Tensionless branes, higher spins and the singleton

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based on hep-th/0701051, with J. Engquist and P. Sundell,
on their previous paper hep-th/0508124
and on work in progress

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1. Introduction
2. The scalar singleton
3. From brane partons to WZW models
4. The $\widehat{\mathfrak{so}}(D-1,2)_{-\epsilon_0}$ WZW model
5. Coset models
6. Conclusions

Motivation

- The group theoretical structure underlying AdS physics is intrinsically different with respect to the flat case. Exploit this in the study of string and brane dynamics in AdS.

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  Understand string theory in the high energy limit?
- Exploit physical intuition from AdS brane dynamics to study a class of noncompact WZW models, interesting in their own right.

Inspiration

Singletons in the AdS/CFT correspondence:

- On the *CFT* side "partonic" description as continuous limit of a discrete spin chain.
  A singleton at each site of the spin chain. (Beisert '03)
  Anything similar on the string side?
- In AdS_5 singletonic dof are pure gauge in the bulk and can be gauged away except at the boundary. Field content = decoupled $U(1)$ of the $U(N)$ $N = 4$ SYM. (Kim, Romans, van Nieuwenhuizen '85)

Singletons and AdS

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- Singletons do not survive the flat space limit. The other irreps can be classified as massless or massive, according to their flat space limit.
- Singletonic particles can be naturally described in the zero radius limit of AdS (Dirac's Hypercone).

Singleton are "fundamental"

... at least in the tensionless limit.

Theorem

singleton $\otimes$ *singleton* $= \bigoplus$ *massless irrep.* *Flato,Frønsdal '78.*

...in general D: Vasiliev '04.

and the product of more than two singletons gives massive representations.
(full classification, to the best or our knowledge, not done yet).

In the bulk of AdS one observes singleton COMPOSITES!

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- The underlying HS algebra is an infinite dimensional extension of $\mathfrak{so}(D-1,2)$, acting on singletons.
- The states of the AdS$_D$ HS models correspond to the tensor product of two singleton modules. (Vasiliev '04).

$\mathfrak{so}(D-1,2)$ algebra

$$[M_{AB}, M_{CD}] = i\eta_{BC}M_{AD} + 3 \;\; perm.$$
$$M_{AB} = -M_{BA} = (M_{AB})^\dagger$$
where $A = 0', 0, 1, ..., D-1$ and $\eta_{AB} = \mathrm{diag}(-,-,+,...,+)$

Maximal compact subalgebra $\mathfrak{so}(2) \oplus \mathfrak{so}(D-1)$:

- $\mathfrak{so}(2)$ (spanned by $E = M_{0'0}$)$\rightarrow$ AdS time translations
- $\mathfrak{so}(D-1)$ (spanned by $J_{rs} = M_{rs}$)$\rightarrow$ AdS rotations
- The rest: ladders $L_r^\pm = M_{0r} \mp iM_{0',r}$$\rightarrow$ AdS spin boosts

In this compact basis, the algebra becomes
$$\left[E, L_r^\pm\right] = \pm L_r^\pm \quad ; \quad \left[J_{rs}, L_t^\pm\right] = 2i\delta_{t[s}L_{r]}^\pm$$
$$\left[L_r^-, L_s^+\right] = 2(iJ_{rs} + \delta_{rs}E)$$
$$[J_{rs}, J_{tu}] = i\delta_{st}J_{ru} + 3 \;\; \mathrm{perm.}$$

LW irrep of $\mathfrak{so}(D-1,2)$

A unitary irreducible lowest weight representation $\mathcal{D}(E_0, \mathbf{j})$ of $\mathfrak{so}(D-1,2)$ characterized by its ***lowest weight state*** $|E_0, \mathbf{j}\rangle$:

- annihilated by L_r^-,
- with $\mathfrak{so}(2)$ energy eigenvalue E_0,
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Unitarity bound:

between energy and spin.
In the scalar case, $E_0 \geq \frac{D-3}{2}$ or $E_0 = 0$.

The scalar singleton

Consider $|\epsilon_0\rangle$ such that

$$E|\epsilon_0\rangle = \epsilon_0|\epsilon_0\rangle \quad ; \quad J_{rs}|\epsilon_0\rangle = 0 \quad ; \quad L_r^-|\epsilon_0\rangle = 0$$

Build the *generalized Verma module*

$$\mathcal{V}(\epsilon_0, 0) \equiv \left\{ L_{r_1}^+ \dots L_{r_n}^+ |\epsilon_0\rangle \right\}_{n=0}^{\infty}$$

For $\epsilon_0 = \frac{D-3}{2}$, saturating the unitarity bound:

- $L_s^+ L_s^+ |\epsilon_0\rangle$ is itself a LWS (it's a singular vector)
- $L_{r_1}^+ \dots L_{r_n}^+ L_s^+ L_s^+ |\epsilon_0\rangle$ is normal to all states (it's a null vector)

Singleton: mod out this "trace-part" from $\mathcal{V}(\epsilon_0, 0)$

$$\mathcal{D}(\epsilon_0, 0) \equiv \left\{ L_{\{r_1}^+ \dots L_{r_n\}}^+ |\epsilon_0\rangle \right\}_{n=0}^{\infty} \quad , \quad \epsilon_0 = \frac{D-3}{2}$$

A lift to (D+1)

Lift the $(D-1)$-covariant condition

$$L_s^+ L_s^+ |\epsilon_0\rangle \sim 0$$

defining the singleton to a $(D+1)$-covariant constraint.
Consider the symmetric traceless bilinear in the currents:

$$V_{AB} = M_{(A}{}^{C} M_{B)C} - \mathit{trace}$$

Then the decoupling condition:

$$\langle \Psi | V_{AB} | \Psi' \rangle = 0$$

is solved only by the scalar (all D) and spinor singletons $(D = 3, 4)$. (Engquist,Sundell,L.T. '07)
Composites? Wait for the affine extension!

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- In $(D+1)$-dimensional ***configuration space***:

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$$S = \frac{1}{2}\int d\tau \left(e^{-1}\dot{X}^2 - eM^2 + \lambda(X^2 + R^2)\right)$$

- In $2(D+1)$-dimensional ***phase-space***:

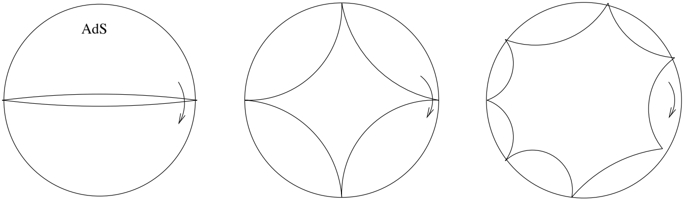
$$S = \int \left(\frac{1}{4}Y^{Ai}DY_{Ai} - \frac{1}{2}\Lambda^{ij}V_{ij}\right)$$

where $Y_i^A = \sqrt{2}(X^A, P_A)$ is an $\mathfrak{sp}(2)$ doublet, Λ^{ij} is an $\mathfrak{sp}(2)$ gauge field, $DY^{Ai} \equiv dY^{Ai} + \Lambda^{ij}Y_j^A$ and $V_{ij} = \mathrm{diag}(M^2, R^2)$.
$\mathfrak{sp}(2)$ invariant phase, $M = R = 0$, $\leftrightarrow$ singletonic particle.

Spiky strings and singletons

String solution in AdS: Folded rotating string with $E \sim S$, (2-cusp string). It can be generalized to N cusps.

Gubser, Klebanov,Polyakov;Kruczenski;Frolov,Tseytlin '02.



- 2-cusp string $\leftrightarrow$ bound state of two partons (singletons).
- Deviation from free singletons measured by effective tension.
- Arbitrary number of cusps $\leftrightarrow$ singleton gas.

Engquist, Sundell '05

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Generalize to folded rotating p-branes.

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Comments:

- Importance of negative cosmological constant. Partonic behavior doesn't occur in flat space.
- Instabilities. The cusps attract each other but randomly distributed configurations are stable.
- Folded rotating branes in AdS perform a stringy motion (they collapse in the transverse directions). Description in terms of a string-like $2d$ sigma model.

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- In particular, a p-brane in AdS can be described by the discretized $(0+1)$-dimensional Nambu-Goto action with N “pieces", or partons.

Are these partons singletons?

To uncover the singletonic nature of the partons, take **tensionless** limit together with **zero radius** in AdS.

$$\Downarrow$$

The system of N partons is described by a singleton gas.

Discretized p-branes

Gauge-fixed Nambu-Goto with rescaled spacial wv. coord.

$$S_p = \int d^{p+1}\sigma \left(\dot{X}^2 - T_p^2 \mathrm{det}' g + \lambda(X^2 + R^2)\right)$$

to be supplemented with the set of constraints:

$$\dot{X}^2 + T_p^2 \mathrm{det}' g = 0 \quad ; \quad \dot{X}^A \partial_r X_A = 0$$

To discretize, replace $X^A(\tau, \sigma^r) \to \{X^A(\tau;\xi)\}_{\xi=1}^N$, but also replace derivatives ∂_r by differences (one needs a prescription!).

Require closure of the algebra of constraints for zero tension and zero radius. Only possible choice for the constraints is:

$$\dot{X}^A(\xi) X_A(\eta) = 0 \quad \forall\, \xi, \eta \qquad (\mathfrak{sp}(2N) \text{ invariant})$$

A heuristic continuum limit

Discretized AdS p-brane in the tensionless and zero radius limit described by the phase-space $\mathfrak{sp}(2N)$-gauged action

$$S = \int d\tau \, Y^{IA} D Y_{IA}, \quad \text{with} \;\; Y_I^A = \left(Y_{i_1}(1) \ldots Y_{i_N}(N)\right), I = 1, \ldots, 2N$$

The limit $N \to \infty$, taken after having first truncated $\mathfrak{sp}(2N) \to \bigoplus_\xi \mathfrak{sp}(2)_\xi$, leads to the $\mathfrak{sp}(2)$-gauged string:

$$S = \frac{1}{2} \int_\Sigma d\tau d\sigma \left(P^A \dot{X}_A + \Lambda^{ij} K_{ij} \right)$$

(K_{ij} is the $\mathfrak{sp}(2)_\xi$ generator), which was shown to be equivalent, in $D = 7$, to a $\widehat{\mathfrak{so}}(6,2)_{-2}$ WZW model. Engquist, Sundell '05

A coset model

- Singleton gas of N partons $\to$ worldline gauged sigma model.
- Target space $=$ N-singleton phase space.
- $N \to \infty$: processes involving an arbitrary number of partons described by a gauged WZW model on the coset

$$\frac{\widehat{\mathfrak{so}}(D-1,2)_{-\epsilon_0}}{\widehat{\mathfrak{h}}_{-\epsilon_0}}\,, \qquad \text{with } \epsilon_0 = (D-3)/2$$

$\mathfrak{h}$ such that the coset contains only singletons and their composites.

Keypoint:

The choice of the level $-\epsilon_0$ (fractional for even D!)$\to$ WZW model not well understood.

Spectrum of the $\widehat{\mathfrak{so}}(D-1,2)_{-\epsilon_0}$ WZW model

A covariant version of the **nontrivial vacuum singular vector** technique.
(Lesage et al, Gaberdiel)

Main observation:

For $k=-\epsilon_0$, $\quad V_{AB}(z)=\left(M_{(A}{}^{C}(z)M_{B)C}(z)\right)-\mathrm{trace}$
is a **KM primary** $\Rightarrow$ the primaries must *decouple* from it

i.e. requiring that the $\mathfrak{so}(D-1,2)$-covariant decoupling condition for the singletons extends to the affine case fixes the KM level to minus the zero point energy of the singleton!

Solutions

The infinite set of scalars

$$|e_0 = P\epsilon_0\rangle, \qquad P = 0, \pm 1, \pm 2, \ldots$$

defined by the **P-twisted** conditions

$$\left(L_r^-\right)_n |e_0\rangle = 0, \quad n \geq -P \quad ; \quad \left(L_r^+\right)_n |e_0\rangle = 0, \quad n \geq P$$
$$\left(J_{rs}\right)_n |e_0\rangle = 0, \quad E_n|e_0\rangle = \delta_{n,0}\, P\epsilon_0|e_0\rangle, \quad n \geq 0$$

Highest (lowest) weight conditions shifted up (down) by P.
($P = 0$ vacuum, $P = 1$ singleton, $P = 2$ massless, $P > 2$ massive,
$P \to -P$: particle $\to$ antiparticle).

Important: for $P > 1$ these are not standard WZW primaries!
However, they are conformal primaries, with $h_P = -\frac{P^2\epsilon_0}{2}$.

Spectral flow

An invariance of the current algebra

affine extension of $\mathfrak{so}(D-1,2)$

$$
\begin{aligned}
&[J_{rs,m}, J_{tu,n}] = i(\delta_{st}J_{ru,m+n} + 3\ \text{perm.}) + 2km\delta_{t[r}\delta_{s]u}\delta_{m+n,0} \\
&[E_m, E_n] = km\delta_{m+n,0} \\
&[L^-_{r,m}, L^+_{s,n}] = 2(iJ_{rs,m+n} + \delta_{rs}E_{m+n}) - 2km\delta_{rs}\delta_{m+n,0} \\
&[L^\pm_{r,m}, L^\pm_{s,n}] = 0 \\
&[E_m, L^\pm_{r,n}] = \pm L^\pm_{r,m+n} \quad ; \quad [J_{rs,m}, L^\pm_{t,n}] = 2i\delta_{t[s}L^\pm_{r],m+n} \\
&[J_{rs,m}, E_n] = 0
\end{aligned}
$$

under the transformations

$$
\tilde{L}^\pm_n = L^\pm_{n\mp w} \quad ; \quad \tilde{E}_n = E_n + kw\delta_{n,0} \quad ; \quad \tilde{J}_{rs,n} = J_{rs,n}
$$

The flow

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- The P-tupletons are all ***connected*** by spectral flow. So they must all be there!
- The GKO conditions for the maximal compact gauging

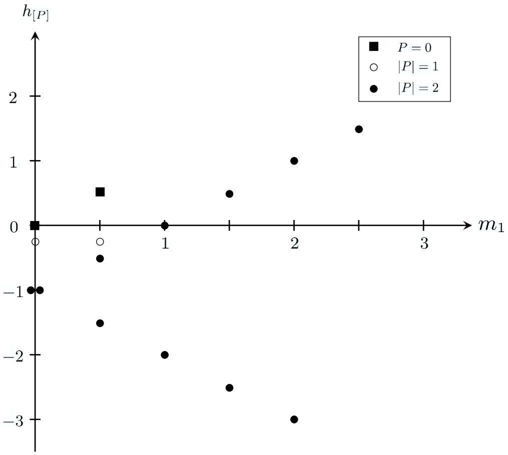
$$E_m|\psi\rangle = 0 \quad , \quad J_{rs,m}|\psi\rangle = 0 \quad (m > 0)$$

are ***invariant*** under spectral flow.
So this choice of the gauging seems to be preferred.

$D = 4$ is special

- The spinor singleton also solves the decoupling condition.
- $\mathfrak{so}(3,2) \sim \mathfrak{sp}(4)$.
- $\widehat{\mathfrak{sp}}(4)_{-1/2}$ WZW model with scalar and spinor singletons plus composites realized in terms of 4 real symplectic bosons.
- a "standard" CFT setting: free fields etc.

The $D = 4$ $P = 0, 1, 2$ sector ground states



$D = 7$ is also special

There is an extra singular bilinear in the currents:

$$W_{ABCD}(z) = \left(M_{[AB}(z)M_{CD]}(z)\right)$$

- No restrictions on the P-twisted primaries.
- Less excited states, at least in the one-singleton sector.

The WZW model in $D = 7$ is the simplest!

Maximal Compact Gauging: P=1, level 1.

$$\frac{\widehat{\mathfrak{so}}(D-1,2)_{-\epsilon_0}}{\widehat{\mathfrak{so}}(D-1)_{-\epsilon_0} \oplus \widehat{\mathfrak{so}}(2)_{-\epsilon_0}} , \qquad \text{with } \epsilon_0 = (D-3)/2$$

conjectured to be topological, i.e. containing only (singleton and singleton-composite) zero modes.

(coset appears also in Mikhailov 04)

Main observations:

- vanishing central charge.
- the singleton primary field has vanishing conformal weight.
- GKO gauging, one-singleton sector: at Virasoro level 1, the states that survive the gauging are KM-singular.

Level 2: The "evil" state

At Virasoro level 2 consider the state

$$|\psi\rangle \equiv L^{-}_{r,-2}|\epsilon_0\rangle$$

- gauge invariant, since $E_1|\psi\rangle$ and $J_{rs,1}|\psi\rangle$ are singular states at level 1 and $E_2|\psi\rangle = 0$.
- not KM singular, since for example

$$L^{+}_{s,2}|\psi\rangle = 2\delta_{sr}\epsilon_0|\epsilon_0\rangle \neq 0 \quad \text{for any } D > 3.$$

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Conclusion:

The GKO gauging of the maximal compact subalgebra does not lead to a topological model, describing only singletons and singletons composites. It contains nontrivial excitations.

Conclusions: The $\widehat{\mathfrak{so}}(D-1,2)_{-\epsilon_0}$ WZW model

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  - A generalized definition of primary fields needed...
  - ... supported by the nontrivial action of the spectral flow.

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- $P = 2$ (massless) sector of interest for HSGT:
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  An affine Lie algebraic setting for HSGT?
- Gauging is needed to remove unphysical negative norm states...

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- ...before the discovery of the evil state at level 2, that is GKO gauge invariant but not KM singular.
- We're trying to use the singular vector V_{AB} (and W_{ABCD} in $D=7$) to find a formula giving all nontrivial excitation at any Virasoro level. A finite number for each level?