Black rings Firenze 23-24 April 2007

Context: hi-d gravity & black holes

Motivation:

- String / M-Theory
- LXD - TeV gravity scenarios: BH's at colliders
  ↓
  D>4: we know little of their physics!
- AdS/CFT: 4D QFT physics from hi-d gravity
- D as a tunable parameter to study gravity & bh's (analogous to "unphysical" Lagrangians in QFT, or N in SU(N))
  what properties are dep/indep of D?
- Natural theoretical interest / Mathematical physics: math of Lorentzian manifolds

Plan

Day 1: - vacuum (neutral): - hi-d bhs
- Ring coords
- Neutral black ring

Day 2 - charges & dipoles

Day 3: susy & micro

References: - review w/ H. Reall hep-th/0608012 (more details in hep-th/0608076 w/ Elvang + Virmani)

- for an alternative view on the susy case Bena + Warner hep-th/0701216

- 4D black holes : virtually <u>all</u> is known (the simplest macroscopic objects in the universe)

Schw (1915) $\quad ds^2 = -\left(1-\frac{\mu}{r}\right)dt^2 + \frac{dr^2}{1-\frac{\mu}{r}} + r^2 d\Omega_2 \qquad \mu = 2GM$

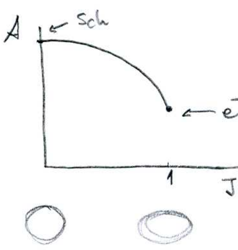
Kerr (1963) $\quad ds^2 = -dt^2 + \frac{\mu r}{\Sigma}\left(dt + a\sin^2\theta\, d\phi\right)^2 + \Sigma\left(\frac{dr^2}{\Delta} + d\theta^2\right) + (r^2+a^2)\sin^2\theta\, d\phi^2$

$\mu = 2GM$

$J = Ma$

$\Sigma = r^2 + a^2\cos^2\theta$

$\Delta = r^2 + a^2 - \mu$



extremal $J = M^2$ regular horizon

fix $M=1$

Horizon at $\Delta = 0$

$r = r_+ = M + \sqrt{M^2 - a^2} \qquad (G=1)$

$= M\left(1 + \sqrt{1 - \frac{J^2}{M^4}}\right)$

- $D > 4$

Tangherlini (1963) : $\quad ds^2 = -\left(1 - \frac{\mu}{r^{D-3}}\right)dt^2 + \frac{dr^2}{1-\frac{\mu}{r^{D-3}}} + r^2 d\Omega_{D-3} \qquad \mu \propto GM$

Easy!
- Stable
- Unique

Myers + Perry (1986) : several rotation planes are possible. $\; t, \underbrace{x_1, x_2}_{r_1, \phi_1}, \underbrace{x_3, x_4}_{r_2, \phi_2}, x_5 \cdots$

(Cartan Torus of $SO(D-1)$ $\; U(1)^{\left[\frac{D-1}{2}\right]}$)

Found solutions in arbitrary D w/ rotation in all planes!

w/ only one rotation

$$ds^2 = -dt^2 + \frac{\mu r^{5-D}}{\Sigma}\left(dt + a\sin^2\theta\, d\phi\right)^2 + \Sigma\left(\frac{dr^2}{\Delta} + d\theta^2\right) + (r^2+a^2)\sin^2\theta\, d\phi^2$$
$$+ r^2\cos^2\theta\, d\Omega_{D-4}$$

$\mu \propto M$

$J \propto Ma$

$\Delta = r^2 + a^2 - \frac{\mu}{r^{D-5}}$

Apparently nothing new, but there are important differences: competition between grav pot and centrifugal

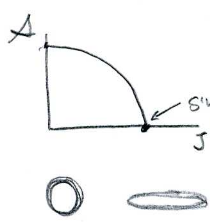
$$\frac{\Delta}{r^2} - 1 = -\underbrace{\frac{\mu}{r^{D-3}}}_{\text{D-dep} \sim \text{grav pot'l}} + \underbrace{\frac{a^2}{r^2}}_{\text{D-indep: confined to plane, centrifugal pot'l}}$$

Horizons : $\Delta = 0 \qquad r_+^2 + a^2 - \frac{\mu}{r_+^{D-5}} = 0$

$D=4$: real solution for $a \le \mu/2$

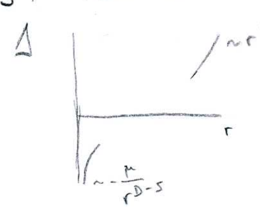
$D=5$: " " " $a^2 \le \mu$

$a^2 = \mu \quad r_+ = 0$ singular extremal limit



$A = \Omega_{D-2}\, \mu\, r_+$

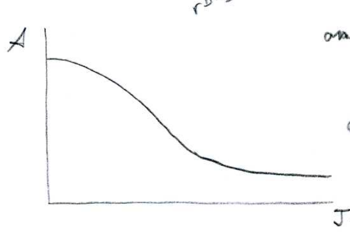
$D > 5$: real solution for all a!



BH's exist for arbitrarily large spin

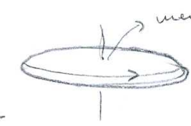
<u>ultra-spinning regime</u>

only in $D \ge 6$ there are black membranes



membrane-like

unstable



$g_{\phi\phi}(r=r_+) = \frac{(\mu\, r_+^{5-D})^2 \sin^2\theta}{\Sigma_+}$

$g_{\theta\theta}(r=r_+) = \Sigma_+$

$\sqrt{g_{\phi\phi}\, g_{\theta\theta}}\big|_{r=r_+} = \mu r_+^{5-D}\sin\theta$

$\sqrt{g_{ang}}\big|_{r=r_+} \doteq \omega_{D-2}\, \mu r_+$

$k = \partial_t + \Omega_H \partial_\phi$

$\Omega_H = -\frac{g_{t\phi}}{g_{\phi\phi}}\Big|_{r_+} = \frac{a}{r_+^2 + a^2}$

$|k|^2 = 0$ at $r = r_+$

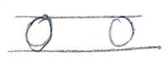
Also: $D > 5$ no inner horizon
singularity spacelike
$r=0$ is singular since $|\partial_t|^2 \to \infty$
Unlike 4D where singu is at $r=0, \theta = \pi/2$

- Are MP bhs all There is? (in AF vacuum gravity)

  No : $\exists$ black rings in all $D \geq 5$

  First example : $D=5$ black rings
  - 5D is actually fairly well-understood by now and we may be close To having a complete catalogue of bh's.
    Sector w/ $U(1)^3$ symmetry is completely integrable
    time + 2 rotations
  - $D>5$ is less well known but progress is being made

- Heuristics of black rings :



black string = $\mathrm{Schw}_{D-1} \times \mathbb{R}$

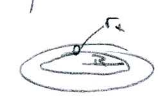
cut and glue

spin up : is it possible To balance forces?

Forces are : grav : D-dependent $\sim \frac{1}{R^{D-3}}$ (potential)

Tension, centrifugal } D-independent

Grav is negligible for Thin rings $R \gg r_+$



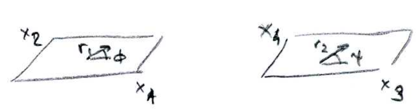
Can spin balance Tension? Yes, $\forall D \geq 5$

- $D=5$ black ring :
- Ring coordinates

  Flat space coords

  $x^1 = r_1 \cos\phi$ $x_3 = r_2 \cos\psi$

  $x^2 = r_2 \sin\phi$ $x_4 = r_2 \sin\psi$



There'll be J_ψ, J_ϕ . Set $J_\phi = 0$

$$ds_4^2 = dr_1^2 + r_1^2 d\phi^2 + dr_2^2 + r_2^2 d\psi^2$$

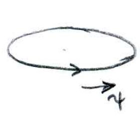
Find adapted coords for a ring-like source : equipotential surfaces

Not scalar source (possible but less convenient)

but 2-form potential $B_{\mu\nu}$

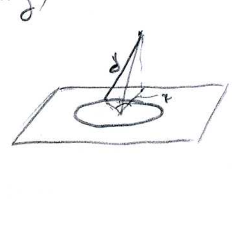
$H = dB$

(eg Test field of a fundamental string)



source at $r_1 = 0$ $r_2 = R$ $0 \leq \psi \leq 2\pi$

$$d^2 = r_1^2 + r_2^2 + R^2 - 2Rr_2 \cos\psi$$



Black ring: like previous, but w/ non-zero curvature encoded in functions

(Get this by double Wick from KK C-metric $\rightarrow$)
has been derived systematically using integrability, solitonic methods)

$$ds^2 = -\frac{F(y)}{F(x)}\left(dt - CR\,\frac{1+y}{F(y)}\,d\psi\right)^2$$
$$+\frac{R^2}{(x-y)^2}F(x)\left[-\frac{G(y)}{F(y)}d\psi^2 - \frac{dy^2}{G(y)} + \frac{dx^2}{G(x)} + \frac{G(x)}{F(x)}d\phi^2\right]$$

$$F(\xi) = 1 + \lambda\xi \qquad C = \sqrt{\lambda(\lambda-\nu)\frac{1+\lambda}{1-\lambda}} \qquad 0 < \nu \le \lambda < 1$$
$$G(\xi) = (1-\xi^2)(1+\nu\xi)$$

$\lambda = \nu = 0$: flat . Horizon at $y = -1/\nu$ (where $G(y)=0$)
$\hookrightarrow$ y = const has topology $S^1\times S^2$: black ring

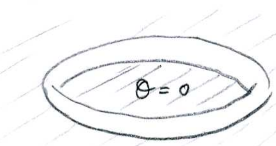
Examine S^2 . make $x = \cos\theta$

$$\frac{dx^2}{(1-x^2)(1+\nu x)} + \frac{1+\nu x}{1+\lambda x}(1-x^2)\,d\phi^2 = \frac{1}{1+\nu\cos\theta}\left(d\theta^2 + \frac{(1+\nu\cos\theta)^2}{1+\lambda\cos\theta}\sin^2\theta\,d\phi^2\right)$$

Near $\theta \simeq 0$ $\quad d\theta^2 + \frac{(1+\nu)^2}{1+\lambda}\theta^2 d\phi^2 \Rightarrow \Delta\phi = 2\pi\frac{\sqrt{1+\lambda}}{1+\nu}$, $\phi \sim \phi + \Delta\phi$

Near $\theta = \pi$ $\quad d\theta^2 + \frac{(1-\nu)^2}{1-\lambda}(\pi-\theta)^2 d\phi^2 \Rightarrow \Delta\phi = 2\pi\frac{\sqrt{1-\lambda}}{1-\nu}$

$\Rightarrow \lambda = \frac{2\nu}{1+\nu^2}$ balance condition



If we impose regularity only at $\theta = \pi$

conical defect "membrane"
Extnl. Force to balance ring
No cones $\Leftrightarrow$ equilibrium

Parameters: R, ν $(\lambda(\nu))$
$\downarrow$ scale (size of S^1)
$\downarrow$ "Thickness" $\sim \frac{R_{S_2}}{R}$

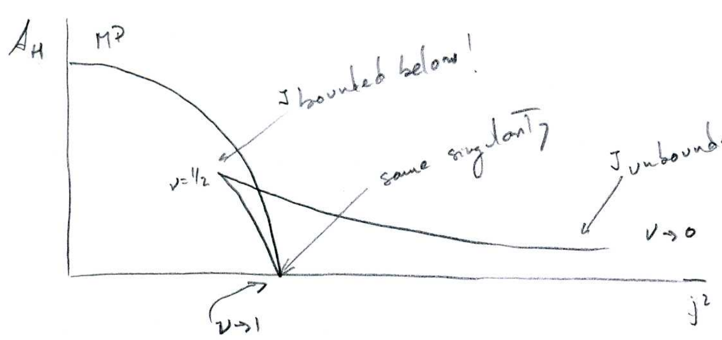
Physically: M, J, R balance fixes one of them in terms of the other two
eg given M, J, then R is fixed

One can measure for this solution M, J (going to asymptotic ∞ [$x \to y \to -1$, similar to change in flat space] and comparing to linearized soln: $g_{tt} \simeq -1 + \frac{GM}{r^2} + \dots \quad g_{t\phi} \simeq \frac{J}{r^3} + \dots$)

Fix $M=1$, then only variable is J. Can plot $A_H(J)$

$$j^2 = \frac{27\pi}{32G}\frac{J^2}{M^3} \qquad \left(\text{or } M = \left(\frac{27\pi}{32G}\right)^{1/3}\right)$$

$$a_H = \frac{3}{16}\sqrt{\frac{3}{\pi}}\frac{A_H}{(GM)^{3/2}}$$



MP
$$a_H = 2\sqrt{2(1-j^2)}$$

Ring
$$a_H = 2\sqrt{\nu(1-\nu)}$$
$$j^2 = \frac{(1+\nu)^2}{8\nu} \qquad 0 < \nu < 1$$

$$\partial_\mu \left(\sqrt{g}\, H^{\mu\nu\rho} \right) = 0$$

$$B_{t\psi} = \frac{R}{2\pi} \int_0^{2\pi} d\psi \, \frac{r_2 \cos\psi}{r_1^2 + r_2^2 + R^2 - 2Rr_2\cos\psi}$$

$$= -\frac{1}{2}\left(1 - \frac{R^2 + r_1^2 + r_2^2}{\Sigma}\right) \qquad \Sigma = \sqrt{(r_1^2 + r_2^2 + R^2)^2 - 4R^2 r_2^2}$$

Construct now gradient surfaces orthogonal to this: electric-magnetic dual potential

In 5D, A_μ is dual to $B_{\mu\nu}$ $\qquad B_{t\psi} \xrightarrow{\text{dual}} A_\phi$: monopole field

$$\partial_\mu\left(\sqrt{-g}\, F^{\mu\nu}\right) = 0$$

$$A_\phi = -\frac{1}{2}\left(1 + \frac{R^2 - r_1^2 - r_2^2}{\Sigma}\right)$$

Now set $B_{t\psi} = -\frac{1}{2}(1+y)$: These define new coords (x, y) in terms of r_1, r_2

$$A_\phi = -\frac{1}{2}(1+x)$$

$$r_1 = R\frac{\sqrt{1-x^2}}{x-y} \qquad r_2 = R\frac{\sqrt{y^2-1}}{x-y}$$

Note
- $-\infty \le y \le -1 \qquad -1 \le x \le 1$
- $y = -\infty$ is ring source $\begin{cases} r_1 = 0 \\ r_2 = R \end{cases}$
- Infinity $\begin{matrix} r_1 \to \infty \\ r_2 \to \infty \end{matrix} \qquad x \to y \to -1$

$$ds^2 = \frac{R^2}{(x-y)^2}\left[(y^2-1)\, d\psi^2 + \frac{dy^2}{y^2-1} + \frac{dx^2}{1-x^2} + (1-x^2)\, d\phi^2 \right]$$

note $x \leftrightarrow y$ symmetry $\to$ elec-mag duality

Can redefine (but still same foliation) $\qquad y = -\frac{R}{r} \qquad x = \cos\theta$

$$0 \le r \le R \qquad 0 \le \theta \le \pi$$

$$ds^2 = \frac{1}{\left(1 + \frac{r\cos\theta}{R}\right)^2}\left[\underbrace{\left(1 - \frac{r^2}{R^2}\right) R^2 d\psi^2}_{S^1} + \frac{dr^2}{1 - \frac{r^2}{R^2}} + \underbrace{r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)}_{S^2} \right]$$

: convenient for $r \ll R$

$ds^2 \simeq R^2 d\psi^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$

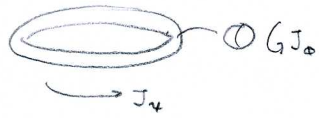
but not for $r \sim R$

$r = \text{const}$ ($y = \text{const}$) are $S^1 \times S^2$: ring-like

Picture of x, y coords

Extensions:

- Two spins J_ψ, J_ϕ



Pomeransky + Senkov

- AdS not yet
- $D > 5$: $S^1 \times S^p$ yes, in progress

  Other Topologies : unknown
- Charges, Susy : Tried early but failed

Charged and susy rings & stringy rings I

Charges and dipoles

$\int R - F^2$

Reissner-Nordström has electric charge Q (electrovac)

Kerr-Newman has " " and rotation $\to$ magnetic dipole, but not independent parameter.

In 5D : Einstein-Maxwell : RN (easy)

"Kerr-Newman"-like : only recently numerically . very hard

Black ring w/ dipole (and no net charge!)

Einstein-Maxwell-Chern-Simons (w/ precise coupling) : BMPV^{susy} and non-susy extension

$\downarrow$ bosonic sector of minimal 5D sugra

Dipole ring (CS term irrelevant)

Susy and non-susy rings

Independent dipoles imply continuous instead of discrete non-uniqueness

N=1 D=5 sugra
8 supercharges
Spin(1,4) = Sp(1,1) = USp(2,2)
irreducible spinor becomes a quaternion doublet
Multiply a solution to the Killing spinor eq by a constant quaternion $\Rightarrow$ general solution has 4 real dof's

We'll be interested in the solution to 5D sugras, and their 11D uplifts

so

$$I = \frac{1}{16\pi G}\int R*1 - \frac{1}{2}F\wedge *F - \frac{1}{6}F\wedge F\wedge A \qquad \text{11D sugra}$$

Reduce on T^6 (could also do it in CY_3)

$$ds_{11}^2 = ds_5^2 + X^1(dz_1^2 + dz_2^2) + X^2(dx_3^2 + dx_4^2) + X^3(dx_5^2 + dx_6^2)$$

$$A = A^1\wedge dz_1\wedge dz_2 + A^2\wedge dz_3\wedge dz_4 + A^3\wedge dz_5\wedge dz_6$$

Assume $X^1X^2X^3 = 1$ (T^6 has constant volume) Volume is in a hypermultiplet and decouples

$$I_5 = \frac{1}{16\pi G_5}\int R*1 - G_{ij}\, dX^i\wedge * dX^j - G_{ij}F^i\wedge *F^j - \frac{1}{6}C_{ijk}F^i\wedge F^j\wedge A^k \qquad F^i = dA^i$$

$$G_{ij} = \frac{1}{2}\begin{pmatrix} (X^1)^{-2} & & \\ & (X^2)^{-2} & \\ & & (X^3)^{-2}\end{pmatrix} \qquad C_{ijk} = \begin{cases}1 & \text{for permutations of } (123)\\ 0 & \text{otherwise}\end{cases}$$

$\Rightarrow$ Minimal 5D sugra + 2 vector supermultiplets

If we set $A^1 = A^2 = A^3$, $X^1 = X^2 = X^3 = \text{const}$ we get minimal 5D sugra

$$\int R*1 - 2F\wedge *F - \frac{8}{3\sqrt{3}}F\wedge F\wedge A \qquad A = \frac{\sqrt{3}}{2}A^1$$

In CY_3 we get N=1 sugra + more vectors (+ hypers which decouple). Things generalize.

(Often take minimal sugra for simplicity of illustration)

Charges: $Q = \frac{1}{16\pi G_5}\int_{S^3} *F$ or $Q_i = \frac{1}{16\pi G_5}\int_{S^3}(X^i)^{-2} * F^i$

This corresponds to having $A_t^i \sim \frac{1}{r^2}$: in 11D this is a membrane charge eg Q_1 is membrane along 012

BPS: $M \geq |Q_1| + |Q_2| + |Q_3|$

Minimal: $Q \neq 0$ membranes

012		Q_1
0	34	Q_2
0	56	Q_3

But we can also have magnetic charges giving dipoles

Saw $B_{t\psi}$ $\xrightarrow{\text{dual}}$ A_ϕ monopole charge

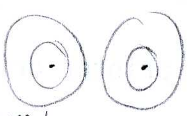
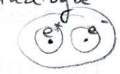
S^2 links ring once

$$q_i = \frac{1}{2\pi}\int_{S^2} F^i$$

$A_\phi = -\frac{1}{2}(1+x) = -\frac{1}{2}(1+\cos\theta)$ gives $q=1$ $\quad x \in [-1,1]$

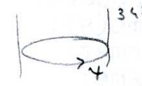
It's a dipole since  have opposite charges

Total magnetic charge is zero : an S^2  gives zero charge

Field is dipole-like  Analogue 

Falls-off faster than monopole

In 11D these correspond to M5 branes $\quad$ Tubular brane



eg q^1 $\quad A^1_\phi \sim \frac{q^1}{2}(1+x)$ is M5 $\quad$ 0 $\quad$ 3 4 5 6 ψ

q^2 $\quad$ 0 1 2 5 6 ψ

q^3 $\quad$ 0 1 2 3 4 ψ

In minimal sugra we'll have the three of them.

- Note: There's a black ring w/ only M5 dipoles, no M2 charges: Dipole black ring

$\Rightarrow$ continuous non-uniqueness: q is not conserved since ψ is contractible

Conserved charges are M, J but need q to specify solution

These are non-susy, since $M > \sum|Q_i| = 0$

- Will focus on susy rings. For simplicity will do minimal sugra

Solving Killing spinor eqs gives

$$ds^2 = -f^2(dt+\omega)^2 + f^{-1} h_{mn}\, dx^m dx^n$$

h_{mn} : hyper-Kähler (will take flat)

f : scalar $f(x_m)$ $\quad$ t-indep $\quad$ $V = \frac{\partial}{\partial t}$ Timelike Killing constructed from Killing spinor

ω : 1-form $\omega_m dx^m$

$$F = \frac{\sqrt{3}}{2}\Big[\underbrace{d\big(f(dt+\omega)\big)}_{\text{electric}} - \underbrace{\Theta}_{\text{magnetic}}\Big]$$

$$\Theta = -\frac{1}{3} f\left(d\omega + *_4 d\omega\right)$$

These conditions are iff for susy

Then impose eoms:

Bianchi: $d\Theta = 0$ $\quad$ Einstein then follows

Maxwell: $\nabla^2 f^{-1} = \Theta^2$

. Will take $h_{mn}\, dx^m dx^n = ds^2(\mathbb{E}_4)$

. Simple solutions: $\Theta = 0$ $\quad \omega = 0$: no dipole, no rotation : susy RN (Strominger-Vafa)

$$\nabla^2 f^{-1} = 0 : \quad f^{-1} = 1 + \frac{Q}{r^2}$$ (harmonic function rules ...)

$$A = \frac{\sqrt{3}}{2} f\, dt$$

- $\Theta = 0$, $\quad d\omega + *_4 d\omega = 0 \quad \omega \neq 0$: no dipole, rotating . BMPV

again $\nabla^2 f^{-1} = 0 \quad f^{-1} = 1 + \frac{Q}{r^2}$

$$\omega = \frac{J}{r^2}(\cos^2\theta\, d\phi + \sin^2\theta\, d\psi) \qquad ds^2(\mathbb{E}_4) = dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\psi^2 + \cos^2\theta\, d\phi^2)$$

$d\omega$ is anti-self-dual 2-form on $\mathbb{R}^4$

- $\Theta \neq 0$, $\omega \neq 0$: susy rotating black ring

more complicated since f^{-1} not harmonic & $d\omega$ not anti-self-dual

Use ring coordinates for $\mathbb{E}_4$: $$ds^2 = \frac{R^2}{(x-y)^2}\left(\frac{dy^2}{y^2-1} + (y^2-1)\,d\psi^2 + (1-x^2)\,d\phi^2 + \frac{dx^2}{1-x^2}\right)$$

This will simplify enormously the solution

ansatz $\Theta = \frac{1}{3} q\,(dy \wedge d\psi + dx \wedge d\phi)$; we know that this is a self-dual 2-form on the base, from our construction of the coordinates!

Recall Θ is related to dipole

so $d\Theta = 0$ $\qquad \left(A = \frac{\sqrt{3}}{2}\left[f(dt+\omega) - \frac{q}{2}[(1+x)\,d\phi + (1+y)\,d\psi]\right]\right)$

Then $$\nabla^2 f^{-1} = \Theta^2 = \frac{1}{2} q^2 \left(\frac{x-y}{R}\right)^4$$

To solve this: first, the homogeneous eqn $\nabla^2 f^{-1} = 0$ is solved by $f^{-1} \propto x - y$ with δ-source at the ring $y = -\infty$

The complete eqn is solved in particular by $$f^{-1} = -\frac{q^2}{4R^2}(x^2 - y^2)$$ (say, by trial and error)

$$f^{-1} = 1 + \frac{Q - q^2}{2R^2}(x-y) - \frac{q^2}{4R^2}(x^2 - y^2)$$

The eq for ω is then also easily solved. $\left(\omega_\phi = \frac{q}{8R^2}(1-x^2)[3Q - q^2(3+x+y)]\right.$, $\left.\omega_\psi = \frac{3}{2}q(1+y) + \frac{q}{8R^2}(1-y^2)[3Q - q^2(3+x+y)]\right)$

Requirements: absence of CTC's near source $\begin{pmatrix} g_{\psi\psi} & g_{\psi\phi} \\ & g_{\phi\phi} \end{pmatrix}$ positive semi-definite

" " Dirac-Misner singularities : ω globally defined

$\omega_\phi(x = \pm 1) = 0$

$\omega_\psi(y = -1) = 0$

Horizon at $y = -\infty$: $S^1 \times S^2$ (where $f \to 0$) $\qquad x = \cos\theta$

Horizon geometry $$ds^2_H = L^2(Q,q,R)\,d\psi^2 + \frac{q^2}{4}(d\theta^2 + \sin^2\theta\, d\phi^2)$$ $S^1 \times S^2$

$A_H = 2\pi L\, 4\pi \frac{q^2}{4} = 2\pi^2 L q^2$

No CTC $\equiv L^2 \geq 0$

S^2 size controlled by dipoles

Near-horizon : locally $AdS_3 \times S^2$

radius q, radius $q/2$ $\rightarrow$ controlled by dipoles, not by charges Q!

- Angular velocities vanish (as they must for a susy AF bh)

Remarks: - BMPV obtained as $R \to 0$. But area jumps discontinuously in limit.

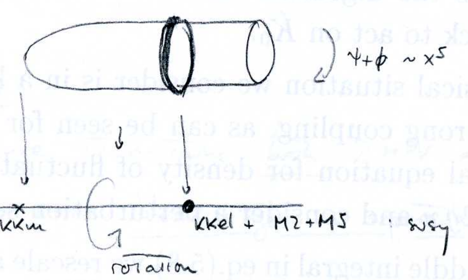
- $J_\psi > J_\phi$: always different parameters than BMPV: $J_\psi = J_\phi$

- $R \to \infty$ limit: 5D susy black string with M2, M5, P charges: among the solutions obtained from the generating sol. of Bertolini + Trigiante

- Non-uniqueness: fix Q_i, J_ψ, J_ϕ (M is fixed by Q_i): 5 parameters
  Solutions have 7 parameters. Two dipoles remain unfixed
  Double-continuous non-uniqueness
  (requiring maximal area fixes uniqueness in susy case, but not in neutral)

- Multi-rings are possible

- Black rings in Taub-NUT



Microscopics of black rings

To identify micro, find a brane configuration w/ same charges.

But focus on what charges: net charges Q_i ?
dipole charges q_i ?

Both are possible, lead to alternative descriptions

$ds_5^2(BR) \times S^1 \times T^4$

Net charges $Q_1\ Q_2\ Q_3$: like BMPV

P | D1-D5 intersection

Can dualize $M2 \perp M2 \perp M2$ to D1-D5-P

Then q_i dualize to $d_1\ d_5$ KKm

This was very successful for BMPV. Also, if only D1-D5 and KKm This is a supertube (zero area)

Less successful for black ring.
The description is important since it contains both BMPV and ring as states of D1-D5-P system. This views black rings as supertubes

If we focus on dipole charges Then we can't include BMPV in the same description since it has none. But we'll be able to count entropy precisely

↳ This means focusing on $M5 \perp M5 \perp M5$ + J as momentum along intersection.
intersect over ring

Recall
$M5 + 1234 \quad \psi$
$M5 + 12 \quad 56 \quad \psi$
$M5 + 3456 \quad \psi$

black rings as circular strings

M2-charges correspond to electric fluxes on the worldvolume of the M5's

Use the same theory that describes $M5 \perp M5 \perp M5 + P$ for a straight intersection (MSW)

$M5 + 1234 \quad z$
$+ 12 \quad 56 \quad z$
$+ \quad 3456 \quad z$

$R \to \infty$ limit of black ring

MSW: smooth out intersection: single M5 wrapping a cycle inside CY_3
small CY_3 $\Rightarrow$ low energy dynamics is $(0,4)$-susy 1+1 σ-model
↳ intersection

$c = 6 n_1 n_2 n_3 \ (+\cdots)$ ⟶ numbers of M5;

$V_6 \ll c$: sugra
$V_6 \gg c$: σ-model

$R \gg V_6^{1/6}$ to get σ-model
$V_6 \gg 1$ to neglect quantum corr.

M2 charges: wv fluxes
Turn them to zero for the moment

$M5^3$ + momentum
$\longrightarrow J_\psi = \hat{q}_0$ oscillator level

$$S_{CFT} = 2\pi\sqrt{\frac{c}{6}\hat{q}_0} = 2\pi\sqrt{n_1 n_2 n_3 J_\psi}$$: for straight string in 5D (4D 4-ch bh)
This is correct

Turn on M2 charges : They give rise to momentum zero modes that contribute to total momentum q_0. So oscillator level (non-zero mode)

$$\hat{q}_0 = q_0 + \frac{1}{2}\left(\frac{N_1 N_2}{n_3} + \text{perm}\right) - \frac{1}{4 n_1 n_2 n_3}\sum_{i=1}^{3}(N_i n_i)^2 + \frac{n_1 n_2 n_3}{4}$$

Setting $q_0 = -J_4$ in $S = 2\pi\sqrt{\frac{c}{6} q_0}$ gives perfect agreement with BH entropy

(Mateos et al)

7-parameter match of the entropy!

Shortcomings : doesn't explain why $Q_1 > q_2 q_3$

R is fixed by equilibrium and not free.

$J_\phi \neq 0$

Doesn't say anything about finite radius effects.

" " microscopic significance of non-uniqueness

$\Rightarrow$ UV description : black rings are supertube-like, rotation has blown them into a tubular structure.

At least shows that string th has the required structure to accommodate different objects w/ the same conserved charges

Black rings seem to be a mixture between a BMPV bh and a supertube (but things are missing)

- Non-susy versions exist, w/ 7 parameters, but would need 9-parameters to be able to go near-susy w/ $3Q\,3q$ (susy limit $3Q\,3q$ is not achieved)

- We may have all susy bhs of 5D sugra. There are solutions w/ arbitrary profile (not circular) 

Have finite area but horizon is not regular (metric is continuous, curvature invariants are finite. But freely falling observers experience infinite tidal forces. Components of Riemann in parallel-prop basis diverge along geodesics

Thm (Reall) says that horizon geometry must be S^3 (or quotients) homogeneously squashed - like BMPV

$S^1 \times S^2$

T^3 (marginal, unlikely)