Dispersive shock waves in interacting one-dimensional systems and edge states of FQHE

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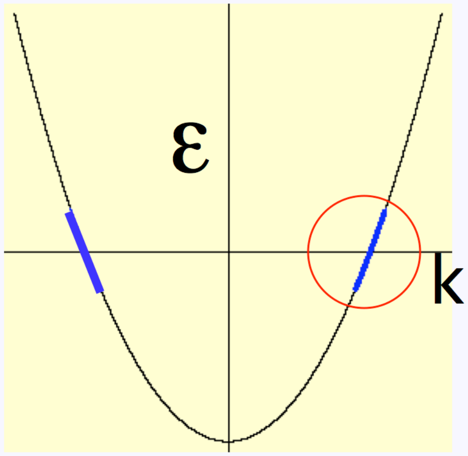
Collaboration: Paul B. Wiegmann, Eldad Bettelheim

Thanks: A. Guliani, M. Kulkarni, ...
ITP, EPFL for hospitality

GGI, Florence, October 13, 2008

Gradient catastrophe

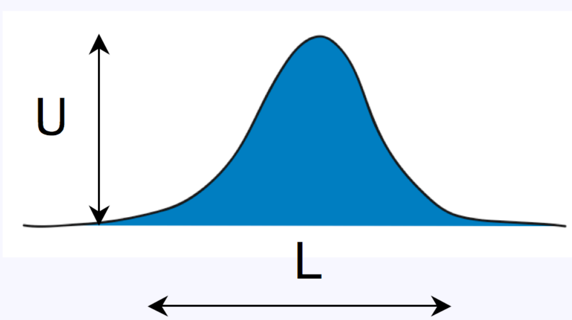


$$\psi_R \propto e^{ik_F x + i\phi_R}$$

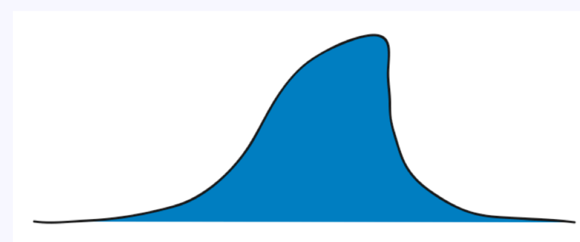
$$u \propto \partial_x \phi_R$$ current

$$u_t + c u_x = 0$$ chiral boson

$$u_t + c u_x + u u_x = 0$$ weak nonlinearity (curvature)



$$t_c \sim L/U$$



Gradient catastrophe in finite time!

Interactions -> Dispersion.

Outline

- Benjamin-Ono equation
- Calogero-Sutherland model
- Dispersive shock waves for CS and FQHE

Phys. Rev. Lett. 95, 076402 (2005)

J. Phys. A: Math.Theor. 40, F193 (2007)
Phys. Rev. Lett. 97, 246401 (2006)
Phys. Rev. Lett. 97, 246402 (2006)

Classical integrable equations

$$u_t + uu_x = 0$$

Riemann

$$u_t + u\,u_x + \epsilon\,u_{xxx} = 0$$

KdV

$$u_t + u\,u_x + \epsilon\,u_{xx} = 0$$

Burgers

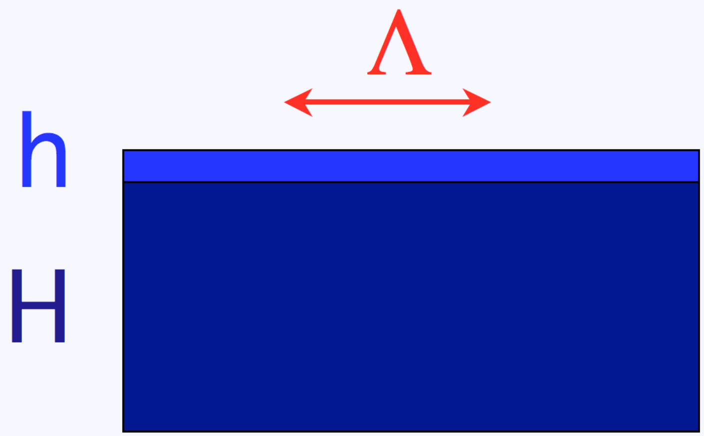
$$u_t + u\,u_x + \epsilon\,u_{xx}^H = 0$$

Benjamin-Ono

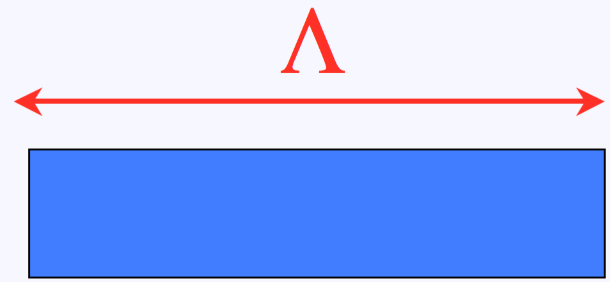
$$\omega = ck - \varepsilon k\,|\,k\,|$$

Classical Benjamin-Ono equation

(internal waves in deep stratified fluid)



$$h \ll \Lambda \ll H$$



$$H \ll \Lambda$$

Benjamin-Ono

$$u_t + u u_x + u^H_{xx} = 0$$

KdV

$$u_t + u u_x + u_{xxx} = 0$$

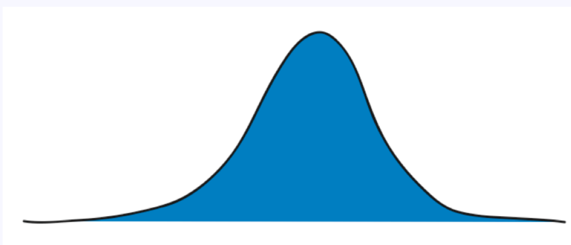
Benjamin 1967
Ono 1975

$$u^H(x) = \frac{1}{\pi} \int_{p.v.} dy \, \frac{u(y)}{y - x} = i(u^+ - u^-)$$

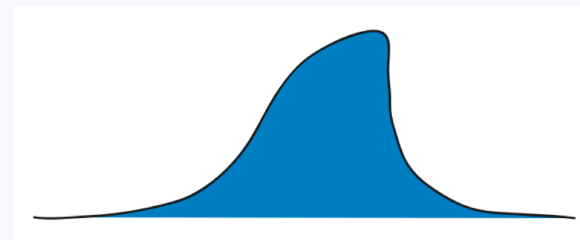
Nonlinearity vs. Dispersion

$$u_t + uu_x = 0$$

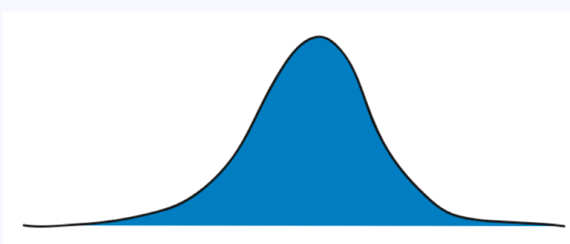
Gradient catastrophy



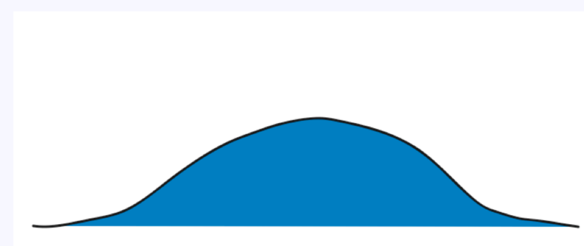
nonlinearity →



$$u_t + u^H_{xx} = 0$$

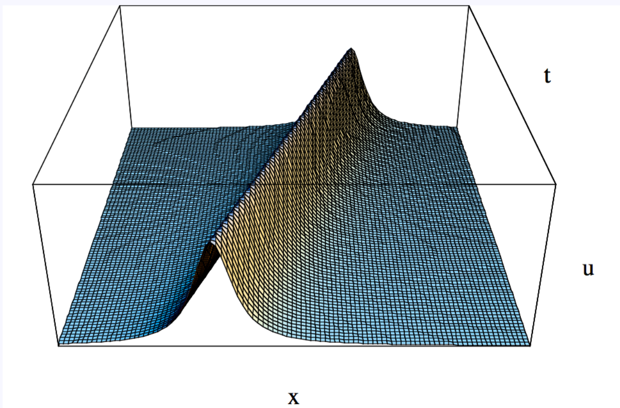


dispersion →



One- and two-soliton solutions of Benjamin-Ono equation

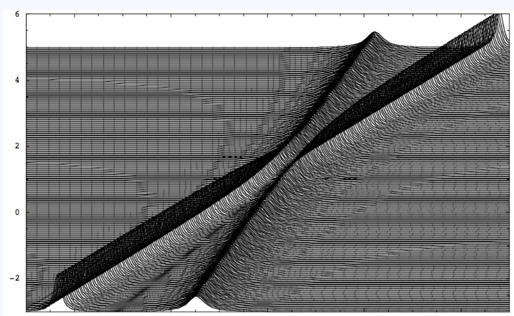
$$u_t + uu_x + u^H_{xx} = 0$$



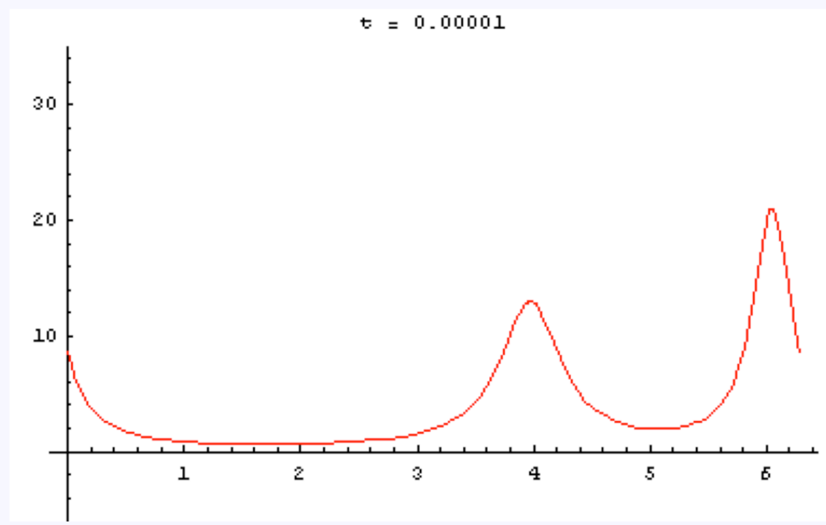
$$u(x,t) = \frac{2V}{V^2(x - x_0 - Vt)^2 + 1}$$

$$\int_{-\infty}^{+\infty} u(x,t)\,dx = 2\pi$$

area “quantization”



solitons do not interact!



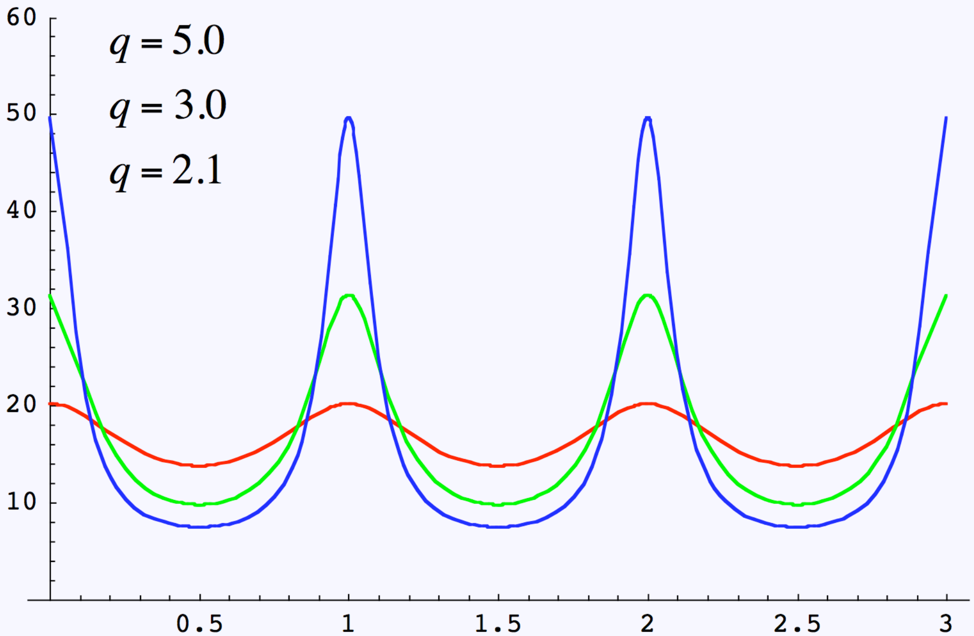
Periodic solutions of BO

$$u_t + uu_x + u_{xx}^H = 0$$

$$u = f(kx - \omega t)$$

$$u = \frac{(p-q)^2}{\frac{p+q}{2} - K - \sqrt{(p-K)(q-K)}\cos\theta} + 2K$$

$$\theta = kx - \omega t = (p-q)x + \frac{1}{2}(p^2 - q^2)t$$



$$p > q > K$$

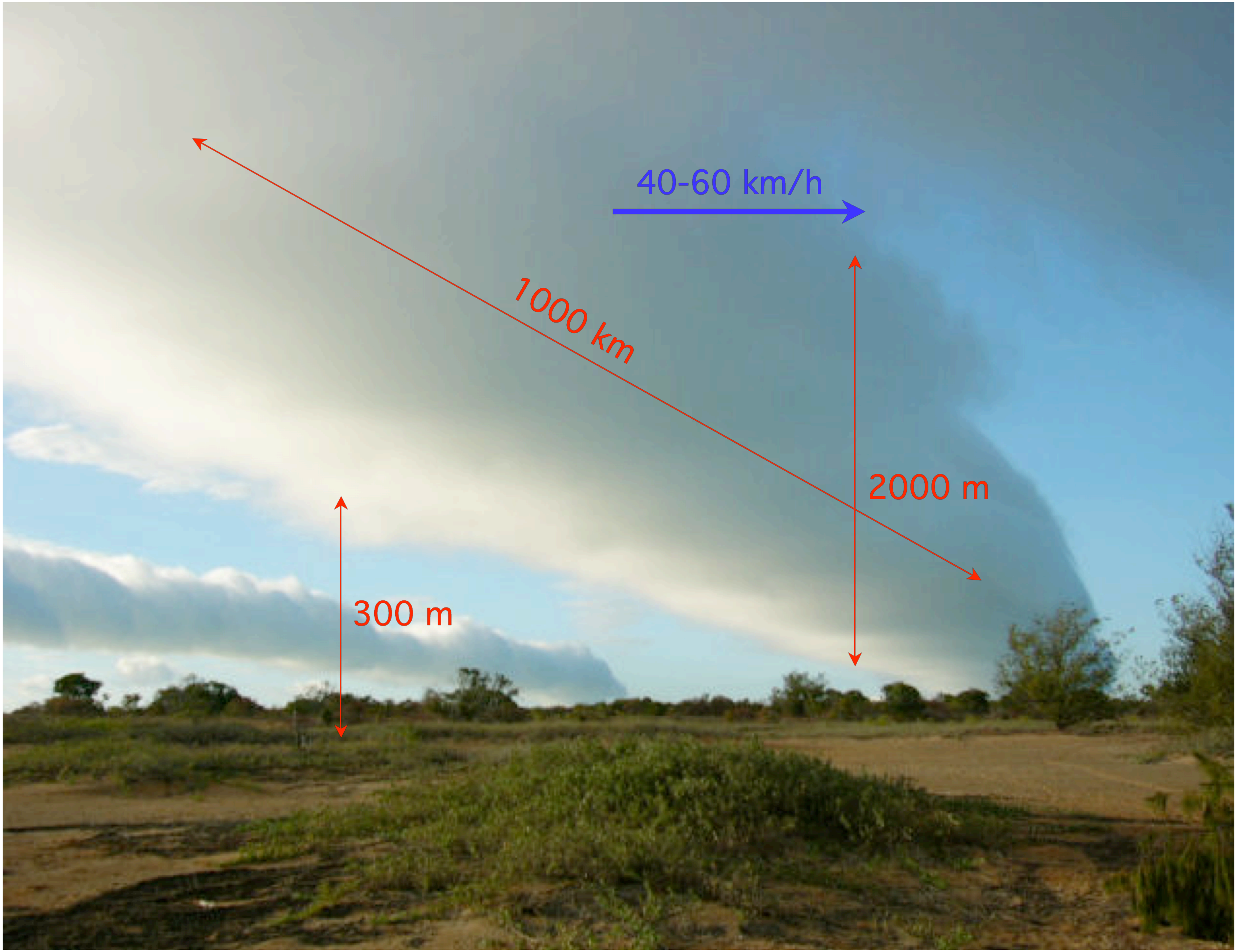
$$K = 2$$

$$p - q = 2\pi = k$$

Applications of Classical BO

- Internal waves in deep stratified fluids
- Atmosphere waves

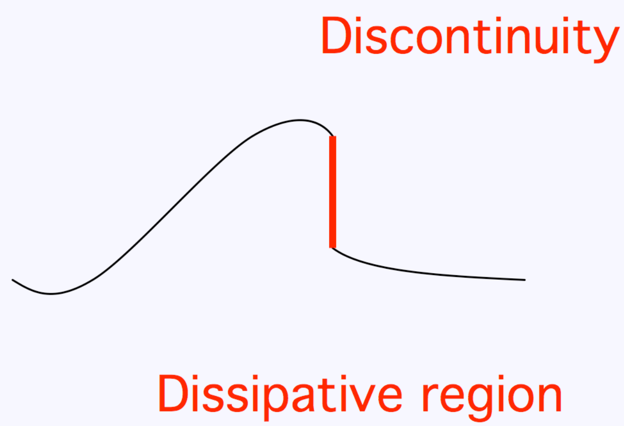
www.dropbears.com



40-60 km/h
1000 km
2000 m
300 m

Conventional shock waves

Solution of Riemann equation + discontinuity

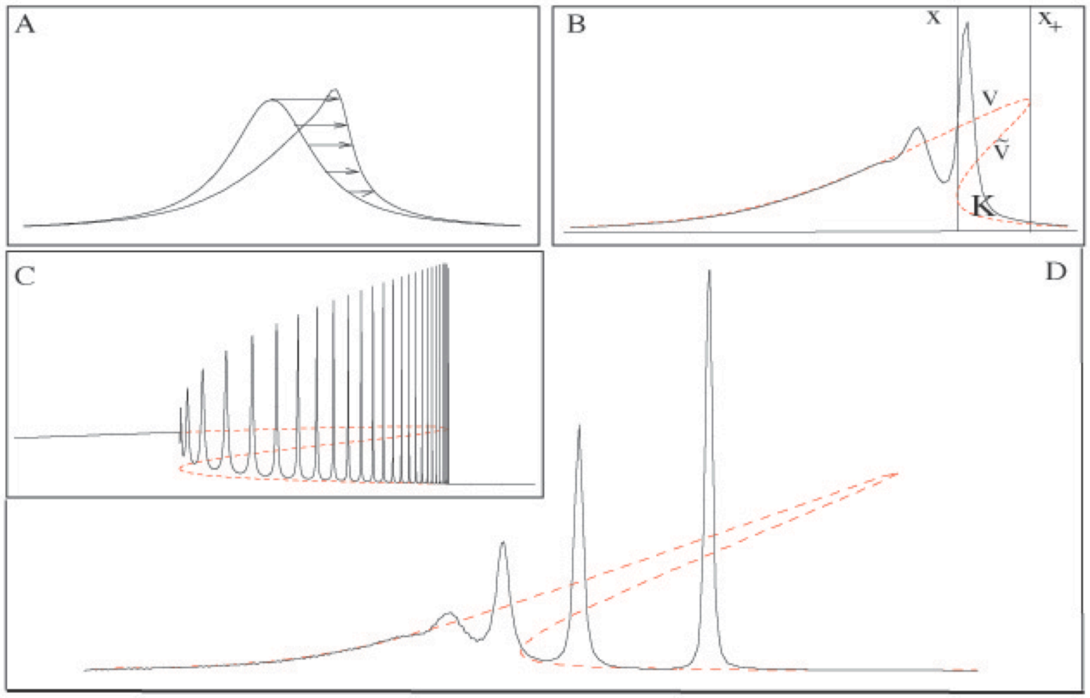


Dissipation → Dispersion

Shock waves → ? Dispersive shock waves

Dispersive shock waves

Solution of Riemann equation + Modulated periodic solution of BO



Bettelheim, AGA, Wiegmann,
Phys. Rev. Lett. 97, 246401 (2006)

Whitham theory

A. Gurevich, L. Pitaevsky (1973)

S. Dobrokhotov, I. Krichever (1991)

Y. Matsuno (1998)

The Whitham theory for BO

$$u_t + uu_x + u_{xx}^H = 0$$

$$u = \frac{(p-q)^2}{\frac{p+q}{2} - K - \sqrt{(p-K)(q-K)}\cos\theta} + 2K$$

p, q, K - moduli

$p > q > K$

$$p(x,t), q(x,t), K(x,t)$$

$$\theta_x = p - q$$

$$\theta_t = \frac{1}{2}(p^2 - q^2)$$

Whitham's equations

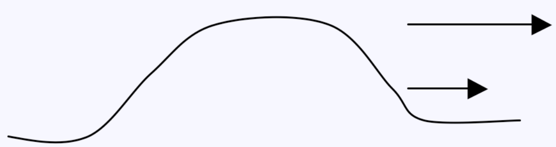
$$\boxed{\begin{aligned} p_t + pp_x &= 0 \\ q_t + qq_x &= 0 \\ K_t + KK_x &= 0 \end{aligned}}$$

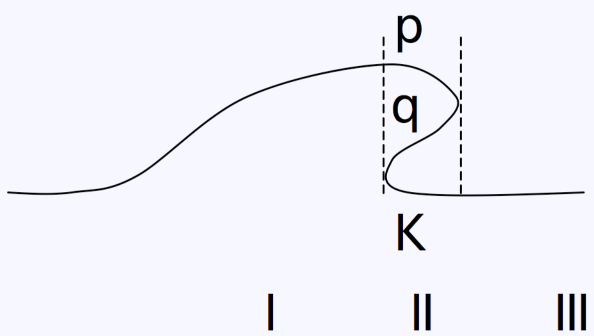
Gurevich-Pitaevskii approach

$$u_t + uu_x + u^H_{xx} = 0$$

$$u_t + uu_x = 0$$

$$u = u_0(x - ut)$$





$$u = \frac{(p-q)^2}{\frac{p+q}{2} - K - \sqrt{(p-K)(q-K)}\cos\theta} + 2K$$

$$\int \frac{dx}{2\pi} u >> 1$$

$$p_t + pp_x = 0$$

$$q_t + qq_x = 0$$

$$K_t + KK_x = 0$$

$$\theta_x = p - q$$

$$\theta_t = \frac{1}{2}(p^2 - q^2)$$

Calogero-Sutherland model

$$H = -\frac{1}{2}\sum_{i=1}^{N}\frac{\partial^2}{\partial x_i^2} + \sum_{i<j}\frac{\lambda(\lambda-1)}{(x_i - x_j)^2}$$

Calogero 1969
Sutherland 1971

$$\Psi_{ground} \sim \prod_{i<j}(x_i - x_j)^{\lambda}$$

λ=0 - free bosons
λ=1 - free fermions

$$k_F = \pi\rho_0$$

$$v_s = \lambda\pi\rho_0$$

$$\epsilon(\rho) = \frac{1}{6}(\lambda\pi\rho)^2$$

Exactly solvable
by Bethe Ansatz

Dispersion of quasiparticles

$$\varepsilon_p(k) = \lambda k_F\,|k|\left(1 + {|k|}\big/{2\lambda k_F}\right)$$

$$\varepsilon_h(k) = \lambda k_F\,|k|\left(1 + {|k|}\big/{2k_F}\right)$$

Hydrodynamics of CS model

$$H = \int dx \left(\frac{\rho v^2}{2} + \rho \varepsilon(\rho) \right)$$

$$[\rho(x), v(y)] = -i\delta'(x-y)$$

$$\varepsilon(\rho) = \frac{1}{6}(\pi\lambda\rho)^2 + \frac{1}{2}\pi\lambda(\lambda-1)\rho_x^H + \frac{(\lambda-1)^2}{8}(\partial_x \log\rho)^2$$

$$\sum_{j<k} \frac{\lambda(\lambda-1)}{(x_j - x_k)^2} \to \int dx \rho(x) \int dy \frac{\lambda(\lambda-1)\rho(y)}{(y-x)^2} \to \pi\lambda(\lambda-1)\int dx \rho\rho_x^H$$

$$\rho_t + (\rho v)_x = 0$$

$$v_t + \left(\frac{v^2}{2} + w[\rho] \right)_x = 0$$

Specific enthalpy $w[\rho] = \frac{\delta(\rho\varepsilon)}{\delta\rho}$

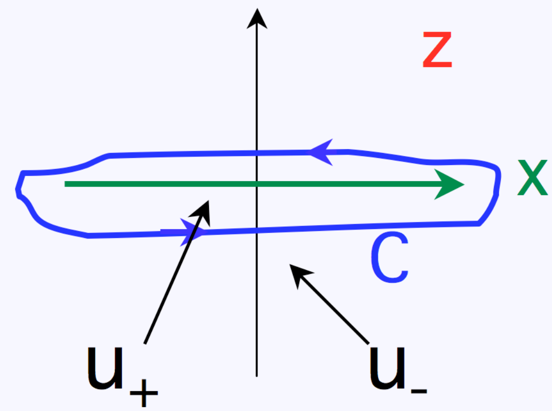
Free fermions $\lambda = 1$

Classical Calogero $\lambda - 1 \to \lambda$

I. Andric, V. Bardek, 1988

Quantum Benjamin-Ono equation

$$u(x \pm i0) = \frac{v}{\sqrt{\lambda}} \mp \sqrt{\lambda}\,\pi\rho + i\alpha_0\,\partial_x \log\rho$$



$$2\alpha_0 = \lambda^{1/2} - \lambda^{-1/2}$$

$\lambda = 1$ Free fermions

$$u(z) = u_+(z) + u_-(z)$$

$$H = \oint_C \frac{dz}{4\pi}\left[\frac{u^3}{3} + i\alpha_0 u \partial(u_+ - u_-)\right]$$

Quantum Benjamin-Ono equation

$$\partial_t u + \partial\left[\frac{u^2}{2} + i\alpha_0\,\partial\left(u_+ - u_-\right)\right] = 0$$

Quantum Benjamin-Ono equation

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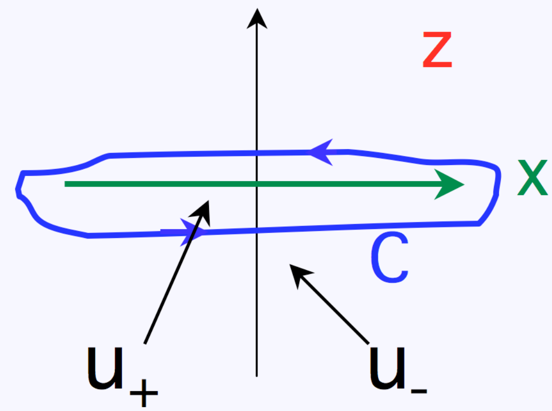


$$2\alpha_0 = \lambda^{1/2} - \lambda^{-1/2}$$

$\lambda = 1$ Free fermions

$$u(z) = u_+(z) + u_-(z)$$

Andric, Bardek

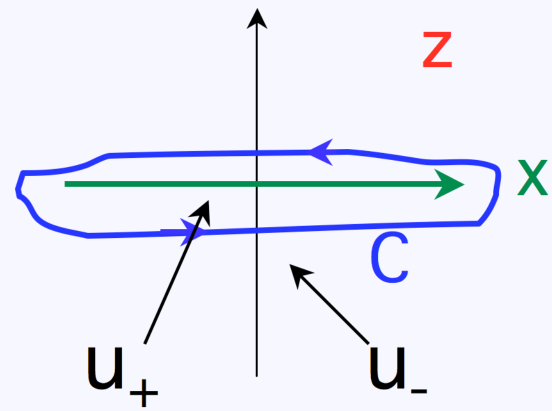
$$\rho_t + \partial_x(\rho v) = 0$$

$$v_t + \partial_x\left[\frac{1}{2}v^2 + w(\rho)\right] = 0$$

$$w = \frac{1}{2}(\pi\lambda\rho)^2 + \pi\lambda(\lambda-1)\rho_x^H - \frac{(\lambda-1)^2}{2}\frac{1}{\sqrt{\rho}}\partial_x^2\sqrt{\rho}$$

Quantum Benjamin-Ono equation

$$u(x \pm i0) = \frac{v}{\sqrt{\lambda}} \mp \sqrt{\lambda}\,\pi\rho + i\alpha_0\,\partial_x \log\rho$$



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Quantum Benjamin-Ono equation

$$\partial_t u + \partial\left[\frac{u^2}{2} + i\alpha_0\,\partial\left(u_+ - u_-\right)\right] = 0$$

Quantum Pole Ansatz

$$H = \oint_C \frac{dz}{4\pi} \left[\frac{u^3}{3} + i\alpha_0 u \partial (u_+ - u_-) \right]$$

$$\Downarrow$$

$$H = -\frac{1}{2} \sum_{i=1}^{N} \frac{\partial^2}{\partial x_i^2} + \sum_{i<j} \frac{\lambda(\lambda - 1)}{(x_i - x_j)^2}$$

$$u_-(z) = -i\sqrt{\lambda} \sum_{j=1}^{N} \frac{1}{z - x_j}$$

$$\sqrt{\lambda}\, u_+(x_j) = \hat{p}_j + i\lambda \sum_i{}' \frac{1}{x_i - x_j}$$

$$\hat{p}_j = -i \frac{\partial}{\partial x_j}$$

Chiral (right) sector

$$\rho_t + (\rho v)_x = 0$$

$$v_t + \left(\frac{v^2}{2} + w[\rho]\right)_x = 0$$

Chiral constraint

$$v = \pi\lambda\rho + \frac{\lambda - 1}{2}\partial_x(\log\rho)^H$$

$$\rho_t + \pi\lambda\partial_x\left[\rho^2 + \frac{\lambda - 1}{2\pi\lambda}\rho\partial_x(\log\rho)^H\right] = 0$$

$$\rho = \rho_0 + \delta\rho$$

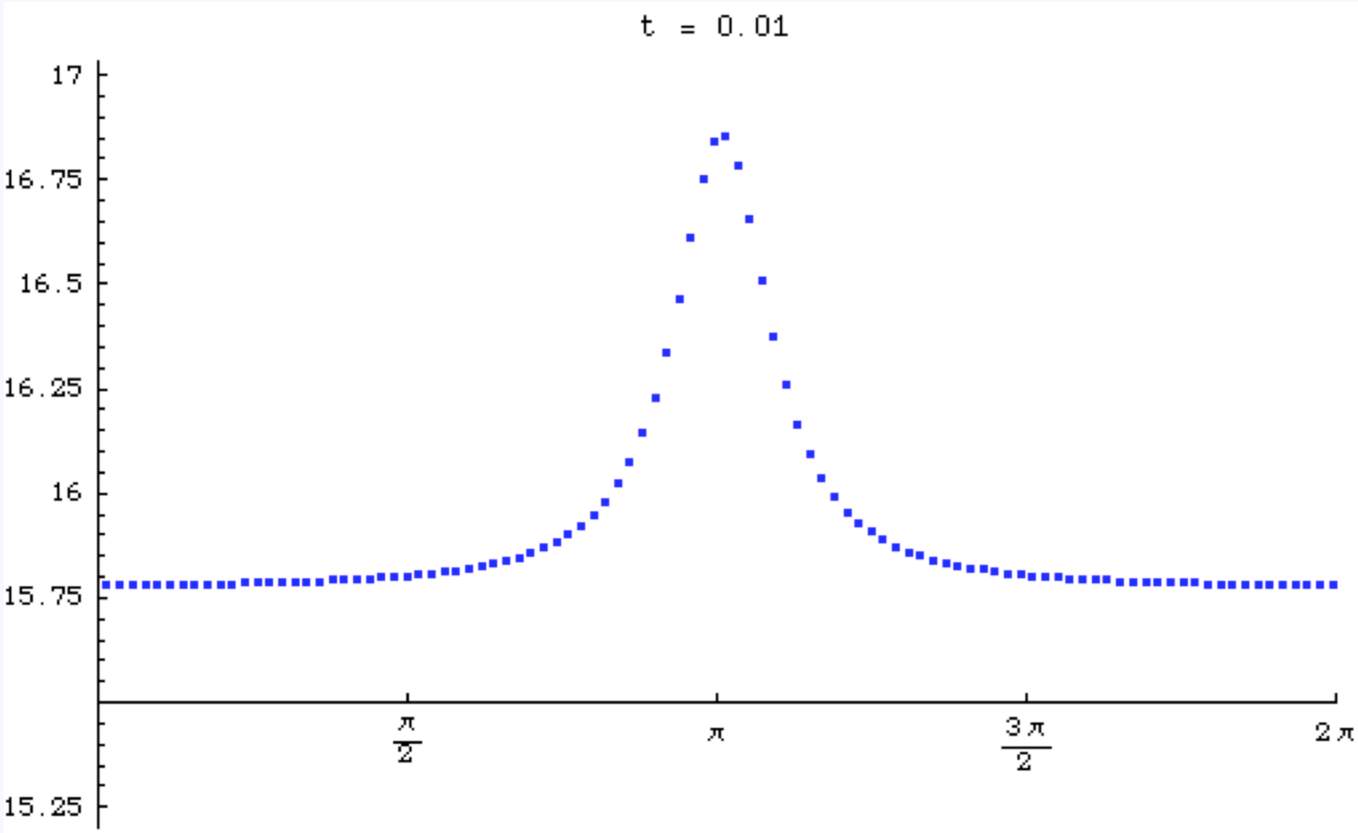
$$u = 2\pi\lambda\delta\rho$$

$$u_t + uu_x + \frac{\lambda - 1}{2}u_{xx}^H = 0$$

Collective motion in Sutherland model

$$\rho = \rho_0\left(1 + \frac{a}{(\pi\rho_0(x - Vt))^2 + a^2}\right)$$

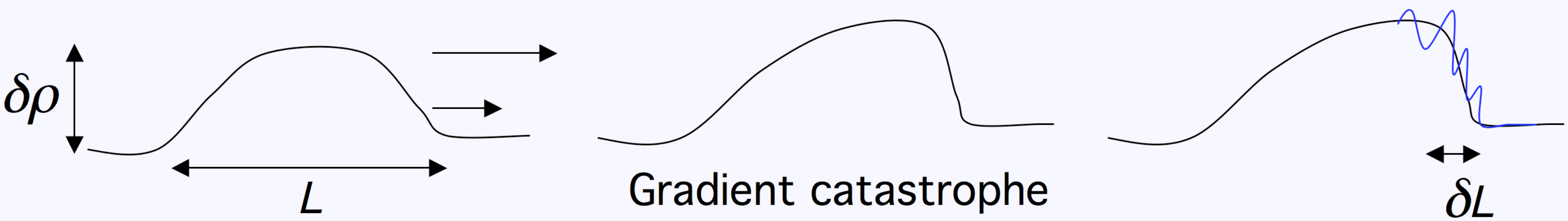
N=100



Shock waves in Sutherland model

$$u_t + uu_x + \frac{\lambda - 1}{2} u^H_{xx} = 0$$

$$\rho = \rho_0 + \delta\rho$$
$$u = 2\pi\lambda\delta\rho$$



Gradient catastrophe

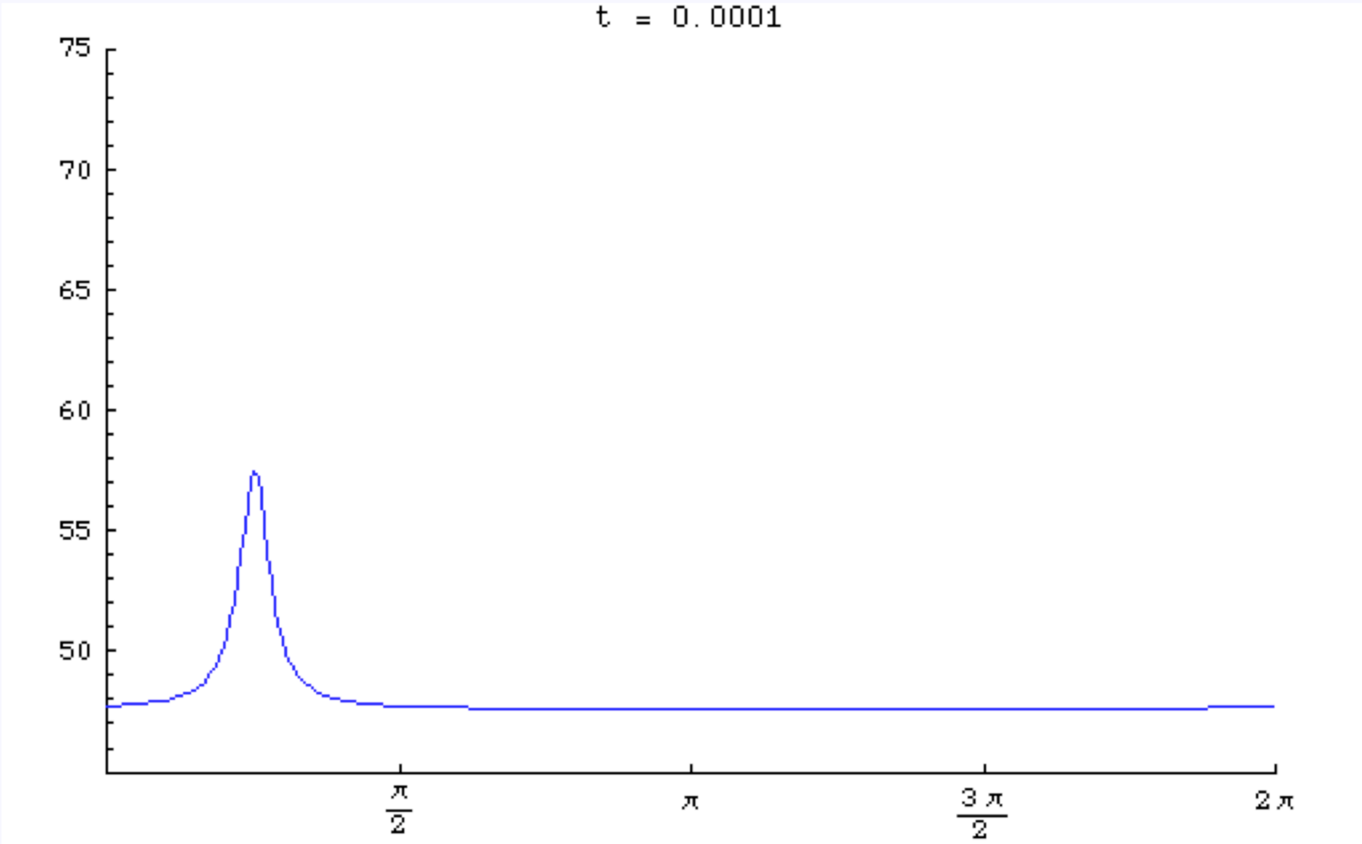
$$t_c \propto \frac{L}{\delta\rho}$$

$$\delta L \propto \frac{\lambda - 1}{\delta\rho}$$

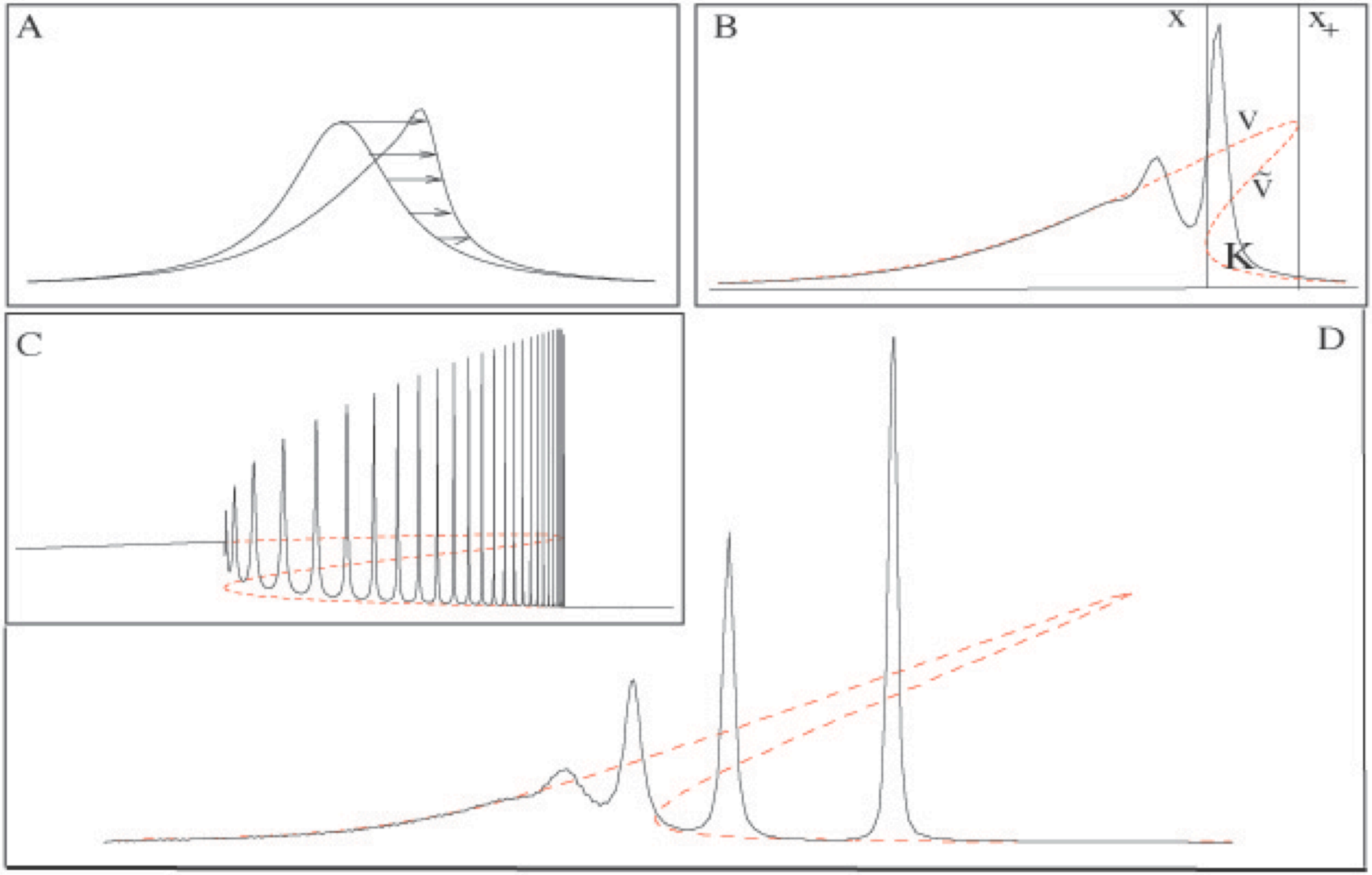
$$\rho_0 \delta L \propto (\lambda - 1)\frac{\rho_0}{\delta\rho} >> 1$$

Collective description is still valid for interacting system

Shock waves in Sutherland model

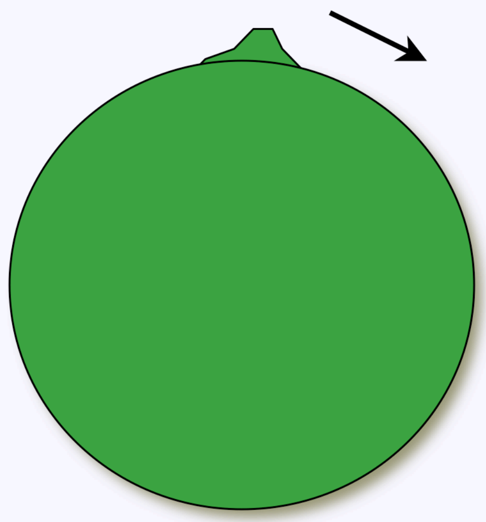


Dynamics of 300 particles on a circle. Initial profile: Lorentzian with area 3.



Bettelheim, AGA, Wiegmann, Phys. Rev. Lett. 97, 246401 (2006)

Edge states of FQHE



$$\eta_t + c\,\eta_x = 0$$

Right wave
Conserved form
Weak nonlinearity
Weak dispersion
Locality

→ KdV

Incompressible in the bulk - no locality!

$$\eta^H_{xx}$$

$$\eta_t + c\,\eta_x + \eta\,\eta_x + \cancel{\eta_{xxx}} = 0$$

BO equation for the edge of FQHE

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Edge of the Laughlin droplet

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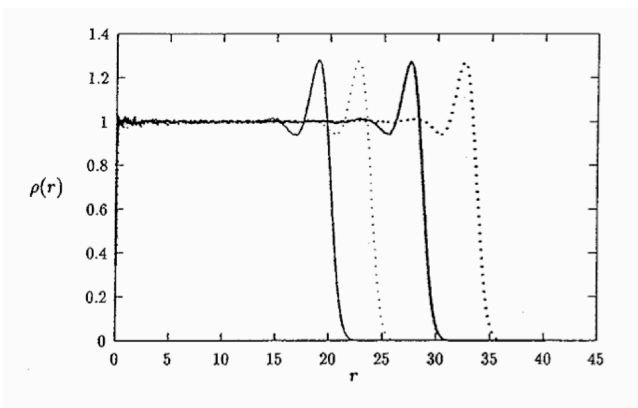
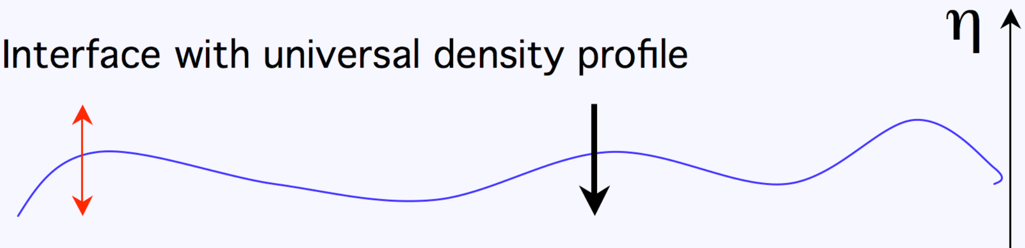


FIG. 3. Plot of $\rho(r)$ vs r for $m=3$ and $N=72$, 100, 144, and 196 electrons.

Interface with universal density profile



Incompressible fluid

Confining field

$$\frac{1}{c}\eta_t + \eta_x + \frac{\beta}{l_0}\eta\eta_x + \frac{\alpha}{2} l_0 \eta^H_{xx} = 0$$

soliton charge $= \dfrac{\alpha}{\beta}\dfrac{1}{m}$

m=3

$$t_c \propto \tau_F(\rho_0 L)\frac{\rho_0}{\delta\rho} \approx 1ns$$

Gradient catastrophe time

$$\alpha = \beta$$

Consequence of the universal density profile!

Conclusions

- Dispersion and nonlinearity in interacting systems -> solitons, dispersive shock waves etc.

- Collective field theory for Calogero-Sutherland -- Benjamin-Ono equation.

- Benjamin-Ono equation - solitons with "quantized" charge.

- Useful for nonlinear phenomena
  - Shock waves in electronic systems and systems of cold atoms,
  - Quantum Hall effect edge states,
  - Tunneling into coherent states,
  - Instantons (rare fluctuations),
  - Correlation functions,
  - Spin-Charge coupling in systems with dispersion,...