Shocks and surprises

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Based on work with Peter Bowcock and Cristina Zambon:

- P. Bowcock, EC, C. Zambon, IJMPA 19 (Suppl) 2004

    (Text of a talk at the Landau Institute 2002)

- P. Bowcock, EC, C. Zambon, JHEP 0401 2004
- EC, C. Zambon, JPA 37L 2004
- P. Bowcock, EC, C. Zambon, JHEP 0508 2005
- EC, C. Zambon, JHEP 0707 2007

See also

- G. Delfino, G. Mussardo, P. Simonetti, PLB 328 1994,
  NPB 432 1994
- R. Konik, A. LeClair, NPB 538 1999

and, for an alternative algebraic setting

- M. Mintchev, E. Ragoucy, P. Sorba, PLB 547 2002

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- eg flow flips from supersonic to subsonic,

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Consider the x-axis with a shock located at x_0

$$\cdots \qquad u(x,t) \qquad x_0 \qquad v(x,t) \qquad \cdots$$

How to sew the two fields together at x_0?

Expect, in a Lagrangian description,

$$\mathcal{L}(u,v) = \theta(x_0 - x)\mathcal{L}(u) + \theta(x - x_0)\mathcal{L}(v) + \delta(x - x_0)\mathcal{B}(u,v),$$

where $\mathcal{B}(u,v)$ could depend on $u, v, u_t, v_t, \ldots$.

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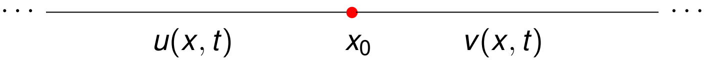

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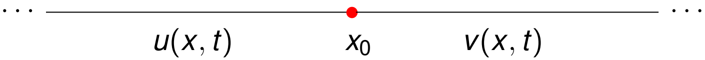

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Example: u, v are free Klein-Gordon fields with mass m

$$\mathcal{B}(u,v) = -\frac{\lambda}{2}uv + \frac{(u_x+v_x)}{2}(u-v)$$

leading to

$$\begin{aligned}
(\partial^2+m^2)u &= 0 \quad x<0\\
(\partial^2+m^2)v &= 0 \quad x>0\\
u &= v \quad x=x_0\\
v_x-u_x &= \lambda u \quad x=x_0
\end{aligned}$$

This is a basic δ-impurity.

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Defects of shock-type

Start with a single selected point on the x-axis, say $x = 0$, and as before denote the field to the left of it ($x < 0$) by u, and to the right ($x > 0$) by v, with field equations in their respective domains:

$$\begin{aligned} \partial^2 u &= -\frac{\partial U}{\partial u}, \quad x < 0 \\ \partial^2 v &= -\frac{\partial V}{\partial v}, \quad x > 0 \end{aligned}$$

- How can the fields be 'sewn' together in a manner preserving integrability?
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• Potential problem: there is a distinguished point, translation symmetry is lost and the conservation laws - at least some of them - (for example, momentum), are violated unless the impurity has the property of adding by compensating terms.

Consider the field contributions to momentum:

$$\mathcal{P} = -\int_{-\infty}^{0} dx\, u_t u_x - \int_{-\infty}^{0} dx\, v_t v_x .$$

Then, using the field equations, $2\dot{\mathcal{P}}$ is given by

$$\begin{aligned} &= -\int_{-\infty}^{0} dx\, \left[u_t^2 + u_x^2 - 2U(u)\right]_x - \int_{0}^{\infty} dx\, \left[v_t^2 + v_x^2 - 2V(v)\right]_x \\ &= -\left[u_t^2 + u_x^2 - 2U(u)\right]_{x=0} + \left[v_t^2 + v_x^2 - 2V(v)\right]_{x=0} \\ &= -2\frac{d\mathcal{P}_s}{dt}\ (?). \end{aligned}$$

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If there are ‘sewing’ conditions for which the last step is valid then $\mathcal{P} + \mathcal{P}_s$ will be conserved, with $\mathcal{P}_s$ a function of u, v, and possibly derivatives, evaluated at $x = 0$.

(Note: this does not happen for a δ-impurity.)

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Next, consider the energy density and calculate

$$\dot{\mathcal{E}} = [u_x u_t]_0 - [v_x v_t]_0.$$

Setting $u_x = v_t + X(u,v), \ \ v_x = u_t + Y(u,v)$ we find

$$\dot{\mathcal{E}} = u_t X - v_t Y.$$

This is a total time derivative provided for some S

$$X = -\frac{\partial S}{\partial u}, \ \ Y = \frac{\partial S}{\partial v}.$$

Then

$$\dot{\mathcal{E}} = -\frac{dS}{dt},$$

and $\mathcal{E} + S$ is conserved, with S a function of the fields evaluated at the shock.

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This argument strongly suggests that the only chance will be sewing conditions of the form

$$u_x = v_t - \frac{\partial S}{\partial u}, \quad v_x = u_t + \frac{\partial S}{\partial v},$$

where S depends on both fields evaluated at $x = 0$, leading to

$$\dot{\mathcal{P}} = v_t \frac{\partial S}{\partial u} + u_t \frac{\partial S}{\partial v} - \frac{1}{2}\left(\frac{\partial S}{\partial u}\right)^2 + \frac{1}{2}\left(\frac{\partial S}{\partial v}\right)^2 + (U - V).$$

This is a total time derivative provided the first piece is a perfect differential and the second piece vanishes. Thus,

$$\frac{\partial S}{\partial u} = -\frac{\partial \mathcal{P}_s}{\partial v}, \quad \frac{\partial S}{\partial v} = -\frac{\partial \mathcal{P}_s}{\partial u} \ldots .$$

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$$\frac{\partial S}{\partial u} = -\frac{\partial \mathcal{P}_s}{\partial v}, \quad \frac{\partial S}{\partial v} = -\frac{\partial \mathcal{P}_s}{\partial u} \ldots.$$

.... and

$$\frac{\partial^2 S}{\partial v^2} = \frac{\partial^2 S}{\partial u^2}, \quad \frac{1}{2}\left(\frac{\partial S}{\partial u}\right)^2 - \frac{1}{2}\left(\frac{\partial S}{\partial v}\right)^2 = U(u) - V(v).$$

- By setting $S = f(u+v) + g(u-v)$ and differentiating the left hand side of the functional equation with respect to u and v one finds:

$$f'''g' = g'''f'.$$

If neither of f or g is constant we also have

$$\frac{f'''}{f'} = \frac{g'''}{g'} = \gamma^2,$$

where γ is constant (possibly zero). Thus....

.... and

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for $\gamma \neq 0$, and quadratic polynomials for $\gamma = 0$. Various choices of the coefficients will provide sine-Gordon, Liouville, massless free ($\gamma \neq 0$); or, massive free ($\gamma = 0$).

In the latter case, setting $U(u) = m^2 u^2/2$, $V(v) = m^2 v^2/2$, the shock function S turns out to be

$$S(u, v) = \frac{m\sigma}{4}(u+v)^2 + \frac{m}{4\sigma}(u-v)^2,$$

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- Note: there is a Lagrangian description of this type of 'shock':

$$\mathcal{L} \;=\; \theta(-x)\mathcal{L}(u) + \delta(x)\left(\frac{uv_t - u_t v}{2} - S(u,v)\right) + \theta(x)\mathcal{L}(v)$$

The usual E-L equations provide both the field equations for u, v in their respective domains and the 'sewing' conditions.

- Note:

In the free case, with a wave incident from the left half-line

$$u = \left(e^{ikx} + Re^{-ikx}\right)e^{-i\omega t}, \quad v = Te^{ikx}e^{-i\omega t}, \quad \omega^2 = k^2 + m^2,$$

we find:

$$R = 0, \qquad T = -\frac{(i\omega - m\sinh\eta)}{(ik + m\cosh\eta)}, \quad \sigma = e^{-\eta}.$$

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$$S(u, v) = 2\left(\sigma \cos\frac{u+v}{2} + \sigma^{-1}\cos\frac{u-v}{2}\right)$$

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• What happens to a soliton when it encounters a shock of this kind?

Consider a soliton incident from $x < 0$ (any point will do), then it will not be possible to satisfy the sewing conditions (in general) unless a similar soliton emerges into the region $x > 0$.

$$e^{iu/2} = \frac{1 + iE}{1 - iE}, \quad e^{iv/2} = \frac{1 + izE}{1 - izE}, \quad E = e^{ax+bt+c},$$
$$a = \cosh\theta, \quad b = -\sinh\theta.$$

Here z is to be determined. As previously, set $\sigma = e^{-\eta}$.

• We find

$$z = \coth\left(\frac{\eta - \theta}{2}\right).$$

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- $\eta < \theta$ implies $z < 0$; ie the soliton emerges as an anti-soliton.

-The final state will contain a discontinuity of magnitude 4π at $x = 0$.

- $\eta = \theta$ implies $z = 0$ and there is no emerging soliton.

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Comments and questions....

• The shock is local so there could be several shocks located at $x = x_1 < x_2 < x_3 < \cdots < x_n$; these behave independently each contributing a factor z_i for a total 'delay' of $z = z_1 z_2 \ldots z_n$.

• When several solitons pass a defect each component is affected separately

- This means that at most one of them can be 'filtered out' (since the components of a multisoliton in the sine-Gordon model must have different rapidities).

• Can solitons be controlled? (Eg see EC, Zambon, 2004.)

• Since a soliton can be absorbed, can a starting configuration with $u = 0$, $v = 2\pi$ decay into a soliton?

- No, there is no way to tell the time at which the decay would occur (and presumably quantum mechanics would be needed to provide the probability of decay as a function of time).

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- Checking integrability

Adapt an idea from Bowcock, EC, Dorey, Rietdijk, 1995.

Two regions overlapping the shock location: $x > a,\ x < b$ with $a < x_0 < b$.

In each region, write down a Lax pair representation:

$$\begin{aligned}
\hat{a}_t^{(a)} &= a_t^{(a)} - \frac{1}{2}\theta(x-a)\left(u_x - v_t + \frac{\partial S}{\partial u}\right)\\
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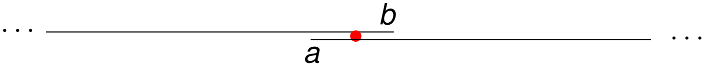

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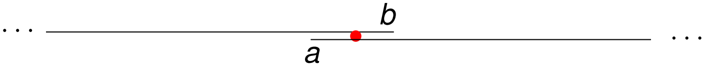

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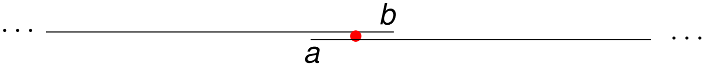

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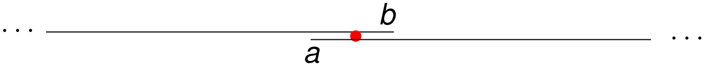

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Where,

$$a_t^{(a)} = u_x H/2 + \sum_i e^{\alpha_i u/2}\left(\lambda E_{\alpha_i} - \lambda^{-1} E_{\alpha_i}\right)$$
$$a_x^{(a)} = u_t H/2 + \sum_i e^{\alpha_i u/2}\left(\lambda E_{\alpha_i} + \lambda^{-1} E_{\alpha_i}\right),$$

$\alpha_0 = -\alpha_1$ are the two roots of the extended $su(2)$ (ie $a_1^{(1)}$) algebra, and H, E_{α_i} are the usual generators of $su(2)$.

There are similar expressions for $a_t^{(b)}, a_x^{(b)}$.

Then

$$\partial_t a_x^{(a)} - \partial_x a_t^{(a)} + \left[a_t^{(a)}, a_x^{(a)}\right] = 0 \;\Leftrightarrow\; \text{sine Gordon}$$

The zero curvature condition for the components of the Lax pairs $\hat{a}_t, \hat{a}_x$ in the two regions imply:

- The field equations for u, v in $x < a$ and $x > b$, respectively,
- The shock conditions at a, b,
- For $a < x < b$ the fields are constant,
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$$\partial_t \kappa = \kappa a_t^{(b)} - a_t^{(a)} \kappa$$

This setup requires the previous expression for $S(u, v)$ when

$$\kappa = e^{-vH/2}\, \tilde{\kappa}\, e^{uH/2} \text{ and } \tilde{\kappa} = |\alpha_1| H + \frac{\sigma}{\lambda}\left(E_{\alpha_0} + E_{\alpha_1}\right).$$

That is

$$S(u, v) = \sigma \sum_0^1 e^{\alpha_i (u+v)/2} + \sigma^{-1} \sum_0^1 e^{\alpha_i (u-v)/2}.$$

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Assume $\sigma > 0$ then...

- Expect Pure transmission compatible with the bulk S-matrix;
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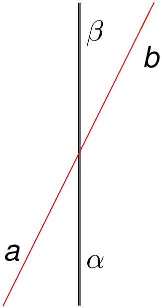

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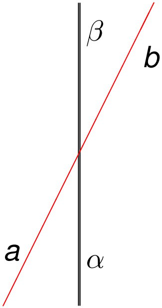

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Schematic triangle relation

$$S^{cd}_{ab}(\Theta)\, T^{f\beta}_{d\alpha}(\theta_a) T^{e\gamma}_{c\beta}(\theta_b) = T^{d\beta}_{b\alpha}(\theta_b) T^{c\gamma}_{a\beta}(\theta_a) S^{ef}_{cd}(\Theta)$$

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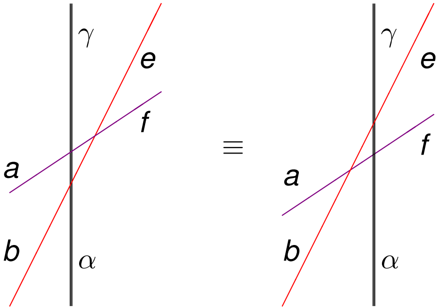

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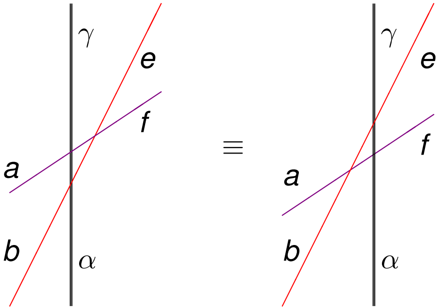

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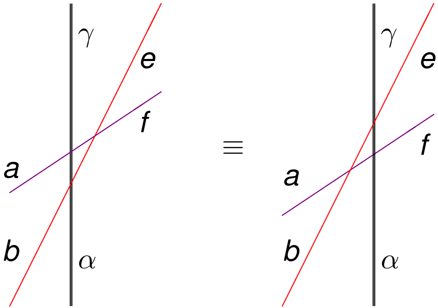

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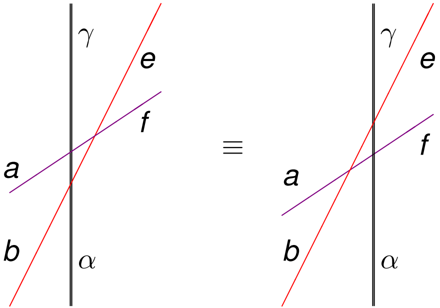

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Zamolodchikov's sine-Gordon S-matrix - reminder

$$S_{ab}^{cd}(\Theta) = \rho(\Theta) \begin{pmatrix} A & 0 & 0 & 0 \\ 0 & C & B & 0 \\ 0 & B & C & 0 \\ 0 & 0 & 0 & A \end{pmatrix}$$

where

$$A(\Theta) = \frac{qx_2}{x_1} - \frac{x_1}{qx_2},\ B(\Theta) = \frac{x_1}{x_2} - \frac{x_2}{x_1},\ C(\Theta) = q - \frac{1}{q}$$

and

$$\begin{aligned} \rho(\Theta) &= \frac{\Gamma(1+z)\Gamma(1-\gamma-z)}{2\pi i} \prod_1^\infty R_k(\Theta) R_k(i\pi - \Theta) \\ R_k(\Theta) &= \frac{\Gamma(2k\gamma + z)\Gamma(1 + 2k\gamma + z)}{\Gamma((2k+1)\gamma + z)\Gamma(1 + (2k+1)\gamma + z)},\ z = i\gamma/\pi. \end{aligned}$$

The Zamolodchikov S-matrix depends on the rapidity variables θ and the bulk coupling β via

$$x = e^{\gamma\theta},\ q = e^{i\pi\gamma},\ \gamma = \frac{8\pi}{\beta^2} - 1,$$

and it is also useful to define the variable

$$Q = e^{4\pi^2 i/\beta^2} = \sqrt{-q}.$$

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$$\bar{f}(q,x) = f(q,qx)$$
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A slightly alternative discussion of these points is given in Bowcock, EC, Zambon, 1995, where most of the properties noted below are also described.

- A 'minimal' solution has the following form

$$f(q,x) = \frac{e^{i\pi(1+\gamma)/4}}{1+ie^{\gamma(\theta-\eta)}}\,\frac{r(x)}{\bar{r}(x)},$$

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Remarks ($\theta > 0$): it is tempting to suppose η (possibly renormalized) is the same parameter as in the classical model.

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• These are the same features we saw in the classical soliton-shock scattering.

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$$\theta = \eta - \frac{i\pi}{2\gamma} \to \eta, \;\; \beta \to 0$$

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Remarks ($\theta > 0$): it is tempting to suppose η (possibly renormalized) is the same parameter as in the classical model.

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$$E = m_s \cosh\theta = m_s(\cosh\eta\cos(\pi/2\gamma) - i\sinh\eta\sin(\pi/2\gamma))$$

and a 'width' proportional to $\sin(\pi/2\gamma)$.

Using this pole and a bootstrap to define ${}^{odd}T$ leads to a non-unitary transmission matrix - interpret as the instability corresponding to the classical feature noted at $\theta = \eta$.

- The Zamolodchikov S-matrix has 'breather' poles corresponding to soliton-anti-soliton bound states at

$$\Theta = i\pi(1 - n/\gamma),\ n = 1, 2, ..., n_{\max};$$

use the bootstrap to define the transmission factors for breathers and find for the lightest breather:

$$T(\theta) = -i\frac{\sinh\left(\frac{\theta-\eta}{2} - \frac{i\pi}{4}\right)}{\sinh\left(\frac{\theta-\eta}{2} + \frac{i\pi}{4}\right)}$$

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Consider the x-axis with a shock located at x_0 and asymptotic values of the fields

$$\cdots \quad u = 2a\pi/\beta \qquad\qquad x_0 \qquad\qquad v = 2b\pi/\beta \quad \cdots$$

Compare $(0,0)$ and (a,b) in functional integral representations:

$$u \to u - 2a\pi/\beta,\ v \to v - 2b\pi/\beta,\ A \to A + \delta A$$

with

$$\delta A = \frac{\pi}{\beta}\int_{-\infty}^{\infty} dt(av_t - bu_t) = \frac{\pi}{\beta}(a\delta v - b\delta u)_{x_0}$$

Soliton: $(a,b) \to (a-1,b-1)$, so $\delta u = \delta v = -2\pi/\beta$

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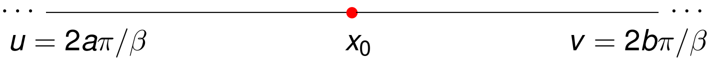

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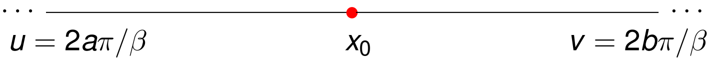

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