Gauge Instantons from Open Strings and D-branes

Alberto Lerda

U.P.O. *"A. Avogadro"* & I.N.F.N. - Alessandria

Firenze, May 9, 2007

Foreword

- This talk is mainly based on:

- M. Billo, M. Frau, I. Pesando, F. Fucito, A. L. and A. Liccardo, JHEP **0302**, 045 (2003), [hep-th/0211250]
- M. Billo, M. Frau, I. Pesando and A. L., JHEP **0405**, 023 (2004), [hep-th/0402160]
- M. Billo, M. Frau, S. Sciuto, G. Vallone and A. L., JHEP **0605**, 069 (2006), [hep-th/0511036]
- M. Billo, M. Frau, F. Fucito and A. L., JHEP **0611**, 012 (2006), [hep-th/0606013]
- R. Argurio, M. Bertolini, G. Ferretti, A. L. and C. Petersson, arXiv:0704.0262 [hep-th].
- M. B. Green and M. Gutperle, JHEP **0002** (2000) 014 [hep-th/0002011] + ...

- Recent literature on the subject:

- R. Blumenhagen, M. Cvetic and T. Weigand, Nucl. Phys. B **771** (2007) 113 [hep-th/0609191].
- L. E. Ibanez and A. M. Uranga, JHEP **0703** (2007) 052 [hep-th/0609213].
- N. Akerblom, R. Blumenhagen, D. Lust, E. Plauschinn and M. Schmidt-Sommerfeld, hep-th/0612132.
- M. Bianchi and E. Kiritsis, hep-th/0702015.
- M. Cvetic, R. Richter and T. Weigand, hep-th/0703028.
- M. Bianchi, F. Fucito and J. F. Morales, arXiv:0704.0784.
- L. E. Ibanez, A. N. Schellekens and A. M. Uranga, arXiv:0704.1079.

Plan of the talk

1. Introduction and motivation

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3. Gauge instantons and D-instantons
   - $\mathcal{N} = 2$ SYM $\longrightarrow$ instanton terms in the prepotential
   - $\mathcal{N} = 1$ SYM $\longrightarrow$ instanton terms in the superpotential

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4. New applications
   - Instantons in closed string backgrounds
   - Stringy instanton effects

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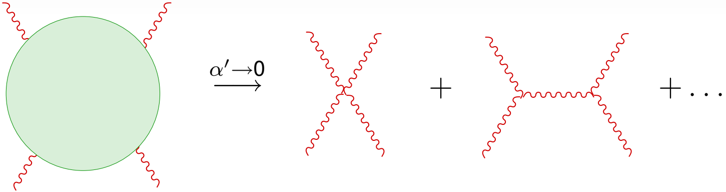
5. Conclusions and perspectives

Introduction

String theory is a very powerful tool to analyze field theories, and in particular gauge theories.

Behind this, there is a rather simple and well-known fact: in the field theory limit $\alpha' \to 0$, a single string scattering amplitude reproduces a sum of different Feynman diagrams



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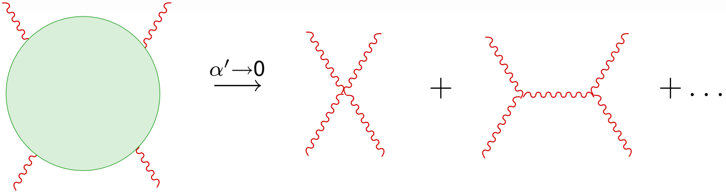

String theory S-matrix elements $\Longrightarrow$ vertices and effective actions in field theory

In general, a N-point string amplitude $\mathcal{A}_N$ is given schematically by

$$\mathcal{A}_N = \int_\Sigma \left\langle V_{\phi_1} \cdots V_{\phi_N} \right\rangle_\Sigma$$

where

- V_{ϕ_i} is the vertex for the emission of the field ϕ_i: $V_{\phi_i} \equiv \phi_i \, \mathcal{V}_{\phi_i}$
- Σ is a Riemann surface of a given topology
- $\left\langle \ldots \right\rangle_\Sigma$ is the v.e.v. with respect to the vacuum defined by Σ.

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The simplest world-sheets Σ are:

spheres for **closed strings** and **disks** for **open strings**

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For any **closed string** field ϕ_{closed}, one has

$$\langle \mathcal{V}_{\phi_{\text{closed}}} \rangle_{\text{sphere}} = 0 \quad \Rightarrow \quad \langle \phi_{\text{closed}} \rangle_{\text{sphere}} = 0$$

and for any **open string** field ϕ_{open}, one has

$$\langle \mathcal{V}_{\phi_{\text{open}}} \rangle_{\text{disk}} = 0 \quad \Rightarrow \quad \langle \phi_{\text{open}} \rangle_{\text{disk}} = 0$$

- Since

$$\langle \phi_{\text{closed}} \rangle_{\text{sphere}} = 0 \quad , \quad \langle \phi_{\text{open}} \rangle_{\text{disk}} = 0$$

spheres and disks can describe only the trivial vacuum around which ordinary perturbation theory is performed

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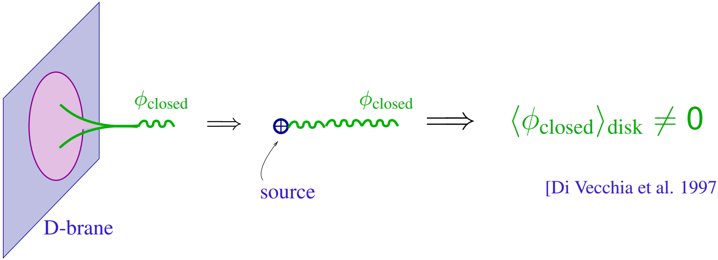
However, after the discovery of D-branes, the perspective has drastically changed, and nowadays also some non-perturbative properties of field theories can be analyzed using string theory!



[Di Vecchia et al. 1997,...]

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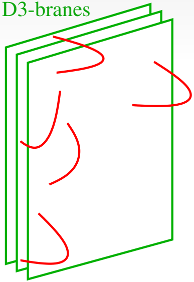
- We will see how instantons in (supersymmetric) gauge theories can be described using open strings and D-branes.

String description of SYM theories and their instantons

The effective action of a SYM theory can be given a simple and calculable string theory realization:

- The gauge degrees of freedom are realized by open strings attached to N D3 branes.



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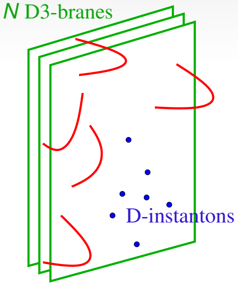


- The instanton sector of charge k is realized by adding k D(–1) branes (D-instantons).

Let us discuss this construction for $\mathcal{N} = 2$ theories

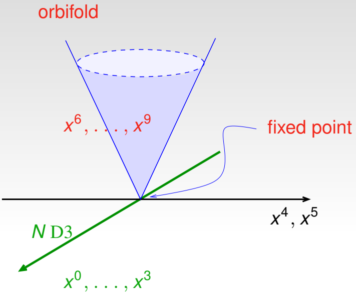
$\mathcal{N} = 2$ SU(N) SYM theory from fractional branes

- It is realized by the massless d.o.f. of open strings attached to N fractional D3-branes in the orbifold background

$$\mathbb{R}^4 \times \mathbb{R}^2 \times \mathbb{R}^4/\mathbb{Z}_2$$

where

$$\mathbb{Z}_2 \; : \{x^6, ..., x^9\} \quad \longrightarrow \quad \{-x^6, ..., -x^9\}$$



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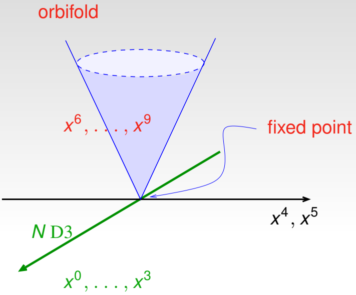

- The orbifold breaks 1/2 SUSY in the bulk, the D3 branes a further 1/2:

$$32 \times \frac{1}{2} \times \frac{1}{2} = 8 \ \text{ real supercharges} \quad \Longrightarrow \quad \mathcal{N} = 2 \text{ SUSY}$$

	0	1	2	3	4	5	6	7	8	9
D3	−	−	−	−	*	*	*	*	*	*

Since

$$\mathrm{SO}(10) \;\to\; \mathrm{SO}(4) \times \mathrm{U}(1) \times \mathrm{SO}(4)$$

the ten dimensional string coordinates X^M, ψ^M and spin fields $S^{\mathcal{A}}$ split as follows

$$X^M \;\longrightarrow\; X^\mu\,,\, X, \bar{X}\,,\, X^i \quad , \quad \psi^M \;\longrightarrow\; \psi^\mu\,,\, \Psi, \bar{\Psi}\,,\, \psi^i$$
$$S^{\mathcal{A}} \;\longrightarrow\; S_\alpha S_- S_A\,,\, S^{\dot\alpha} S^+ S^A\,,\, S_\alpha S_+ S_{\dot A}\,,\, S^{\dot\alpha} S^- S^{\dot A}$$

For example

$$\psi^\mu \;\in\; ((\mathbf{2},\mathbf{2}), 0, (\mathbf{1},\mathbf{1})) \quad , \quad \bar{\Psi} \in ((\mathbf{1},\mathbf{1}), -1, (\mathbf{1},\mathbf{1}))$$
$$S_\alpha S_- S_A \;\in\; ((\mathbf{2},\mathbf{1}), -1/2, (\mathbf{2},\mathbf{1})) \quad , \quad S_\alpha S_+ S_{\dot A} \in ((\mathbf{2},\mathbf{1}), +1/2, (\mathbf{1},\mathbf{2}))$$

String vertex operators and fields

- String vertex operators:

$$V_A \simeq A_\mu \, \psi^\mu \, \mathrm{e}^{ip\cdot X} \, \mathrm{e}^{-\varphi}$$

$$V_\Lambda \simeq \Lambda^{\alpha A} \, S_\alpha S^- S_A \, \mathrm{e}^{ip\cdot X} \, \mathrm{e}^{-\frac{1}{2}\varphi}$$

$$V_\phi \simeq \phi \, \overline{\Psi} \, \mathrm{e}^{ip\cdot X} \, \mathrm{e}^{-\varphi}$$

with all polarizations in the adjoint of SU(N)

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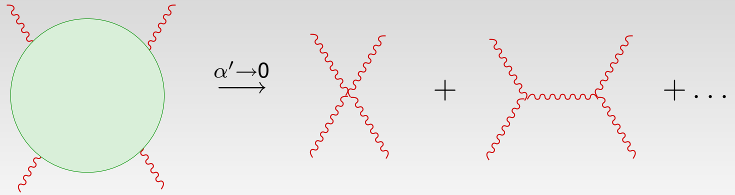
- Field content: $\mathcal{N} = 2$ vector superfield

$$\Phi(x,\theta) = \phi(x) + \theta\Lambda(x) + \frac{1}{2}\,\theta\sigma^{\mu\nu}\theta \, F^+_{\mu\nu}(x) + \cdots$$

Gauge action from disks on D3's



- String amplitudes on disks attached to the D3 branes in the limit

$$\alpha' \to 0 \text{ with gauge quantities fixed}$$

give rise to the $\mathcal{N} = 2$ SYM action

$$S_{\rm SYM} = \int d^4x \, \mathrm{Tr}\Big\{\frac{1}{2} F_{\mu\nu}^2 + 2\, D_\mu\bar\phi\, D^\mu\phi - 2\,\bar\Lambda_A \bar{\not{D}}\,\Lambda^A$$
$$+ \, i\sqrt{2}\, g\, \bar\Lambda_A \epsilon^{AB}\big[\phi, \bar\Lambda_B\big] + i\sqrt{2}\, g\, \Lambda^A \epsilon_{AB}\big[\bar\phi, \Lambda^B\big] + g^2\big[\phi, \bar\phi\big]^2\Big\}$$

Effective action

- We are interested in the low-energy effective action on the Coulomb branch parametrized by the v.e.v.'s of the adjoint chiral superfields:

$$\langle \Phi_{uv} \rangle \equiv \langle \phi_{uv} \rangle = a_{uv} = a_u \, \delta_{uv} \quad , \quad u, v = 1, ..., N \quad , \quad \sum_u a_u = 0$$

breaking $\mathrm{SU}(N) \to \mathrm{U}(1)^{N-1}$

- Up to two-derivatives, $\mathcal{N} = 2$ SUSY constrains the effective action for Φ to be of the form

$$S_{\text{eff}}[\Phi] = \int d^4x \, d^4\theta \, \mathcal{F}(\Phi) + \mathrm{c.c}$$

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- We will now discuss how to obtain the instanton corrections $\mathcal{F}^{(k)}$ to the prepotential $\mathcal{F}$ in our string set-up.

Instanton calculus in string theory

Instantons and D-instantons

- Consider the effective action for a stack of N D3 branes

$$\text{D. B. I.} + \int_{\text{D3}} \left[C_4 + \frac{1}{2} C_0 \, \text{Tr}\left(F \wedge F \right) \right]$$

The topological density of an instanton configuration corresponds to a localized source for the R-R scalar C_0, i.e., to a distribution of D-instantons inside the D3's.

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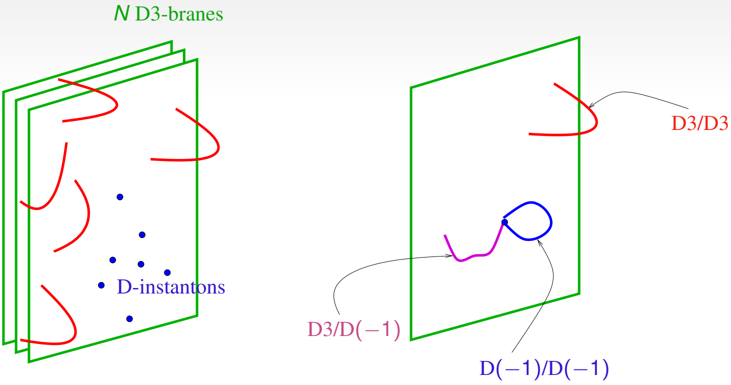
- Instanton-charge k solutions of SU(N) gauge theories correspond to k D-instantons inside N D3-branes.

[Witten 1995, Douglas 1995, Dorey 1999, ...]

Stringy description of gauge instantons

	0	1	2	3	4	5	6	7	8	9
D3	−	−	−	−	∗	∗	∗	∗	∗	∗
D(−1)	∗	∗	∗	∗	∗	∗	∗	∗	∗	∗



Moduli vertices and instanton parameters

Open strings with at least one end on a D(–1) carry no momentum: they are moduli, rather than dynamical fields.

	ADHM	Meaning	Vertex	Chan-Paton
D(–1)/D(–1) (NS)	a'_μ	*centers*	$\psi^\mu \, \mathrm{e}^{-\varphi}$	adj. U(k)
	χ	*aux.*	$\overline{\Psi} \, \mathrm{e}^{-\varphi(z)}$	⋮
	D_c	*Lagrange mult.*	$\bar{\eta}^c_{\mu\nu} \psi^\nu \psi^\mu$	⋮
D(–1)/D(–1) (R)	$M^{\alpha A}$	*partners*	$S_\alpha S_- S_A \, \mathrm{e}^{-\frac{1}{2}\varphi}$	⋮
	$\lambda_{\dot\alpha A}$	*Lagrange mult.*	$S^{\dot\alpha} S^+ S^A \, \mathrm{e}^{-\frac{1}{2}\varphi}$	⋮

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D(–1)/D3 (NS)	$w_{\dot{\alpha}}$	*sizes*	$\Delta S^{\dot{\alpha}} \, \mathrm{e}^{-\varphi}$	$k \times N$
	$\bar{w}_{\dot{\alpha}}$	*sizes*	$\overline{\Delta} S^{\dot{\alpha}} \, \mathrm{e}^{-\varphi}$	$N \times k$
D(–1)/D3 (R)	μ^A	*partners*	$\Delta S_- S_A \mathrm{e}^{-\frac{1}{2}\varphi}$	$k \times N$
	$\bar{\mu}^A$	⋮	$\overline{\Delta} S_- S_A \, \mathrm{e}^{-\frac{1}{2}\varphi}$	$N \times k$

Super-coordinates and centered moduli

- Among the D(–1)/D(–1) moduli we can single out the instanton center x_0^μ and its super-partners $\theta^{\alpha A}$:

$$
\begin{aligned}
a'^{\mu} &= x_0^\mu \, 1\!\!1_{k\times k} + y_c^\mu \, T^c \quad (T^c = \text{generators of SU}(k)) \\
M^{\alpha A} &= \theta^{\alpha A} \, 1\!\!1_{k\times k} + \zeta_c^{\alpha A} \, T^c
\end{aligned}
$$

  The moduli x_0^μ and $\theta^{\alpha A}$ decouple from many interactions and play the rôle of $\mathcal{N} = 2$ superspace coordinates.

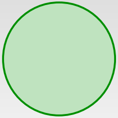
- We will distinguish the moduli $\mathcal{M}_{(k)}$ into

$$
\mathcal{M}_{(k)} \to \left\{ x_0, \theta \, ; \, \widehat{\mathcal{M}}_{(k)} \right\}
$$

  where $\widehat{\mathcal{M}}_{(k)}$ are the so-called centered moduli.

Disk amplitudes and effective actions

D3 disks



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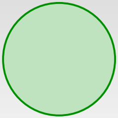
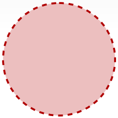


D(–1) disks



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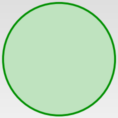

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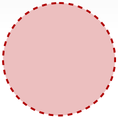
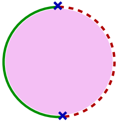


Mixed disks



Disk amplitudes and effective actions

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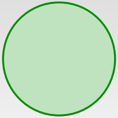

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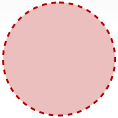

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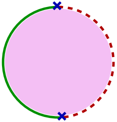


Disk amplitudes

↓ field theory limit $\alpha' \to 0$

Effective actions

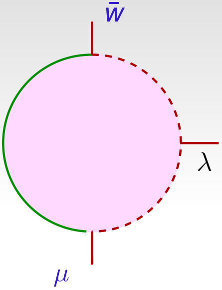
D3 disks ↙ ↘ D(–1) and mixed disks

$\mathcal{N} = 2$ SYM action

ADHM measure

An example of a mixed disk amplitude

Consider the following mixed disk diagram



which corresponds to the following amplitude

$$\left\langle\!\!\left\langle V_\lambda \, V_{\bar{w}} \, V_\mu \right\rangle\!\!\right\rangle \equiv C_0 \int \frac{\prod_i dz_i}{dV_{\text{CKG}}} \times \langle V_\lambda(z_1) \, V_{\bar{w}}(z_2) \, V_\mu(z_3) \rangle = \ldots = \mathrm{tr}_k \left\{ \mathrm{i} \, \lambda^{\dot{\alpha}}_A \; \bar{w}_{\dot{\alpha}} \; \mu^A \right\}$$

where $C_0 = 8\pi^2/g^2$ is the disk normalization.

The action for the instanton moduli

From all D(–1) and mixed disk diagrams with insertion of all moduli vertices, we can extract the ADHM moduli action (at fixed k)

$$\mathcal{S}_{\text{mod}} = \mathcal{S}_{\text{bos}}^{(k)} + \mathcal{S}_{\text{fer}}^{(k)} + \mathcal{S}_{\text{c}}^{(k)}$$

with

$$\begin{aligned}
\mathcal{S}_{\text{bos}}^{(k)} &= \mathrm{tr}_k\Big\{ -2\,[\chi^\dagger, a'_\mu][\chi, a'^\mu] + \chi^\dagger \bar{w}_{\dot\alpha} w^{\dot\alpha}\chi + \chi \bar{w}_{\dot\alpha} w^{\dot\alpha}\chi^\dagger \Big\} \\
\mathcal{S}_{\text{fer}}^{(k)} &= \mathrm{tr}_k\Big\{ \mathrm{i}\,\frac{\sqrt{2}}{2}\,\bar{\mu}^A \epsilon_{AB}\mu^B \chi^\dagger - \mathrm{i}\,\frac{\sqrt{2}}{4}\, M^{\alpha A}\epsilon_{AB}[\chi^\dagger, M^B_\alpha] \Big\} \\
\mathcal{S}_{\text{c}}^{(k)} &= \mathrm{tr}_k\Big\{ -\mathrm{i}D_c\big(\bar{w}_{\dot\alpha}(\tau^c)^{\dot\alpha\dot\beta} w_{\dot\beta} + \mathrm{i}\bar{\eta}^c_{\mu\nu}\,[a'^\mu, a'^\nu]\big) \\
&\qquad -\mathrm{i}\lambda^{\dot\alpha}_A\big(\bar{\mu}^A w_{\dot\alpha} + \bar{w}_{\dot\alpha}\mu^A + [a'_{\alpha\dot\alpha}, M'^{\alpha A}]\big)\Big\}
\end{aligned}$$

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with

$$\mathcal{S}_{\text{bos}}^{(k)} = \text{tr}_k\Big\{ -2\,[\chi^\dagger, a'_\mu][\chi, a'^\mu] + \chi^\dagger \bar{w}_{\dot\alpha} w^{\dot\alpha}\chi + \chi \bar{w}_{\dot\alpha} w^{\dot\alpha}\chi^\dagger \Big\}$$

$$\mathcal{S}_{\text{fer}}^{(k)} = \text{tr}_k\Big\{ \mathrm{i}\,\frac{\sqrt{2}}{2}\,\bar\mu^A \epsilon_{AB}\mu^B\chi^\dagger - \mathrm{i}\,\frac{\sqrt{2}}{4}\,M^{\alpha A}\epsilon_{AB}[\chi^\dagger, M^B_\alpha]\Big\}$$

$$\mathcal{S}_{\text{c}}^{(k)} = \text{tr}_k\Big\{ -\mathrm{i}D_c\big(\bar{w}_{\dot\alpha}(\tau^c)^{\dot\alpha\dot\beta}w_{\dot\beta} + \mathrm{i}\bar\eta^c_{\mu\nu}\,[a'^\mu, a'^\nu]\big) - \mathrm{i}\lambda^{\dot\alpha}_A\big(\bar\mu^A w_{\dot\alpha} + \bar{w}_{\dot\alpha}\mu^A + [a'_{\alpha\dot\alpha}, M'^{\alpha A}]\big)\Big\}$$

- In $\mathcal{S}_{\text{c}}^{(k)}$ the bosonic and fermionic ADHM constraints appear

Take for simplicity $k = 1$ ($\longrightarrow [\ ,\ ] = 0$). The bosonic "equations of motion"

$$w_{u\dot{\alpha}}\,\chi = 0 \quad , \quad \bar{w}_{\dot{\alpha}u}(\tau^c)^{\dot{\alpha}\dot{\beta}}w_{u\dot{\beta}} = 0$$

determine the classical vacua.

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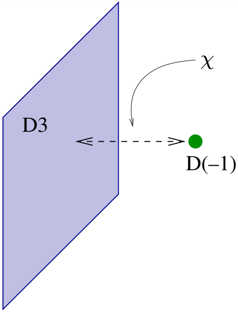
There are two types of solutions:

$$\chi \neq 0 \ , \ w_{u\dot\alpha} = 0$$



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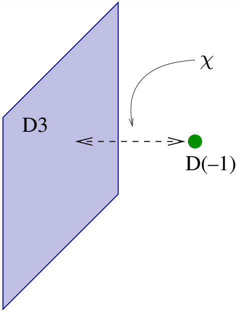
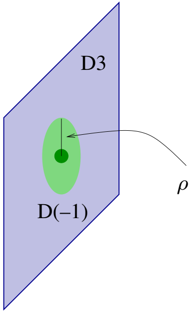


$$\chi = 0 \ , \ \ w_{u\dot\alpha} = \rho \begin{pmatrix} 1_{2\times 2} \\ 0_{(N-2)\times 2} \end{pmatrix}$$



Properties of the moduli action $\mathcal{S}_{\text{mod}}$

- $\mathcal{S}_{\text{mod}}$ depends only on the centered moduli $\widehat{\mathcal{M}}_{(k)}$ but does not depend on the center x_0^μ and its super-partners $\theta^{\alpha A}$

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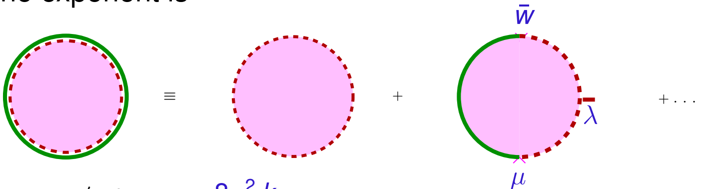
- $\mathcal{S}_{\text{mod}}$ depends only on the centered moduli $\widehat{\mathcal{M}}_{(k)}$ but does not depend on the center x_0^{μ} and its super-partners $\theta^{\alpha A}$

- Integration over all moduli leads to the instanton partition function

[Polchinski 1994, ..., Dorey et al. 1999, ...]

$$Z^{(k)} = \int d^4x_0\, d^4\theta\, d\widehat{\mathcal{M}}_{(k)}\ \mathrm{e}^{-\frac{8\pi^2 k}{g^2} - \mathcal{S}_{\text{mod}}(\widehat{\mathcal{M}}_{(k)})}$$

where the exponent is



$$\overset{\alpha'\to 0}{\simeq} \quad -\frac{8\pi^2 k}{g^2} \quad - \quad \mathcal{S}_{\text{mod}}(\widehat{\mathcal{M}}_{(k)})$$

Instanton classical solution

- The mixed disks are the sources for a non-trivial gauge field whose profile is exactly that of the classical instanton!!

[Billó et al. 2002,...]

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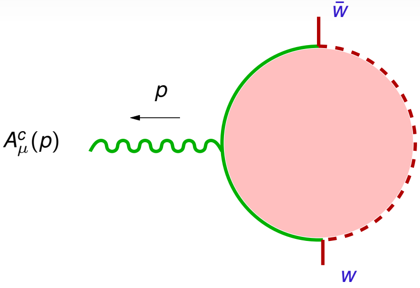
Let us consider the following mixed-disk amplitude:



$$\equiv \left\langle \, \mathcal{V}_{A_\mu^c(p)} \, \right\rangle_{\text{mixed disk}}$$

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[Billó et al. 2002,...]

Using the explicit expressions of the vertex operators, for $SU(2)$ with $k = 1$ one finds

$$\left\langle \mathcal{V}_{A^c_\mu(p)} \right\rangle_{\text{mixed disk}} \equiv \left\langle V_{\bar{w}}\, \mathcal{V}_{A^c_\mu}(p)\, V_w \right\rangle$$

$$= -i\, p^\nu\, \bar{\eta}^c_{\mu\nu} \left(\bar{w}^{\dot{\alpha}}\, w_{\dot{\alpha}} \right) \mathrm{e}^{-i\, p \cdot x_0} \equiv A^c_\mu(p; w, x_0)$$

- On this mixed disk the gauge vector field has a non-vanishing tadpole!

- Taking the Fourier transform of $A^c_\mu(p; w, x_0)$, after inserting the free propagator $1/p^2$, we obtain

$$A^c_\mu(x) \equiv \int \frac{d^4p}{(2\pi)^2}\; A^c_\mu(p; w, x_0)\,\frac{1}{p^2}\,\mathrm{e}^{i\,p\cdot x} = 2\,\rho^2\,\bar{\eta}^c_{\mu\nu}\,\frac{(x-x_0)^\nu}{(x-x_0)^4}$$

  where we have used the solution of the ADHM constraints and defined $\bar{w}^{\dot\alpha}\,w_{\dot\alpha} = 2\,\rho^2$.

- This is the leading term in the large distance expansion of an $\mathrm{SU}(2)$ instanton with size ρ and center x_0 in the singular gauge!!

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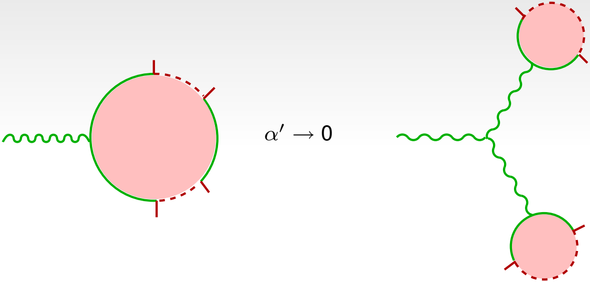
- This is the leading term in the large distance expansion of an $\mathrm{SU}(2)$ instanton with size ρ and center x_0 in the singular gauge!!
- In fact

$$\begin{aligned} A^c_\mu(x)\Big|_{\text{instanton}} &= 2\,\rho^2\,\bar\eta^c_{\mu\nu} \, \frac{(x-x_0)^\nu}{(x-x_0)^2\,\left[(x-x_0)^2+\rho^2\right]} \\ &= 2\,\rho^2\,\bar\eta^c_{\mu\nu} \, \frac{(x-x_0)^\nu}{(x-x_0)^4} \left(1 - \frac{\rho^2}{(x-x_0)^2} + \ldots\right) \end{aligned}$$

- The subleading terms in the large distance expansion can be obtained from mixed disks with more insertions of moduli.

- For example, at the next-to-leading order we have to consider the following mixed disk which can be easily evaluated for $\alpha' \to 0$

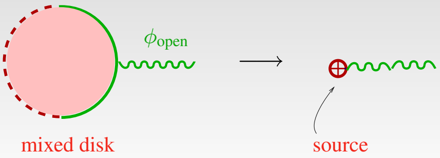


- Its Fourier transform gives precisely the 2nd order in the large distance expansion of the instanton profile

$$A^c_\mu(x)^{(2)} = -2\,\rho^4\,\bar{\eta}^c_{\mu\nu}\,\frac{(x-x_0)^\nu}{(x-x_0)^6}$$

Summary

- Mixed disks are sources for open strings



$$\longrightarrow \Big\langle \mathcal{V}_{\phi_{\text{open}}} \Big\rangle_{\text{mixed disk}} \neq 0$$

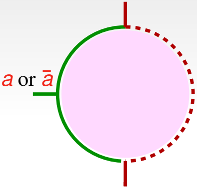
- The gauge field emitted from mixed disks is precisely that of the classical instanton

$$\Big\langle \mathcal{V}_{A_\mu} \Big\rangle_{\text{mixed disk}} \quad \Leftrightarrow \quad A_\mu \Big|_{\text{instanton}}$$

- This procedure can be easily generalized to the SUSY partners of the gauge boson.

Introducing v.e.v.'s for the scalars

- One can introduce v.e.v.'s a and $\bar{a}$ for the scalars of the gauge multiplet by computing mixed disk diagrams with insertions of $V_{\phi=a}$ and $V_{\bar{\phi}=\bar{a}}$



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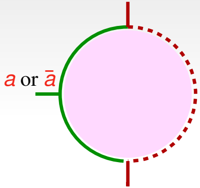


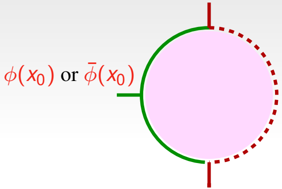
- This amounts to the following shifts

$$\chi \to \chi - a \quad , \quad \chi^\dagger \to \chi^\dagger - \bar{a}$$

- In the resulting action the v.e.v.'s a and $\bar{a}$ are not on the same footing: a does not appear in the fermionic part of the action.

Field-dependent moduli action (I)

- Exploiting the broken translational symmetry, we can promote a (or $\bar{a}$) to the full dynamical field $\phi(x_0)$ (or $\bar{\phi}(x_0)$) through diagrams like



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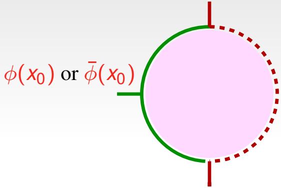


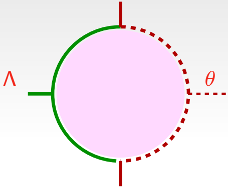
  where x_0 is the instanton center (denoted x from now on)

- Thus, the field-dependent action $\mathcal{S}_{\text{mod}}(\phi, \bar{\phi}; \widehat{\mathcal{M}}_{(k)})$ is thus simply obtained from $\mathcal{S}_{\text{mod}}(a, \bar{a}; \widehat{\mathcal{M}}_{(k)})$ by

$$a \;\longrightarrow\; \phi(x) \qquad , \qquad \bar{a} \;\longrightarrow\; \bar{\phi}(x)$$

Field-dependent moduli action (II)

- Exploiting the broken SUSY, we can promote $\phi(x)$ to the full chiral superfield $\Phi(x, \theta)$ through diagrams like



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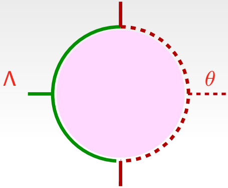


- Thus, the superfield-dependent action $\mathcal{S}_{\text{mod}}(\Phi,\bar{\Phi};\widehat{\mathcal{M}}_{(k)})$ is thus simply obtained from $\mathcal{S}_{\text{mod}}(a,\bar{a};\widehat{\mathcal{M}}_{(k)})$ by

$$a \quad\rightarrow\quad \phi(x) \quad\rightarrow\quad \Phi(x,\theta) \qquad , \qquad \bar{a} \quad\rightarrow\quad \bar{\phi}(x) \quad\rightarrow\quad \bar{\Phi}(x,\theta)$$

Instanton contributions to the prepotential

Integrating over the moduli one gets the instanton induced effective action for Φ:

$$\begin{aligned} S_{\text{eff}}^{(k)}[\Phi] &= \int d^4x\, d^4\theta\, d\widehat{\mathcal{M}}_{(k)}\, \mathrm{e}^{-\frac{8\pi k}{g^2} - \mathcal{S}_{\text{mod}}(\Phi,\bar{\Phi};\widehat{\mathcal{M}}_{(k)})} \\ &= \int d^4x\, d^4\theta\, \mathcal{F}^{(k)}(\Phi) \end{aligned}$$

Correspondingly, the prepotential $\mathcal{F}^{(k)}$ for the low energy $\mathcal{N} = 2$ theory is given by the centered instanton partition function

$$\mathcal{F}^{(k)}(\Phi) = \int d\widehat{\mathcal{M}}_{(k)}\, \mathrm{e}^{-\frac{8\pi k}{g^2} - \mathcal{S}_{\text{mod}}(\Phi,\bar{\Phi};\widehat{\mathcal{M}}_{(k)})}$$

- For example, for the $\mathcal{N}=2$ SYM theory with $\mathrm{SU}(2)$ (broken to $\mathrm{U}(1)$), one finds

$$\mathcal{F}^{(k)}(\Phi) = c_k\,\Phi^2\,\left(\frac{\Lambda}{\Phi}\right)^{4k}$$

where Λ is the dynamically generated scale, and the coefficients c_k can be obtained by evaluating the integral over the instanton moduli:

$$c_1 = \frac{1}{2} \quad , \quad c_2 = \frac{5}{16} \quad , \quad \cdots$$

(in perfect agreement with the Seiberg-Witten exact solution of the theory).

- ...

Now let us study instanton effects in $\mathcal{N}=1$ SYM theories.

$\mathcal{N} = 1$ SYM theories from fractional branes

- They can be realized by the massless d.o.f. of open strings attached to fractional D3-branes in the orbifold background

$$\mathbb{R}^4 \times \mathbb{R}^6 / (\mathbb{Z}_2 \times \mathbb{Z}_2)$$

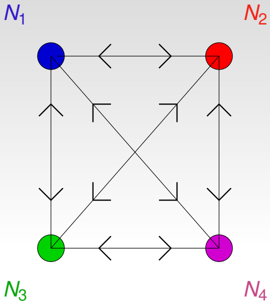
where the orbifold group acts as

$$\begin{array}{lcccc} g_1 & : & \{x^6, x^7, x^8, x^9\} & \longrightarrow & \{-x^6, -x^7, -x^8, -x^9\} \\ g_2 & : & \{x^4, x^5, x^8, x^9\} & \longrightarrow & \{-x^4, -x^5, x^8, -x^9\} \end{array}$$

- This orbifold breaks 1/4 SUSY in the bulk, the D3 branes a further 1/2:

$$32 \times \frac{1}{4} \times \frac{1}{2} = 4 \text{ real supercharges} \quad \Longrightarrow \quad \mathcal{N} = 1 \text{ SUSY}$$

- In this orbifold there 4 types of fractional D3 branes, giving rise to the following quiver gauge theory



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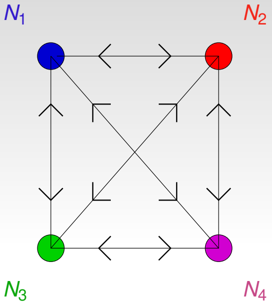
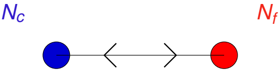
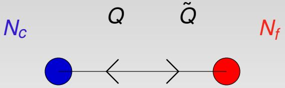


- If we take $N_1 = N_c$, $N_2 = N_f$, $N_3 = N_4 = 0$, the theory living on the N_c D3 branes is

$\mathcal{N} = 1$ SQCD with gauge group $\mathrm{SU}(N_c)$ and N_f flavors



Now let us add the D(–1) branes to incorporate gauge instanton effects. Then, the quiver diagram



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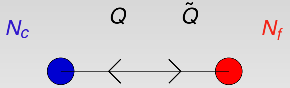
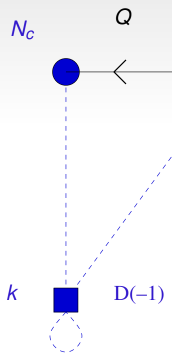
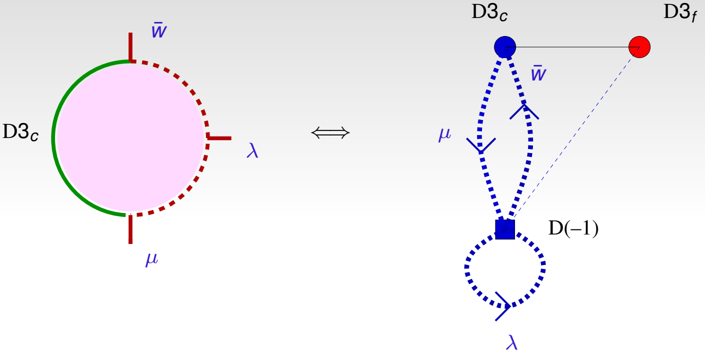


becomes



where the dashed lines represent the ADHM instanton moduli $\{x_0, \theta, \widehat{W}_{(k)}\}$.

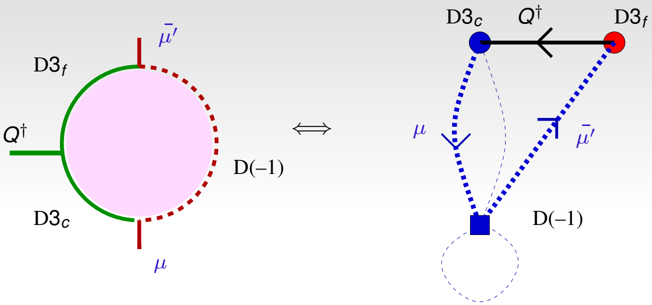
To obtain the moduli action, we have to compute all (mixed) disk diagrams. As before, we have, for example,



which leads to

$$\left\langle\!\left\langle V_\lambda \, V_{\bar{w}} \, V_\mu \right\rangle\!\right\rangle = \mathrm{i}\, \lambda^{\dot\alpha} \, \bar{w}_{\dot\alpha} \, \mu$$

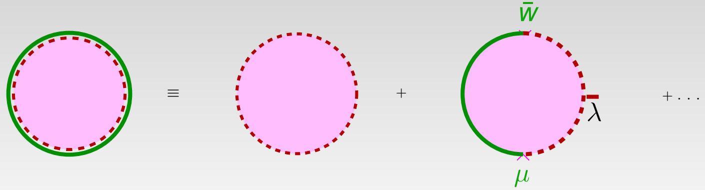
Now, however, there are new kinds of diagrams. For example, we have



which leads to

$$\left\langle\!\left\langle V_{\mu'}\, V_{Q^\dagger}\, V_{\mu} \right\rangle\!\right\rangle = \frac{i}{2}\,\bar{\mu}'\, Q^\dagger\, \mu$$

Putting together all these (mixed) diagrams



one finds (for $k = 1$)

$$\frac{8\pi^2}{g^2} + \mathcal{S}(Q, \tilde{Q}) + \mathcal{S}_c$$

where

$$\mathcal{S}(Q, \tilde{Q}) = \frac{1}{2}\,\bar{w}_{\dot\alpha}\big(Q\,Q^\dagger + \tilde{Q}^\dagger\,\tilde{Q}\big)\,w^{\dot\alpha} + \frac{i}{2}\,\bar\mu\,\tilde{Q}^\dagger\,\mu' - \frac{i}{2}\,\bar\mu'\,Q^\dagger\,\mu$$

$$\mathcal{S}_c = \Big\{ -\,\mathrm{i}D_c\big(\bar{w}_{\dot\alpha}(\tau^c)^{\dot\alpha\dot\beta}w_{\dot\beta}\big) - \mathrm{i}\lambda^{\dot\alpha}\big(\bar\mu\,w_{\dot\alpha} + \bar{w}_{\dot\alpha}\,\mu\big)\Big\}$$

Thus, the 1-instanton induced effective action is

$$S^{(1)}_{\text{eff}}[Q,\tilde{Q}] = \int d^4x\, d^2\theta\; d\widehat{\mathcal{M}}_{(1)}\; \mathrm{e}^{-\frac{8\pi}{g^2}-\mathcal{S}(Q,\tilde{Q})-\mathcal{S}_c}$$

and the superpotential is

$$\begin{aligned} W &= \int d\widehat{\mathcal{M}}_{(1)}\; \mathrm{e}^{-\frac{8\pi}{g^2}-\mathcal{S}(Q,\tilde{Q})-\mathcal{S}_c} \\ &= \int \cdots d^{N_c}\mu\, d^{N_c}\bar{\mu}\, d^{N_f}\mu'\, d^{N_f}\bar{\mu}'\, d^2\lambda \cdots \\ &\quad \times\; \cdots\; \mathrm{e}^{\frac{i}{2}\bar{\mu}\,\tilde{Q}^\dagger\,\mu' - \frac{i}{2}\bar{\mu}'\,Q^\dagger\,\mu - \mathrm{i}\lambda^{\dot{\alpha}}\left(\bar{\mu}\,w_{\dot{\alpha}} + \bar{w}_{\dot{\alpha}}\,\mu\right)} \end{aligned}$$

- Integrating over $\lambda^{\dot{\alpha}}$ yields $\sim (\bar{\mu}\,\mu)$
- Integrating over $\bar{\mu}'$ and μ' yields $\sim (\bar{\mu}\,\mu)^{N_f}$
- Integration over $\bar{\mu}$ and μ is non-vanishing iff

$$N_c = N_f + 1$$

Setting $N_c = N_f + 1$, the remaining integrations over the bosonic moduli lead to

$$W = \frac{\Lambda^{2N_c+1}}{\det(\tilde{Q}\,Q)}$$

which is the VTY-ADS superpotential !

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- This result can be generalized to other classical gauge groups ($\mathrm{SO}(N_c)$ and $\mathrm{Usp}(N_c)$) by adding an orientifold projection to the orbifold

- This result has been recently obtained also using intersecting brane models

[Akerblom, Blumenhagen et al. 2006,...]

New applications

Gauge theories in closed string background

Gauge theories in closed string backgrounds are very interesting because, in general, they are characterized by

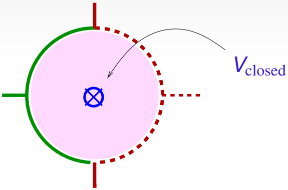
- new geometry in (super)space-time
- new mathematical structures
- new types of interactions and couplings

Closed string backgrounds produce deformations in the gauge theories. For instance:

- non-commutative theories arise from NSNS background $B_{\mu\nu}$
- non-anticommutative theories from specific RR backgrounds
- ...

- The instanton calculus through mixed disks can be easily generalized in the presence of a non-trivial closed string background

- One simply computes mixed disks with one or more insertions of closed string vertex operators



- These new disks produce new terms in the ADHM moduli action and suitably “deform” the instanton calculus.

A very interesting example:

Instanton calculus in a graviphoton background of $\mathcal{N} = 2$ SUGRA

[Billò et al. 2004]

This RR background allows:

- to find the gravitational corrections to the prepotential of the $\mathcal{N} = 2$ SYM theory
- to deform the ADHM measure in such a way that the instanton contributions can be computed via localization techniques
- to clarify a recent conjecture by N. Nekrasov on the so-called Ω-background
- to establish a nice correspondence with the topological string

Another interesting application:

Stringy instantons

[Blumenhagen et al. 2006, Ibanez-Uranga 2006, Argurio et al. 2007, ...]

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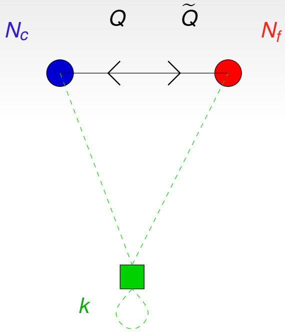

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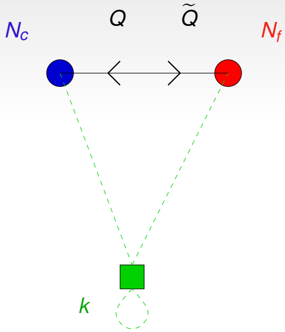


- As a consequence, there are more than 4 mixed ND directions. These stringy instantons are a generalization of the usual D(–1)/D3 systems.

The distinctive features of these exotic D-instantons are:

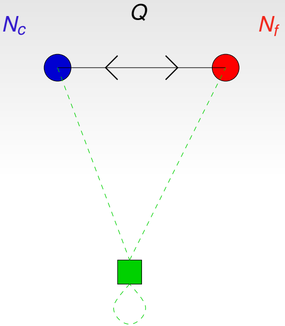
- there are no bosonic moduli from the mixed sectors ($\rightarrow$ no w's and $\bar{w}$'s)
- there are no ADHM-like constraints
- they may give rise to new interesting superpotential terms in orientifold models

The distinctive features of these exotic D-instantons are:

- there are no bosonic moduli from the mixed sectors ($\rightarrow$ no w's and $\bar{w}$'s)
- there are no ADHM-like constraints
- they may give rise to new interesting superpotential terms in orientifold models
- many possible applications...

Example: $\mathcal{N} = 1$ Usp(N) SYM with N flavors

- Introduce the orientifold projection in the $\mathbb{R}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orbifold so that the gauge group on the color D3 branes becomes $\mathrm{Usp}(N_c)$.



For $N_c = N_f$ this configuration leads to

$$W_{\text{exotic}} = \int d\mu \; \mathrm{e}^{-\frac{8\pi^2}{g^2} + i\mu^t\, Q\, \mu} = c \det(Q)$$

- The full non-perturbative superpotential is

$$W_{\text{non-pert}} = W_{\text{VTY-ADS}} + W_{\text{exotic}} = \frac{\Lambda^{2N+3}}{\det(Q)} + c \det(Q)$$

- The run-away behavior is stabilized!!

Conclusions and perspectives

- The D3/D(–1) system provides a very efficient string set-up to perform instanton calculations in gauge theories
- The instanton corrections to the prepotential in $\mathcal{N} = 2$ SYM theories and to the superpotential in $\mathcal{N} = 1$ SYM can be computed from (mixed) disk diagrams
- Non-trivial closed string backgrounds can be easily incorporated
- Generalizations of the gauge instantons to truly stringy configurations are possible and lead to very interesting effects

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- Further developments are under considerations:
  - role of stringy instantons in Dynamical Susy Breaking
  - role of stringy instantons in magnetized brane models

    [Billó, Di Vecchia, Frau, A.L., Marotta, Pesando in progress]
  - ...