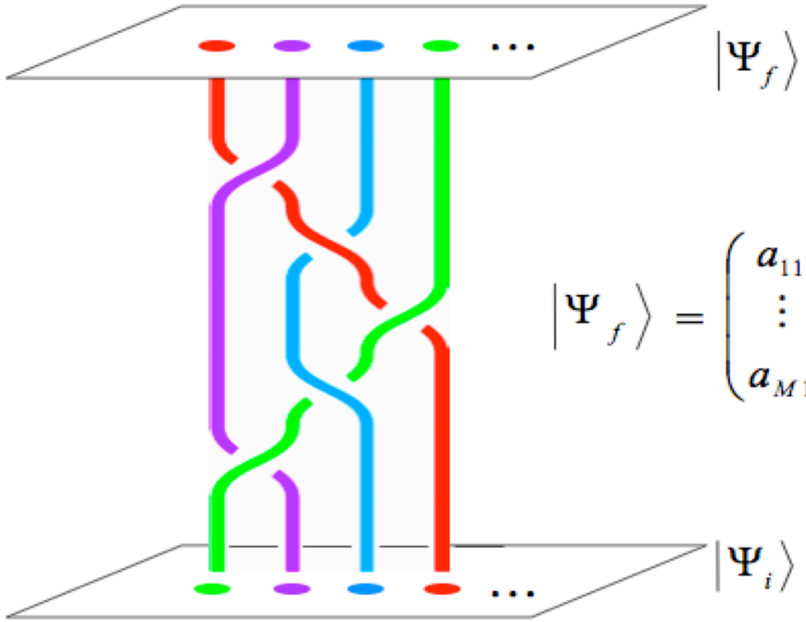


$$\left|\psi_{\text{final}}\right\rangle$$

$$\left|\psi_{\text{initial}}\right\rangle$$

Quantum braids (1)

figures
N. Bonesteel

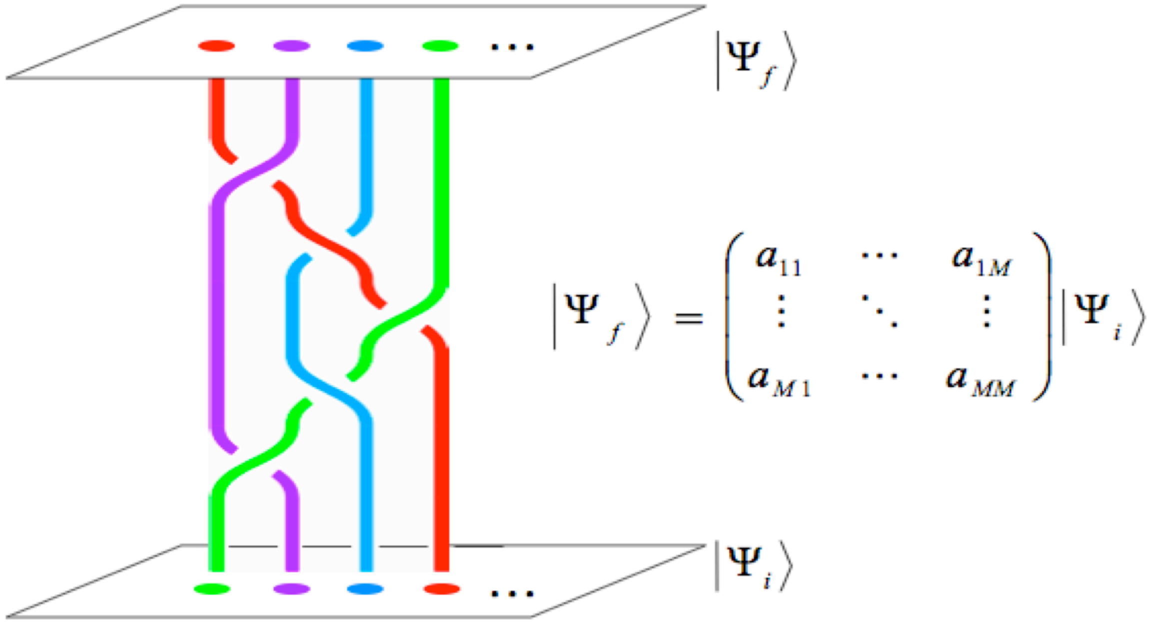


$$|\Psi_f\rangle = \begin{pmatrix} a_{11} & \cdots & a_{1M} \\ \vdots & \ddots & \vdots \\ a_{M1} & \cdots & a_{MM} \end{pmatrix} |\Psi_i\rangle$$

quantum register

if the particles are such that the wave functions $|\psi_i\rangle$ and $|\psi_f\rangle$ are multi-component, the many-body state can be used as a quantum register

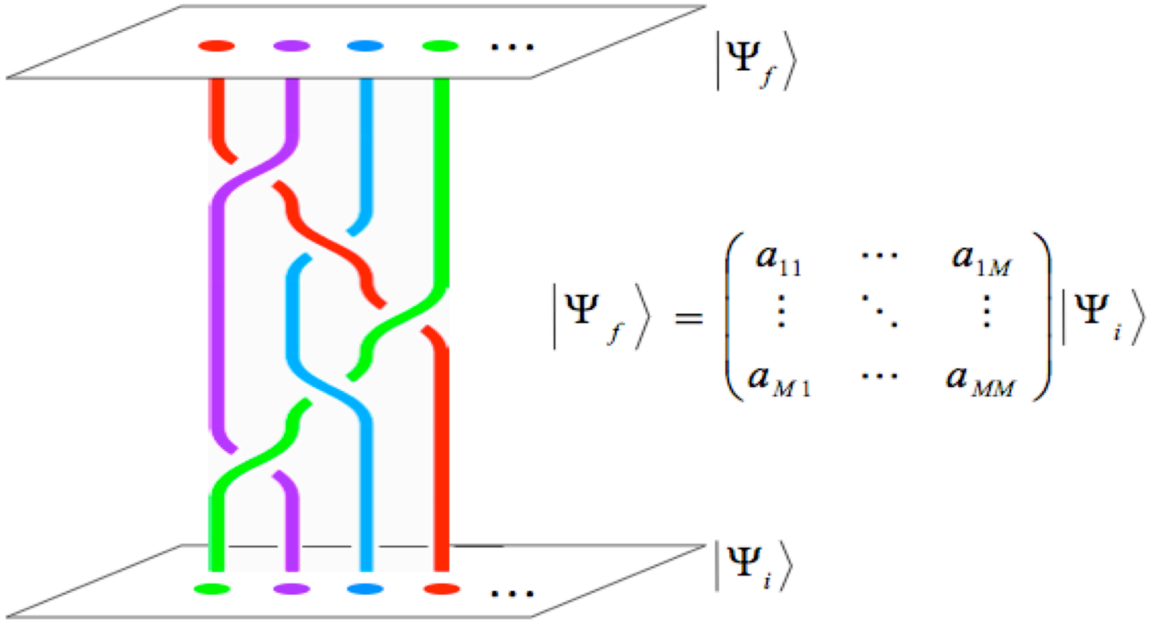
Quantum braids (2)



quantum logical gates

if $|\psi_i\rangle$ and $|\psi_f\rangle$ are multi-component, the braiding is represented by an $M \times M$ matrix; successive braidings do **not** commute (non-abelian braiding)

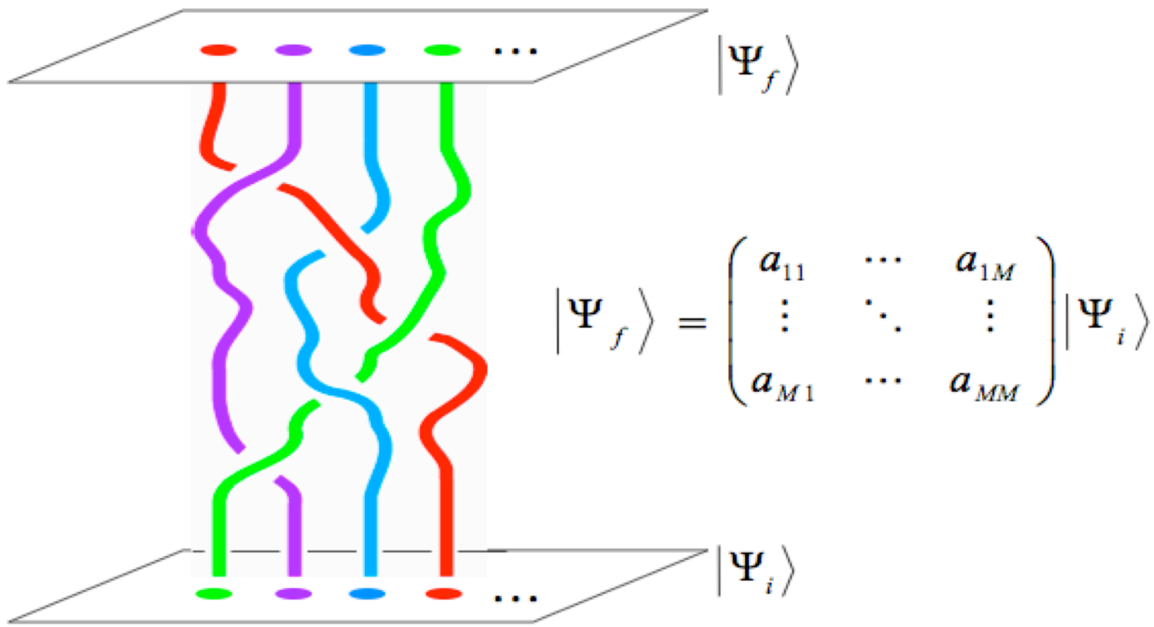
Quantum braids (3)



topological stability

perturbations of the particles (other than braiding) do **not** affect the final state $|\psi_f\rangle$

Quantum braids (3)



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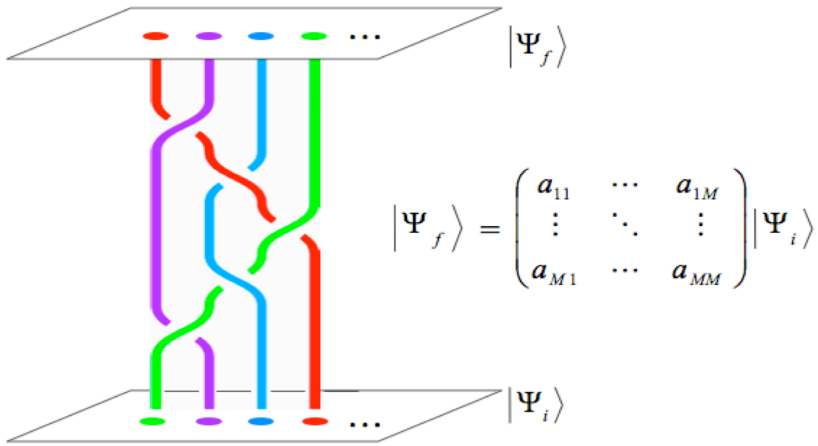
Anyons for quantum computation

to obtain a non-trivial quantum register, quasi-particles in 2D quantum matter system should be such that the many-particle wave functions are multi-cpt



→ need non-abelian anyons

Anyons for quantum computation

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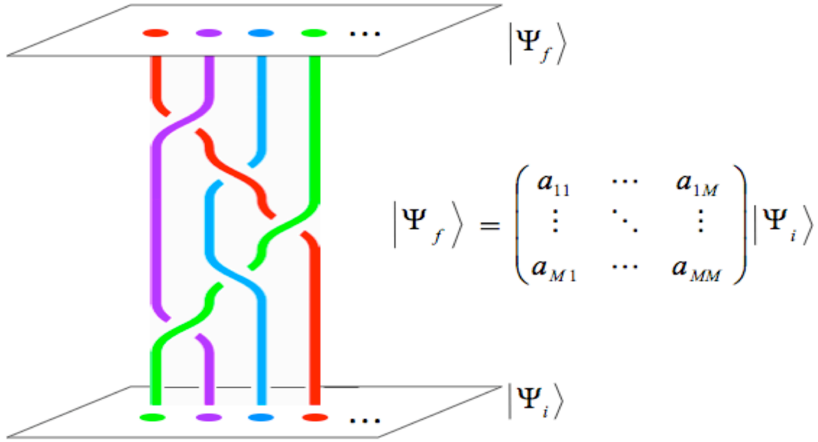


→ need non-abelian anyons

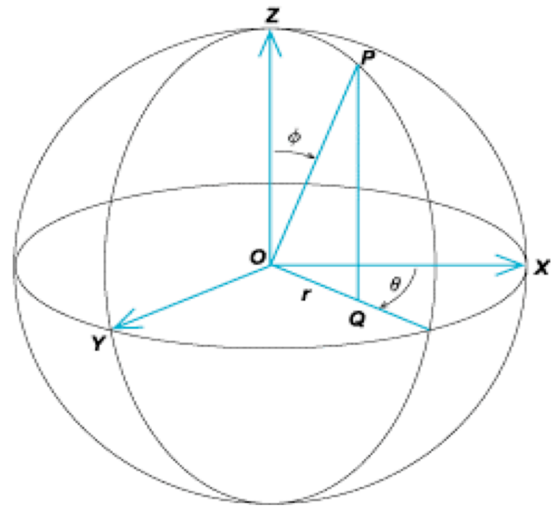
compare:

- bosons, fermions: single-cpt, braiding gives +/- signs
- (abelian) anyons: single-cpt, braiding gives complex phases

Anyons for quantum computation

analogy spin qubit vs. topological qubit

for spin qubit: need particles such that spatial rotations are represented by matrices acting on multi-component wavefunctions, that end up acting as quantum register



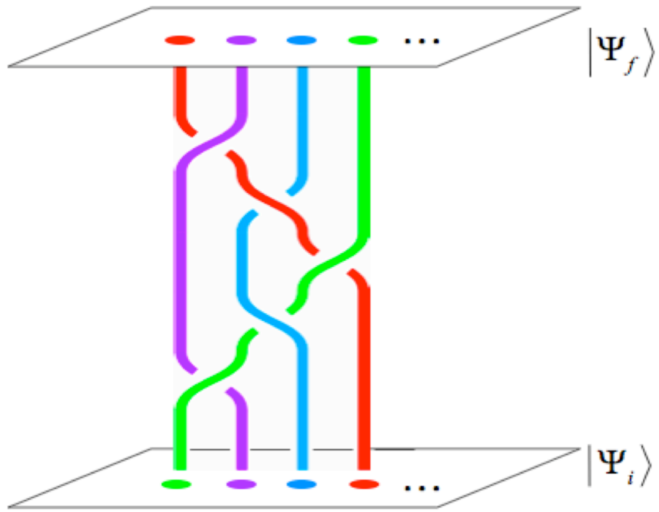
QM allows representations of dimension

$D_n = (2S+1)^n$ (n particles of spin S)

Anyons for quantum computation

analogy spin qubit vs. topological qubit

for topological qubit: need particles such that braids on *n* particles are represented by matrices acting on multi-component wavefunctions of dimension D_n , that end up acting as quantum register



QM allows ….

Anyons for quantum computation

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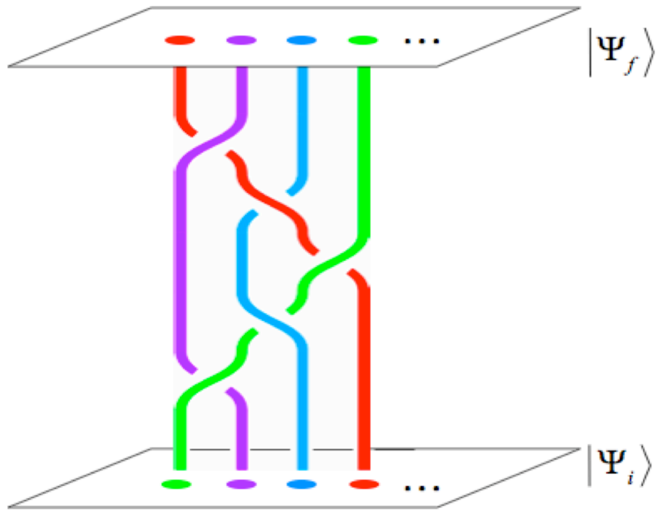

QM allows ….

$$D_n = 2^{(n-2)/2}$$ (Ising anyons)

Anyons for quantum computation

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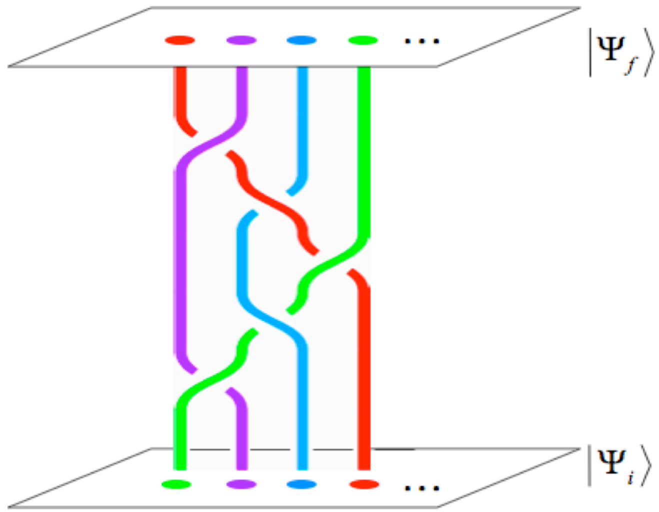

QM allows ….

$$D_n = \mathrm{Fibo}_n = 1, 1, 2, 3, 5,..$$ (Fibonacci anyons)

Anyons for quantum computation

analogy spin qubit vs. topological qubit

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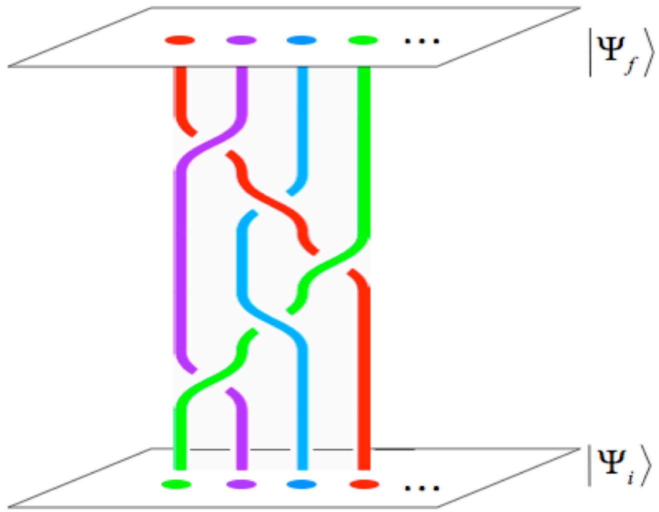


QM allows ….

$$D_n = \ldots$$ (etc.)

Non-abelian anyons

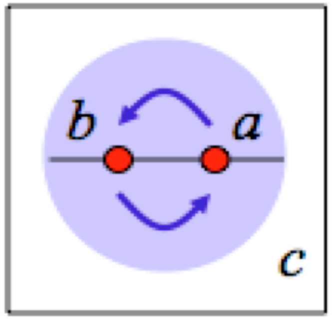
issues

- what are consistent possibilities for non-abelian anyons?
- what is the dimension of the *n* - particle quantum register?
- what are the braid matrices? Are they suitable for (universal) quantum computation?

- **where do we find them?**

Non-abelian anyons

formalism



• non-abelian anyons characterized by fusion and braiding relations

• degenerate ground states in 1-1 correspondence with fusion channels

• algebraic framework (`topological modular functors'); relations among fusion and braiding matrices (pentagon and hexagon identities)

Non-abelian anyons

Fibonacci anyons

particles of type **`0'** and **`1'** with fusion rules

$$0 \times 0 = 0, \quad 0 \times 1 = 1, \quad 1 \times 1 = 0 + 1$$

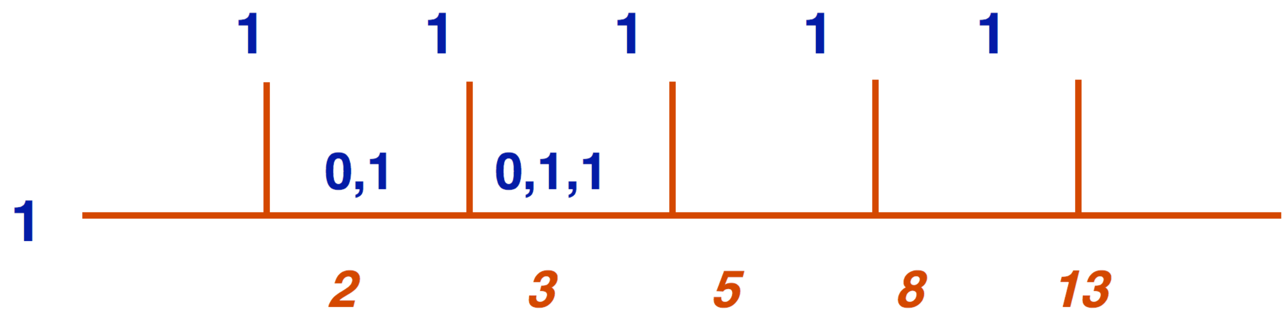
Non-abelian anyons

Fibonacci anyons

particles of type `0’ and `1’ with fusion rules

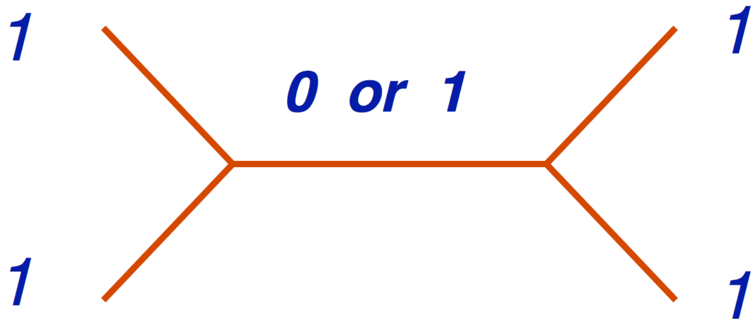
0 x 0 = 0,   0 x 1 = 1,   1 x 1 = 0 + 1

For collection of type `1’ particles, ground state degeneracies follow the Fibonacci numbers



Non-abelian anyons

n = 4 **Fibonacci particles of type `1'**

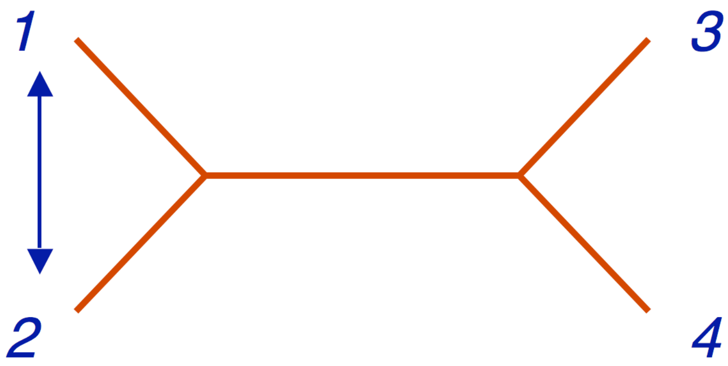


$|\psi_0\rangle$ or $|\psi_1\rangle$

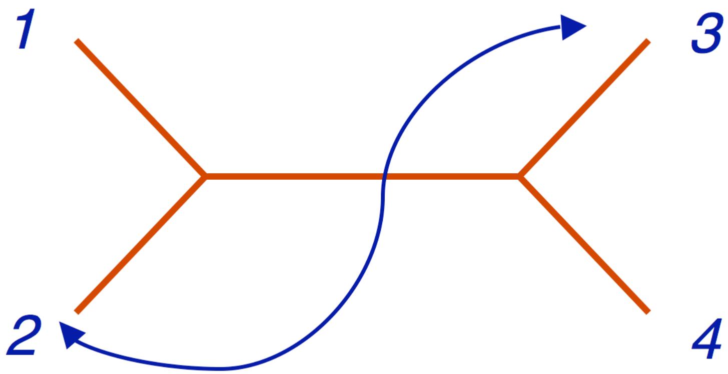
- 2 fusion channels
- quantum register 2-dimensional (qubit!)
- braiding represented as 2 x 2 matrices

Non-abelian anyons

braiding *n = 4* Fibonacci particles



$$U_{1\leftrightarrow 2} = \sigma_1 = \begin{pmatrix} (-1)^{4/5} & 0 \\ 0 & (-1)^{-3/5} \end{pmatrix}$$



$$U_{2\leftrightarrow 3} = \sigma_2 = \begin{pmatrix} \tau & \sqrt{\tau} \\ \sqrt{\tau} & -\tau \end{pmatrix}$$

$$\tau = \frac{1}{2}\left(\sqrt{5} - 1\right)$$

Non-abelian anyons

quantum gates with Fibonacci anyons

with well-chosen iterations of σ_1 and σ_2, logical gates can be approximated to any desired precision!

Non-abelian anyons

quantum gates with Fibonacci anyons

with well-chosen iterations of σ_1 and σ_2, logical gates can be approximated to any desired precision!

$$\sigma_1^{-2}\sigma_2^{-4}\sigma_1^{4}\sigma_2^{-2}\sigma_1^{2}\sigma_2^{2}\sigma_1^{-2}\sigma_2^{4}\sigma_1^{-2}\sigma_2^{4}\sigma_1^{2}\sigma_2^{-4}\sigma_1^{2}\sigma_2^{-2}\sigma_1^{2}\sigma_2^{-2}\sigma_1^{-2} \approx \left(\begin{array}{cc|c} 0 & i & 0 \\ i & 0 & 0 \\ \hline 0 & 0 & 1 \end{array}\right)$$

figure: NOT gate with accuracy better than 10^{-3}

Bonesteel et al, 2005

Non-abelian anyons

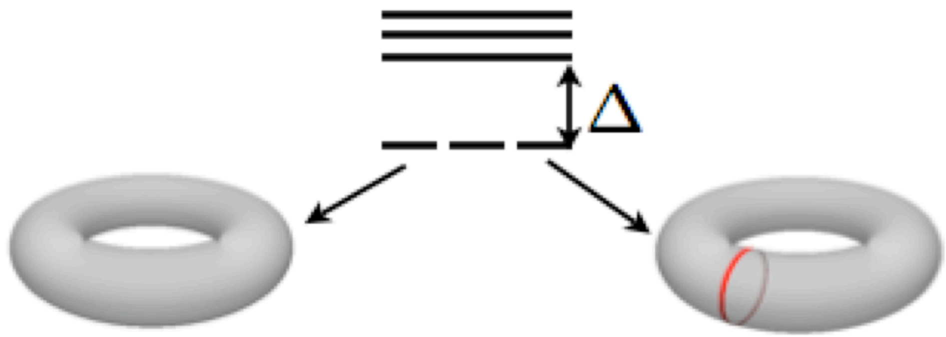
where do we find them?

Non-abelian anyons

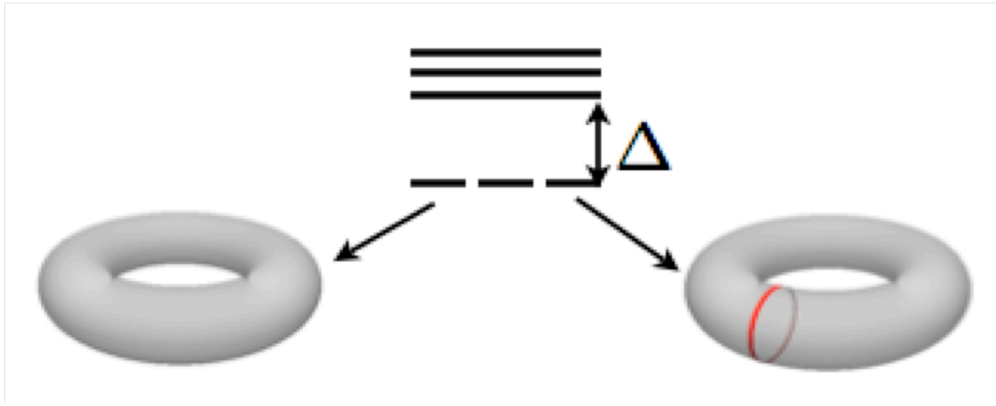
where do we find them?

key feature of qantum matter systems supporting (non-abelian) anyons: topological order

- gapped spectrum
- ground state degeneracy on torus or punctured plane
- ground states are locally indistinguishable
- excitations carrying fractional charges

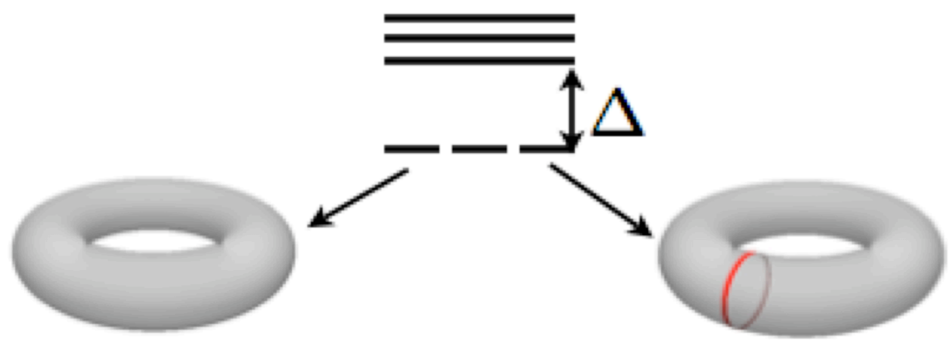


Non-abelian anyons



topological phases

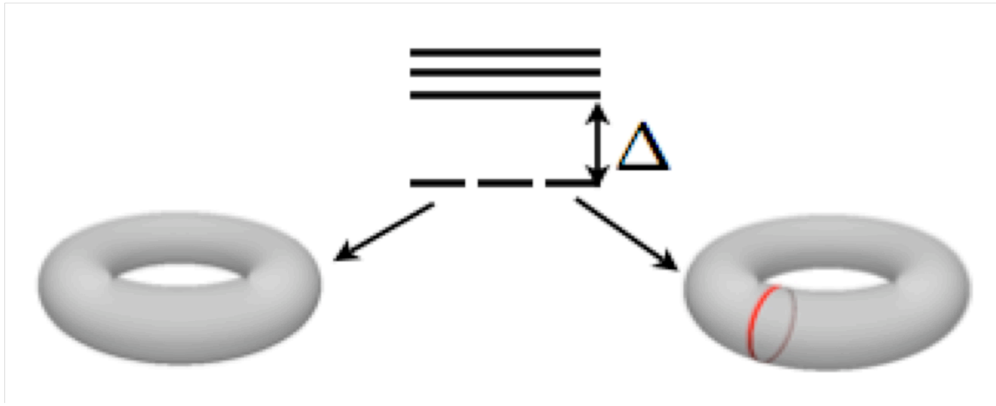
Non-abelian anyons



topological phases

- prototype: the fractional quantum Hall liquids

Non-abelian anyons



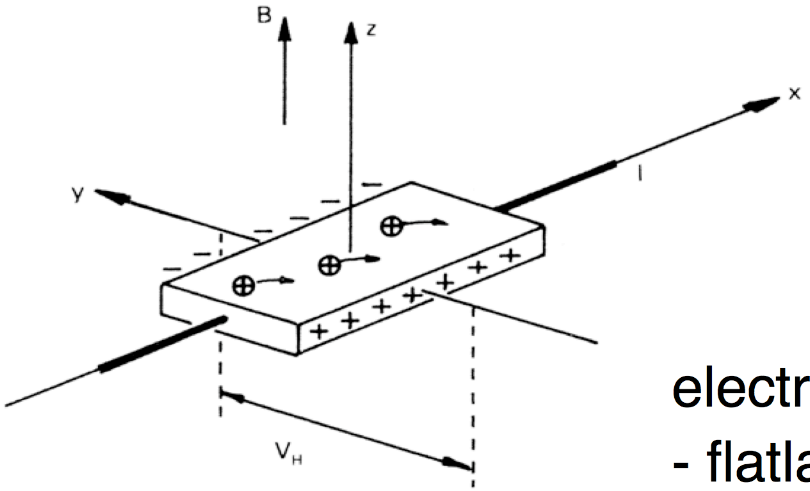
topological phases

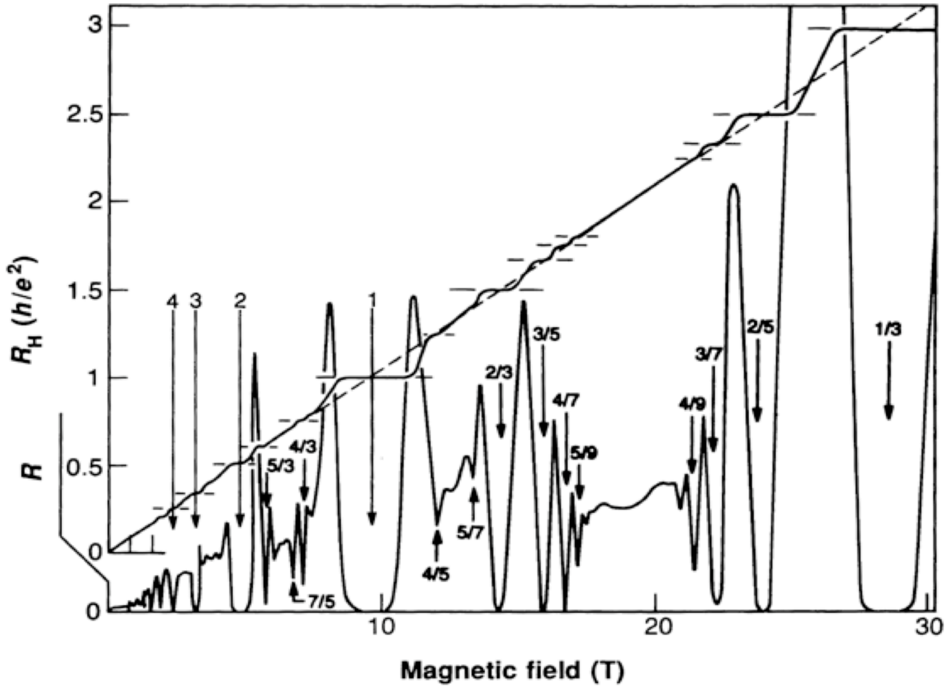
- prototype: the fractional quantum Hall liquids
- other: specific lattice models [Kitaev]

  Josephson junction networks [Douçot et al]

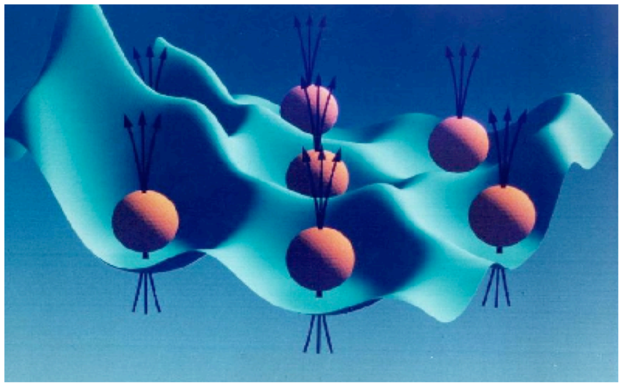
  ...

Quantum Hall systems





electrons in
- flatland
- strong B-field
- low temperature



fractional quantum Hall liquids

Quantum Hall systems

Fractional quantum Hall liquids are known to possess topological order; can they be used for topological quantum computation?

Quantum Hall systems

Fractional quantum Hall liquids are known to possess topological order; can they be used for topological quantum computation?

Issues

- do they support anyonic excitations?
- can these be non-abelian?
- can the excitations be of Fibonacci type ?
- can the necessary fusion and braid operations be implemented?

Quantum Hall systems

do fractional qH states support anyonic excitations?

Quantum Hall systems

do fractional qH states support anyonic excitations?

- for some simple fractional qH states, fractional charge q=1/3, q=1/5 of fundamental excitations has been demonstrated experimentally

- indirect demonstration of fractional statistics (via hierarchy scheme)

- recent results on interference experiments

Camino et al, 2006

Quantum Hall systems

can qH excitations exhibit non-abelian statistics?

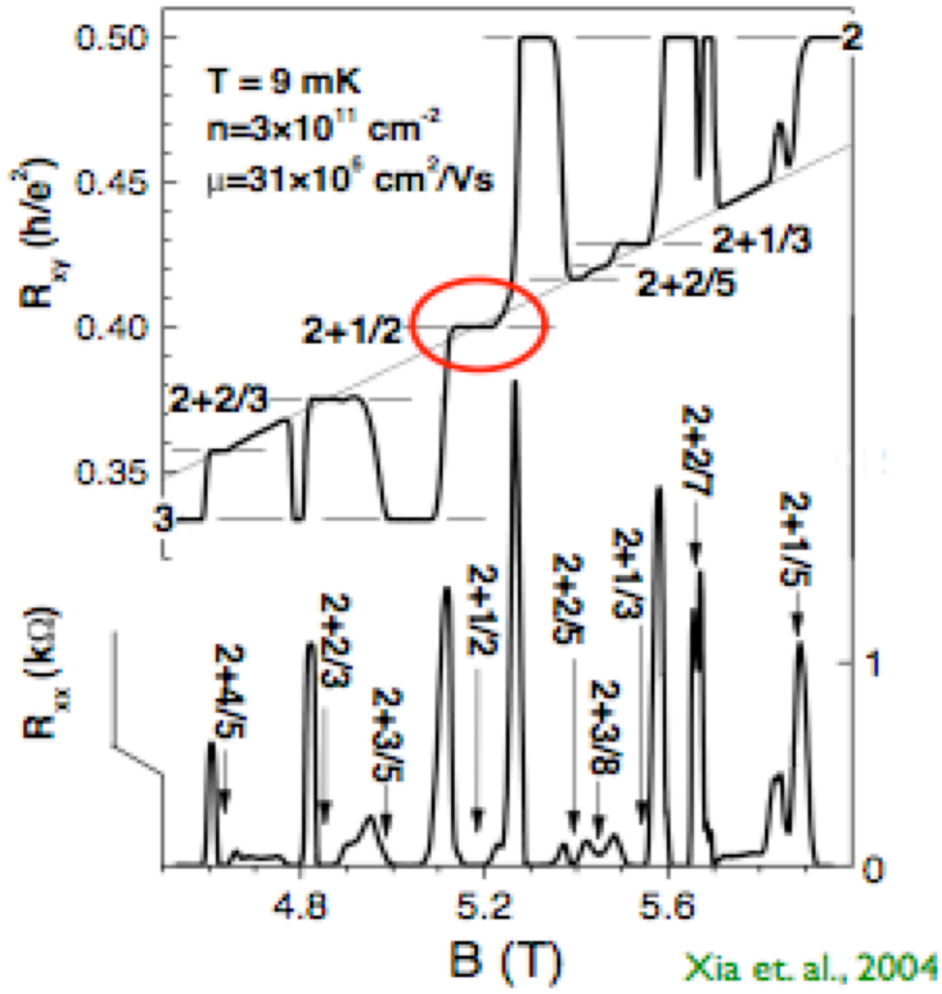
Quantum Hall systems

can qH excitations exhibit non-abelian statistics?

- strong evidence that the 2+1/2 fractional qH plateau is due to so-called Moore-Read (pfaffian) state [Moore-Read 1991]

- excitations over Moore-Read state are non-abelian, but **not** suitable for universal TQC

- experimental tests forthcoming



Quantum Hall systems

can qH excitations be Fibonacci anyons?

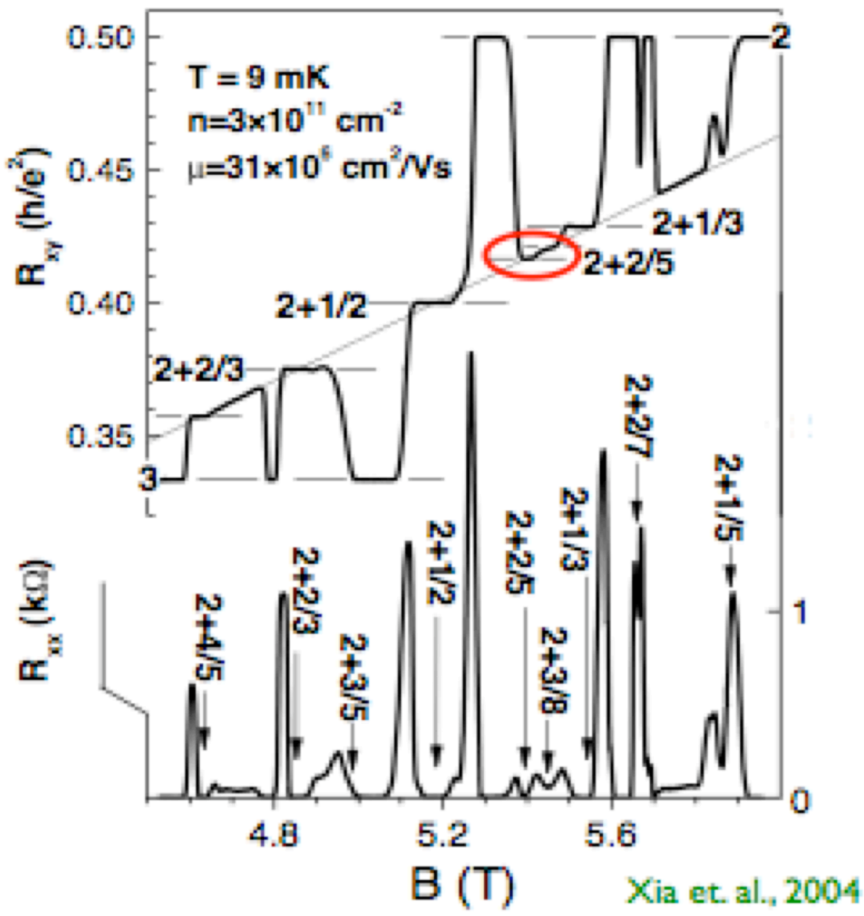
Quantum Hall systems

can qH excitations be Fibonacci anyons?

- they can at the level of model wavefunctions: the k=3 clustered state [Read-Rezayi 1999] and the k=2 non-abelian spin-singlet (NASS) state [Ardonne-KjS 1999]

- (some) evidence that the fractional qH plateau at 2+2/5 is due to k=3 Read-Rezayi state

- experimental tests proposed



Xia et. al., 2004

Quantum Hall systems

can the necessary fusion and braid operations be implemented?

Quantum Hall systems

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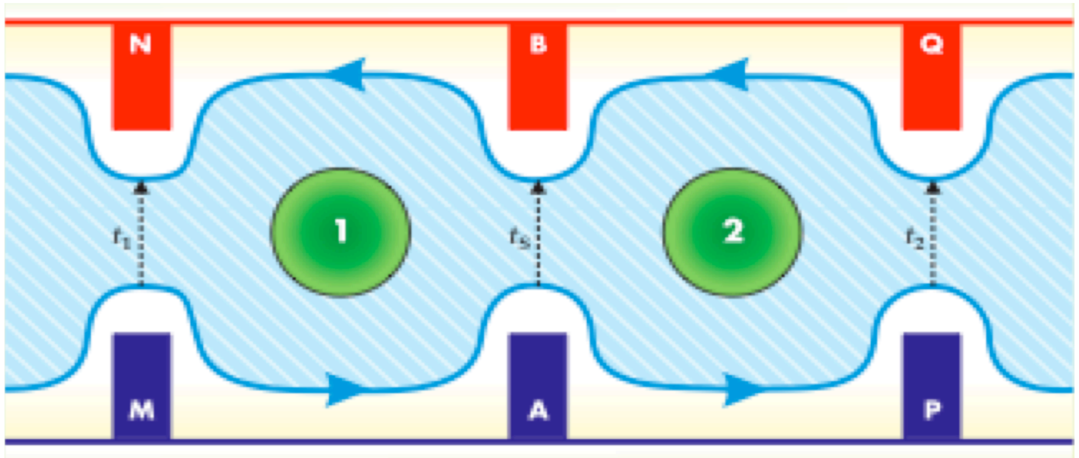

• proposed protocol with controled tunneling of quasi-particles on, off and in between quantum dots in background of suitable quantum Hall state

[das Sarma-Nayak-Freedman 2005]

Quantum Hall quantum registers

Issues and current research

Quantum Hall quantum registers

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- catalogue and analyze relevant qH states

  [excitations, braiding, edges and interfaces, …]

Quantum Hall quantum registers

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- investigate (and engineer) physical settings where these qH states can be realized
  [high mobility qH, multicpt. and multilayer, rotating BEC, cold atoms in optical lattices, …]

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- experiments!

Quantum Hall quantum registers

For now

- brief discussion of MR and NASS states

MR state: wavefunction

$$\Psi_{\mathrm{MR}}(z_1,\dots,z_N) = \mathrm{Pf}\left(\frac{1}{z_i - z_j}\right)\prod_{i<j}(z_i - z_j)^{M+1}\exp\left(-\frac{|z|^2}{4l^2}\right)$$

- quantum Hall state at filling fraction 1/(M+1)
- Pfaffian factor: p-wave pairing of composite fermions, as in BCS superconductor
- *M=1* : MR state for 5/2 qHe

MR state: pairing

$$\Psi_{MR}(z_1,..,z_N) = \Psi_{\text{boson}}(z_1,..,z_N)\prod_{i<j}(z_i - z_j)^M \exp\left(-\frac{|z|^2}{4l^2}\right)$$

$$\Psi_{\text{boson}}(z_1,..,z_N) = \mathrm{Pf}\,\left(\frac{1}{z_i - z_j}\right)\prod_{i<j}(z_i - z_j)$$

Pairing property

$$z_1 = z_2 \qquad \Psi_{\text{boson}} \neq 0$$

$$z_1 = z_2 = z_3 \qquad \Psi_{\text{boson}} = 0$$

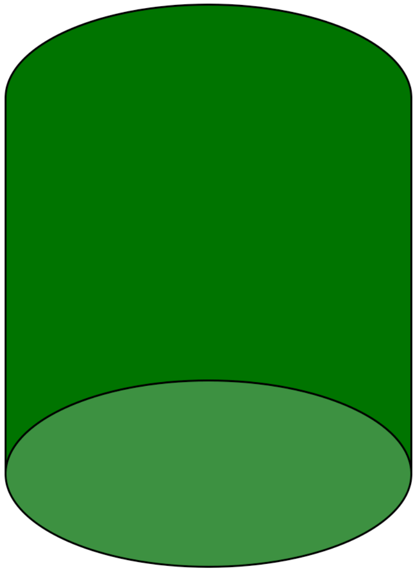
M=0 MR wavefunction: maximal density E=0 eigenstate of hamiltonian

$$H = V \sum_{i_1 <..< i_3} \delta^2(z_{i_1} - z_{i_2})\delta^2(z_{i_2} - z_{i_3})$$

The qH-CFT connection

Chern Simons Landau Ginzburg theory in 2+1 dimensions:

bulk excitations, topological order



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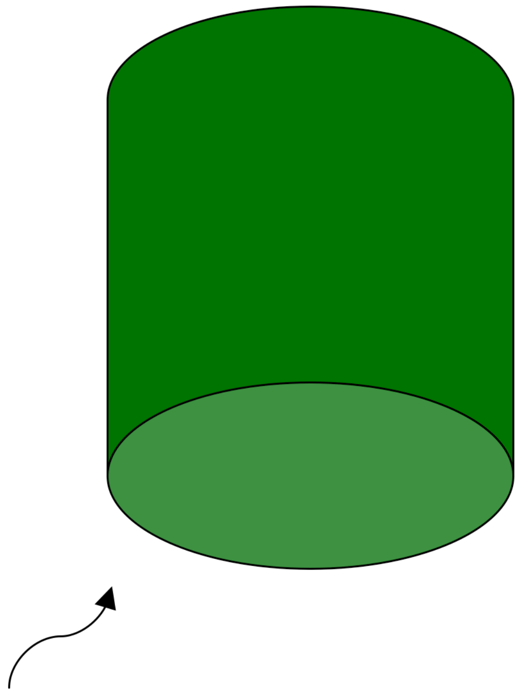

quantum Hall disc, CFT in D=2+0:

qH wave functions <-> CFT correlators

The qH-CFT connection

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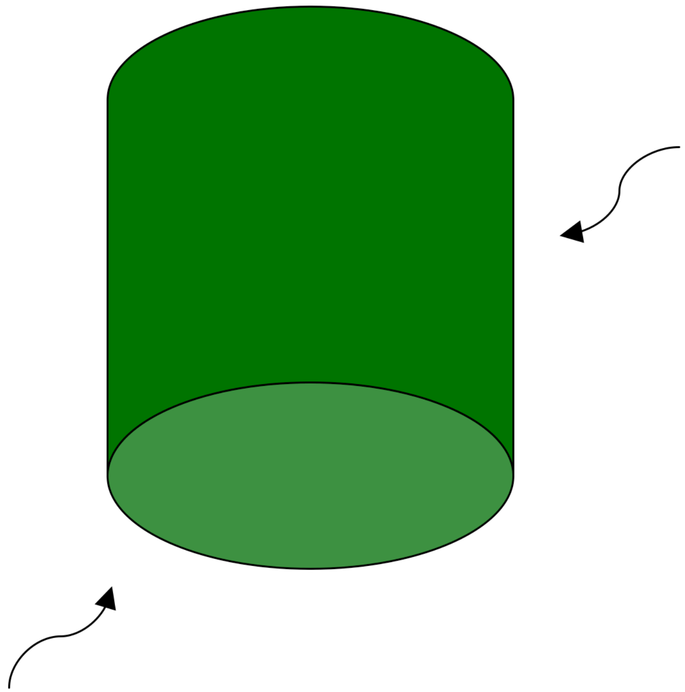


On the cylinder: quantum Hall edge theory, CFT in 1+1 dimensions:

massless edge excitations

quantum Hall disc, CFT in D=2+0:

qH wave functions <-> CFT correlators

qH wavefunctions from CFT

ground state wave function

$$\Psi_{GS}(z_1,\ldots,z_N) \cong \left\langle \psi_e(z_1)\ldots\psi_e(z_N)\psi_{\text{background}}(z_\infty)\right\rangle_{CFT}$$

electron (boson)
condensate operator

neutralizing background
charge

quasi-hole excitations : fixed by

$$\phi_{qh}(w)\psi_e(z_1) = (z-w)^{\text{integer}}\left[\phi_2(w) + \ldots\right]$$

excited state wave function:

$$\Psi_{qh}(w_1,w_2,\ldots;z_1,z_2,\ldots) \cong \left\langle \phi_{qh}(w_1)\phi_{qh}(w_2)\ldots\psi_e(z_1)\psi_e(z_2)\ldots\right\rangle_{CFT}$$

MR state: bulk operators

$$\psi_{\mathrm{e}}(z) = \psi \; e^{i\sqrt{2}\varphi}(z)$$

$$\phi_{\mathrm{qh}}(w) = \sigma \, e^{\frac{i}{2\sqrt{2}}\varphi}(w)$$

neutral operators from Ising (c=1/2) CFT:

$$\psi(z)\psi(w) = (z-w)^{-1} I + \ldots$$

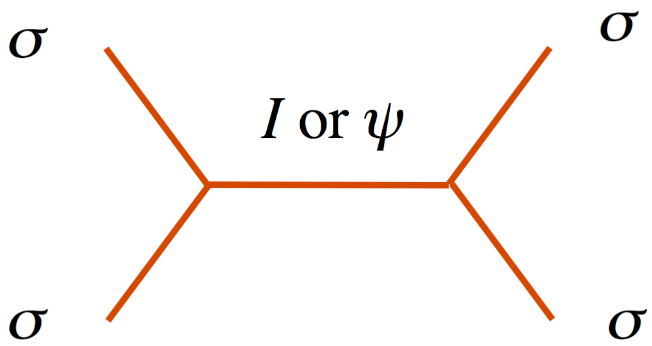
$$\psi(z)\sigma(w) = (z-w)^{-1/2} \sigma(w) + \ldots$$

$$\sigma(z)\sigma(w) = (z-w)^{-1/8} I + (z-w)^{3/8} \psi(w) + \ldots$$

vertex operators describing charge

MR state: 4 quasi-hole wavefunctions

fusion channels *(0,1)*



$\Psi^{(0)}$ or $\Psi^{(1)}$

$$\Psi^{(0,1)}(w_1, w_2, w_3, w_4; z_1, z_2, \ldots) =$$

$$A^{(0,1)}(\{w_i\})\Psi_{[12,34]}(\{w_i; z_j\}) + B^{(0,1)}(\{w_i\})\Psi_{[13,24]}(\{w_i; z_j\})$$

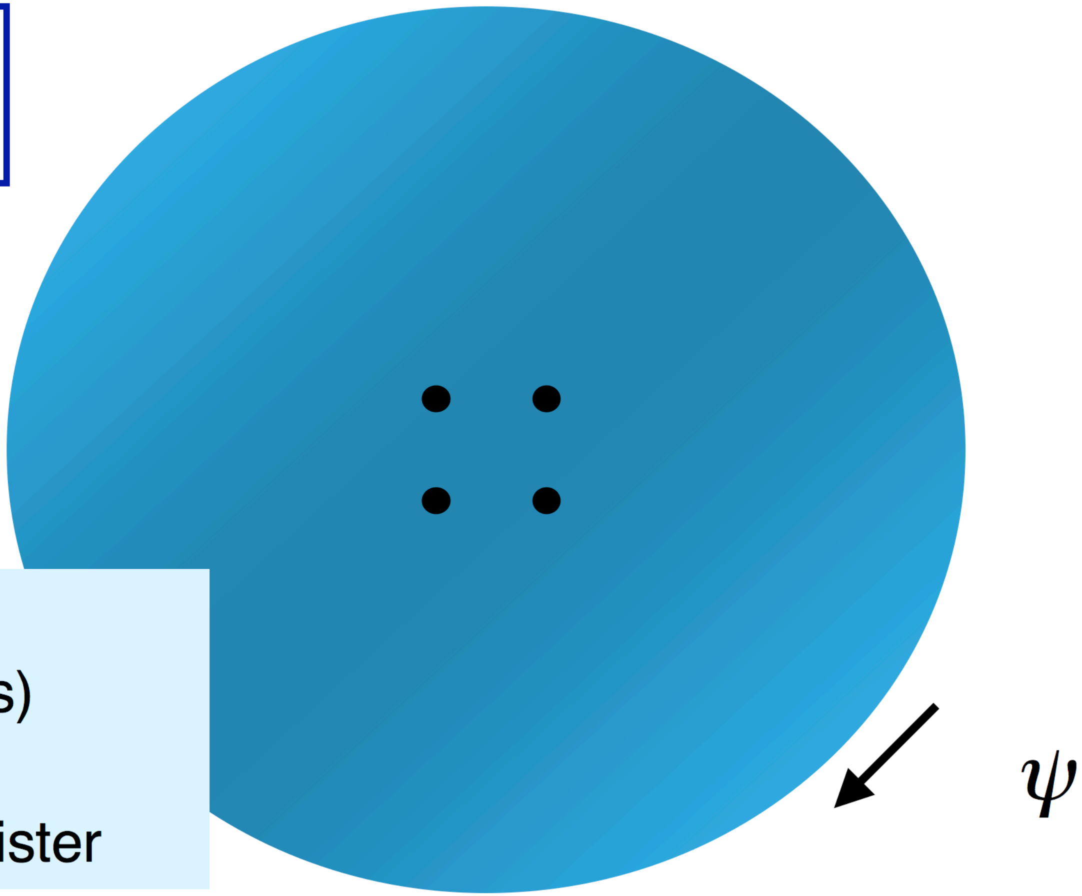
pre-factors depending on fusion channel *(0,1)* and on quasi-hole locations w_i

basis for two-fold degenerate internal register; polynomial in w_i , z_j

Nayak and Wilczek, 1996

MR state: bulk and edge

MR state
at filling 1/2



quasi-holes
(Ising anyons)
spanning a
quantum register

neutral edge mode:
Majorana fermions
(c=1/2)

Non-abelian spin-singlet (NASS) states

Ardonne-KjS 1999

Extend pairing analysis based on

$$H = V \sum_{i_1 < .. < i_3} \delta^2(z_{i_1} - z_{i_2})\delta^2(z_{i_2} - z_{i_3})$$

to spin unpolarized states for spin-1/2 particles:

NASS states at filling factor 4/3 [M=0], 4/7 [M=1], etc.

Explicit example [M=0, N=4]:

$$\tilde{\Psi}_{\mathrm{NASS}}(z_1^\uparrow, z_2^\uparrow, z_1^\downarrow, z_2^\downarrow) = (z_1^\uparrow - z_1^\downarrow)(z_2^\uparrow - z_2^\downarrow) + (z_1^\uparrow - z_2^\downarrow)(z_2^\uparrow - z_1^\downarrow)$$

NASS state: CFT and bulk operators

Underlying CFT is that of charge and spin bosons together with $SU(3)_2$ parafermions (central charge c=1+1+6/5=16/5).

Spin-up and spin-down electrons: $\psi_1(z),\ \psi_2(w)$

Quasi-holes over the 4/7 NASS state come in 3 types

- spin 0, charge 2/7: $\sigma_3(z)$
- spin-1/2, charge 1/7: $\sigma_\uparrow(z), \sigma_\downarrow(z)$

NASS state: quasi-hole wavefunctions

To study braiding properties, we explicitly compute the wavefunction for four spin-less quasiholes in the M=0 NASS state

The qH-CFT correspondence gives

$$\Psi_{3333}(w_1, w_2, \ldots; z_1^{\uparrow}, z_2^{\uparrow}, \ldots, z_{1'}^{\downarrow}, z_{2'}^{\downarrow}, \ldots) =$$

$$\left\langle \sigma_3(w_1)\sigma_3(w_2)\ldots\psi_1(z_1^{\uparrow})\psi_1(z_2^{\uparrow})\ldots\psi_2(z_{1'}^{\downarrow})\psi_2(z_{2'}^{\downarrow})\ldots\right\rangle_{\mathrm{CFT}}$$

$$\times\left[\Psi^{221}\left(\left\{z_i^{\uparrow}, z_{j'}^{\downarrow}\right\}\right)\right]^{1/2}\prod_{i,j}\left(z_i^{\uparrow}-w_j\right)^{1/2}\prod_{i,j}\left(z_{i'}^{\downarrow}-w_j\right)^{1/2}\prod_{i<j}\left(w_i-w_j\right)^{1/3}$$

NASS state: quasi-hole wavefunctions

Going into the wavefunction for 4 spin-less quasi-holes

$$\left\langle \sigma_3(w_1)\sigma_3(w_2)\ldots\psi_1(z_1^{\uparrow})\psi_1(z_2^{\uparrow})\ldots\psi_2(z_{1'}^{\downarrow})\psi_2(z_{2'}^{\downarrow})\ldots \right\rangle_{\mathrm{CFT}}$$

SU(3)$_2$ parafermion algebra

$$\psi_1(z)\psi_1(w) = (z-w)^{-1} I + \ldots$$

$$\psi_2(z)\psi_2(w) = (z-w)^{-1} I + \ldots$$

$$\psi_1(z)\psi_2(w) = (z-w)^{-1/2} \psi_{12} + \ldots$$

and the spin-field OPE, with two independent fusion channels

$$\sigma_3(z)\sigma_3(w) = (z-w)^{-1/5} I + (z-w)^{2/5} \rho_3(w) + \ldots$$

NASS state: quasi-hole wavefunctions

Step1.

In absence of quasi-holes, we have the following expression for the wavefunction [Cappelli et al 2001, Ardonne et al 2002]

$$\Psi_{\mathrm{GS}} = \frac{1}{N} \sum_{\{S_1, S_2\}} \Psi_{S_1}^{221}\left(z_i^{\uparrow}, z_{j'}^{\downarrow}\right) \Psi_{S_2}^{221}\left(z_i^{\uparrow}, z_{j'}^{\downarrow}\right)$$

with particles in subsets S_1, S_2 each forming a Halperin 221 state

$$\tilde{\Psi}^{221}(z_1^{\uparrow}, \ldots, z_N^{\uparrow}, z_{1'}^{\downarrow}, \ldots, z_{N'}^{\downarrow}) =$$

$$\prod_{i<j}\left(z_i^{\uparrow} - z_j^{\uparrow}\right)^2 \prod_{i<j}\left(z_{i'}^{\downarrow} - z_{j'}^{\downarrow}\right)^2 \prod_{i,j'}\left(z_i^{\uparrow} - z_{j'}^{\downarrow}\right)$$

NASS state: quasi-hole wavefunctions

Step 2.

Basis for 4 quasi-hole state obtained by distributing the quasi-holes over de sets S_1, S_2 ; two independent choices for this are $\Psi_{[12,34]}$ and $\Psi_{[13,24]}$

$$\Psi_{[12,34]} = \frac{1}{N} \sum_{\{S_1,S_2\}} \left[\prod_{i,j' \in S_1} \left(z_i^{\uparrow} - w_1\right)\left(z_i^{\uparrow} - w_2\right)\left(z_{j'}^{\downarrow} - w_1\right)\left(z_{j'}^{\downarrow} - w_2\right) \right] \Psi_{S_1}^{221}\left(z_i^{\uparrow}, z_{j'}^{\downarrow}\right)$$

$$\times \left[\prod_{i,j' \in S_2} \left(z_i^{\uparrow} - w_3\right)\left(z_i^{\uparrow} - w_4\right)\left(z_{j'}^{\downarrow} - w_3\right)\left(z_{j'}^{\downarrow} - w_4\right) \right] \Psi_{S_2}^{221}\left(z_i^{\uparrow}, z_{j'}^{\downarrow}\right)$$

$$\Psi_{[13,24]} = \ldots$$

NASS state: quasi-hole wavefunctions

Step 3.

Decompose wavefunction over $\Psi_{[12,34]}$ and $\Psi_{[13,24]}$ and impose consistency upon fusing some of the parafermions $\psi_{1,2}$ with the σ_3.

This requires Operator Products Expansions (OPE), and 4-point functions in the *SU(3)*$_2$ WZW model [Knizhnik-Zamolodchikov, 1984]

Building blocks are hypergeometric functions

$$F_1^{(0)} = x^{-8/15}(1-x)^{1/15} F(\frac{1}{5},\frac{-1}{5},\frac{2}{5},x)$$

$$F_2^{(0)} = \frac{1}{2} x^{7/15}(1-x)^{1/15} F(\frac{6}{5},\frac{4}{5},\frac{7}{5},x)$$

$$F_1^{(1)} = x^{1/15}(1-x)^{1/15} F(\frac{2}{5},\frac{4}{5},\frac{8}{5},x)$$

$$F_2^{(1)} = -3x^{1/15}(1-x)^{1/15} F(\frac{2}{5},\frac{4}{5},\frac{3}{5},x)$$

$$x = \frac{(w_1 - w_2)(w_3 - w_4)}{(w_1 - w_4)(w_3 - w_2)}$$

NASS state: quasi-hole wavefunctions

Final result

$$\Psi^{(0,1)}_{3333}(w_1,w_2,w_3,w_4;z_1^{\uparrow},z_2^{\uparrow},\ldots,z_{1'}^{\downarrow},z_{2'}^{\downarrow},\ldots) =$$

$$A^{(0,1)}_{3333}(\{w_i\})\Psi_{[12,34]}(\{w_i;z_i,z_{j'}\}) + B^{(0,1)}_{3333}(\{w_i\})\Psi_{[13,24]}(\{w_i;z_i,z_{j'}\})$$

$$A^{(0)}_{3333} = [w_{12}w_{34}]^{4/5} x^{-2/15}(1-x)^{2/3} F_2^{(0)}(x)$$

$$B^{(0)}_{3333} = [w_{12}w_{34}]^{4/5} x^{-2/15}(1-x)^{2/3} F_1^{(0)}(x)$$

$$A^{(1)}_{3333} = [w_{12}w_{34}]^{4/5}(-1)^{8/5} C\, x^{-2/15}(1-x)^{2/3} F_2^{(1)}(x)$$

$$B^{(1)}_{3333} = [w_{12}w_{34}]^{4/5}(-1)^{8/5} C\, x^{-2/15}(1-x)^{2/3} F_1^{(1)}(x)$$

$$C^2 = \frac{1}{9}\frac{\Gamma^3\left(\frac{2}{5}\right)\Gamma\left(\frac{4}{5}\right)}{\Gamma^3\left(\frac{3}{5}\right)\Gamma\left(\frac{1}{5}\right)}$$

NASS state: quasi-hole braiding

Explicit expressions for 4 quasi-hole wavefunction:

$$\Psi^{(0,1)}_{3333}(w_1,w_2,w_3,w_4;z_1^{\uparrow},z_2^{\uparrow},\ldots,z_{1'}^{\downarrow},z_{2'}^{\downarrow},\ldots)$$
$$= A^{(0,1)}_{3333}(\{w_i\})\Psi_{[12,34]}(\{w_i;z_i,z_{j'}\})$$
$$+ B^{(0,1)}_{3333}(\{w_i\})\Psi_{[13,24]}(\{w_i;z_i,z_{j'}\})$$

$$A^{(0)}_{3333} = [w_{12}w_{34}]^{4/5}x^{-2/15}(1-x)^{2/3}F_2^{(0)}(x)$$
$$B^{(0)}_{3333} = [w_{12}w_{34}]^{4/5}x^{-2/15}(1-x)^{2/3}F_1^{(0)}(x)$$
$$A^{(1)}_{3333} = [w_{12}w_{34}]^{4/5}(-1)^{8/5}C\,x^{-2/15}(1-x)^{2/3}F_2^{(1)}(x)$$
$$B^{(1)}_{3333} = [w_{12}w_{34}]^{4/5}(-1)^{8/5}C\,x^{-2/15}(1-x)^{2/3}F_1^{(1)}(x)$$

Example of braiding: w_2 <--> w_3.

this swaps $\Psi_{[12,34]}$ and $\Psi_{[13,24]}$; furthermore

$$F_2^{(0)}(1-x) = C_0^0 F_1^{(0)}(x) + C_1^0 F_1^{(1)}(x), \quad \text{etc.}$$
$$C_0^0 = \frac{1}{2}\left(\sqrt{5}-1\right) = \tau\,, \quad C_1^0/C = -\sqrt{\tau}$$

$$\Leftrightarrow$$

$$U_{2\leftrightarrow 3} = (-1)^{4/5}\begin{pmatrix} \tau & \sqrt{\tau} \\ \sqrt{\tau} & -\tau \end{pmatrix}$$

NASS state: quasi-hole braiding

Full set of braiding relations on the 4 quasi-hole wavefunctions at M=0

$$U_{1\leftrightarrow 2} = (-1)^{-2/3}\begin{pmatrix} (-1)^{4/5} & 0 \\ 0 & (-1)^{-3/5} \end{pmatrix}$$

$$U_{2\leftrightarrow 3} = (-1)^{4/5}\begin{pmatrix} \tau & \sqrt{\tau} \\ \sqrt{\tau} & -\tau \end{pmatrix}$$

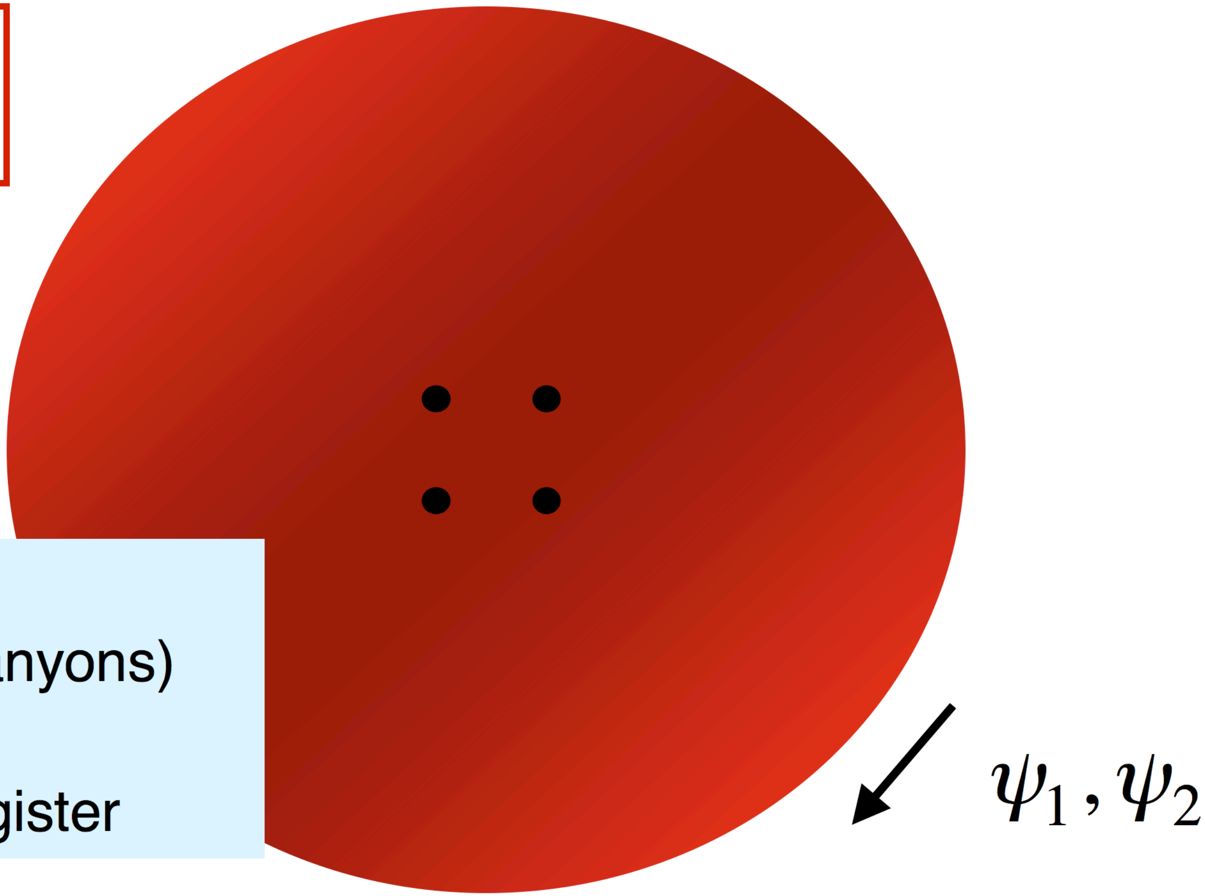
$$U_{1\leftrightarrow 3} = (-1)^{8/15}\begin{pmatrix} \tau & (-1)^{-3/5}\sqrt{\tau} \\ (-1)^{-3/5}\sqrt{\tau} & (-1)^{-1/5}\tau \end{pmatrix}$$

This shows that the NASS quasi-holes are Fibonacci anyons indeed!

Ardonne-KjS, 2007

NASS states: bulk and edge

NASS state
at filling 4/7

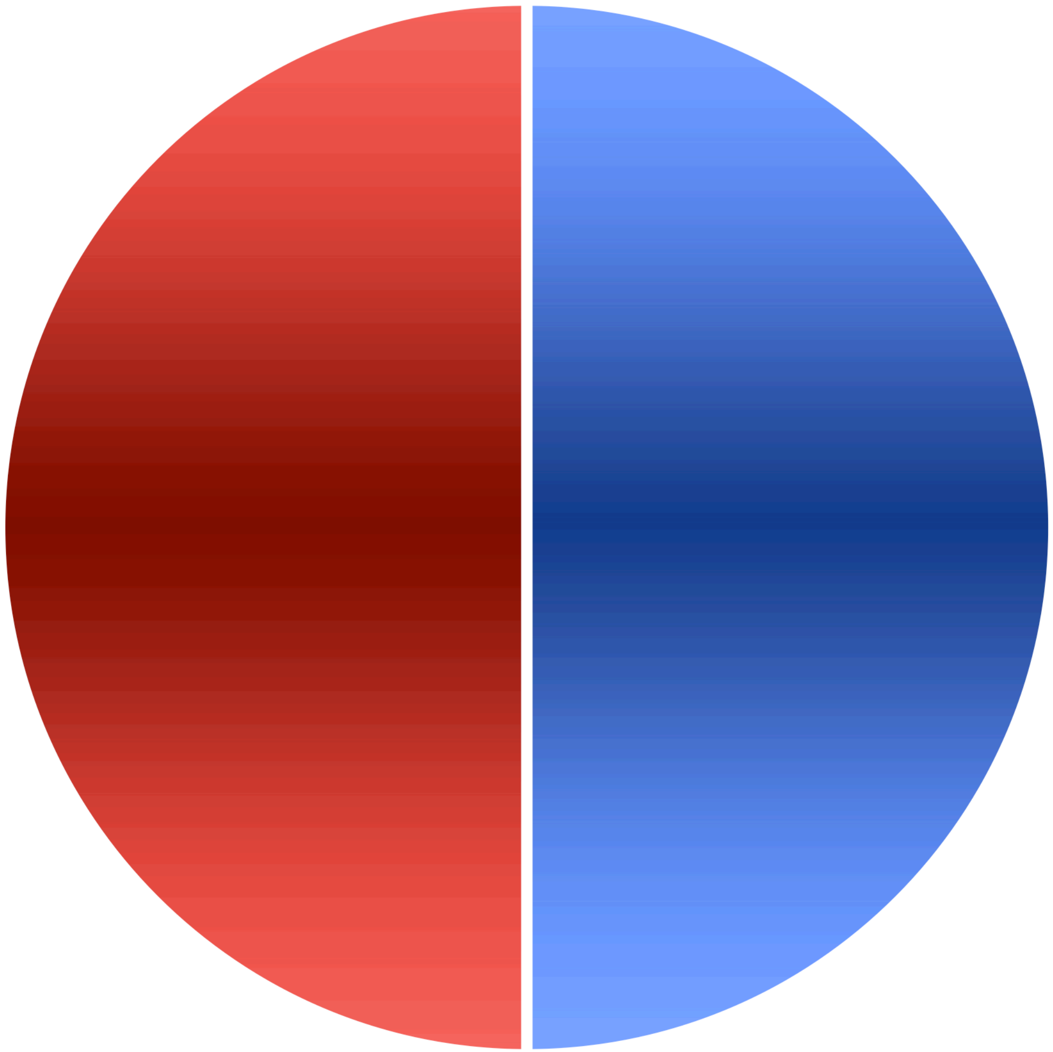


quasi-holes
(Fibonacci anyons)
spanning a
quantum register

neutral edge
modes: $SU(3)_2$
parafermions

Fibonacci meets Ising: NASS / MR

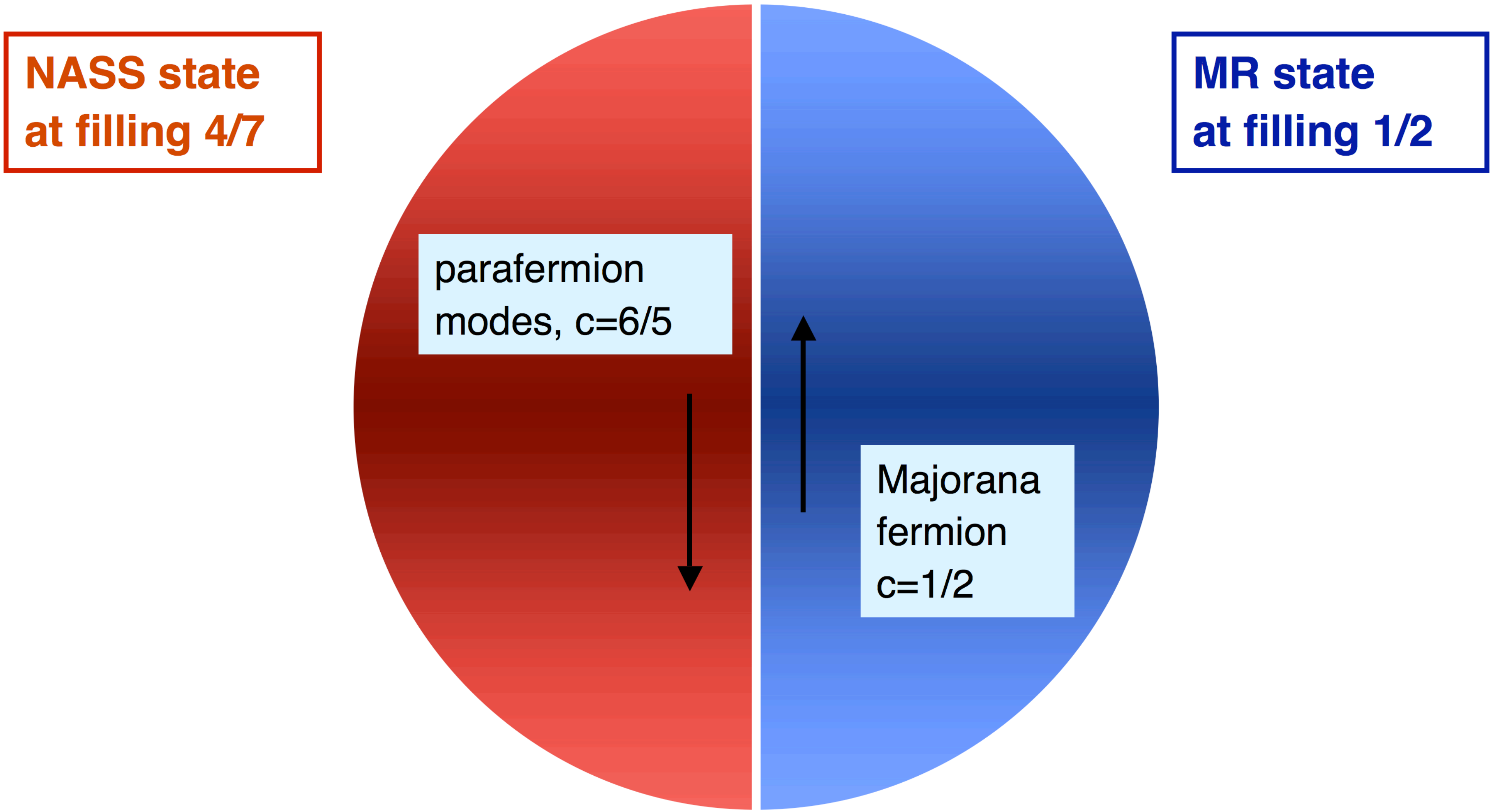
NASS state
at filling 4/7



MR state
at filling 1/2

Grosfeld - S, 2008

Fibonacci meets Ising: NASS / MR

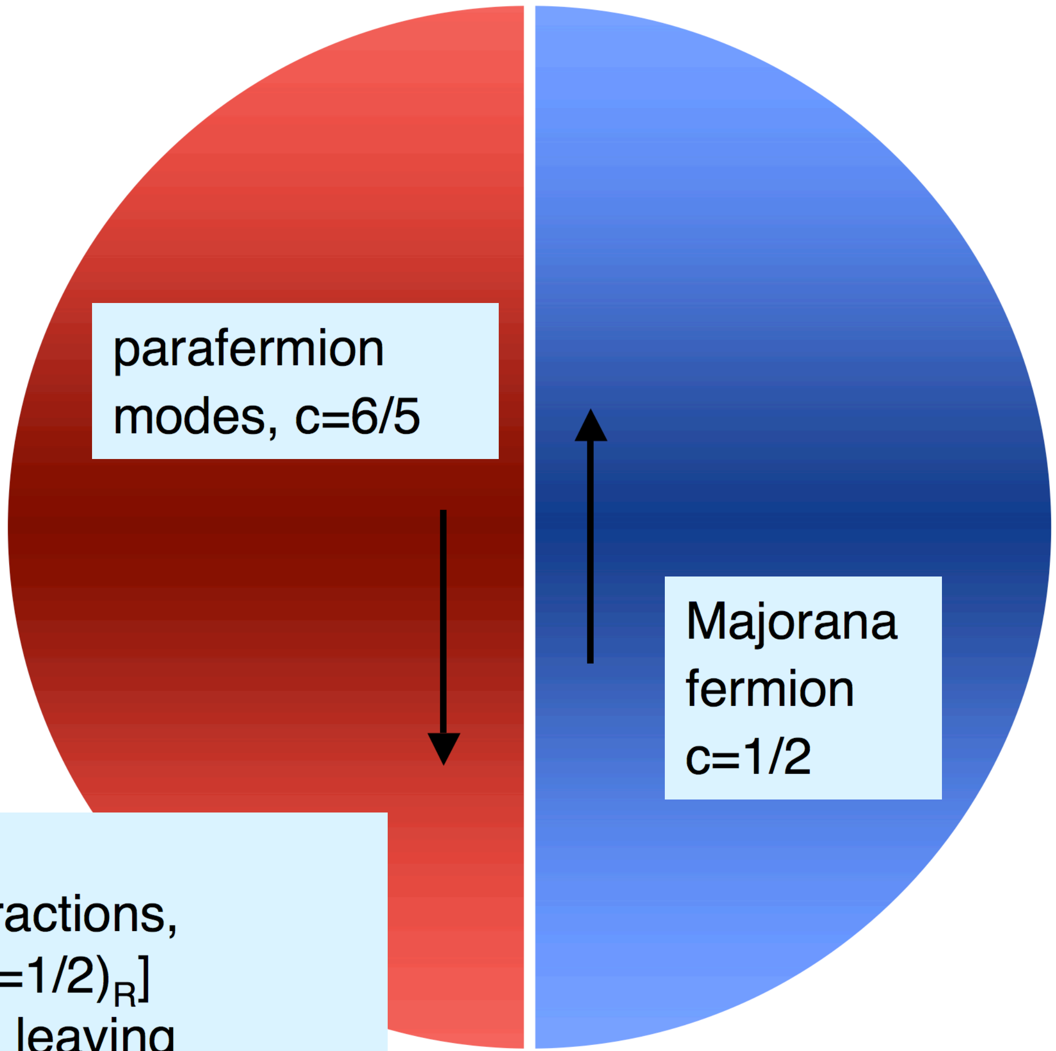


Grosfeld - S, 2008

Fibonacci meets Ising: NASS / MR

NASS state at filling 4/7

MR state at filling 1/2



Mechanism:
due to inter-edge interactions,
a factor $[(c=1/2)_L \times (c=1/2)_R]$
develops a mass gap, leaving
a net $[(c=7/10)_R]$

Grosfeld - S, 2008

Fibonacci meets Ising: NASS / MR