Quantum information processing with cold Fermi gases in the fast pairing regime

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September 19, 2008

Quantum information theory ...

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$$|\text{State}\rangle = c_1|\text{Alive}\rangle + c_2|\text{Not}\rangle, \quad c_{1,2} \in \mathbb{C}, \quad |c_1|^2 + |c_2|^2 = 1.$$

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- Effects of time-correlated noise ?

$$C(t,t') = V e^{-|t-t'|/\tau}$$

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- Decay channels

$$E_{0,1} = \pm\sqrt{\epsilon^2 + |\Delta|^2}, \quad E_2, E_3, \ldots$$

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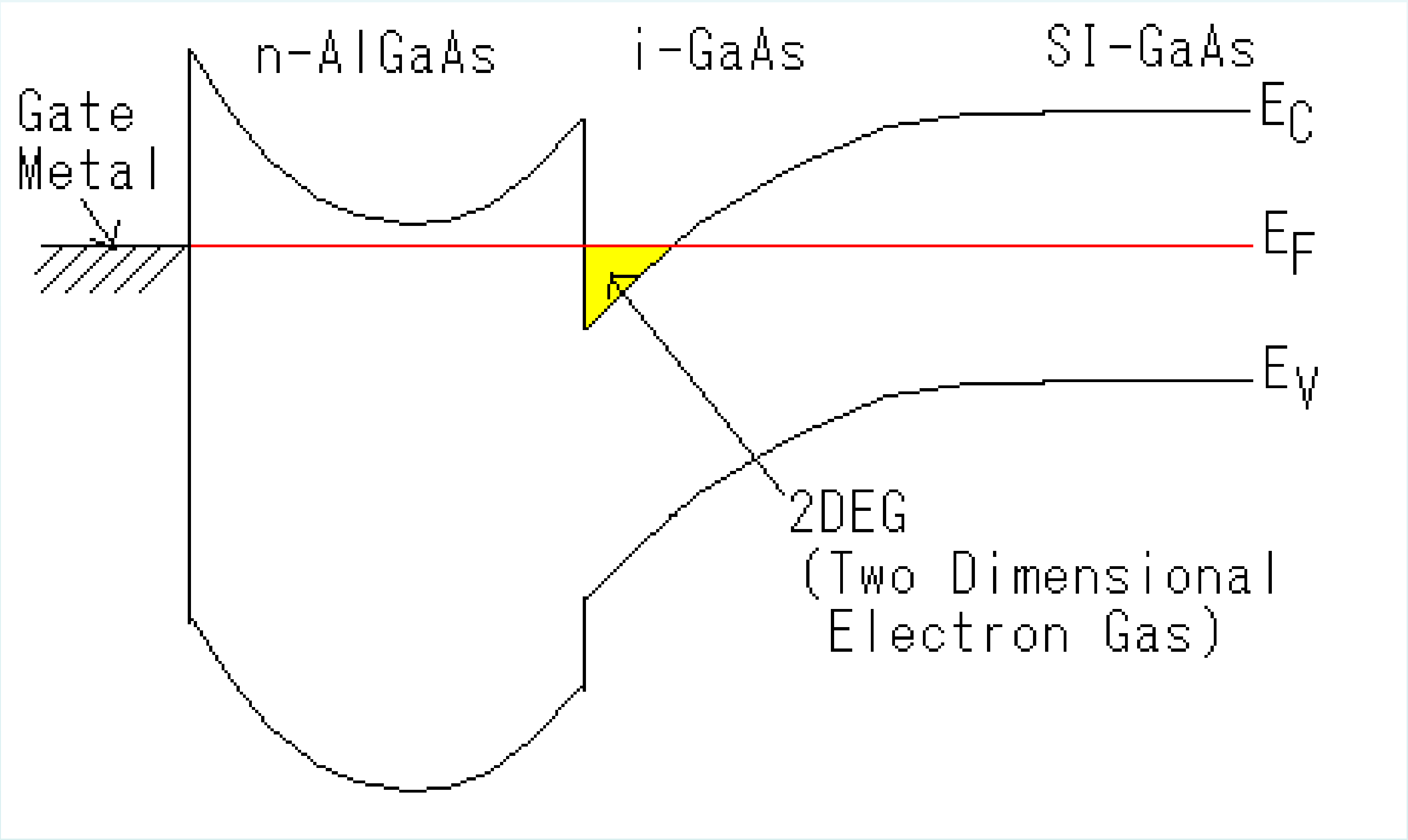

Fractional Quantum Hall Effect: a refresher

- Berry phase, Aharonov-Bohm: $\Psi(\vec{r}) \sim \exp\left[\frac{ie}{\hbar c}\int_{\vec{r}_0}^{\vec{r}} \vec{A}\cdot d\vec{\ell}\right]$

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- Quantization (Streda's formula): $\sigma = \frac{\delta A}{\delta J} \sim \frac{\delta N_\phi}{\delta N_e} = \nu$

- The $\nu = 5/2$ state: Laughlin's wavefunction:

$$\Psi_{GS}(z_1, \ldots z_{2n}) = \mathrm{Pf}[(z_i - z_j)^{-1}]\prod_{k<l}(z_k - z_l)^2\prod_j e^{-|z_j|^2/4},$$
$$\mathrm{Pf}(A) = \sqrt{\mathrm{Det}(A)}$$

Two-point functions in the $\nu = 5/2$ state

$$\Psi_{2qh}(z_1, z_2; z_3, \ldots z_{2n}) = \mathrm{Pf}\left[\frac{z_1 - z_2}{z_1 z_2 (z_i - z_j)}\right] \prod_m z_m \prod_{k<l} (z_k - z_l)^2 \prod_j e^{-|z_j|^2/4},$$

$$\Psi(z_2, z_1) = \Psi(z_1, z_2) e^{\frac{2i\pi}{\kappa}}, \quad \kappa \in \mathbb{Z}.$$

- $\kappa = 1$: bosons
- $\kappa = 2$: fermions
- $\kappa = 4$: Pfaffians (for $\nu = 5/2$)

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$$J^i = \frac{\delta W}{\delta A_i} = \left(\bar{\psi}\gamma^i\psi\right) = \sum_{-\infty}^{\infty} J_n^i z^{-n-1},$$

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$$c = \frac{3k}{k+2}, \quad k = 1 \to c = 1;\; k = 2 \to c = \frac{3}{2} = 1 + \frac{1}{2}.$$

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$$S = k \cdot \frac{1}{4\pi} \left[\mathrm{Tr} \int d^2\xi \frac{1}{2} \partial^\mu g^{-1} \partial_\mu g + \epsilon^{\mu\nu} \int_0^1 d\tau \int d^2\xi g^{-1} \partial_\tau g g^{-1} \partial_\mu g g^{-1} \partial_\nu g \right]$$

- Equations for field g (K-Z equations):

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- Equations for field g (K-Z equations):

$$\left[(k+2) \frac{\partial}{\partial z_i} + \sum_{j \neq i}^{n} \frac{\tau_i^a \tau_j^a}{z_i - z_j} \right] \langle g(z_1) g(z_2) ... g(z_n) \rangle = 0.$$

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- Dense representations

Is there such a physical system within reach ?

The pairing model

Richardson (1964), Gaudin (1972)

$$\hat{H} = \sum_{\mathbf{p},\sigma} \epsilon_{\mathbf{p}} \hat{c}^{\dagger}_{\mathbf{p},\sigma} \hat{c}_{\mathbf{p},\sigma} - g \sum_{\mathbf{p},\mathbf{k}} \hat{c}^{\dagger}_{\mathbf{p}\,\uparrow} \hat{c}^{\dagger}_{-\mathbf{p}\,\downarrow} \hat{c}_{-\mathbf{k}\,\downarrow} \hat{c}_{\mathbf{k}\,\uparrow}$$

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$$\boldsymbol{c} \to \boldsymbol{t}, \quad [t_i^3, t_j^{\pm}] = \pm\delta_{ij} t_j^{\pm}, \quad [t_i^+, t_j^-] = 2\delta_{ij} t_j^3,$$

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$$H_P = \sum_{l\in\Lambda} 2\epsilon_l t^3_l - g \sum_{l,l'} t^+_l t^-_{l'} = \sum_{l\in\Lambda} 2\epsilon_l t^3_l - g \mathbf{t}^+ \cdot \mathbf{t}^-$$

Solution of the Richardson model

$$H_P = 2 \sum_{l \in \Lambda} \epsilon_l R_l + \text{const}, \quad R_l = t_l^3 - \frac{g}{2} \sum_{l' \neq l} \frac{\mathbf{t}_l \cdot \mathbf{t}_{l'}}{\epsilon_l - \epsilon_{l'}}$$

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$$|\Psi\rangle = \prod_{k=1}^{N} b_k^\dagger |0\rangle, , \quad b_k^\dagger = \sum_l \frac{t_l^\dagger}{2\epsilon_l - e_k}, \frac{1}{g} = \sum_{p \neq k} \frac{2}{e_k - e_p} + \sum_l \frac{1}{2\epsilon_l - e_k}$$

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$$R_k\Psi = 0, \quad H_P\Psi = 0.$$

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$$(k+2)\frac{\partial \Psi}{\partial \epsilon_i} + \widetilde{R}_i \Psi = 0, \quad \widetilde{R}_i \sim \sum_{i \neq j} \tau_i^a \tau_j^a \zeta(\epsilon_i - \epsilon_j)$$

$$\boxed{c = \frac{3k}{k+2} \rightarrow -\infty}$$

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- Same commutation relations like $k = 2$, but different level of $SU(2)_k$
- Fractional level representations: Gaberdiel et. al.

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- Dense representations $k \neq 1, 2, 4$

Semiclassical approximation

$$H_{MF} = \sum_{l \in \Lambda} \epsilon_l S_l^3 - \frac{g}{4} |J^-|^2, \quad \mathbf{J} = \sum_{l \in \Lambda} \mathbf{S}_l$$

Commutators give Poisson brackets, nonlinear Bloch equations

$$\{S_i^\alpha, \, S_j^\beta\} = 2\epsilon^{\alpha\beta\gamma} S_i^\gamma \delta_{ij}, \qquad \boxed{\dot{\vec{S}}_i = 2(-\vec{\Delta} + \epsilon_i \hat{z}) \times \vec{S}_i}$$

Classical gap parameter, constants of motion:

$$\vec{\Delta} = \frac{1}{2}(gJ_x, gJ_y, 0), \quad r_i = \frac{1}{2}\left[S_i^z - \frac{g}{2} \sum_{j \neq i} \frac{\vec{S}_i \cdot \vec{S}_j}{\epsilon_i - \epsilon_j} \right]$$

Elliptic solutions

Levitov-Barankov-Spivak Ansatz:

$$\epsilon_k = -\epsilon_{-k}, \quad S_k^{y,z} = -S_{-k}^{y,z}, \quad S_k^x = S_{-k}^x$$

$$S_k^x = A_k \Omega, \quad S_k^y = B_k \dot{\Omega}, \quad S_k^z = C_k + D_k \Omega^2, \quad \Omega = |\Delta|$$

$$|\vec{S}_k| = 1 \Rightarrow (\dot{\Omega})^2 = (\Omega^2 - \Omega_-^2)(\Omega_+^2 - \Omega^2).$$

Constants $\Omega_\pm$ fixed by initial conditions and

$$1 = g \sum_k A_k$$

Multi-frequency (multi-cut) hyperelliptic solutions

Dubrovin equations:

$$i\frac{\dot{\Omega}}{\Omega} = \sum_{k=1}^{n-1} u_k, \quad \dot{u}_i = \frac{2iy(u_i)}{\prod_{j\neq i}(u_i - u_j)},$$

Spectral curve:

$$y^2(\lambda) = \prod_{i=1}^{n}(\lambda - \epsilon_i)^2 \det\left[\sigma_3 + \frac{g}{2}\sum_{i=0}^{n}\frac{\vec{S}_i \cdot \vec{\sigma}}{\lambda - \epsilon_i}\right] = \prod_{i=0}^{2n}(\lambda - E_i)$$

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