Low-dimensional QFTs and Applications

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$\mathcal{N} = 6$ Chern-Simons theory S-matrix and all-loop Bethe ansatz equations

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Based on joint work with Changrim Ahn

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Introduction

[Maldacena '97, ...]

AdS_5		CFT_4
type IIB string in $AdS_5 \times S^5$ with N units of 5-form flux		$\mathcal{N}=4\ SU(N)$ Yang-Mills in 3+1 dims
$g_s = \frac{\lambda}{4\pi N} \qquad T \equiv \frac{R^2}{2\pi\alpha'} = \frac{\sqrt{\lambda}}{2\pi}$		$\lambda \equiv g_{YM}^2 N$
$E(\lambda)$	$=$	$\Delta(\lambda)$
(energy of one-particle states)		(scaling dims of local, gauge-invariant, single-trace operators)

Problem: how to compute for all λ ?

Problem becomes tractable (integrable) in an interesting limit:

AdS_5

free string

$(g_s \to 0)$

large angular momentum J

classical sigma model is integrable

[Bena, Polchinski & Roiban '03, KMMZ '05, ...]

CFT_4

't Hooft planar limit

$(g_{YM} \to 0, N \to \infty, \lambda = \text{fixed})$

long operators

dilatation operator is an integrable quantum spin chain Hamiltonian

[Minahan & Zarembo '02, BS '03, BKS '03, ...]

Systematic approach: (not historical!)

factorized S-matrix [Staudacher '04, Beisert '05, ...]

↓

Bethe-Yang equation for particles on a circle

↓

all-loop asymptotic Bethe ansatz equations (BAEs)
& anomalous dimensions

[Beisert & Staudacher '05]

May also

- solve wrapping problem (short operators) [Bajnok & Janik '08]
- lead to quantization of string theory

Exciting recent development:
similar story may hold in one dimension less

Exciting recent development:
similar story may hold in one dimension less

[Aharony, Bergman, Jafferis & Maldacena '08]

AdS_4	CFT_3
type IIA string in $AdS_4 \times CP^3$	planar $\mathcal{N}=6$ Chern-Simons in 2+1 dims
classical sigma model is integrable	2-loop dilatation operator in scalar sector is an integrable quantum spin chain Hamiltonian
[Arutyunov & Frolov '08, Stefanski '08, ...]	[Minahan & Zarembo '08]

all-loop BAEs proposed [Gromov & Vieira '08]

Goals

- Find factorized S-matrix
- Derive (via Bethe-Yang equations) all-loop BAEs

Outline

- $\mathcal{N} = 6$ Chern-Simons theory
- 2-loop BAEs
- Symmetries & elementary excitations
- S-matrix
- Bethe-Yang eqs & all-loop BAES
- Discussion

$\mathcal{N} = 6$ Chern-Simons theory

[ABJM '08]

$$S = \frac{k}{4\pi}\int d^3x \,\mathrm{tr}\Big[\epsilon^{\mu\nu\lambda}\big(A_\mu\partial_\nu A_\lambda + \frac{2}{3}A_\mu A_\nu A_\lambda$$

gauge

$$-\hat{A}_\mu\partial_\nu\hat{A}_\lambda - \frac{2}{3}\hat{A}_\mu\hat{A}_\nu\hat{A}_\lambda\big)$$

$+D_\mu Y_A^\dagger D^\mu Y^A + Y^6$ terms

scalars

$+$ fermions $\Big]$

k integer

gauge symmetry: $SU(N) \times SU(N)$

$A_\mu \qquad \hat{A}_\mu$

R - symmetry: $SU(4) \supset SU(2) \times SU(2)$

$$A_a \qquad B_{\dot{a}}$$

$$(N, \bar{N}; 2) \quad (\bar{N}, N; 2)$$

$$Y^A = (A_1, A_2, B_{\dot{1}}^\dagger, B_{\dot{2}}^\dagger) \qquad (N, \bar{N}; 4)$$

$$Y_A^\dagger = (A_1^\dagger, A_2^\dagger, B_{\dot{1}}, B_{\dot{2}}) \qquad (\bar{N}, N; \bar{4})$$

Scale invariant: $\Delta_0(A_\mu) = \Delta_0(\hat{A}_\mu) = 1, \quad \Delta_0(Y) = 1/2$ 3 dims!

Has $\mathcal{N} = 6$ superconformal symmetry for $k > 2$

$$Osp(2,2|6) \supset SO(2,3) \times SO(6)$$

$$\uparrow$$

also isometry group of $AdS_4 \times CP^3$

Planar limit: $N, k \to \infty, \quad \lambda \equiv N/k = \text{fixed}$

2-loop BAEs

[Minahan & Zarembo '08]

- Scalar sector

Local, gauge-invariant, single-trace operators:

$$\mathrm{tr}\, Y^{A_1}(x) Y^{\dagger}_{B_1}(x) Y^{A_2}(x) Y^{\dagger}_{B_2}(x) \cdots Y^{A_L}(x) Y^{\dagger}_{B_L}(x)$$

$\longleftrightarrow$

states of closed $SU(4)$ quantum spin chain with $2L$ sites

$$|A_1 B_1 A_2 B_2 \cdots A_L B_L\rangle$$

alternating $4\ \bar{4}\ \cdots$

Explicit computation $\Rightarrow$

2-loop dilatation operator (mixing matrix) is an integrable quantum spin chain Hamiltonian

Eigenvalues (anomalous dimensions) given by Bethe ansatz:

$$\gamma = \lambda^2 \left(\sum_{j=1}^{M_u} \frac{1}{u_j^2 + 1/4} + \sum_{j=1}^{M_v} \frac{1}{v_j^2 + 1/4} \right)$$

$\{u_j, v_j, r_j\}$ are solutions of BAEs:

$$e_1(u_j)^L = \prod_{\substack{k=1\\k\neq j}}^{M_u} e_2(u_j - u_k) \prod_{k=1}^{M_r} e_{-1}(u_j - r_k)$$

$$1 = \prod_{k=1}^{M_r} e_2(r_j - r_k) \prod_{k=1}^{M_u} e_{-1}(r_j - u_k) \prod_{k=1}^{M_v} e_{-1}(r_j - v_k)$$

$$e_1(v_j)^L = \prod_{\substack{k=1\\k\neq j}}^{M_u} e_2(v_j - v_k) \prod_{k=1}^{M_r} e_{-1}(v_j - r_k)$$

where

$$e_n(u) \equiv \frac{u + in/2}{u - in/2}$$

For integrable spin chain with simple Lie group symmetry,

$$e_{V_l}(u_j^{(l)})^L = \prod_{\substack{k=1 \\ k\neq j}}^{M^{(l)}} e_{A_{l,l}}(u_j^{(l)} - u_k^{(l)}) \prod_{l'\neq l} \prod_{k=1}^{M^{(l')}} e_{A_{l,l'}}(u_j^{(l)} - u_k^{(l')})$$

[Ogievetsky & Wiegmann ‘86]

$A_{l,l'}$ Cartan matrix

V_l Dynkin labels of representation

$SU(4):$

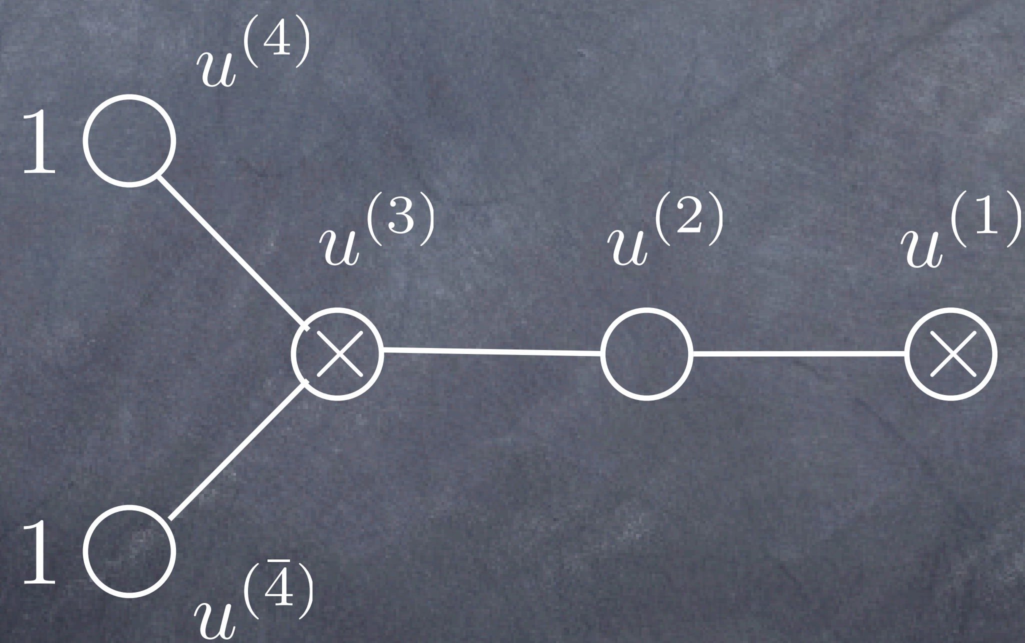
$$A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$$

Reproduces above set of BAEs

- Full theory: "guess" from group theory

$$Osp(2,2|6)$$

Dynkin diagram not unique. Two fermionic roots:



$\Rightarrow$ BAEs

$$
\begin{aligned}
e_1(u_j^{(4)})^L &= \prod_{\substack{k=1\\k\neq j}}^{M^{(4)}} e_2(u_j^{(4)}-u_k^{(4)}) \prod_{k=1}^{M^{(3)}} e_{-1}(u_j^{(4)}-u_k^{(3)}) \\
e_1(u_j^{(\bar{4})})^L &= \prod_{\substack{k=1\\k\neq j}}^{M^{(\bar{4})}} e_2(u_j^{(\bar{4})}-u_k^{(\bar{4})}) \prod_{k=1}^{M^{(3)}} e_{-1}(u_j^{(\bar{4})}-u_k^{(3)}) \\
1 &= \prod_{k=1}^{M^{(4)}} e_{-1}(u_j^{(3)}-u_k^{(4)}) \prod_{k=1}^{M^{(\bar{4})}} e_{-1}(u_j^{(3)}-u_k^{(\bar{4})}) \prod_{k=1}^{M^{(2)}} e_1(u_j^{(3)}-u_k^{(2)}) \\
1 &= \prod_{\substack{k=1\\k\neq j}}^{M^{(2)}} e_{-2}(u_j^{(2)}-u_k^{(2)}) \prod_{k=1}^{M^{(3)}} e_1(u_j^{(2)}-u_k^{(3)}) \prod_{k=1}^{M^{(1)}} e_1(u_j^{(2)}-u_k^{(1)}) \\
1 &= \prod_{k=1}^{M^{(2)}} e_1(u_j^{(1)}-u_k^{(2)})
\end{aligned}
$$

Symmetries & elementary excitations

Global symmetry is partially broken by vacuum!

Analogy: XXX ferromagnet

Vacuum: $|\uparrow\cdots\uparrow\rangle$

Breaks $SU(2)\rightarrow U(1)$

Elementary excitations:

$$\sum_n e^{ipn}|\uparrow\cdots\uparrow\overset{n}{\downarrow}\uparrow\cdots\uparrow\rangle$$ "magnons"

$\mathcal{N}=6$ Chern-Simons:

[Nishioka & Takayanagi '08,
Gaiotto, Giombi & Yin '08,
Grignani, Harmark & Orselli '08]

Vacuum: $\mathrm{tr}\,(A_1 B_{\dot{1}} A_1 B_{\dot{1}} \cdots A_1 B_{\dot{1}})$

$$\gamma = 0$$

(True to all orders in λ , since is chiral primary operator.)

$$\Delta = L = J \quad \Rightarrow \quad \Delta - J = 0$$

$$J(A_1) = J(B_{\dot{1}}) \quad = \quad 1/2$$
$$J(A_2) = J(B_{\dot{2}}) \quad = \quad 0$$

Breaks $SU(4) \to SU(2)$ (rotates $A_2, B_{\dot{2}}^\dagger$)

$Osp(2,2|6) \to SU(2|2)$

Elementary excitations:

$$\sum_n e^{ipn} |A_1 B_{\dot 1} \cdots \overset{n}{\chi} B_{\dot 1} \cdots A_1 B_{\dot 1}\rangle$$

"A - particles"

$$\chi \in \{A_2, B^\dagger_{\dot 2}, \text{fermions}\}$$

$$\sum_n e^{ipn} |A_1 B_{\dot 1} \cdots A_1 \overset{n}{\chi} \cdots A_1 B_{\dot 1}\rangle$$

"B - particles"

$$\chi \in \{A^\dagger_2, B_{\dot 2}, \text{fermions}\}$$

$$\Delta_0 - J = 1$$

Fundamental reps $(2|2)$ of $SU(2|2)$

Elementary excitations:

ZF operators:

$$\sum_n e^{ipn} |A_1 B_{\dot 1} \cdots \overset{n}{\chi} B_{\dot 1} \cdots A_1 B_{\dot 1}\rangle$$

$$\chi \in \{A_2, B_{\dot 2}^\dagger, \text{fermions}\}$$

"A - particles" $A_i^\dagger(p)$

$$\sum_n e^{ipn} |A_1 B_{\dot 1} \cdots A_1 \overset{n}{\chi} \cdots A_1 B_{\dot 1}\rangle$$

$$\chi \in \{A_2^\dagger, B_{\dot 2}, \text{fermions}\}$$

"B - particles" $B_i^\dagger(p)$

$i = 1, \ldots, 4$

$$\Delta_0 - J = 1$$

Acting on $|0\rangle$ create asymptotic particle states of momentum p

Fundamental reps $(2|2)$ of $SU(2|2)$

S-matrix

- A-A scattering:

$$A_i^\dagger(p_1)\, A_j^\dagger(p_2) = S^{AA}{}_{i\,j}^{\,i'j'}(p_1,p_2)\, A_{j'}^\dagger(p_2)\, A_{i'}^\dagger(p_1)$$

Associativity $\Rightarrow$ Yang-Baxter equation

$$S_{12}^{AA}(p_1,p_2)\, S_{13}^{AA}(p_1,p_3)\, S_{23}^{AA}(p_2,p_3) = S_{23}^{AA}(p_2,p_3)\, S_{13}^{AA}(p_1,p_3)\, S_{12}^{AA}(p_1,p_2)$$

$$S_{12}^{AA} = S^{AA} \otimes \mathbb{I},\ S_{23}^{AA} = \mathbb{I} \otimes S^{AA},\ S_{13}^{AA} = \mathcal{P}_{12}\, S_{23}^{AA}\, \mathcal{P}_{12}$$

$SU(2|2)$ symmetry determines S^{AA} up to scalar factor

[Beisert '05, AFZ '06]

$$S^{AA}(p_1,p_2) = S_0(p_1,p_2)\, \widehat{S}(p_1,p_2)$$

Besides YBE, $\widehat{S}(p_1, p_2)$ satisfies unitarity:

$$\widehat{S}_{12}(p_1, p_2)\, \widehat{S}_{21}(p_2, p_1) = \mathbb{I}$$

and crossing: [Janik '06]

$$\widehat{S}^{t_2}_{12}(p_1, p_2)\, C_2\, \widehat{S}_{12}(p_1, \bar{p}_2)\, C_2^{-1} = \widehat{S}^{t_1}_{12}(p_1, p_2)\, C_1\, \widehat{S}_{12}(\bar{p}_1, p_2)\, C_1^{-1} = f(p_1, p_2)\, \mathbb{I}$$

$$f(p_1, p_2) = \frac{\left(\frac{1}{x_1^+} - x_2^-\right)(x_1^+ - x_2^+)}{\left(\frac{1}{x_1^-} - x_2^-\right)(x_1^- - x_2^+)}$$

$$x^{\pm}(\bar{p}) = \frac{1}{x^{\pm}(p)}$$

- B-B scattering:

$$B_i^\dagger(p_1)\,B_j^\dagger(p_2) = S^{BB\;\;i'j'}_{\;\;\;\;i\,j}(p_1,p_2)\,B_{j'}^\dagger(p_2)\,B_{i'}^\dagger(p_1)$$

- A-B and B-A scattering:

$$A_i^\dagger(p_1)\,B_j^\dagger(p_2) = S^{AB\;\;i'j'}_{\;\;\;\;i\,j}(p_1,p_2)\,B_{j'}^\dagger(p_2)\,A_{i'}^\dagger(p_1)$$

N.B. reflectionless

Symmetry suggests

$$\begin{aligned} S^{BB}(p_1,p_2) &= S^{AA}(p_1,p_2) = S_0(p_1,p_2)\,\widehat{S}(p_1,p_2) \\ S^{AB}(p_1,p_2) &= S^{BA}(p_1,p_2) = \tilde{S}_0(p_1,p_2)\,\widehat{S}(p_1,p_2) \end{aligned}$$

"matrix part" fixed - remains only to determine scalar factors $S_0,\ \tilde{S}_0$

Assume unitarity:

$$S_{12}^{AA}(p_1,p_2)\,S_{21}^{AA}(p_2,p_1) = S_{12}^{AB}(p_1,p_2)\,S_{21}^{AB}(p_2,p_1) = \mathbb{I}$$

$\Rightarrow$

$$S_0(p_1,p_2)\,S_0(p_2,p_1) = 1\,, \qquad \tilde{S}_0(p_1,p_2)\,\tilde{S}_0(p_2,p_1) = 1$$

Assume crossing:

$$S_{12}^{AA\ t_2}(p_1,p_2)\,C_2\,S_{12}^{AB}(p_1,\bar{p}_2)\,C_2^{-1} = S_{12}^{AA\ t_1}(p_1,p_2)\,C_1\,S_{12}^{AB}(\bar{p}_1,p_2)\,C_1^{-1} = \mathbb{I}$$

$\Rightarrow$

$$S_0(p_1,p_2)\,\tilde{S}_0(p_1,\bar{p}_2) = S_0(p_1,p_2)\,\tilde{S}_0(\bar{p}_1,p_2) = \frac{1}{f(p_1,p_2)}$$

Satisfied by

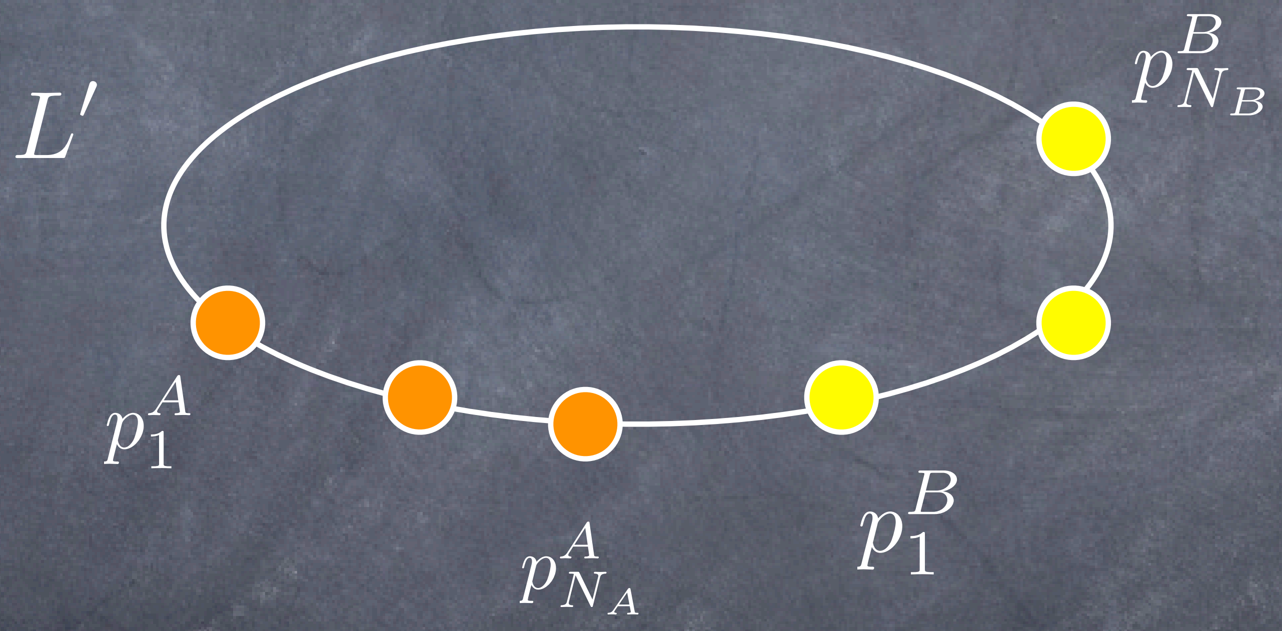
$$\begin{aligned} S_0(p_1\,,p_2) &= \frac{1-\frac{1}{x_1^+x_2^-}}{1-\frac{1}{x_1^-x_2^+}}\sigma(p_1\,,p_2) \\ \tilde{S}_0(p_1\,,p_2) &= \frac{x_1^- - x_2^+}{x_1^+ - x_2^-}\sigma(p_1\,,p_2) \end{aligned}$$

$\sigma(p_1\,,p_2)$ dressing factor

[Beisert, Eden & Staudacher ‘06]

Bethe-Yang eqs and all-loop BAEs

Consider a set of A - particles $\{p_1^A, \ldots, p_{N_A}^A\}$ and a set of B - particles $\{p_1^B, \ldots, p_{N_B}^B\}$ that are widely separated on a ring of length L'



Periodic boundary conditions on wavefunction

$\Rightarrow$ quantization conditions for momenta

A - particles:

$$e^{-ip_k^A L'} = \Lambda(\lambda = p_k^A, \{p_i^A, p_i^B\})$$

$\Lambda(\lambda, \{p_i^A, p_i^B\})$ are eigenvalues of transfer matrix

$$t(\lambda, \{p_i^A, p_i^B\}) = \mathrm{str}_a\, S^{AA}_{a\,1}(\lambda, p_1^A) \cdots S^{AA}_{a\,N_A}(\lambda, p^A_{N_A})\, S^{AB}_{a\,N_A+1}(\lambda, p_1^B) \cdots S^{AB}_{a\,N_A+N_B}(\lambda, p^B_{N_B})$$

$$= (\text{ scalar factors })(\text{ “matrix part” })$$

Eigenvalues of ``matrix part”: [Martins & Melo ‘07]

$$\widehat{\Lambda}(\lambda, \{p_i^A, p_i^B\}; \{\lambda_j, \mu_j\}) = \prod_{i=1}^{N_A} \left[\frac{x^+(\lambda) - x^-(p_i^A)}{x^-(\lambda) - x^+(p_i^A)} \frac{\eta(p_i^A)}{\eta(\lambda)} \right] \prod_{i=1}^{N_B} \left[\frac{x^+(\lambda) - x^-(p_i^B)}{x^-(\lambda) - x^+(p_i^B)} \frac{\eta(p_i^B)}{\eta(\lambda)} \right]$$

$$\times \prod_{j=1}^{m_1} \left[\eta(\lambda) \frac{x^-(\lambda) - x^+(\lambda_j)}{x^+(\lambda) - x^+(\lambda_j)} \right] + \text{ terms which vanish if } \lambda = p_k^A$$

$$\eta(\lambda) \equiv e^{i\lambda/2}$$

where $\{\lambda_j, \mu_j\}$ are solutions of BAEs

$$e^{i(P^A+P^B)/2}\prod_{i=1}^{N_A}\frac{x^+(\lambda_j)-x^-(p_i^A)}{x^+(\lambda_j)-x^+(p_i^A)}\prod_{i=1}^{N_B}\frac{x^+(\lambda_j)-x^-(p_i^B)}{x^+(\lambda_j)-x^+(p_i^B)}=\prod_{l=1}^{m_2}\frac{x^+(\lambda_j)+\frac{1}{x^+(\lambda_j)}-\tilde{\mu}_l+\frac{i}{2g}}{x^+(\lambda_j)+\frac{1}{x^+(\lambda_j)}-\tilde{\mu}_l-\frac{i}{2g}}$$

$$\prod_{j=1}^{m_1}\frac{\tilde{\mu}_l-x^+(\lambda_j)-\frac{1}{x^+(\lambda_j)}+\frac{i}{2g}}{\tilde{\mu}_l-x^+(\lambda_j)-\frac{1}{x^+(\lambda_j)}-\frac{i}{2g}}=\prod_{\substack{k=1\\k\neq l}}^{m_2}\frac{\tilde{\mu}_l-\tilde{\mu}_k+\frac{i}{g}}{\tilde{\mu}_l-\tilde{\mu}_k-\frac{i}{g}} \qquad (1)$$

Taking into account also scalar factors, Bethe-Yang eqs. for A-particles become:

$$e^{ip_k^A L}=\prod_{\substack{i=1\\i\neq k}}^{N_A}\left[\frac{x^+(p_k^A)-x^-(p_i^A)}{x^-(p_k^A)-x^+(p_i^A)}\right]\left[\frac{1-\frac{1}{x^+(p_k^A)x^-(p_i^A)}}{1-\frac{1}{x^-(p_k^A)x^+(p_i^A)}}\sigma(p_k^A,p_i^A)\right]$$

$$\times\prod_{i=1}^{N_B}\sigma(p_k^A,p_i^B)\prod_{j=1}^{m_1}\left[\frac{x^-(p_k^A)-x^+(\lambda_j)}{x^+(p_k^A)-x^+(\lambda_j)}\right] \qquad (2)$$

Bethe-Yang eqs. for B-particles:

$$e^{ip_k^B L} = \prod_{\substack{i=1\\i\neq k}}^{N_B} \left[\frac{x^+(p_k^B) - x^-(p_i^B)}{x^-(p_k^B) - x^+(p_i^B)}\right] \left[\frac{1 - \frac{1}{x^+(p_k^B)x^-(p_i^B)}}{1 - \frac{1}{x^-(p_k^B)x^+(p_i^B)}} \sigma(p_k^B\,, p_i^B)\right]$$

$$\times \prod_{i=1}^{N_A} \sigma(p_k^B\,, p_i^A) \prod_{j=1}^{m_1} \left[\frac{x^-(p_k^B) - x^+(\lambda_j)}{x^+(p_k^B) - x^+(\lambda_j)}\right] \qquad (3)$$

Can map (1)-(3) to all-loop BAEs:

$$\begin{aligned}
x^\pm(p_k^A) &= x^\pm_{4,k}\,, \quad k = 1, \ldots, K_4 \equiv N_A\,, \\
x^\pm(p_k^B) &= x^\pm_{\bar{4},k}\,, \quad k = 1, \ldots, K_{\bar{4}} \equiv N_B\,, \\
x^+(\lambda_j) &= \frac{1}{x_{1,j}}\,, \quad j = 1, \ldots, K_1\,, \\
x^+(\lambda_{K_1+j}) &= x_{3,j}\,, \quad j = 1, \ldots, K_3\,, \quad K_1 + K_3 \equiv m_1\,, \\
\tilde{\mu}_j &= \frac{u_{2,j}}{g}\,, \quad j = 1, \ldots, K_2 \equiv m_2
\end{aligned}$$

$$e^{ip_{4,k}L} = \prod_{\substack{j=1\\j\neq k}}^{K_4} \frac{u_{4,k}-u_{4,j}+i}{u_{4,k}-u_{4,j}-i}\,\sigma(u_{4,k},u_{4,j}) \prod_{j=1}^{K_{\bar{4}}} \sigma(u_{4,k},u_{\bar{4},j})$$

$$\times \prod_{j=1}^{K_1} \frac{1-\frac{1}{x^-_{4,k}x_{1,j}}}{1-\frac{1}{x^+_{4,k}x_{1,j}}} \prod_{j=1}^{K_3} \frac{x^-_{4,k}-x_{3,j}}{x^+_{4,k}-x_{3,j}}$$

$$e^{ip_{\bar{4},k}L} = \prod_{\substack{j=1\\j\neq k}}^{K_{\bar{4}}} \frac{u_{\bar{4},k}-u_{\bar{4},j}+i}{u_{\bar{4},k}-u_{\bar{4},j}-i}\,\sigma(u_{\bar{4},k},u_{\bar{4},j}) \prod_{j=1}^{K_4} \sigma(u_{\bar{4},k},u_{4,j})$$

$$\times \prod_{j=1}^{K_1} \frac{1-\frac{1}{x^-_{\bar{4},k}x_{1,j}}}{1-\frac{1}{x^+_{\bar{4},k}x_{1,j}}} \prod_{j=1}^{K_3} \frac{x^-_{\bar{4},k}-x_{3,j}}{x^+_{\bar{4},k}-x_{3,j}}$$

$$\prod_{i=1}^{K_4} \frac{1-\frac{1}{x_{1,j}x^-_{4,i}}}{1-\frac{1}{x_{1,j}x^+_{4,i}}} \prod_{i=1}^{K_{\bar{4}}} \frac{1-\frac{1}{x_{1,j}x^-_{\bar{4},i}}}{1-\frac{1}{x_{1,j}x^+_{\bar{4},i}}} = \prod_{l=1}^{K_2} \frac{u_{1,j}-u_{2,l}+\frac{i}{2}}{u_{1,j}-u_{2,l}-\frac{i}{2}}$$

$$\prod_{i=1}^{K_4} \frac{x_{3,j}-x^-_{4,i}}{x_{3,j}-x^+_{4,i}} \prod_{i=1}^{K_{\bar{4}}} \frac{1-\frac{1}{x_{3,j}x^-_{\bar{4},i}}}{1-\frac{1}{x_{3,j}x^+_{\bar{4},i}}} = \prod_{l=1}^{K_2} \frac{u_{3,j}-u_{2,l}+\frac{i}{2}}{u_{3,j}-u_{2,l}-\frac{i}{2}}$$

$$\prod_{\substack{j=1\\j\neq l}}^{K_2} \frac{u_{2,l}-u_{2,j}+i}{u_{2,l}-u_{2,j}-i} = \prod_{j=1}^{K_1} \frac{u_{2,l}-u_{1,j}+\frac{i}{2}}{u_{2,l}-u_{1,j}-\frac{i}{2}} \prod_{j=1}^{K_3} \frac{u_{2,l}-u_{3,j}+\frac{i}{2}}{u_{2,l}-u_{3,j}-\frac{i}{2}}$$

Same as

[Gromov & Vieira ’08]

!

Weak-coupling limit:

$$g \to 0\,, \quad x \to \frac{u}{g}\,, \quad x^{\pm} \to \frac{1}{g}(u \pm i/2)$$

Recover 2-loop BAEs ✓

Good check on S-matrix

Discussion

- Alternative S-matrix with A - B reflection?

Tensor product structure $SU(2) \otimes SU(2|2)$

$$A^\dagger_{a\,i}(p_1)\, A^\dagger_{b\,j}(p_2) = S_0(p_1,p_2)\, S_{a\,b}^{a'\,b'}(p_1,p_2)\, \widehat{S}_{i\,j}^{i'\,j'}(p_1,p_2)\, A^\dagger_{b'\,j'}(p_2)\, A^\dagger_{a'\,i'}(p_1)$$

Does not lead to correct BAEs.

- Many assumptions - more checks needed!

Controversy on all-loop BAEs

[McLoughlin & Roiban; Alday et al; Krishnan; Gromov & Mikhaylov; McLoughlin et al '08]

Finite-size corrections to dispersion relation of giant magnons

[Bombardelli & Fioravanti '08, Lukowski & Sax '08]