The E_{11} origin of gauged maximal supergravities

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based on work with Peter West

arXiv:0705.0752

Introduction

Massless maximal supergravities all arise from dimensional reduction of 11-dimensional and IIB supergravities.

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The scalars parametrise the manifold G/H, where H is the maximal compact subgroup of G.

Introduction

D	G
10A	$\mathbb{R}^+$
10B	$SL(2,\mathbb{R})$
9	$SL(2,\mathbb{R}) \times \mathbb{R}^+$
8	$SL(3,\mathbb{R}) \times SL(2,\mathbb{R})$
7	$SL(5,\mathbb{R})$
6	$SO(5,5)$
5	$E_{6(+6)}$
4	$E_{7(+7)}$
3	$E_{8(+8)}$

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$\longrightarrow$ Massive theories

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$\longrightarrow$ Massive theories

Although some of these theories can be seen as Scherk-Schwarz compactifications, or as reductions with fluxes turned on, in general a complete understanding of such theories in terms of higher dimensional ones is lacking

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Simplest example: Romans' massive IIA (not actually a gauged theory)

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This theory does not arise from 11-dimensional supergravity

E_{11} provides an 11-dimensional origin of all maximal supergravities (and much more...)

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- More about supergravities

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- An introduction to E_{11}

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- The fields of E_{11}

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- Conclusions

More about supergravities

In a series of papers, all the gauged maximal supergravities in $D = 7, 6, \ldots, 3$ have been classified

de Wit, Samtleben and Trigiante, hep-th/0212239, hep-th/0412173, hep-th/0507289

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Gauging:

$$D_\mu = \partial_\mu - A_\mu^M \Theta_M{}^\alpha t_\alpha$$

The embedding tensor Θ belongs to a reducible representation of G

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The fact that the gauge symmetry is a Lie group, as well as supersymmetry, pose constraints on Θ

More about supergravities

Example: $D = 5$

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$A_{\mu,M}$ belongs to the $\mathbf{27}$ of E_6

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The embedding tensor belongs to

$$\overline{\mathbf{27}} \otimes \mathbf{78} = \overline{\mathbf{27}} \oplus \overline{\mathbf{351}} \oplus \overline{\mathbf{1728}}$$

The Jacobi identities and the constraints from supersymmetry restrict the embedding tensor to be in the $\overline{\mathbf{351}}$

More about supergravities

Field strength:

$$\partial_\mu A_{\nu,M} - \frac{1}{2} A_{\mu,N} \Theta^N{}_\alpha (t^\alpha)_M{}^P A_{\nu,P} - 2 Z_{MN} A_{\mu\nu a}{}^N$$

where

$$Z_{MN} = Z_{[MN]} \qquad Z_{MN} \Theta^N_\alpha = 0$$

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The vectors that do not belong to the adjoint of the gauge group are gauged away, *i.e.* dualised to 2-forms. The 2-forms are massive and satisfy massive self-duality conditions

More about supergravities

This result is more general: some dualisations are needed in order to determine the most general embedding tensor.
Simple examples: $D = 4$ and $D = 3$

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In $D = 9$ all the gauged supergravities have been classified via a case-by-case analysis

Bergshoeff, de Wit, Gran, Linares, Roest, hep-th/0209205

More about supergravities

D	G	Masses
9	$SL(2,\mathbb{R}) \times \mathbb{R}^+$	$\mathbf{2} \oplus \mathbf{3}$
8	$SL(3,\mathbb{R}) \times SL(2,\mathbb{R})$	?
7	$SL(5,\mathbb{R})$	$\mathbf{15} \oplus \mathbf{40}$
6	$SO(5,5)$	$\overline{\mathbf{144}}$
5	$E_{6(+6)}$	$\overline{\mathbf{351}}$
4	$E_{7(+7)}$	$\mathbf{912}$
3	$E_{8(+8)}$	$\mathbf{1} \oplus \mathbf{3875}$

More about supergravities

Supersymmetry algebra of IIB: democratic formulation. All the fields appear together with their magnetic duals

Bergshoeff, de Roo, Kerstan, F.R., hep-th/0506013

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The forms one gets are

$$A_2^{\alpha} \qquad A_4 \qquad A_6^{\alpha} \qquad A_8^{(\alpha\beta)} \qquad A_{10}^{(\alpha\beta\gamma)} \qquad A_{10}^{\alpha}$$

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$$A_2^\alpha \qquad A_4 \qquad A_6^\alpha \qquad A_8^{(\alpha\beta)} \qquad A_{10}^{(\alpha\beta\gamma)} \qquad A_{10}^\alpha$$

The 9-branes belong to a non-linear doublet out of the quadruplet

Bergshoeff, de Roo, Kerstan, Ortin, F.R., hep-th/0601128

This leads to an $SL(2,R)$-invariant formulation of brane effective actions

Bergshoeff, de Roo, Kerstan, Ortin, F.R., hep-th/0611036

More about supergravities

Same analysis for IIA

Bergshoeff, Kallosh, Ortin, Roest, Van Proeyen, hep-th/0103233

Bergshoeff, de Roo, Kerstan, Ortin, F.R., hep-th/0602280

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The algebra describes both massless and massive IIA

If $m \neq 0$ the algebra does not arise from 11-dimensions

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The D8-branes are electrically charged with respect to the 9-form

An introduction to E_{11}

Starting point: gravity as a non-linear realisation

Borisov, Ogievetsky, 1974

$$g = exp(x^a P_a)\ exp(h_a{}^b K^a{}_b)$$

with

$$[K^a{}_b, K^c{}_d] = \delta^c_b K^a{}_d - \delta^a_d K^c{}_b \quad [K^a{}_b, P_c] = \delta^a_c P_b$$

Gravity is formulated as the non-linear realisation of the closure of this group with the conformal group

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The theory is invariant under

$$g \to g_0 g h^{-1}$$

An introduction to E_{11}

Maurer-Cartan form:

$$\mathcal{V} = g^{-1}dg - \omega$$

ω: spin connection. It transforms as

$$\omega \to h\omega h^{-1} + hdh^{-1}$$

As a result, $\mathcal{V}$ transforms as

$$\mathcal{V} \to h\mathcal{V}h^{-1}$$

An introduction to E_{11}

Maurer-Cartan form:

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As a result, $\mathcal{V}$ transforms as

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One gets

$$\mathcal{V} = dx^{\mu} (e_{\mu}{}^{a} P_a + \Omega_{\mu a}{}^{b} K^{a}{}_{b})$$

An introduction to E_{11}

Similar analysis for the bosonic sector of 11-dimensional supergravity:

$$[R^{abc}, R^{def}] = R^{abcdef}$$

group element:

$$g = exp(x^a P_a)\ exp(h_a{}^b K^a{}_b)\ exp(A_{abc} R^{abc} + A_{abcdef} R^{abcdef})$$

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Field equations: duality relations

West, hep-th/0005270

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Field equations: duality relations

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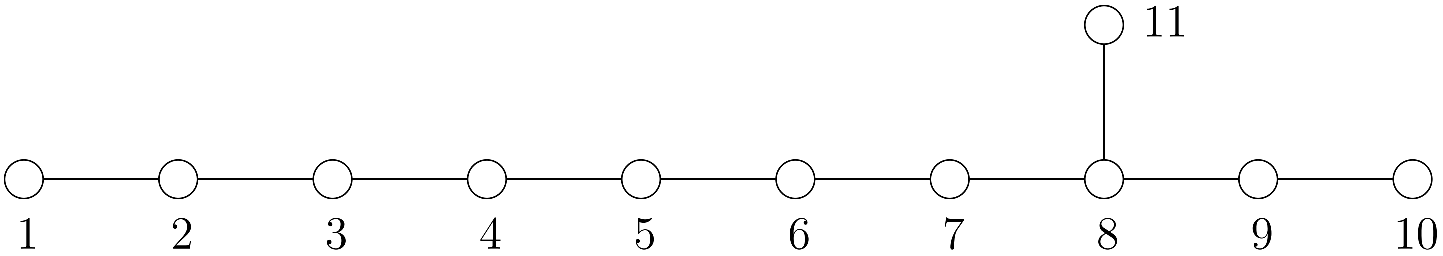
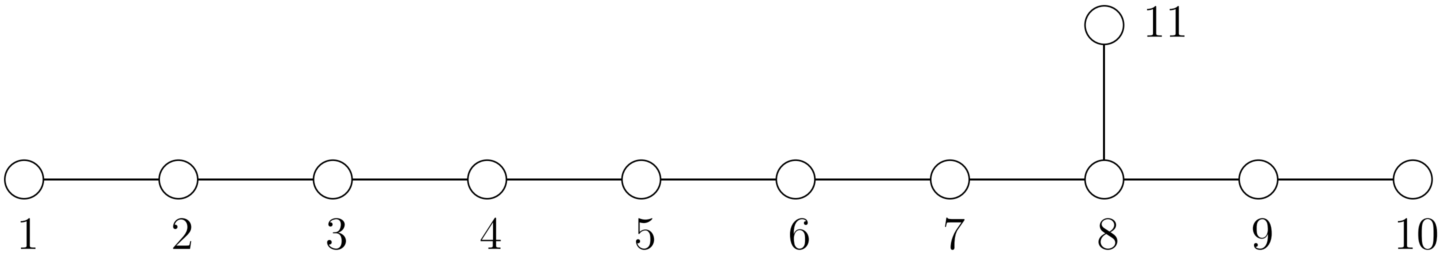
E_{11} is the smallest Kac-Moody group that contains this group

West, hep-th/0104081

An introduction to E_{11}

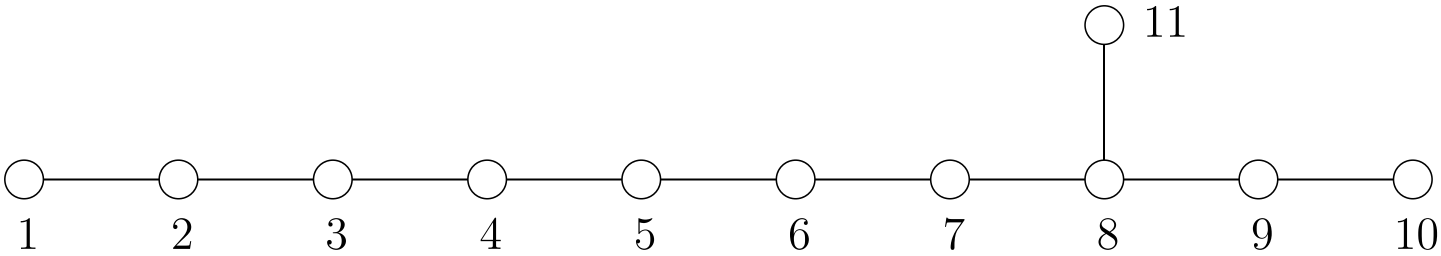


An introduction to E_{11}



Cartan matrix with Minkowskian signature $\rightarrow$ The algebra is infinite-dimensional

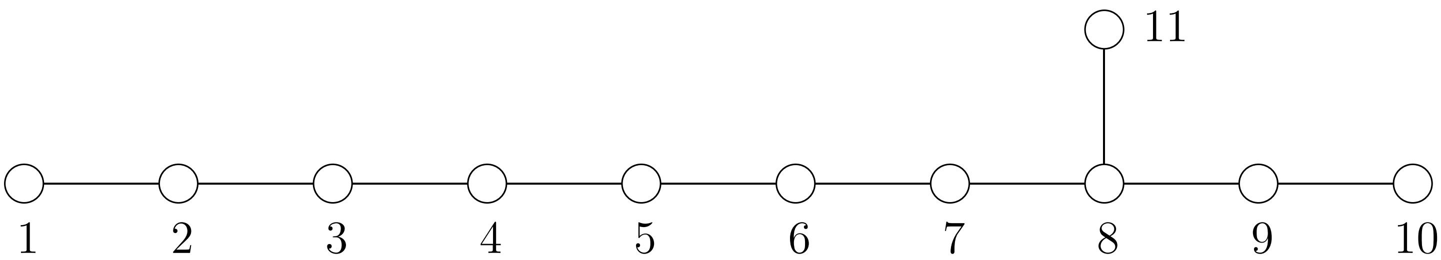
An introduction to E_{11}



Cartan matrix with Minkowskian signature $\longrightarrow$ The algebra is infinite-dimensional

A complete list of the generators is lacking, not to mention other representations...

An introduction to E_{11}



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Idea: write each positive root in terms of the simple roots of A_{10} and the simple root α_{11}

$$\alpha = \sum_{i=1}^{10} n_i \alpha_i + l\alpha_{11} \qquad l = \text{level}$$

An introduction to E_{11}

A necessary condition for the occurrence of a representation of A_{10} with highest weight $\sum_j p_j \lambda_j$ is that this weight arises in a root of E_{11}. One then gets

$$\alpha^2 = -\frac{2}{11} l^2 + \sum_{i,j} p_i (A_{ij})^{-1} p_j$$

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The fact that E_{11} is a Kac-Moody algebra with symmetric Cartan matrix imposes the constraint

$$\alpha^2 = 2, 0, -2, -4 \ldots$$

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$$\alpha^2 = 2, 0, -2, -4 \dots$$

We can solve this level by level

An introduction to E_{11}

Solutions, using $q_j = p_{11-j}$:

$$K^a{}_b \;\; l = 0$$

$$R^{abc} \;\; l = 1, \;\; q_3 = 1$$

$$R^{a_1 \ldots a_6}, \;\; l = 2, \;\; q_6 = 1$$

$$R^{a_1 \ldots a_8, b}, \;\; l = 3, \;\; q_1 = 1, \;\; q_8 = 1$$

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The (8,1) generator is associated to the dual graviton

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The (8,1) generator is associated to the dual graviton

All the generators arise from multiple commutators of R^{abc}

The level is the number of times R^{abc} occurs

An introduction to E_{11}

Non-linear realisation: To each positive level generator we associate a gauge field

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At level 4 one gets the solution $q_{10} = 1, q_1 = 2$ corresponding to the gauge field

$$A_{10,1,1}$$

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Dimensional reduction $\rightarrow A_9$, that is Romans theory!

Schnakenburg and West, hep-th/0204207

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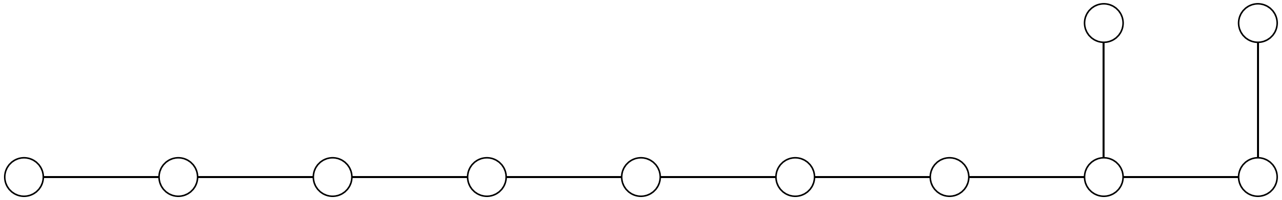
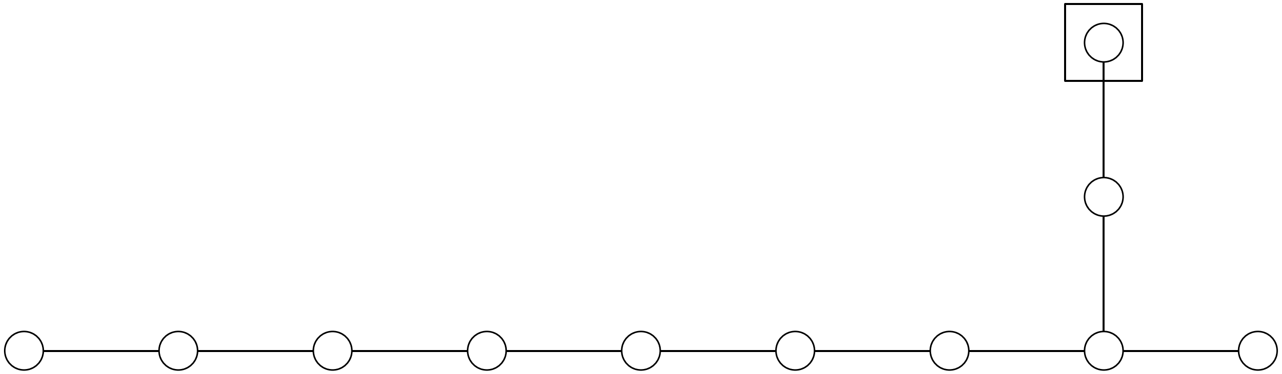
The theory is unique, gravity emerges from the choice of the background

compare with: Julia, hep-th/9805083

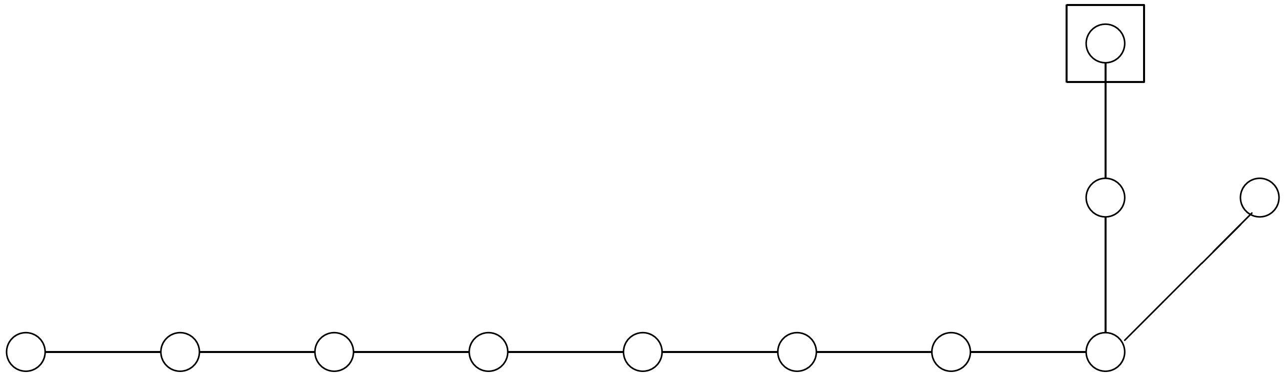
$D = 10A$



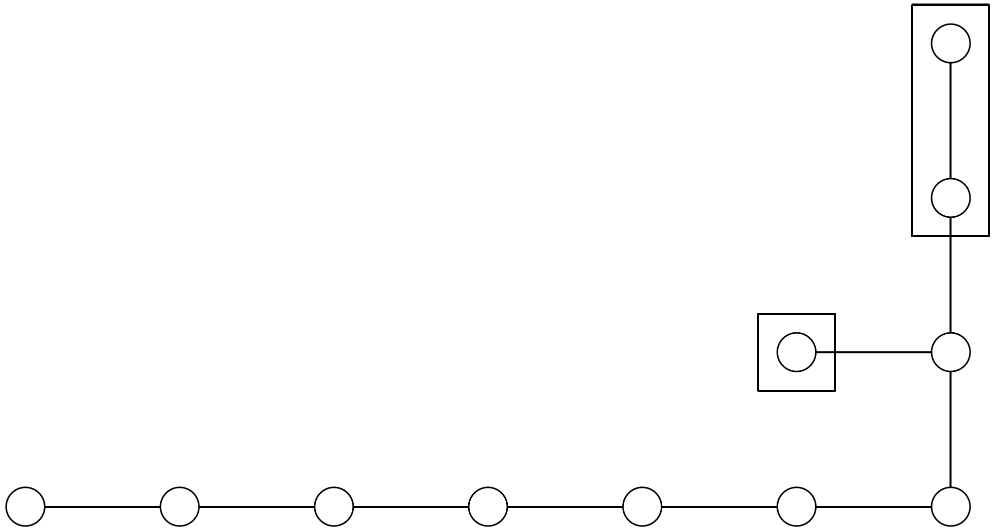
$D = 10B$



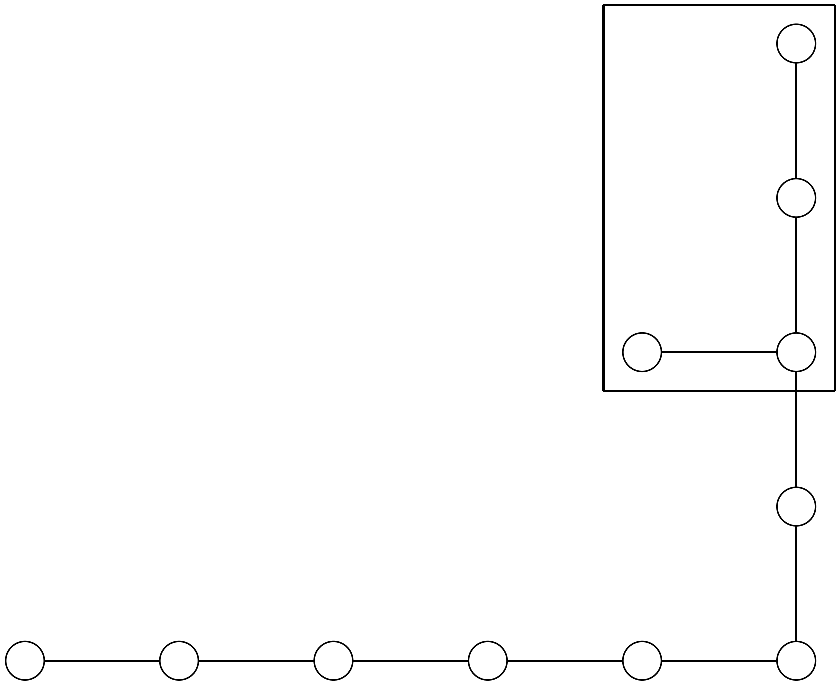
$D = 9$



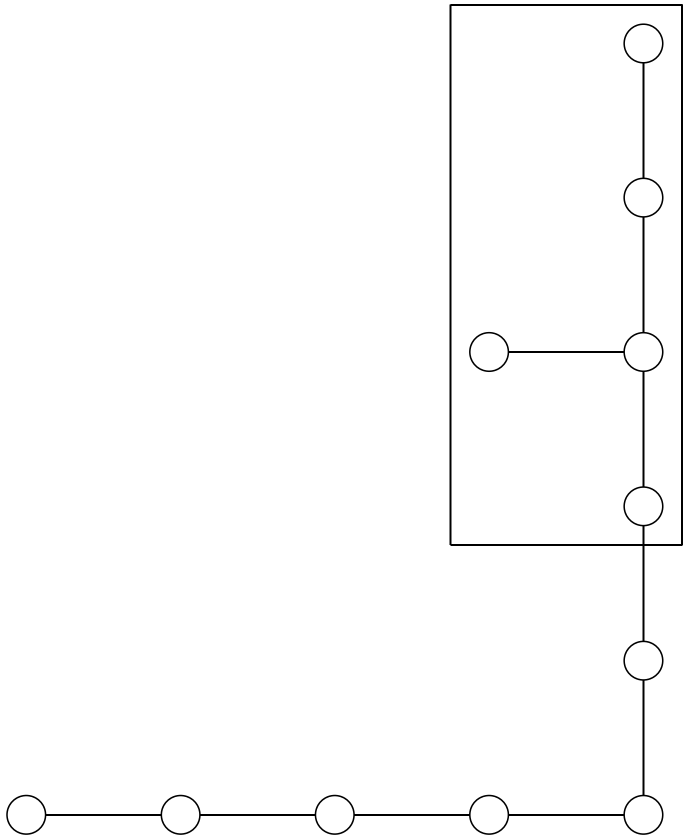
$D = 8$



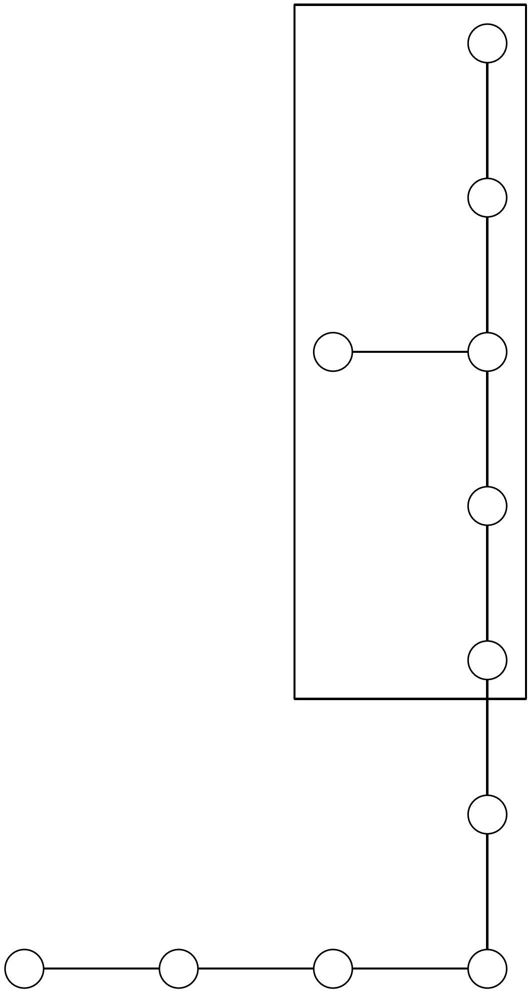
$D = 7$



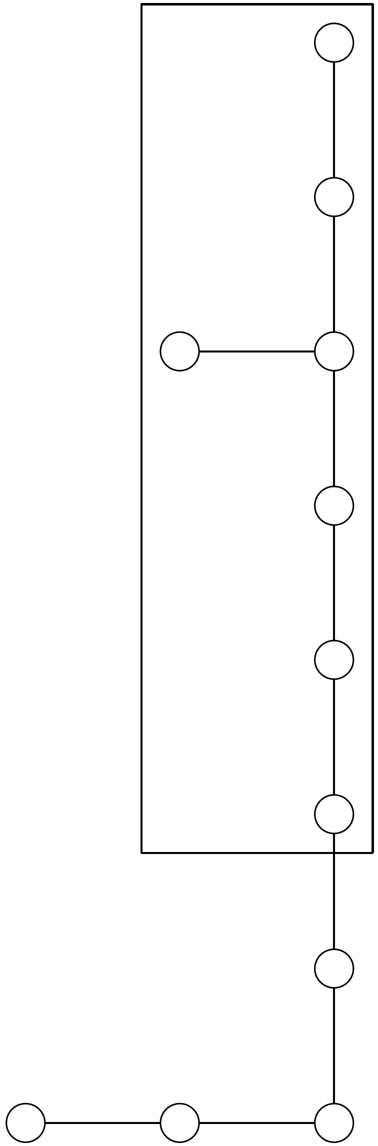
$D = 6$



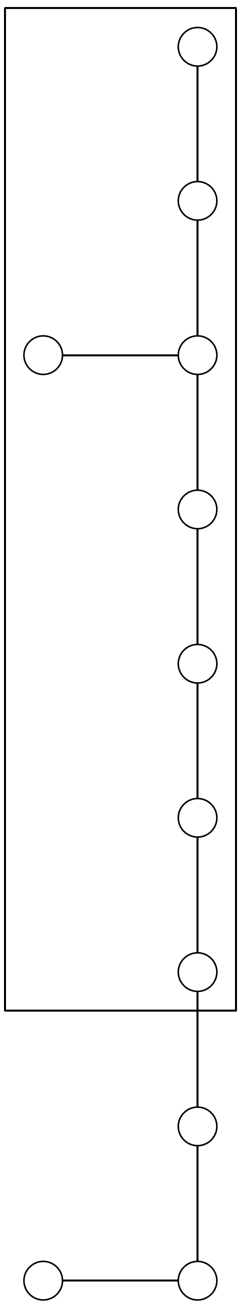
$D = 5$



$D = 4$



$D = 3$



The fields of E_{11}

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Basic idea: the sum of the indices of each field has to be equal to $3l$:

$$11n + \sum_j j q_j = 3l$$

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Basic idea: the sum of the indices of each field has to be equal to $3l$:

$$11n + \sum_j j q_j = 3l$$

Propagating fields have $n = q_{10} = 0$. One gets

$$A_{9,9,\ldots,9,3} \qquad A_{9,9,\ldots,9,6} \qquad A_{9,9,\ldots,9,8,1}$$

That is we get infinitely many dual descriptions of the same fields

The fields of E_{11}

We want to determine all the forms that arise from dimensional reduction.

The propagating fields in lower dimensions arise from the propagating fields in $D = 11$. We study the dimensional reduction to D.

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Finally, in order to determine the D-forms, we also need to consider $q_{10} = q_9 = 0 \; n = 1$

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Finally, in order to determine the D-forms, we also need to consider $q_{10} = q_9 = 0 \; n = 1$

Remarkably, there are only a finite number of 11-dimensional fields that give rise to forms in any dimension above two

The fields of E_{11}

D	field
10	$\hat{g}^1{}_1$ $\hat{A}_3$ $\hat{A}_6$ $\hat{A}_{8,1}$
8	$\hat{A}_{9,3}$
5	$\hat{A}_{9,6}$
3	$\hat{A}_{9,8,1}$

The fields of E_{11}

D	field
10	$\hat{A}_{10,1,1}$
7	$\hat{A}_{10,4,1}$
5	$\hat{A}_{10,6,2}$
4	$\hat{A}_{10,7,1}$ $\hat{A}_{10,7,4}$ $\hat{A}_{10,7,7}$
3	$\hat{A}_{10,8}$ $\hat{A}_{10,8,2,1}$ $\hat{A}_{10,8,3}$ $\hat{A}_{10,8,5,1}$ $\hat{A}_{10,8,6}$ $\hat{A}_{10,8,7,2}$ $\hat{A}_{10,8,8,1}$ $\hat{A}_{10,8,8,4}$ $\hat{A}_{10,8,8,7}$

The fields of E_{11}

D	field	μ
10	$\hat{A}_{11,1}$	1
8	$\hat{A}_{11,3,1}$	1
7	$\hat{A}_{11,4}$	1
	$\hat{A}_{11,4,3}$	1
6	$\hat{A}_{11,5,1,1}$	1
5	$\hat{A}_{11,6,1}$	2
	$\hat{A}_{11,6,3,1}$	1
	$\hat{A}_{11,6,4}$	1
	$\hat{A}_{11,6,6,1}$	1
4	$\hat{A}_{11,7}$	1
	$\hat{A}_{11,7,2,1}$	1
	$\hat{A}_{11,7,3}$	2
	$\hat{A}_{11,7,4,2}$	1
	$\hat{A}_{11,7,5,1}$	1
	$\hat{A}_{11,7,6}$	2
	$\hat{A}_{11,7,6,3}$	1
	$\hat{A}_{11,7,7,2}$	1
	$\hat{A}_{11,7,7,5}$	1

E_{11} and dimensional reduction

Consider the 7-dimensional example

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Consider the 7-dimensional example

6-forms:

$$\hat{A}_6 \to \mathbf{1} \qquad \hat{A}_{8,1} \to \overline{\mathbf{4}} \oplus \overline{\mathbf{20}}$$

$$\hat{A}_{9,3} \to \mathbf{6} \oplus \overline{\mathbf{10}} \qquad \hat{A}_{10,1,1} \to \mathbf{10} \qquad \hat{A}_{10,4,1} \to \mathbf{4}$$

of $SL(4,R)$. This is $\overline{\mathbf{15}} \oplus \overline{\mathbf{40}}$ of $SL(5,R)$

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$$\hat{A}_{11,1} \rightarrow \mathbf{4} \qquad \hat{A}_{11,3,1} \rightarrow \mathbf{15} \qquad \hat{A}_{11,4} \rightarrow \mathbf{1} \qquad \hat{A}_{11,4,3} \rightarrow \overline{\mathbf{4}}$$

that is $\mathbf{5} \oplus \mathbf{45} \oplus \mathbf{70}$ of $SL(5,R)$

E_{11} and dimensional reduction

D	G	1-forms	2-forms	3-forms	4-forms	5-forms	6-forms	7-forms	8-forms	9-forms	10-forms
10A	$\mathbb{R}^+$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$		$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$ $\mathbf{1}$
10B	$SL(2,\mathbb{R})$		$\mathbf{2}$		$\mathbf{1}$		$\mathbf{2}$		$\mathbf{3}$		$\mathbf{4}$ $\mathbf{2}$
9	$SL(2,\mathbb{R}) \times \mathbb{R}^+$	$\mathbf{2}$ $\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$ $\mathbf{1}$	$\mathbf{3}$ $\mathbf{1}$	$\mathbf{3}$ $\mathbf{2}$	$\mathbf{4}$ $\mathbf{2}$ $\mathbf{2}$	
8	$SL(3,\mathbb{R}) \times SL(2,\mathbb{R})$	$(\overline{\mathbf{3}},\mathbf{2})$	$(\mathbf{3},\mathbf{1})$	$(\mathbf{1},\mathbf{2})$	$(\overline{\mathbf{3}},\mathbf{1})$	$(\mathbf{3},\mathbf{2})$	$(\mathbf{8},\mathbf{1})$ $(\mathbf{1},\mathbf{3})$	$(\mathbf{6},\mathbf{2})$ $(\overline{\mathbf{3}},\mathbf{2})$	$(\mathbf{15},\mathbf{1})$ $(\mathbf{3},\mathbf{3})$ $(\mathbf{3},\mathbf{1})$ $(\mathbf{3},\mathbf{1})$		
7	$SL(5,\mathbb{R})$	$\overline{\mathbf{10}}$	$\mathbf{5}$	$\overline{\mathbf{5}}$	$\mathbf{10}$	$\mathbf{24}$	$\overline{\mathbf{40}}$ $\overline{\mathbf{15}}$	$\mathbf{70}$ $\mathbf{45}$ $\mathbf{5}$			
6	$SO(5,5)$	$\mathbf{16}$	$\mathbf{10}$	$\overline{\mathbf{16}}$	$\mathbf{45}$	$\mathbf{144}$	$\mathbf{320}$ $\overline{\mathbf{126}}$ $\mathbf{10}$				
5	$E_{6(+6)}$	$\mathbf{27}$	$\overline{\mathbf{27}}$	$\mathbf{78}$	$\mathbf{351}$	$\overline{\mathbf{1728}}$ $\overline{\mathbf{27}}$					
4	$E_{7(+7)}$	$\mathbf{56}$	$\mathbf{133}$	$\mathbf{912}$	$\mathbf{8645}$ $\mathbf{133}$						
3	$E_{8(+8)}$	$\mathbf{248}$	$\mathbf{3875}$ $\mathbf{1}$	?							

E_{11} and dimensional reduction

D	G	1-forms	2-forms	3-forms	4-forms	5-forms	6-forms	7-forms	8-forms	9-forms	10-forms
10A	$\mathbb{R}^+$	1	1	1		1	1	1	1	1	1 1
10B	$SL(2,\mathbb{R})$		2		1		2		3		4 2
9	$SL(2,\mathbb{R})\times\mathbb{R}^+$	2 1	2	1	1	2	2 1	3 1	3 2	4 2 2	
8	$SL(3,\mathbb{R})\times SL(2,\mathbb{R})$	$(\overline{3},2)$	$(3,1)$	$(1,2)$	$(\overline{3},1)$	$(3,2)$	$(8,1)$ $(1,3)$	$(6,2)$ $(\overline{3},2)$	$(15,1)$ $(3,3)$ $(3,1)$ $(3,1)$		
7	$SL(5,\mathbb{R})$	$\overline{10}$	5	$\overline{5}$	10	24	$\overline{40}$ $\overline{15}$	70 45 5			
6	$SO(5,5)$	16	10	$\overline{16}$	45	144	320 $\overline{126}$ 10				
5	$E_{6(+6)}$	27	$\overline{27}$	78	351	$\overline{1728}$ $\overline{27}$					
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3-forms in 3 dimensions: $\mathbf{248 \oplus 3875 \oplus 147250}$

Bergshoeff, De Baetselier, Nutma, arXiv:0705.1304

Some dynamics

Consider the 5-dimensional example. $G = E_6$

$$[R^{a,M}, P_b] = \delta^a_b \, \Theta^M{}_\alpha R^\alpha$$

$$[R^{ab}{}_M, P_c] = Z_{MN}(\delta^a_c R^{b,N} - \delta^b_c R^{a,N})$$

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The term in the Cartan form proportional to the 1-form generator $R^{a,M}$ is

$$\partial_\mu A_{a,M} - \frac{1}{2} A_{\mu,N} \Theta^N{}_\alpha (D^\alpha)_M{}^P A_{a,P} - 2 Z_{MN} A_{\mu a}{}^N$$

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The scalar sector is the Cartan form of the gauged supergravity coset space
The equations are first order duality relations. This reproduces the supergravity results

Conclusions

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- It would be interesting to determine the D-forms in any dimension D below 10 from supersymmetry
- Gauged supergravities in $D = 8$
- Many other physical implications: uplifting of D8-branes, Horava-Witten...