Cold atoms in 2D optical lattices under staggered rotation

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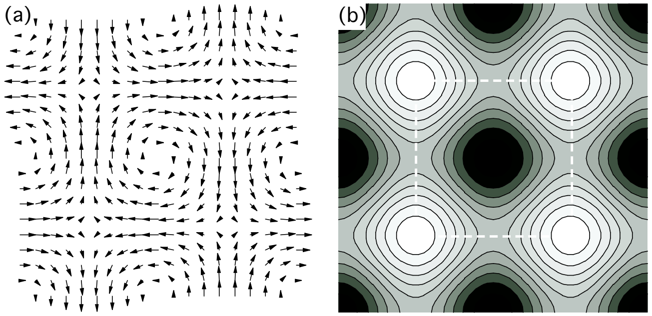
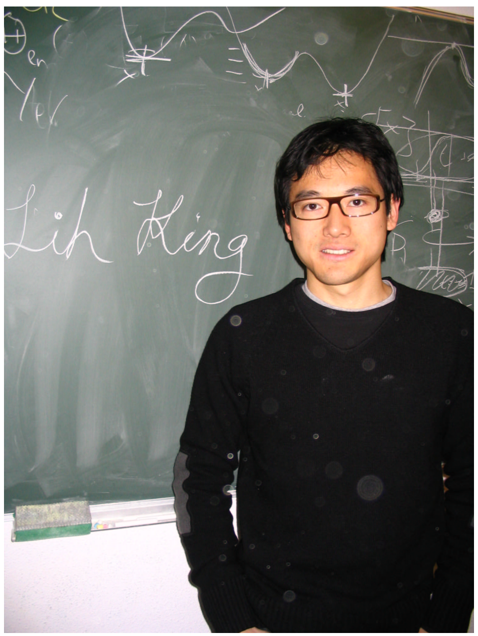


Collaborators

Lih-King Lim and Andreas Hemmerich



Outline

Low-D systems: observation of quantum effects

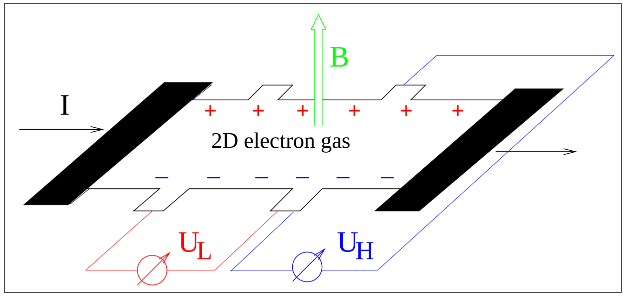
2DES in a $B_\perp$: several interesting quantum phases

- electron-liquid phases: Laughlin, Moore-Read, Read-Rezayi
- electron-solid phases: Wigner crystals, bubbles, stripes
- nematic phases, BEC of excitons in bilayers, etc...

2D Systems: cond-mat

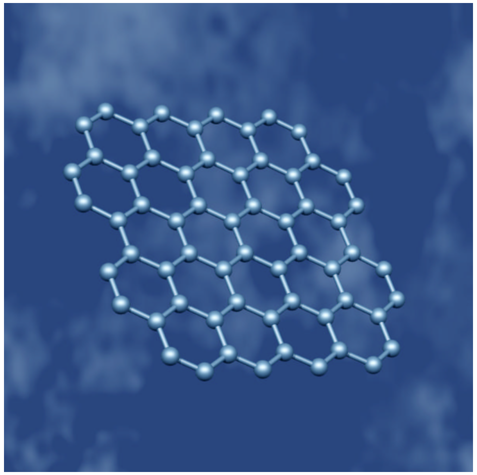
Uniform

- GaAs, Si-MOSFETs



Lattice

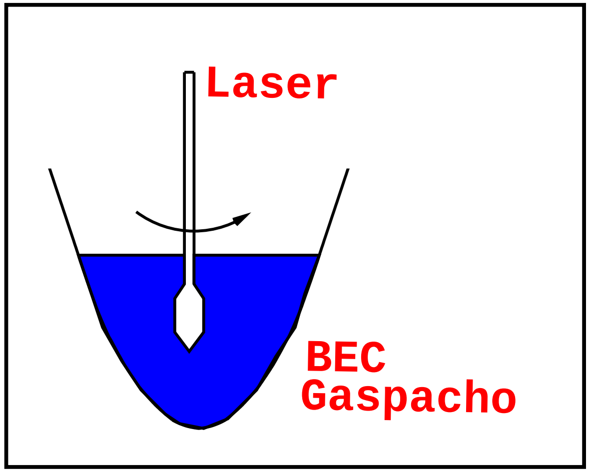
- graphene



2D Systems: cold atoms

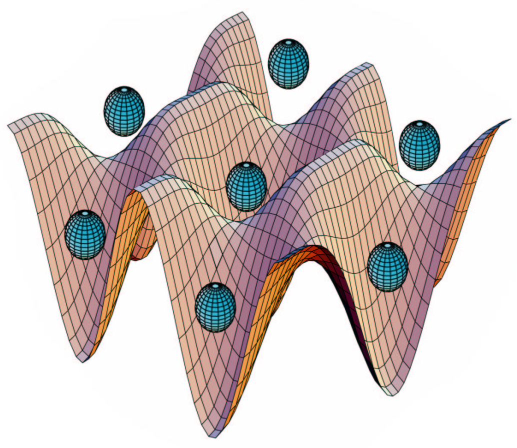
Uniform

- rotating BECs



Lattice

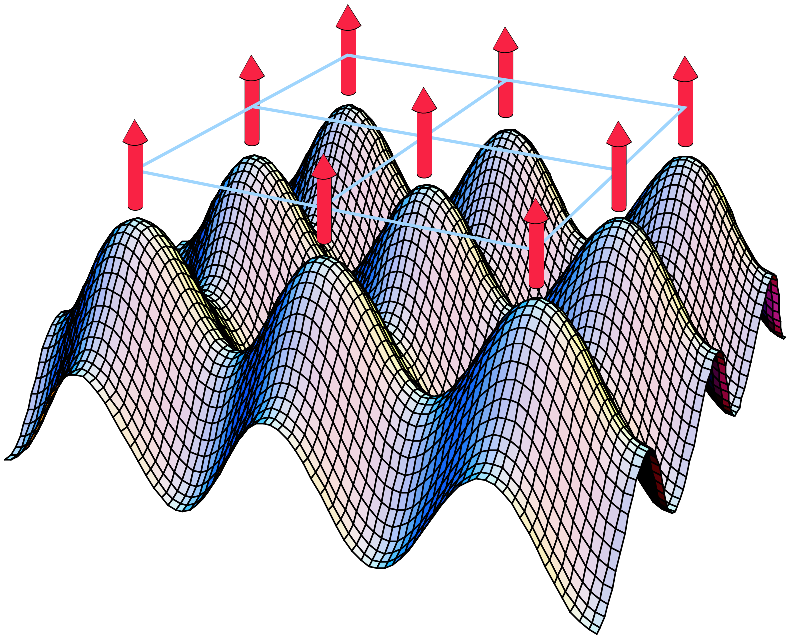
- rotating BECs in optical lattices



Cold atoms in optical lattices

Sorensen, Demler, Lukin, PRL 94, 086803 (2005)

quadrupolar potential + tunneling for bosons



FQHE Laughlin state: 95% overlap

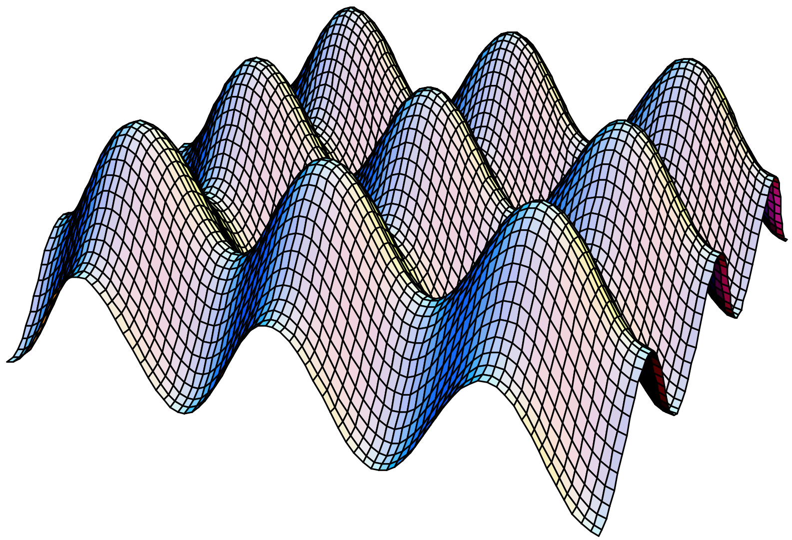
larger gap than in harmonically trapped BECs because interaction energy is larger

Cold atoms in optical lattices

3D: M. Greiner et al., Nature 419, 51 (2002)

2D: Phillips group, PRL 100, 120404 (2008)

Superfluid-Mott insulator transition



Theoretical description: Bose-Hubbard model

$$H = -J \sum_{<i,j>} a_i^\dagger a_j + H.c. + U \sum_i n_i(n_i - 1)$$

Cold atoms in optical lattices

- simulators of cond-mat systems
  - full control of lattice parameters
  - load with bosons or fermions
  - control of interactions (Feshbach resonance)
  - no disorder
- generate NEW situations
  - alternating magnetic fields

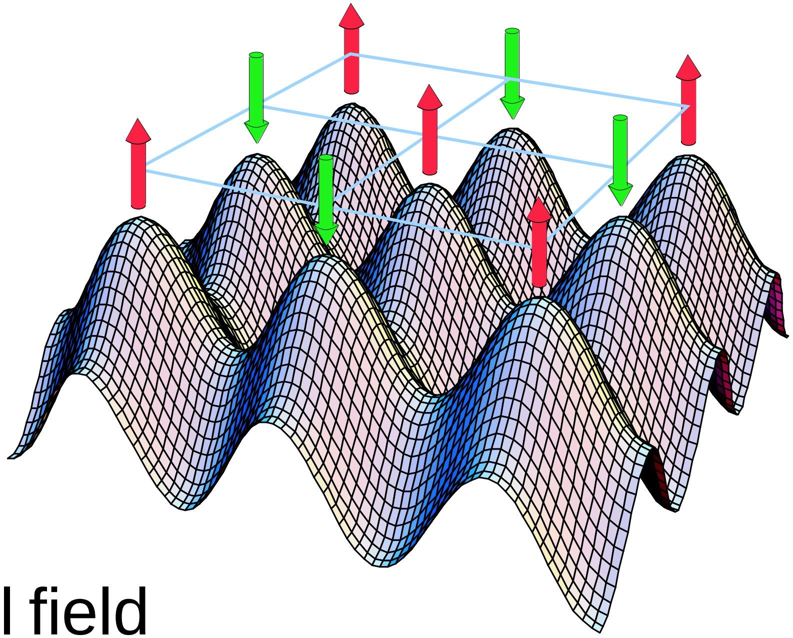
Haldane 1988: graphene with zero net magnetic field per plaquette

QHE: no uniform B, but break time-reversal symmetry

Cold atoms in optical lattices

A. Hemmerich and C.M.S., PRL 99, 113002 (2007)

Bosons and fermions



Staggered rotational field

Novel phases

Staggered Rotational Field

A. Hemmerich and C.M.S., PRL 99, 113002 (2007)

How to realize it experimentally?

Linearly polarized bichromatic light-field

$$E_1(\mathbf{r},t) \equiv A_1 e^{i(\omega+\Omega)t} |\phi(x,y)| e^{iS(x,y)}$$

$$E_2(\mathbf{r},t) \equiv A_2 e^{i\omega t} |\phi(x,y)| e^{-iS(x,y)}$$

We assume $|\Omega| \ll \omega$,

$|\phi(x,y)|^2 = \sin^2(kx) + \sin^2(ky)$

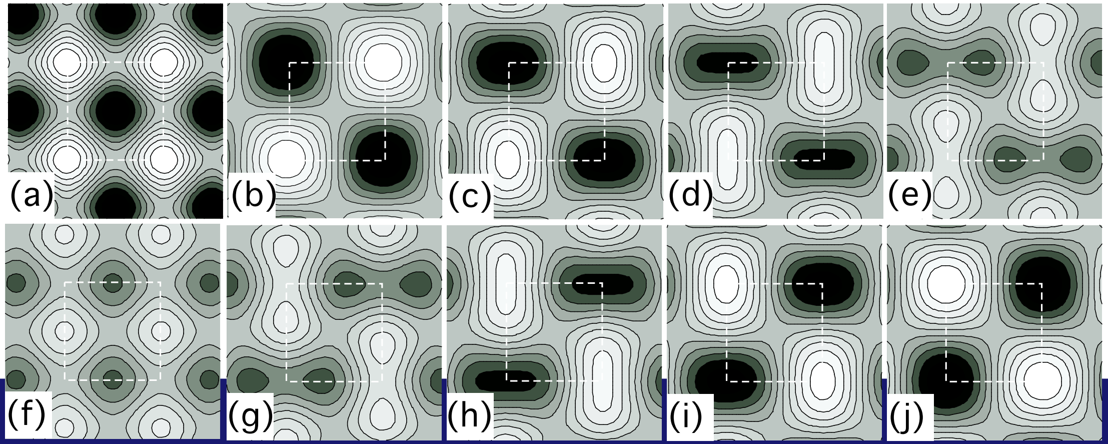
$$S(x,y) = \arctan\left[\frac{\sin(kx) - \sin(ky)}{\sin(kx) + \sin(ky)}\right]$$

Staggered Rotational Field

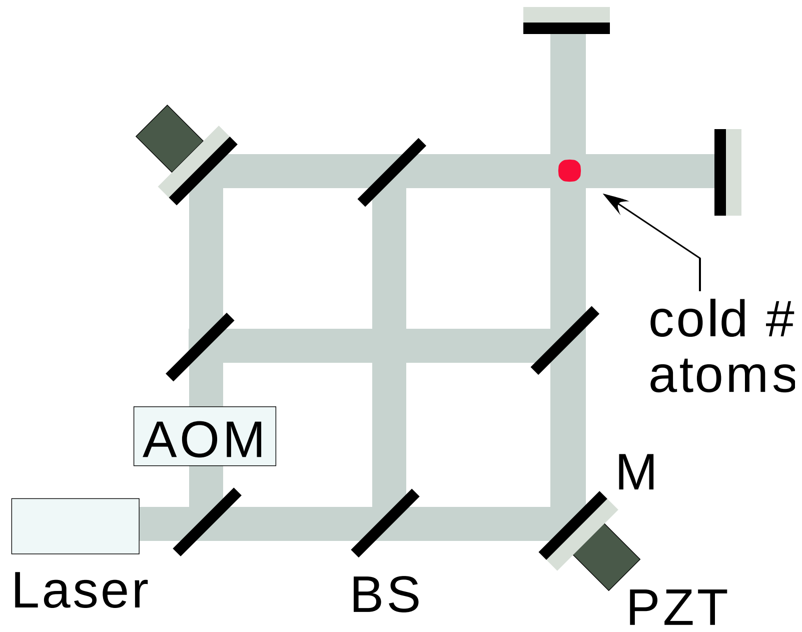
A. Hemmerich and C.M.S., PRL 99, 113002 (2007)

$$I(x,y,t) \equiv \mathbf{E}(\mathbf{r},t)\mathbf{E}^*(\mathbf{r},t) = I_L(x,y) + I_R(x,y,t)$$

- Stationary term $I_L(x,y) = (A_1 + A_2)^2 |\phi(x,y)|^2$
- Time-dependent term
  $I_R(x,y,t) = 2A_1A_2|\phi(x,y)|^2 \cos\left(2S(x,y) - \Omega t\right)$



Optical Setup



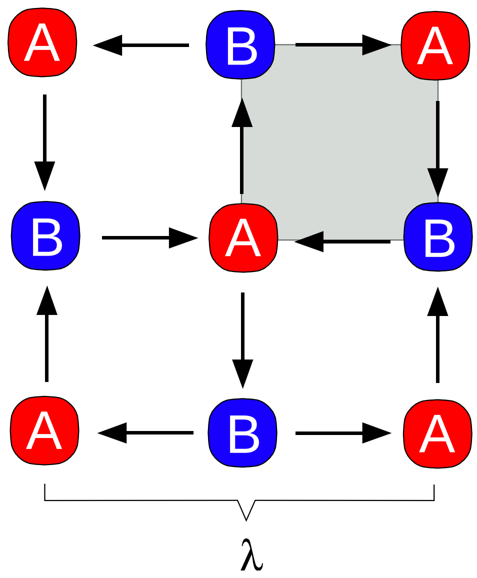
Two nested Michelson interferometers

PZT: piezoelectric transducer

M: mirror BS: beam splitter

AOM: acousto-optic frequency shifter

Time-Dependent Bose-Hubbard Model



$$\hat{H}(t) = -\sum_{\mathbf{r}\in A, l=1-4} J_l(t) \left\{ \hat{a}^\dagger_{\mathbf{r}}\, \hat{a}_{\mathbf{r}+\mathbf{e}_l} + \text{H.c.} \right\}$$

$$+ \sum_{\mathbf{r}\in A\oplus B} \epsilon_{\mathbf{r}}(t)\, \hat{n}_{\mathbf{r}} + \frac{1}{2} U \sum_{\mathbf{r}\in A\oplus B} \hat{n}_{\mathbf{r}} \left(\hat{n}_{\mathbf{r}} - 1 \right)$$

where $J_l(t) = J + (-1)^l \kappa V_0 \chi_1 \sin(\Omega t)$

anisotropic time-varying hopping

$$\epsilon_{\mathbf{r}\in A,B}(t) = \pm 2\kappa V_0 \chi_2 \cos(\Omega t)$$

time-varying energy offset

Effective Hamiltonian

Hamiltonian is periodic: $\hat{H}(t) = \hat{H}(t+\tau_n)$ with $\tau_n \equiv 2n\pi/\Omega$

Dyson Series: $\hat{U}(\tau_n) = e^{-\frac{i}{\hbar}\hat{H}_{\text{eff}}\,\tau_n}$

$$\hat{U}(\tau_n) = 1 + \left(\frac{-i}{\hbar}\right)\int_0^{\tau_n} dt\hat{H}(t) + \left(\frac{-i}{\hbar}\right)^2 \int_0^{\tau_n} dt \int_0^t dt'\hat{H}(t)\hat{H}(t')$$

Effective Hamiltonian

$$\hat{H}_{\text{eff}} \approx -\sum_{\mathbf{r}\in A, l=1-4} \left\{ |c| \mathrm{e}^{i\theta(-1)^l} \hat{a}^\dagger_{\mathbf{r}}\ \hat{a}_{\mathbf{r}+\mathbf{e}_l} + \text{H.c.} \right\} + \frac{1}{2} U \sum_{\mathbf{r}\in A\oplus B} \hat{n}_{\mathbf{r}}(\hat{n}_{\mathbf{r}}-1)$$

$$|c| = \sqrt{J^2+W^2} \qquad \theta = \tan^{-1}\left(\frac{W}{J}\right) \qquad W = \frac{2\kappa^2 V_0^2 \chi_1 \chi_2}{\hbar\Omega}$$

.

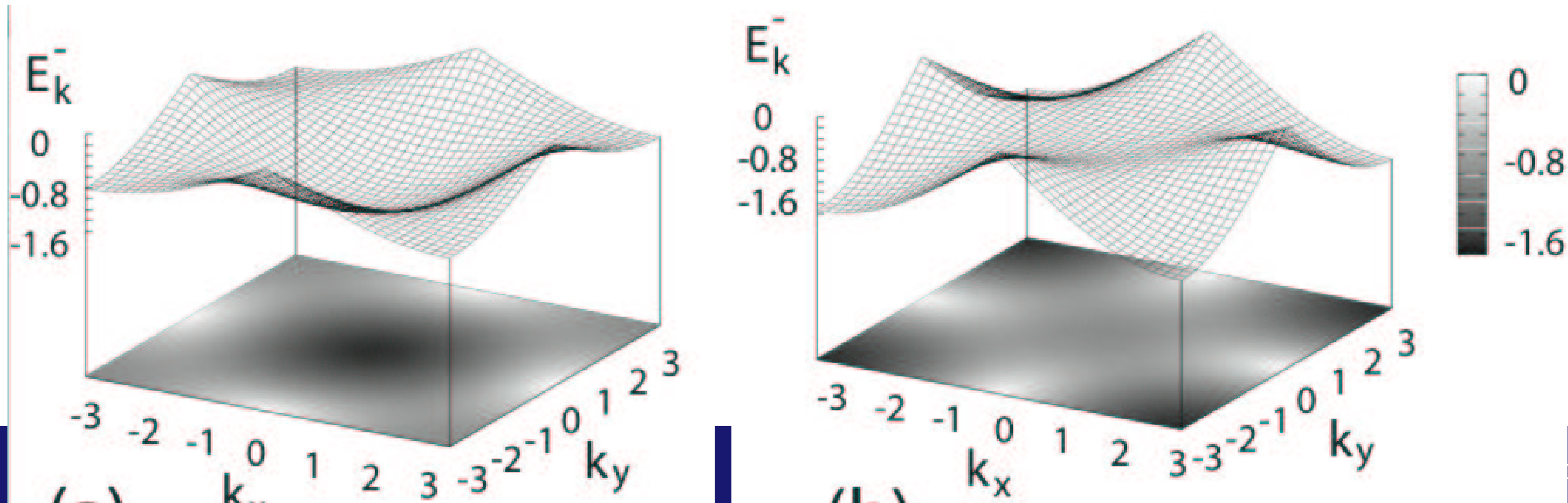
Single-particle spectrum

Write $\hat{H}$ in $k-$space

$$\hat{H}_0 = \sum_{\mathbf{k}} E^-_{\mathbf{k}} \hat{\beta}^\dagger_{\mathbf{k}} \hat{\beta}_{\mathbf{k}} + E^+_{\mathbf{k}} \hat{\alpha}^\dagger_{\mathbf{k}} \hat{\alpha}_{\mathbf{k}}$$

$$E^\pm_{\mathbf{k}} = \pm|\epsilon_{\mathbf{k}}| = \pm 2|c|[\cos^2 k^+ + \cos^2 k^- + 2\cos k^+ \cos k^- \cos(2\theta)]^{1/2}$$

$$k^+ = (k_x + k_y)/2 \qquad k^- = (k_x - k_y)/2$$



Bosons: Mean Field Theory

$$Z = \int \mathcal{D}a^* \mathcal{D}a \, \exp\{-S[a^*, a]/\hbar\}$$

$$S[a^*, a] = \int_0^{\hbar\beta} d\tau \left[\sum_{\mathbf{r}} a^*_{\mathbf{r}}(\tau) \left(\hbar\partial_\tau - \mu\right) a_{\mathbf{r}}(\tau) + H_{\text{eff}} \right]$$

Mott regime

- Hubbard-Stratonovich field $\psi_{\mathbf{r}}(\tau)$ to decouple the hopping term
- Integrate out the boson fields (a^*, a)
- Effective action (quadratic order in $\psi^A_{\omega,\mathbf{k}}$, $\psi^B_{\omega,\mathbf{k}}$)

Mean Field Theory - Effective Action

$$S^{(2)}[\psi^*,\psi] = -\sum_{\omega,\mathbf{k}} \begin{pmatrix} \psi^A_{\omega,\mathbf{k}} \\ \psi^B_{\omega,\mathbf{k}} \end{pmatrix}^\dagger \begin{pmatrix} \epsilon^2_{\mathbf{k}} f_\omega & \epsilon_{\mathbf{k}} \\ \epsilon^*_{\mathbf{k}} & (\epsilon^*_{\mathbf{k}})^2 f_\omega \end{pmatrix} \begin{pmatrix} \psi^A_{\omega,\mathbf{k}} \\ \psi^B_{\omega,\mathbf{k}} \end{pmatrix}$$

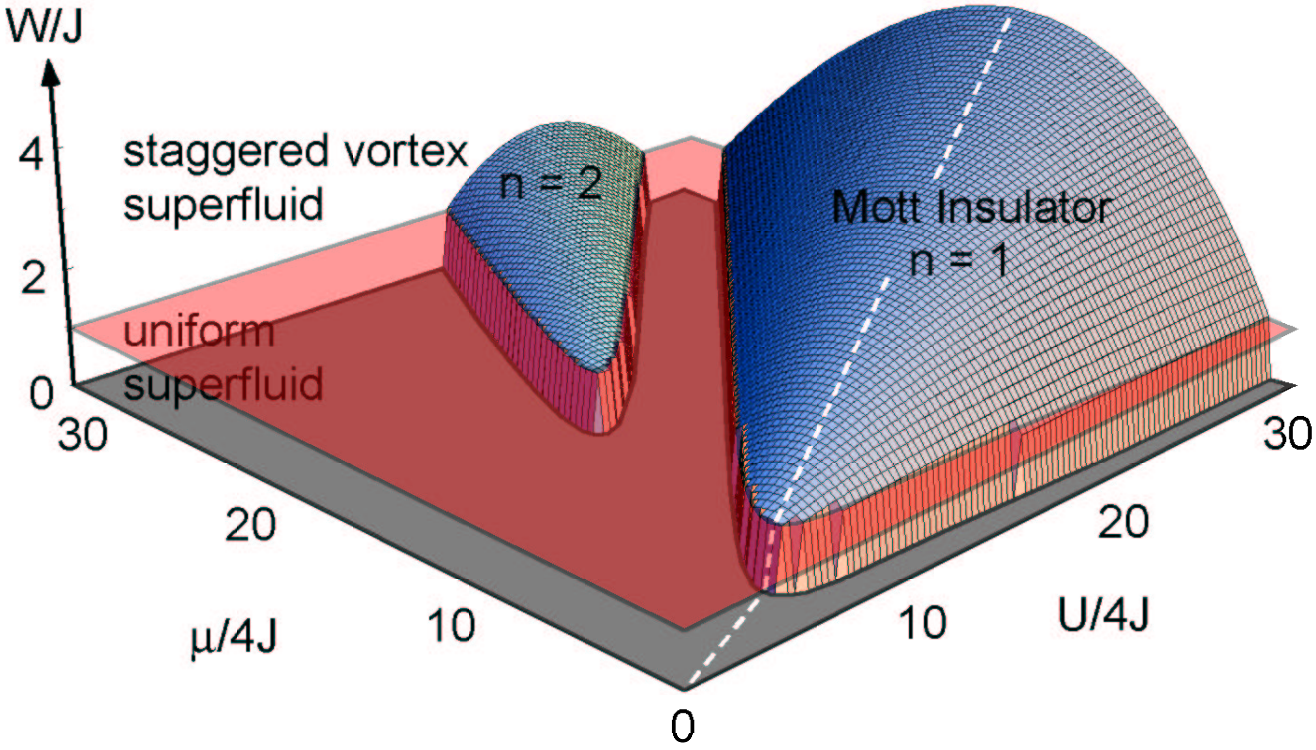
Real frequencies ($i\omega \to \omega$), $T = 0$
Quasi-particle (hole) energy dispersion

$$\epsilon^{qp,qh}_{\mathbf{k}} = -\mu + \frac{U}{2}(2n-1) - \frac{|\epsilon_{\mathbf{k}}|}{2} \pm \frac{1}{2}\hbar\,\omega_{\mathbf{k}}$$

where $\hbar\omega_{\mathbf{k}} = \sqrt{|\epsilon_{\mathbf{k}}|^2 - (4n+2)|\epsilon_{\mathbf{k}}|U + U^2}$: energy for creating a quasiparticle-quasihole pair.

Phase Diagram

L.-K. Lim, C.M.S., and A. Hemmerich, PRL 100, 130402 (2008)



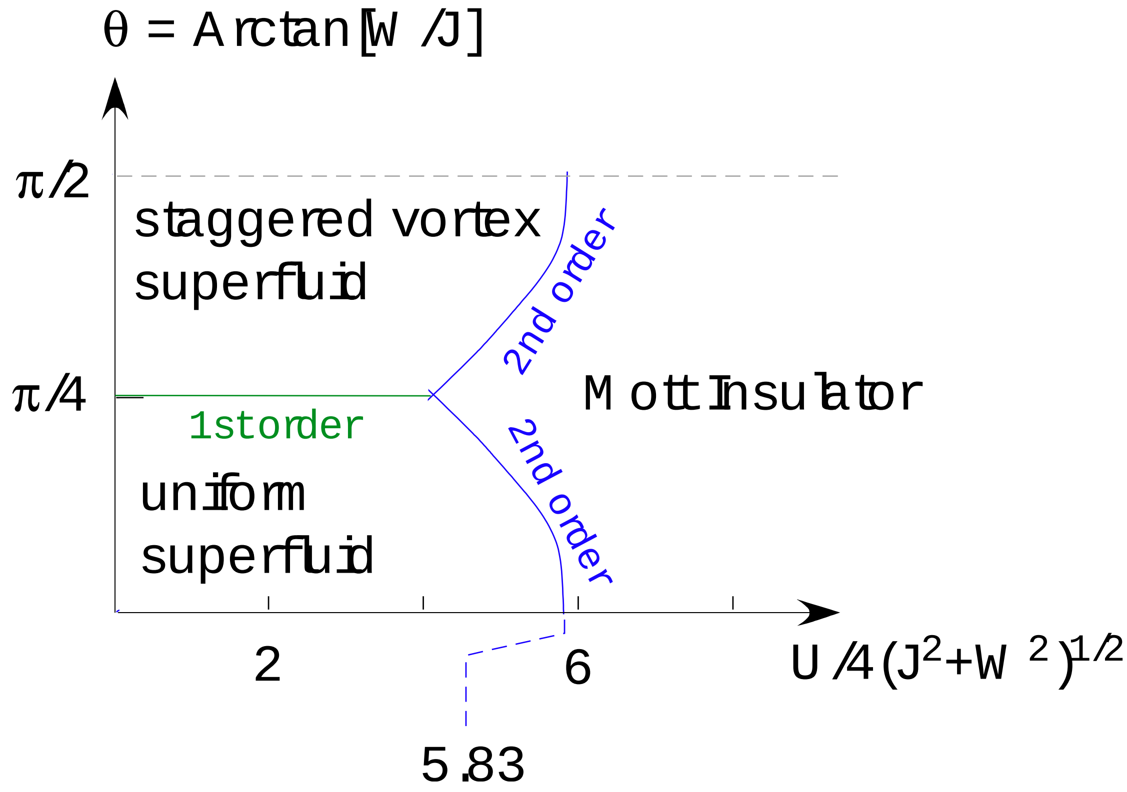
$(\mu/U = 2 - \sqrt{2})$-plane: dashed white line

Phase Diagram

Bosons: BEC at lowest single-particle state

$\theta < \pi/4$: min at $\mathbf{k} = (0,0)$ GS is uniform SF

$\theta > \pi/4$: min at $\mathbf{k} = (\pi,\pi)$ GS is finite k SF



Superfluid Phases

Variational mean-field ansatz for the ground state:

$|\xi,\sigma\rangle = (e^{-i\xi/2}\cos(\sigma)\hat{\beta}_0^\dagger + e^{i\xi/2}\sin(\sigma)\hat{\beta}_\pi^\dagger)^N|0\rangle$

$\theta < \pi/4$: $\sigma = \sigma_0 = 0$

$\theta > \pi/4$: $\sigma = \sigma_0 = \pi/2$

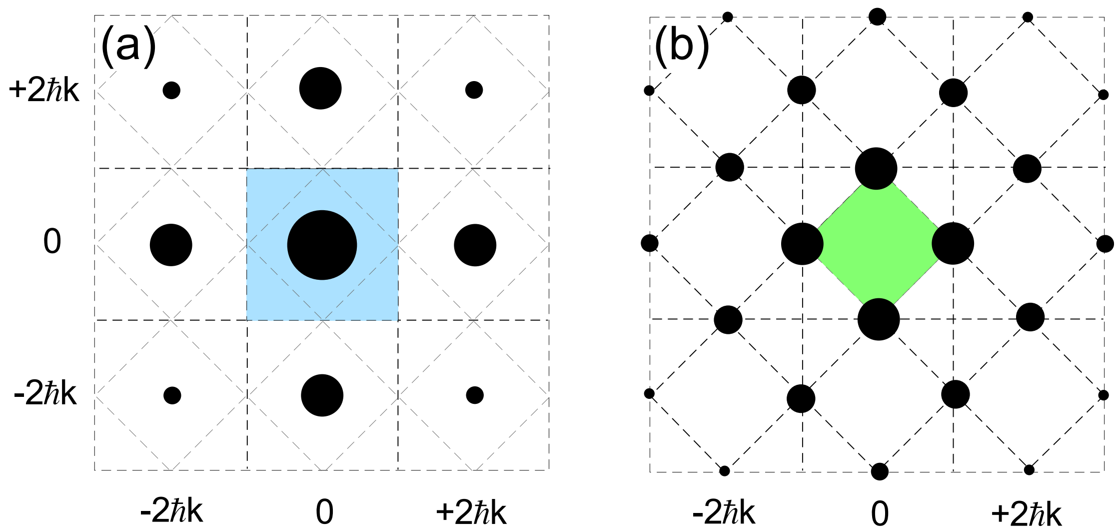
Order parameter σ_0 changes discontinuously by $\pi/2$ at $\theta = \pi/4$

Finite momentum SF: analogies with Abrikosov lattice and "FFLO states"

Experimental Detection

Momentum distribution:

$$\langle \Psi^\dagger(\mathbf{k})\Psi(\mathbf{k})\rangle = |W(\mathbf{k})|^2\, S_B(\mathbf{k})\, S_P(\mathbf{k})$$



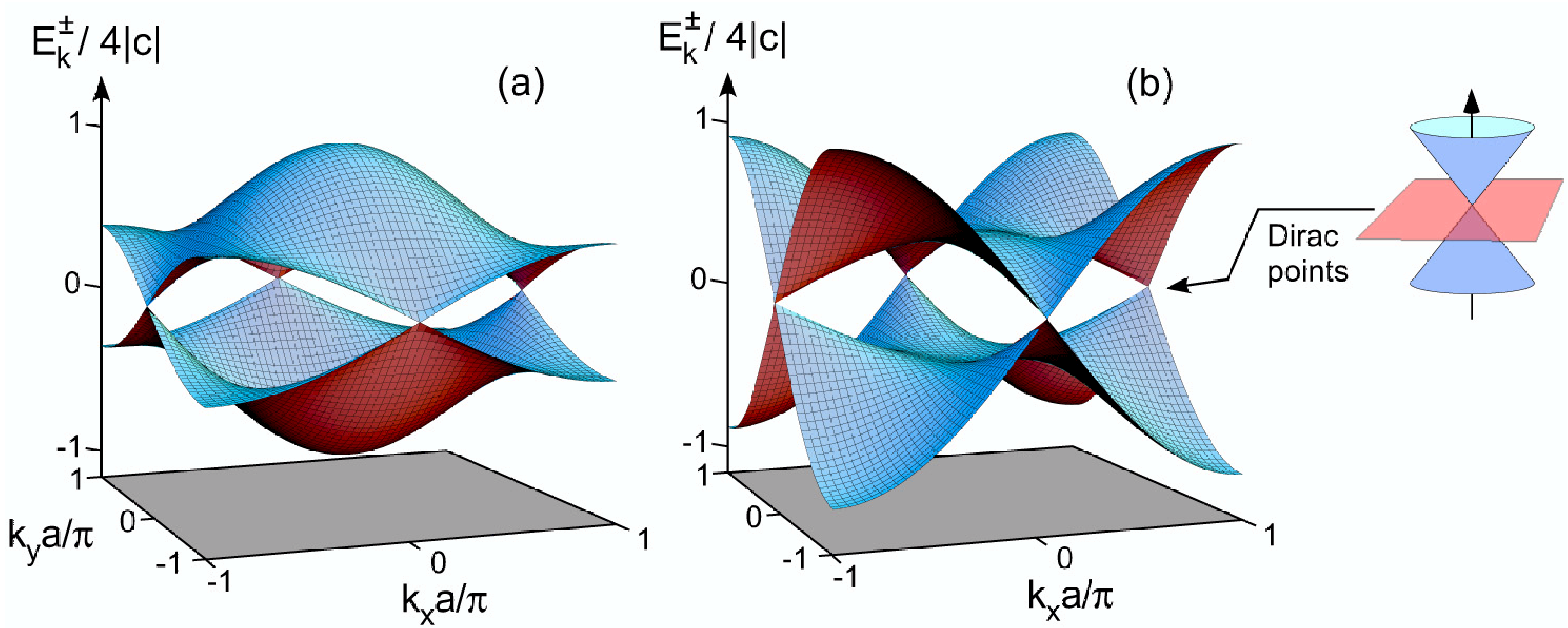
(a) Uniform SF

(b) Staggered vortex SF

$W(\mathbf{k})$: Fourier transform of Wannier function

$S(\mathbf{k})$: structure factor (B: Bravais lattice, P: plaquette)

Fermions in optical lattices



At half-filling: anisotropic Dirac cones

Graphene under uniaxial pressure

At $\theta = \pi/4$: staggered-π flux phase (HTSC)

Simulating Graphene

2 ineq. points: $K = (\pm\pi, 0)$ $K' = (0, \pm\pi)$

Long-wavelength expansion around K and K'

$\epsilon_k \sim -2a|c|[k_x \cos\theta + ik_y \sin\theta]$

$\epsilon_k \sim -2a|c|[k_y \cos\theta + ik_x \sin\theta]$

$$H = -\sum_k \epsilon_k^* a_k^\dagger b_k + H.c.$$

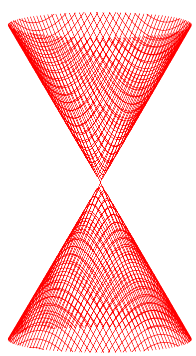
Tight-binding model for graphene ($\theta = \pi/4$)

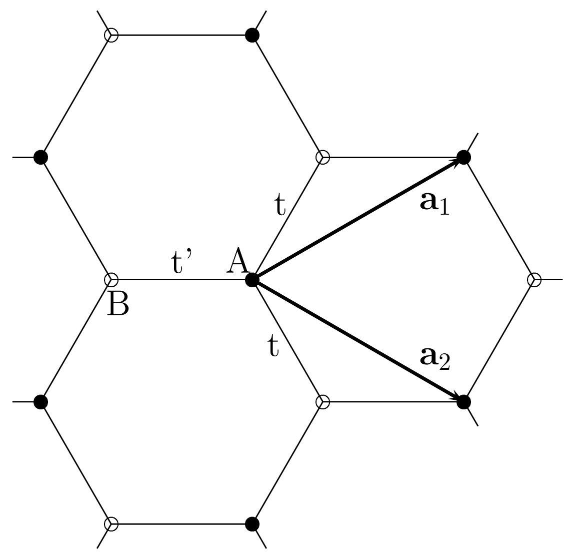
$$H = \sum_k (k_x - ik_y) a_{F,+}^\dagger(\mathbf{k}) b_{F,+}(\mathbf{k}) + (-k_x - ik_y) a_{F,-}^\dagger(\mathbf{k}) b_{F,-}(\mathbf{k})$$

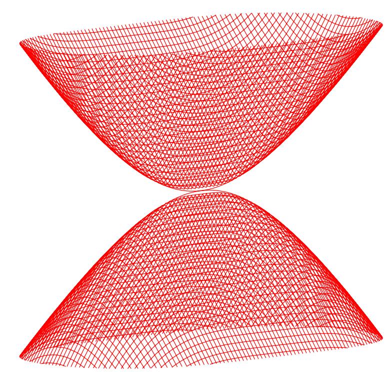
Graphene under uniaxial pressure

Cheol-Hwan Park et al., Nature Physics 4, 213 (2008)

P. Dietl, F. Piechon, G. Montambaux, cond-mat/0707.0219







$t = t'$

$t' = 2t$

Fermions and Bosons

$$S = S_B + S_F + S_{\rm int}$$

$$S_B = \int d\tau \Big\{ \sum_{i,j} a_B^\dagger(i,\tau) T_{ij} b_B(j,\tau) + H.c.$$

$$+ \sum_i a_B^\dagger(i,\tau) \;\; (\hbar\partial_\tau - \mu_B) a_B(i,\tau) + \sum_i b_B^\dagger(i,\tau) \;\; (\hbar\partial_\tau - \mu_B) b_B(i,\tau)$$

$$+ \frac{U}{2} \sum_i \Big[a_B^\dagger(i,\tau) a_B^\dagger(i,\tau) a_B(i,\tau) a_B(i,\tau) + b_B^\dagger(i,\tau) b_B^\dagger(i,\tau) b_B(i,\tau) b_B(i,\tau) \Big]$$

Fermions: same terms, replace B by F, neglect U terms (spin polarized fermions: neglect p-wave collisions at low-T)

Fermions and Bosons

$$
\begin{aligned}
S_I &= T_{BF} \int d\tau \left\{ \sum_i a_F^\dagger(i,\tau) a_F(i,\tau) a_B^\dagger(i,\tau) a_B(i,\tau) \right. \\
&+ \left. \sum_i b_F^\dagger(i,\tau) b_F(i,\tau) b_B^\dagger(i,\tau) b_B(i,\tau) \right\}
\end{aligned}
$$

Bosons condense at k_0: integrate them out

New “phonon-mediated” interaction which couples fermions in A-B sublattices

$V(r)$: Yukawa-like interaction

MF decoupling: generate Mass term for fermions

See Kekulé distortion for graphene $\to$ C. Mudry

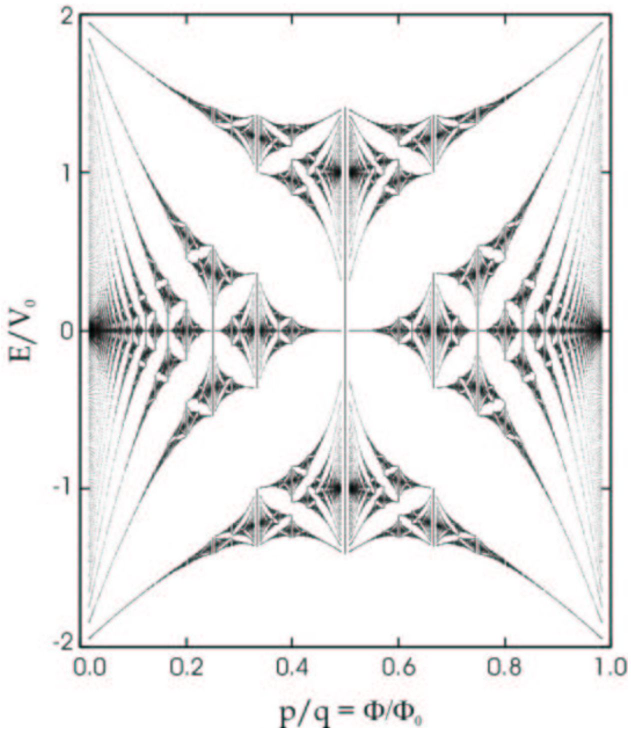
Conclusions

Cold atoms in optical lattices under staggered rotation:

- Superfluid / Mott-insulator transition
- Staggered flux phase (bosons)
  finite momentum condensate
- Simulate graphene under pressure (fermions)
- Simulate staggered-π flux phase for high-Tc superconductors at $\theta = \pi/4$
- Simulate Kekulé distortion in graphene (fermions and bosons)
- Novel Phases? QHE, bilayers, supersolids, etc...

Perspectives

Hofstadter butterfly



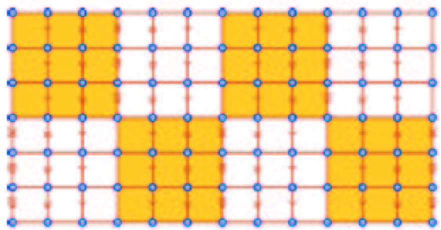
ϕ: flux/plaquette

cond-mat: $B = 10^4$T

uniform B

Perspectives

- Staggered $N\sqrt{2} \times N\sqrt{2}$ field



- role of interactions
- ring-exchange and longer-range int.
- triangular, hexagonal geometries