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Phase diagram and frustration of decoherence in Y-shaped Josephson junction networks

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Main idea

Y-Shaped network of Josephson junction chains (YJJN) with a magnetic frustration⇒Finite-coupling fixed point (FFP) in the phase diagram;

YJJN working near the FFP ⇒ Frustration of decoherence in the emerging two-level quantum system (2LQS);

Application: engineering of a reduced-decoherence 2LQS.

Technology: renormalization group+boundary conformal field theory.

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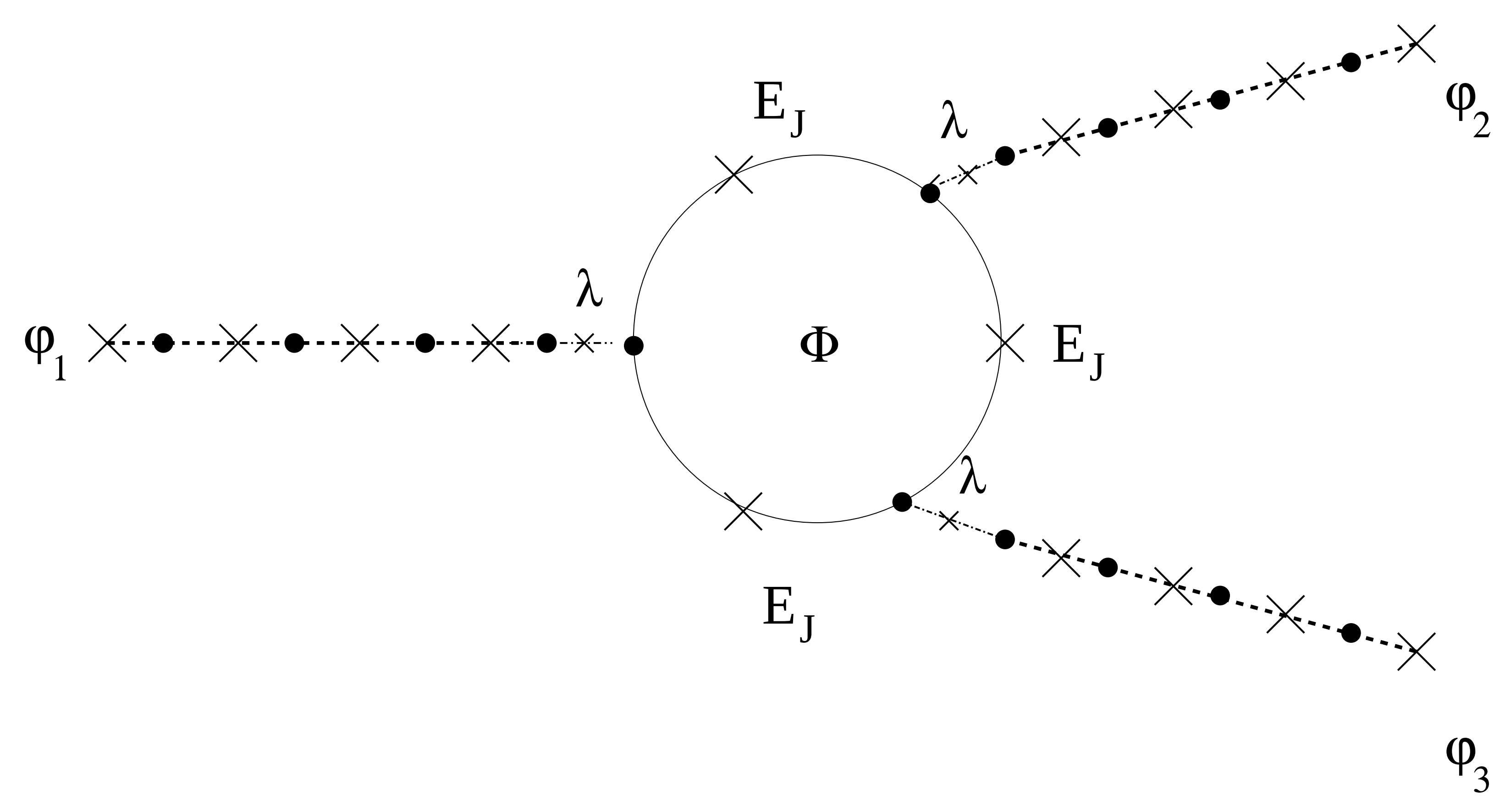
Plan of the talk:

1. The YJJN as a junction of charged, one-dimensional, bosonic systems;
2. The parameters and the phase diagram of the YJJN: weakly coupled and strongly coupled fixed points;
3. Emergence of a FFP in the phase diagram;
4. Current's patter in the YJJN near the fixed points: the YJJN as a "quantum switch";
5. Spectral density and frustration of decoherence in the YJJN working near the FFP;
6. Conclusions, possible applications, perspectives.

1. The YJJN and its field-theoretical description



Central region Hamiltonian

$$H_\Delta = \frac{E_C}{2}\sum_{i=1}^{3}\left[-i\frac{\partial}{\partial \phi_i^{(0)}} - e^* W_g\right]^2 - \frac{E_J}{2}\sum_{i=1}^{3}\left[e^{i(\phi_i^{(0)} - \phi_{i+1}^{(0)} + \varphi/3)} + h.c.\right]$$

$E_C >> E_J \Rightarrow$ Effective (3)-spin Hamiltonian

$$[S_i^{(0)}]^z = -i\frac{\partial}{\partial \phi_i^{(0)}} - N - \frac{1}{2}$$

$$[S_i^{(0)}]^+ = e^{i\phi_i^{(0)}}$$

$$e^* W_g = N + h + \frac{1}{2}$$

$$H_\Delta = -h\sum_{i=1}^{3}[S_i^{(0)}]^z - \frac{E_J}{2}\sum_{i=1}^{3}\left\{[S_i^{(0)}]^+[S_{i+1}^{(0)}]^- e^{i\varphi/3} + h.c.\right\}$$

Low-energy eigenstates ($h>E_J$)

$$\left|\uparrow\uparrow\uparrow\right\rangle \qquad \varepsilon_0 = -\tfrac{3}{2}h$$

$$\frac{1}{\sqrt{3}}\left[\left|\uparrow\uparrow\downarrow\right\rangle + \left|\uparrow\downarrow\uparrow\right\rangle + \left|\downarrow\uparrow\uparrow\right\rangle\right] \qquad \varepsilon_{11} = -\tfrac{1}{2}h - \frac{E_J}{2}\cos\left(\frac{\varphi}{3}\right)$$

$$\frac{1}{\sqrt{3}}\left[\left|\uparrow\uparrow\downarrow\right\rangle - e^{-i\frac{\pi}{3}}\left|\uparrow\downarrow\uparrow\right\rangle - e^{i\frac{\pi}{3}}\left|\downarrow\uparrow\uparrow\right\rangle\right] \qquad \varepsilon_{12} = -\tfrac{1}{2}h - \frac{E_J}{2}\cos\left(\frac{\varphi-\pi}{3}\right)$$

$$\frac{1}{\sqrt{3}}\left[\left|\uparrow\uparrow\downarrow\right\rangle - e^{i\frac{\pi}{3}}\left|\uparrow\downarrow\uparrow\right\rangle - e^{-i\frac{\pi}{3}}\left|\downarrow\uparrow\uparrow\right\rangle\right] \qquad \varepsilon_{13} = -\tfrac{1}{2}h - \frac{E_J}{2}\cos\left(\frac{\varphi+\pi}{3}\right)$$

Only these states will be kept in the effective theory

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Charge tunneling at the "inner boundaries"

$$H_T = -\lambda \sum_{i=1}^{3} \cos[\phi_i^{(0)} - \phi_i^{(1)}]$$

"Weak tunneling" limit: $\lambda << h, E_J$ ⇒ Schrieffer-Wolff transformation ⇒ Boundary interaction term

$$H_B = -E_W \sum_{i=1}^{3} [e^{i(\phi_i^{(1)} - \phi_{i+1}^{(1)})} e^{i\gamma} + h.c.]$$

$$E_W \approx \frac{\lambda^2 E_J}{24 h^2} \sqrt{[\cos^2(\frac{\varphi}{3}) + 9 \sin^2(\frac{\varphi}{3})}$$

$$\gamma = \arctan[3 \tan(\frac{\varphi}{3})]$$

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Effective field theory of a JJ-chain

(L. I. Glazman and A. I. Larkin, PRL 79, 3736 (1997),

D. Giuliano and P. Sodano, NPB 711, 480 (2005))

$$H_0 = \sum_{k=1}^{3} \left\{ \frac{E_C}{2} \sum_j \left[-i \frac{\partial}{\partial \phi_j^{(k)}} - N \right]^2 - E_J \sum_j \cos\left[\phi_j^{(k)} - \phi_{j+1}^{(k)}\right] + \left(E_Z - \frac{3}{16} \frac{E_J^2}{E_C} \right) \sum_j n_j^{(k)} n_{j+1}^{(k)}$$

(N=n+1/2)

Mapping onto spin chain+Jordan-Wigner fermions+Bosonization ⇒Luttinger liquid (LL) effective Hamiltonian

$$H_{LL} = \sum_{k=1}^{3} \left\{ \frac{g}{4\pi} \int_0^L u \left[\left(\frac{\partial \Phi^{(k)}}{\partial x} \right)^2 + \frac{1}{u^2} \left(\frac{\partial \Phi^{(k)}}{\partial t} \right)^2 \right] dx \right\}$$

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LL parameters and boundary conditions

Weak boundary coupling (E_W/E_J <<1) ⇒Neumann boundary conditions at the inner boundary ($\partial\Phi^{(k)}(0)/\partial x=0$);

Coupling to the bulk superconductors ⇒Dirichlet boundary conditions at the outer boundary $\Phi^{(k)}(L)=\sqrt{2}(2\pi n^{(k)}+\varphi^{(k)})$;

$$g=\sqrt{\frac{v_F+g_2-g_4}{v_F+g_2+g_4}}$$

$$u=\sqrt{(v_F+g_2)^2-g_4^{\,2}}$$

$$g_2=g_4=4\pi a\Delta[1-\cos(2k_F a)]$$

$$(\Delta=E^z-\frac{3}{16}\frac{J^2}{E_C})$$

"Normal" fields

$$X(x) = \frac{1}{\sqrt{3}} \sum_{i=1}^{3} \Phi_i(x)$$

$$\chi_1(x) = \frac{1}{\sqrt{2}} [\Phi_1(x) - \Phi_2(x)]$$

$$\chi_2(x) = \frac{1}{\sqrt{6}} [\Phi_1(x) + \Phi_2(x) - 2\Phi_3(x)]$$

$$\chi_1(L) = \varphi_1 - \varphi_2 + 2\pi n_{12}$$

$$\chi_2(L) = \frac{\phi_1 + \phi_2 - 2\phi_3}{\sqrt{3}} + 2\pi n_{13}$$

Boundary Hamiltonian

$$H_{Bou} = -E_W \sum_{i=1}^{3} \exp[i(\vec{\alpha} \bullet \vec{\chi}(0) + \gamma)] + h.c.$$

$$\left(\alpha_1 = (1,0); \alpha_2 = (-\frac{1}{2}, \frac{\sqrt{3}}{2}); \alpha_3 = (-\frac{1}{2}, -\frac{\sqrt{3}}{2}) \right)$$

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(Dynamical) boundary conditions at the inner boundary

$$\frac{ug}{2\pi}\frac{\partial\vec{\chi}}{\partial x}-2\overline{E}_W\sum_i\vec{\alpha}_i\sin[\vec{\alpha}_i\bullet\vec{\chi}(0)+\gamma]=0$$

Weakly coupled FP

$$\frac{\partial\vec{\chi}}{\partial x}=0$$

Strongly coupled FP

Minimum of H_{Bou}

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2. Phase diagram of the YJJN: weakly and strongly coupled fixed points

Weakly coupled fixed point

Mode expansion for the plasmon fields

$$\chi_i(x,t) = \xi_i + \sqrt{\frac{2}{g}} \sum_n \cos\left[\frac{\pi}{L}\left(n+\frac{1}{2}\right)x\right] \frac{\alpha_i(n)}{n+\frac{1}{2}} e^{i\frac{\pi}{L}(n+\frac{1}{2})ut}$$

$$\xi_1 = \varphi_1 - \varphi_2$$

$$\xi_2 = \frac{\phi_1 + \phi_2 - 2\phi_3}{\sqrt{3}}$$

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O.P.E. between boundary vertices:

$$:e^{i\vec{\alpha}_i \bullet \vec{\chi}(\tau)}::e^{i\vec{\alpha}_j \bullet \vec{\chi}(\tau')}:\approx [\tau-\tau']^{-2/g} :e^{-i\vec{\alpha}_k \bullet \vec{\chi}(\tau')}:$$

$$(i \neq j \neq k)$$

Dimensionless boundary coupling $G(L)=LE_W(a/L)^{1/g}$

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Second-order renormalization group equations

$$\frac{d[G(L)e^{i\gamma}]}{d\ln\left(L/L_0\right)} = [1-\frac{1}{g}]G(L)e^{i\gamma} + 2G^2(L)e^{-2i\gamma}$$

$$\frac{dG(L)}{d\ln(L/L_0)} = [1-\frac{1}{g}]G(L) + 2\cos(3\gamma)G^2(L)$$

$$\frac{d\gamma(L)}{d\ln(L/L_0)} = -2\sin(3\gamma)G^2(L)$$

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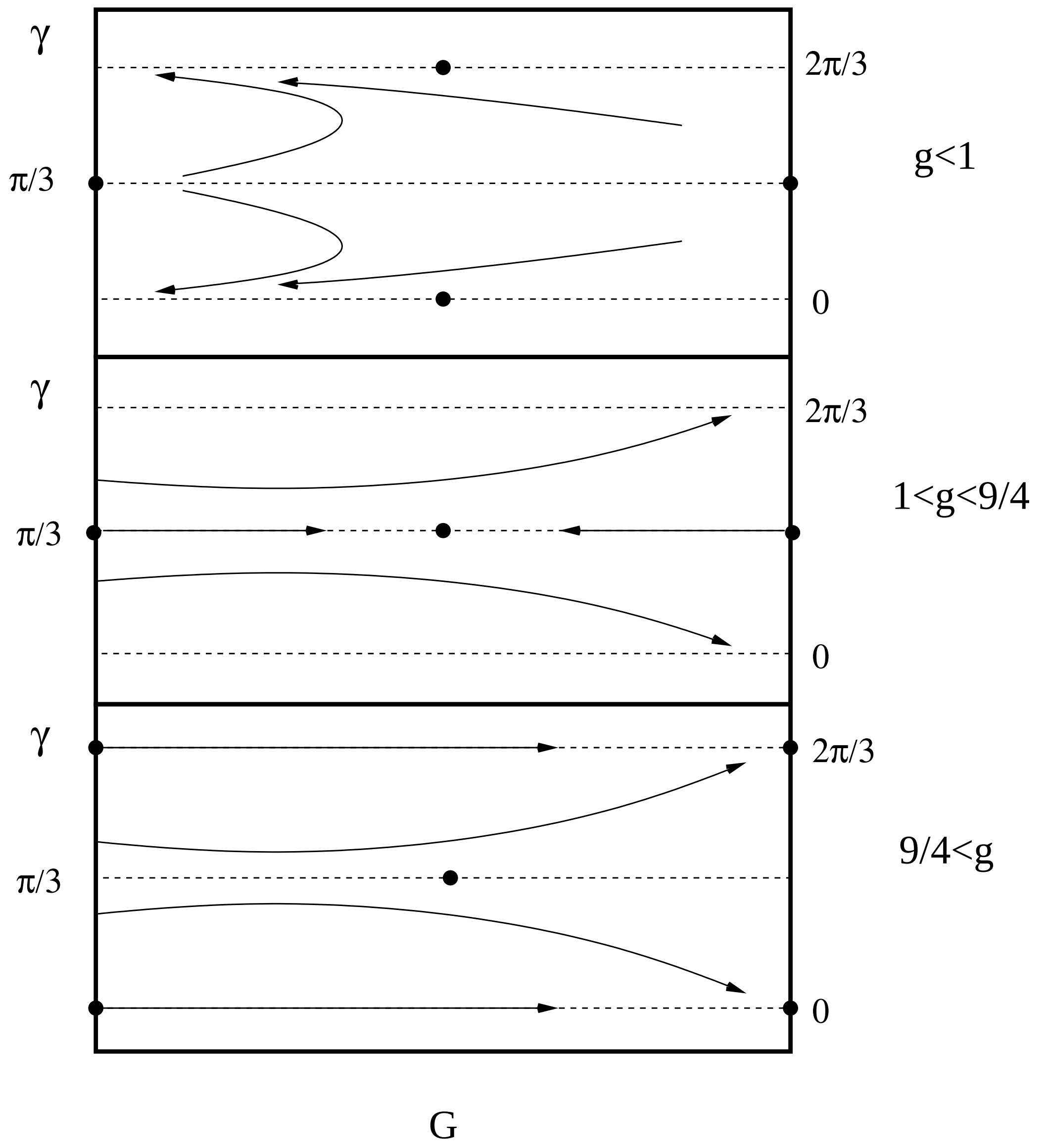
Phase diagram



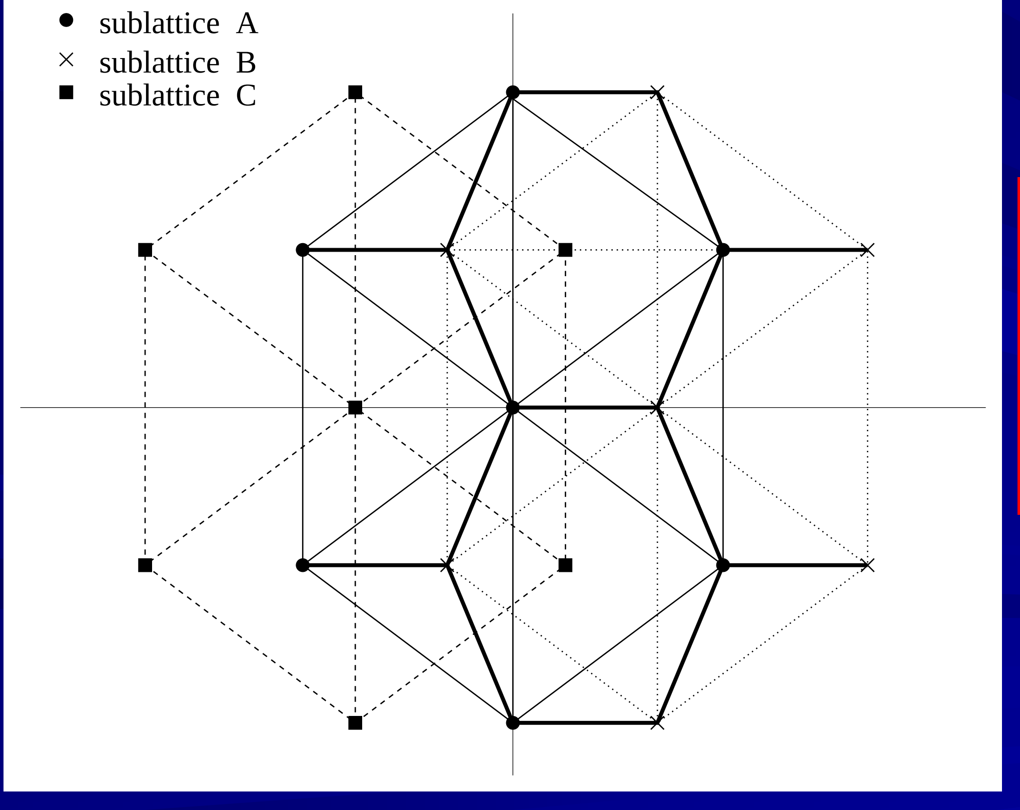
*** g<1:stable fixed point at G=γ=0; fixed lines at γ=0,π/3,2π/3.**

***1<g<9/4:strongly coupled fixed point for γ≠π/3; finite coupling fixed point for γ=π/3.**

***9/4<g: strongly coupled stable FP**

Strongly coupled fixed point

G->∞ ⇒Dirichlet boundary conditions at the inner boundary, as well. ($\chi_1(0)$, $\chi_2(0)$) span a triangular lattice, depending on the value of γ



For γ=π/3 the minima lie on a honeycomb lattice (merging of two triangular sublattices)

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Mode expansion of the plasmon fields at the SFP

$$\chi_i(x,t) = \xi_i + \sqrt{\frac{2}{g}}\left[-P_i\frac{\pi x}{L} - \sum_n \sin[\frac{\pi}{L}nx]\frac{\alpha_i(n)}{n}e^{i\frac{\pi}{L}nut}\right]$$

Dual fields

$$\psi_i(x,t) = \sqrt{2g}\left\{\theta_i + \frac{\pi v t}{L}P_i + \frac{\pi x}{L} + i\sum_n \cos[\frac{\pi}{L}nx]\frac{\alpha_i(n)}{n}e^{-i\frac{\pi}{L}nut}\right\}$$

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For γ≠kπ+ π /3 the minima span only one of the three sublattices : in this case, the leading boundary perturbation is given by a combination of "long" V-instantons.

$$H_S = -Y \sum_{i=1}^{3} \overline{V}_i(0) + h.c.$$

$$\overline{V}_j(\tau) =: \exp\left[\pm i2\sqrt{\frac{2}{3}} \vec{\rho}_j \bullet \vec{\psi}(\tau) \right]:$$

$$\vec{\rho}_1 = (0,1); \vec{\rho}_2 = (\frac{\sqrt{3}}{2}, -\frac{1}{2}); \vec{\rho}_3 = (-\frac{\sqrt{3}}{2}, -\frac{1}{2})$$

The "V-instanton" operators have conformal dimension $h_S(g)=4g/3$: for ¾<g<1 (and for γ≠kπ+π/3) both the weakly coupled and the strongly coupled fixed point is stable (repulsive FFP).

3. Emergence of a stable finite coupling fixed point

For γ=kπ+ π /3 two triangular sublattices become degenerate in energy: they merge to form a honeycomb lattice. In this case, the leading boundary perturbation is given by a combination of "short" W-instanton.

$$H_F = -\varsigma \sum_{i=1}^{3} \tau^{+} \overline{W}_i(0) + h.c.$$

$$\overline{W}_j(\tau) =: \exp\left[\pm i \frac{2}{3} \vec{\alpha}_j \bullet \vec{\psi}(\tau) \right]:$$

τ^{+},τ^{-} are effective isospin operators, connecting sites on inequivalent sublattices

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Perturbative renormalization group equation for the running coupling strength

$$\frac{d\zeta}{d\ln(L/L_0)} = \left[1 - \frac{4g}{9}\right]\zeta - 2\zeta^3$$

The "W-instanton" operators have conformal dimension $h_F(g)=4g/9$: for $1<g<9/4$ neither the weakly coupled, or the strongly coupled fixed point is stable: the IR behavior of the system is driven by an attractive FFP

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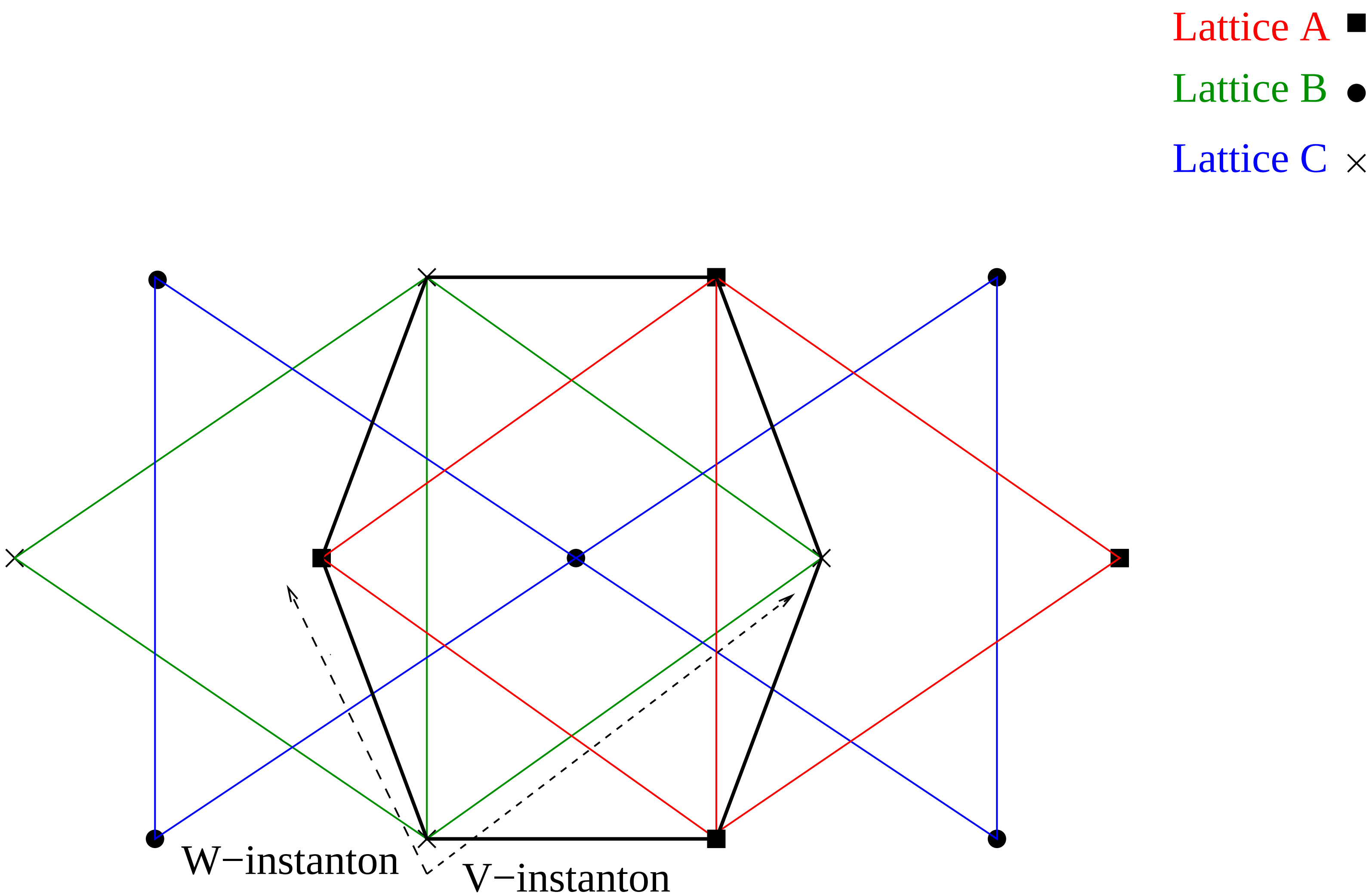

4. Current pattern near the (W and S) fixed points

Current: logarithmic derivatives of the partition function Z

$$I_1 = \frac{e^*}{g\beta}\left[\frac{1}{\sqrt{2}}\frac{\partial \ln Z}{\partial \beta_1} + \frac{1}{\sqrt{6}}\frac{\partial \ln Z}{\partial \beta_2}\right]$$

$$I_2 = \frac{e^*}{g\beta}\left[-\frac{1}{\sqrt{2}}\frac{\partial \ln Z}{\partial \beta_1} + \frac{1}{\sqrt{6}}\frac{\partial \ln Z}{\partial \beta_2}\right]$$

$$I_3 = -\frac{e^*}{g\beta}\sqrt{\frac{2}{3}}\frac{\partial \ln Z}{\partial \beta_2}$$

$$\beta_1 = \frac{\varphi_1 - \varphi_2}{\sqrt{2}}$$

$$\beta_2 = \frac{\varphi_1 + \varphi_2 - 2\varphi_3}{\sqrt{6}}$$

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Weakly coupled fixed point

Perturbative calculation: the result is the "typical" sinusoidal behavior, as a function of the applied phase differences

$$I_1 = \frac{2e^*G}{gL}\left[\sin(\vec{\alpha}_1 \bullet \vec{\beta} + \gamma) - \sin(\vec{\alpha}_3 \bullet \vec{\beta} + \gamma)\right]$$

$$I_2 = \frac{2e^*G}{gL}\left[\sin(\vec{\alpha}_2 \bullet \vec{\beta} + \gamma) - \sin(\vec{\alpha}_1 \bullet \vec{\beta} + \gamma)\right]$$

$$I_3 = \frac{2e^*G}{gL}\left[\sin(\vec{\alpha}_3 \bullet \vec{\beta} + \gamma) - \sin(\vec{\alpha}_2 \bullet \vec{\beta} + \gamma)\right]$$

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Strongly coupled fixed point

Zero-mode contribution to the energy eigenvalues

$$E = E[n_{12}, n_{13}] + E_{osc}$$

$$E[n_{12}, n_{13}] = \frac{\pi v g}{L}\left\{\left[n_{12} + \frac{\beta_1}{2\pi} + \varepsilon_l\right]^2 + \left[n_{13} + \frac{n_{12}}{2} + \frac{\sqrt{3}}{2}\frac{\beta_2}{2\pi}\right]^2\right\}$$

$$\varepsilon_A = 0, \varepsilon_B = 1, \varepsilon_C = -1$$

On a finite-size system this breaks the degeneracy between the minima of the boundary potential (labelled by the n's)

The main contribution to the total current comes from the zero-mode term in the total energy: this implies abrupt jumps (perturbatively rounded by V-instantons) at the degeneracy between two eigenvalues

$$I_1 = \frac{e^* vg}{L}\left[\frac{1}{\sqrt{2}}\left(\frac{\beta_1}{2\pi}+n_{12}\right)+\frac{1}{\sqrt{6}}\left(\frac{\beta_2}{2\pi}+\frac{2n_{13}+n_{12}}{\sqrt{3}}\right)\right]$$

$$I_2 = \frac{e^* vg}{L}\left[-\frac{1}{\sqrt{2}}\left(\frac{\beta_1}{2\pi}+n_{12}\right)+\frac{1}{\sqrt{6}}\left(\frac{\beta_2}{2\pi}+\frac{2n_{13}+n_{12}}{\sqrt{3}}\right)\right]$$

$$I_3 = -\frac{e^* vg}{L}\frac{\sqrt{2}}{3}\left(\frac{\beta_2}{2\pi}+\frac{2n_{13}+n_{12}}{\sqrt{3}}\right)$$

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Tuning two eigenstates of the zero-mode operator near by a degeneracy ⇒ effective two-level quantum device

For instance: setting

$$-\frac{1}{6} < \frac{\beta_1}{2\pi} < \frac{1}{6}$$

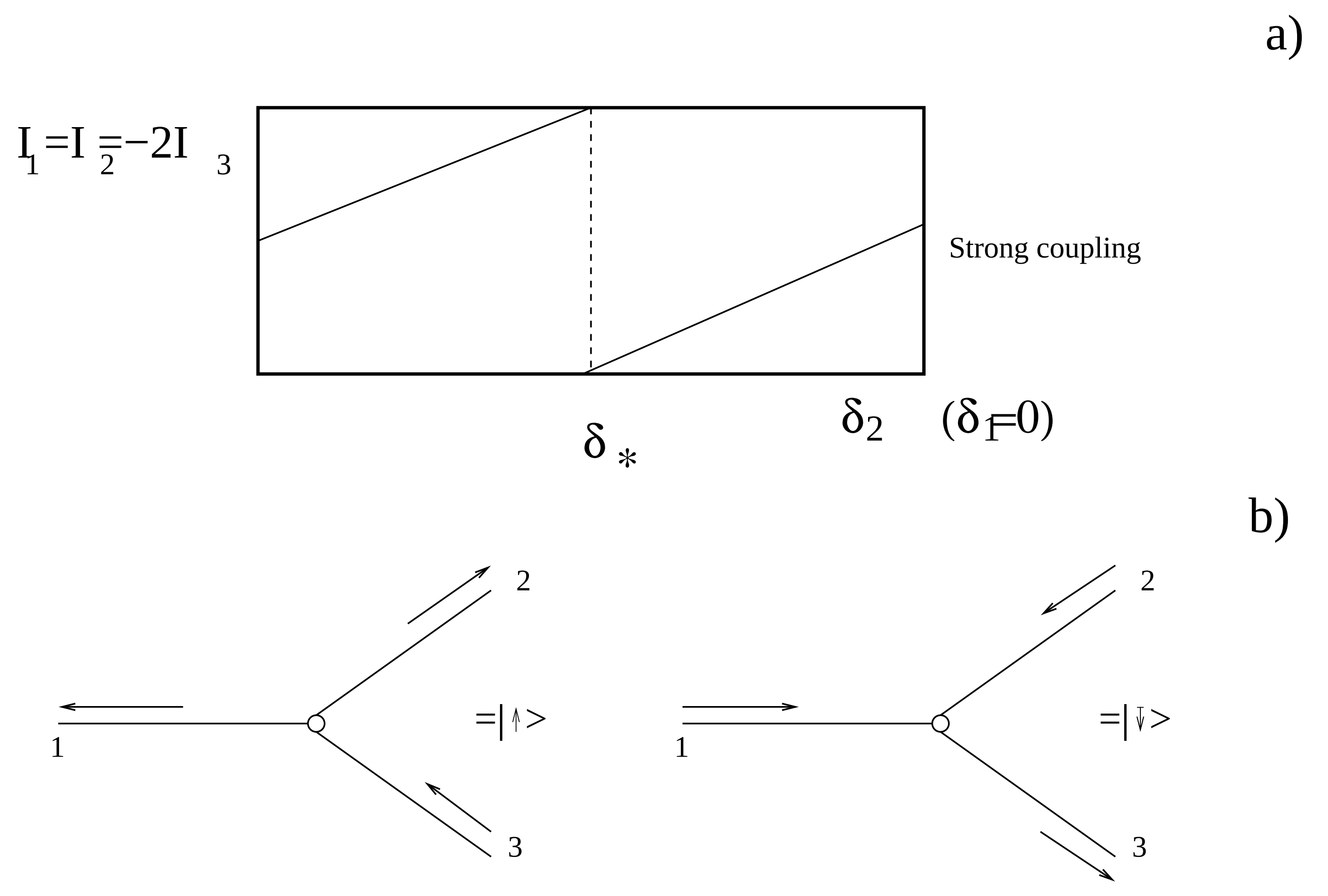
$$\frac{\beta_2}{2\pi} = -\frac{1}{\sqrt{3}} + \delta$$

$$\left(\left(\frac{\delta}{\pi}\right) << 1\right)$$

The following two states define an effective 2LQD

$$|0,0>_A ; |0,1>_A \equiv |\uparrow>, |\downarrow>$$

Operating the system as a quantum switch



δ_2 measures the detuning off the degeneracy: acting on this parameter one makes the system "switch" between the two states

5. The system working near the FFP: current pattern and frustration of decoherence

The current pattern near the FFP

Though it is possible to set up a self-consistent formalism to formally derive the current pattern near the FFP, a closed-formula can be given only for g=9/4-ε, with ε<<1. In this case, one may set the parameters as

$$\alpha = \frac{g}{6\pi}$$

$$\beta_1 \approx \beta_1 * + \delta = -\frac{\pi}{3} + \delta$$

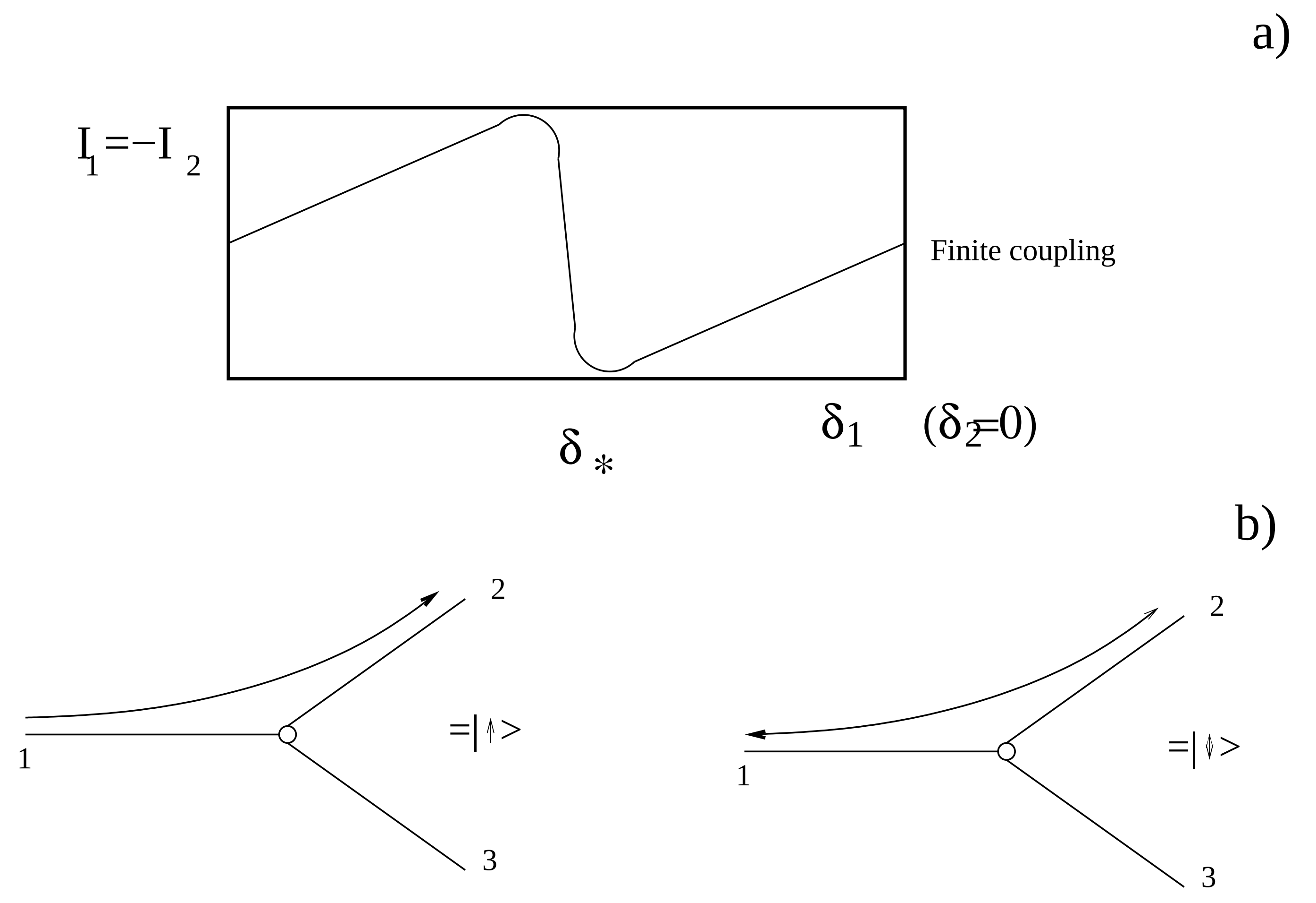
$$\zeta_* \approx \varepsilon^{\frac{1}{2}}$$

Current across the three arms

$$I_1 = \frac{e^* v}{2\pi L}\left[\frac{\delta}{\sqrt{2}}\left(-1+\frac{4\pi^2\alpha^2}{\sqrt{\alpha^2\beta^2+3(\zeta^*)^2}}\right)-\frac{\beta_2}{\sqrt{6}}\right]$$

$$I_2 = \frac{e^* v}{2\pi L}\left[-\frac{\delta}{\sqrt{2}}\left(-1+\frac{4\pi^2\alpha^2}{\sqrt{\alpha^2\beta^2+3(\zeta^*)^2}}\right)-\frac{\beta_2}{\sqrt{6}}\right]$$

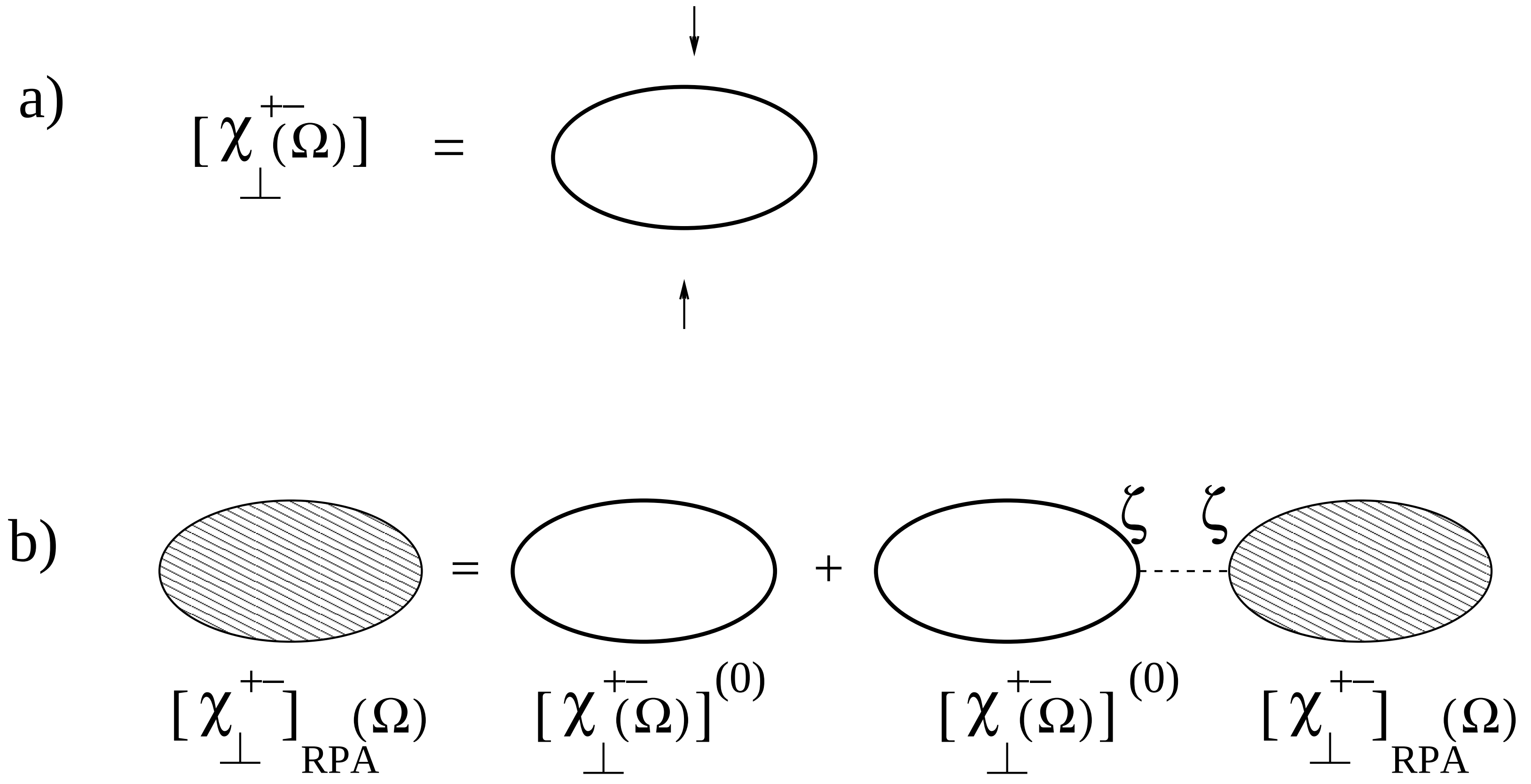
$$I_3 = \frac{e^* vg}{L}\sqrt{\frac{2}{3}}\beta_2$$



Again, this is a smoothened sawtooth-like behavior but, now, it is associated to a stable FP

We relate the decoherence to the entanglement of the system with the plasmon bath ⇒ spectral density of states of the effective 2LQD, Χ″(Ω)/Ω

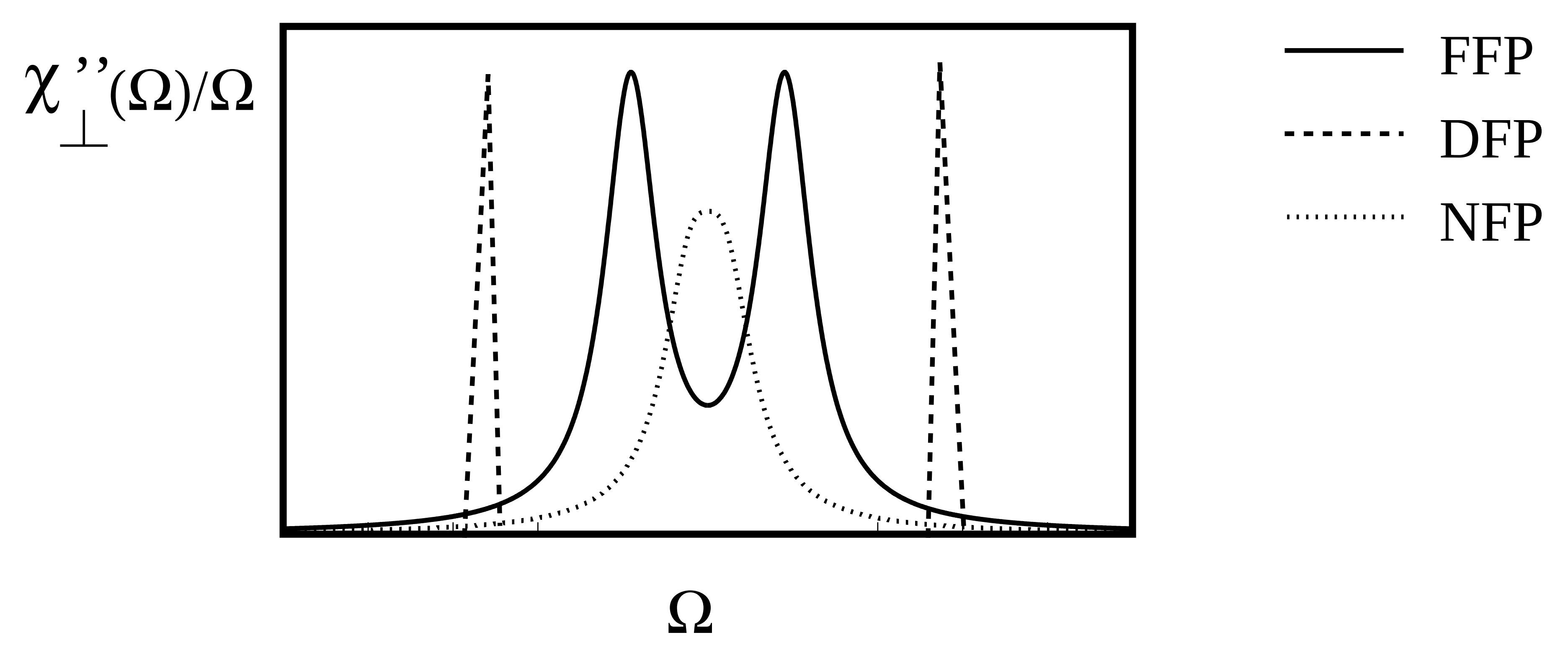
(E. Novais et al., Phys. Rev. B 72, 014417 (2005))



$$\chi^{RPA}{}_{\perp}(\Omega) = \frac{1}{\Omega - \Delta_*(\vec{\beta}) - \varsigma^2 \Gamma[-1-\frac{9}{8}\varepsilon][-\Omega]^{1+\frac{9}{8}\varepsilon}}$$

$$\Delta_*(\vec{\beta}) = \sqrt{[(E(\vec{\beta}))^2 + (\varsigma_* / L)^2]}$$

Using the RPA approximation sketched above yields



<u>Near the SFP</u>: no entanglement between the 2LQD and the bath, but no quantum tunneling between the states either (no energy renormalization);

<u>Near the WFP</u>: full entanglement between the 2LQD and the bath (full decoherence);

<u>Near the FFP</u>: consistent (and robust) tunnel splitting of the two states, with an accettable level of (frustrated) decoherence

6. Conclusions

1. A Y-shaped JJ-network may exhibit a FFP in its phase diagram;

2. At the FFP an effective 2-level quantum devices emerges, with enhanced quantum coherence;

3. Relevant issues for a practical realization: stabilizing γ, applying the external phases, controlling other sources of noise…