Alternatives to Dark Energy and Dark Matter and their implications

Evidence for Dark Energy and Dark Matter

Modified Gravity Models and their observational implications

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International Workshop on Advances in Precision Tests and Experimental Gravitation in Space

28-30 September, Florence, Italy

General Relativity
($\gamma = \beta = 1$)

- **GR has survived all tests so far…**

  [C. Will, gr-qc/0510072; S. Turyshev, M. Shao, K. Nordtvedt, gr-qc/0601035]
  [O.B., J. Páramos, S. Turyshev, gr-qc/0602016]

- **Parametrized Post-Newtonian Formalism (U-gravitational potential, v_i velocity)**

$$g_{00} = -1 + 2U - 2\beta U^2 + \ldots, \quad g_{ij} = (1 + 2\gamma U)\delta_{ij} + \ldots, \quad g_{0i} = -\frac{1}{2}(4\gamma + 3)v_i + \ldots$$

- **Local (solar system) tests**

Mercury´s perihelion shift: $|2\gamma - \beta - 1| < 3\times10^{-3}$ **[Shapiro 1990]**

Lunar Laser Ranging: $4\beta - \gamma - 3 = (4.4 \pm 4.5)\times10^{-4}$ **[Williams, Turyshev, Boggs 2004]**

LBLI light deflection: $|\gamma - 1| < 4\times10^{-4}$ **[Eubanks et al. 1997]**

Cassini Experiment: $\gamma - 1 = (2.1 \pm 2.3)\times10^{-5}$ **[Bertotti, Iess, Tortora 2003]**

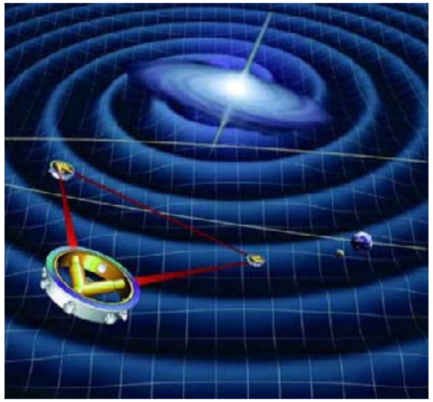
Cassini-Huygens Radiometric Experiment

B. Bertotti, L. Iess and P. Tortora, Nature **425** (2003) 374

(Partially) Unconfirmed predictions:

Gravitational waves – PSR B1913+16

(LIGO, …, LISA)



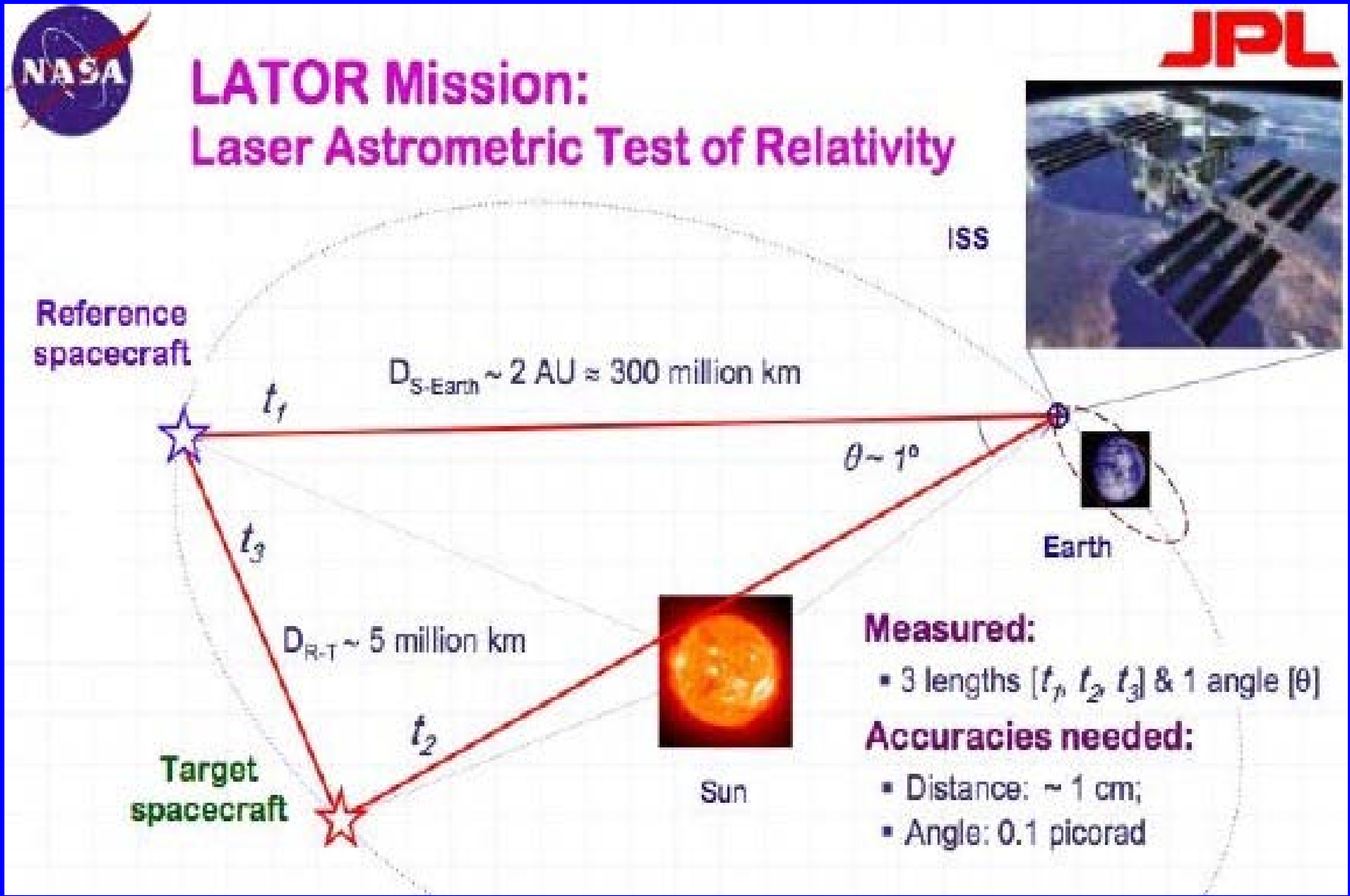
Lense-Thirring Effect

(Gravity Probe-B)

BepiColombo Mission to Mercury (ESA/ISAS)

$$\frac{\Delta J_2}{J_2} < 10^{-9}, \frac{\Delta \gamma}{\gamma} < 2.5 \times 10^{-6}, \frac{\Delta \beta}{\beta} < 5 \times 10^{-6}$$

$$\frac{\Delta \eta_1}{\eta_1} < 2 \times 10^{-5}, \eta_1 = -1 - \beta + 2\gamma$$



Turyshev et al., gr-qc/0506104

Cosmological Tests of General Relativity

- **Outstanding challenges (GR + Quantum Field Theory)**
  - **Singularity Problem**
  - **Cosmological Constant Problem**
  - **Underlying particle physics theory for Inflation**

- **Theory provides in the context of the Big Bang model an impressive picture of the history of the Universe**
  - **Nucleosynthesis** **(** $N_\nu < 4$ **,** $\Omega_B h^2 = 0.023 \pm 0.001$ **)**
  - **Cosmic Microwave Background Radiation**
  - **Large Scale Structure**
  - **Gravitational lensing**
  - …

- **Required entities (missing links):**
  - **Dark Matter**
  - **Dark Energy**

Dark Matter

- **Evidence:**

  Flatness of the rotation curve of galaxies
  Large scale structure
  Gravitational lensing
  N-body simulations and comparison with observations
  Merging galaxy cluster 1E 0657-56

- **Cold Dark Matter (CDM) Model**

  Weakly interacting non-relativistic massive particle at decoupling

- **Candidates:**

  Neutralinos (SUSY WIMPS), axions, scalar fields, self-interacting scalar particles, *etc.*

Dark Energy

- **Evidence:**

  Dimming of type Ia Supernovae with *z > 0.35*

  Accelerated expansion (negative deceleration parameter): $q_0 \equiv -\frac{\ddot{a}a}{\dot{a}^2} \leq -0.47$

  [Perlmutter et al. 1998; Riess et al. 1998, …]

- **Homogeneous and isotropic expanding geometry**

  Driven by the vacuum energy density Ω_Λ and matter density Ω_M

  Equation of state: $p = \omega\rho \qquad \omega \leq 1$

- **Friedmann and Raychaudhuri equations imply:** $q_0 = \frac{1}{2}(3\omega + 1)\Omega_m - \Omega_\Lambda$

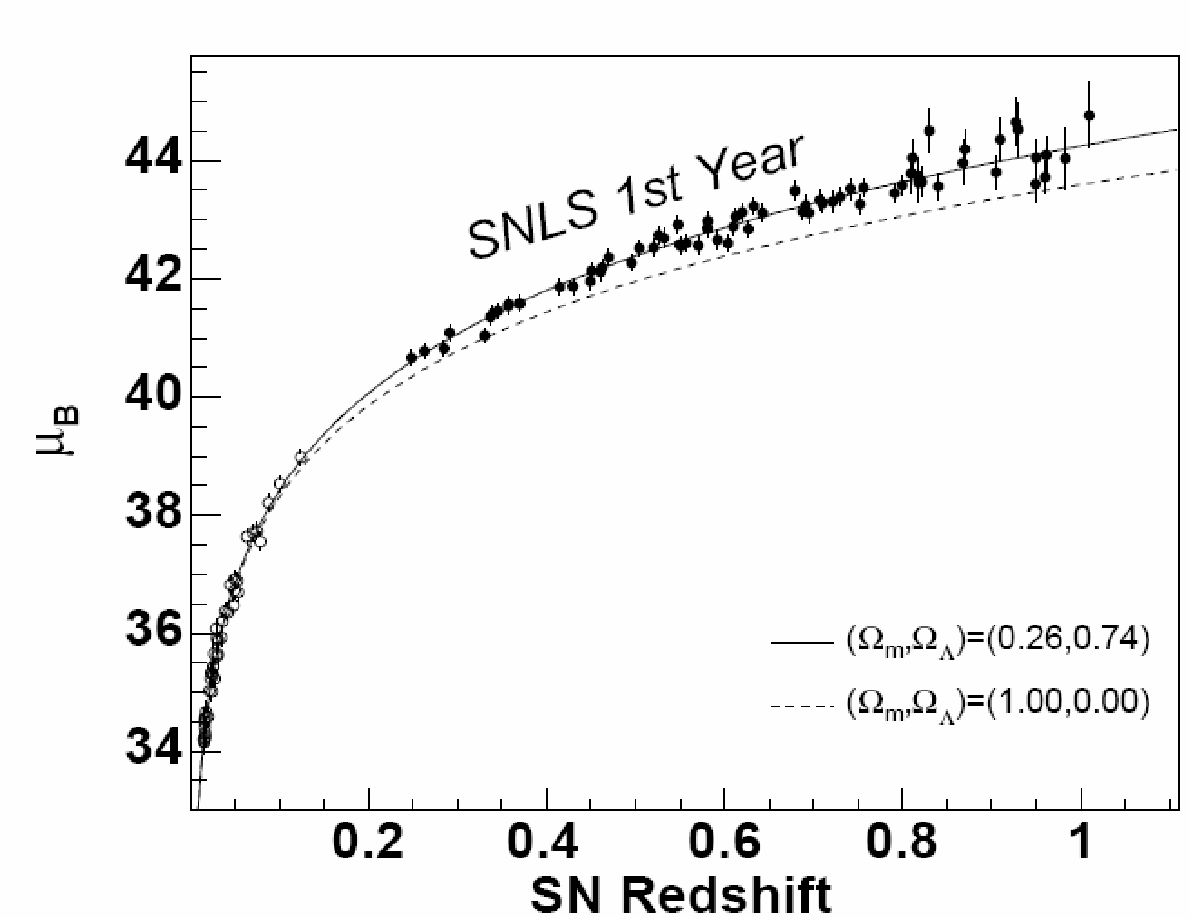
  $q_0 < 0$ **suggests an invisible smooth energy distribution**

- **Candidates:**

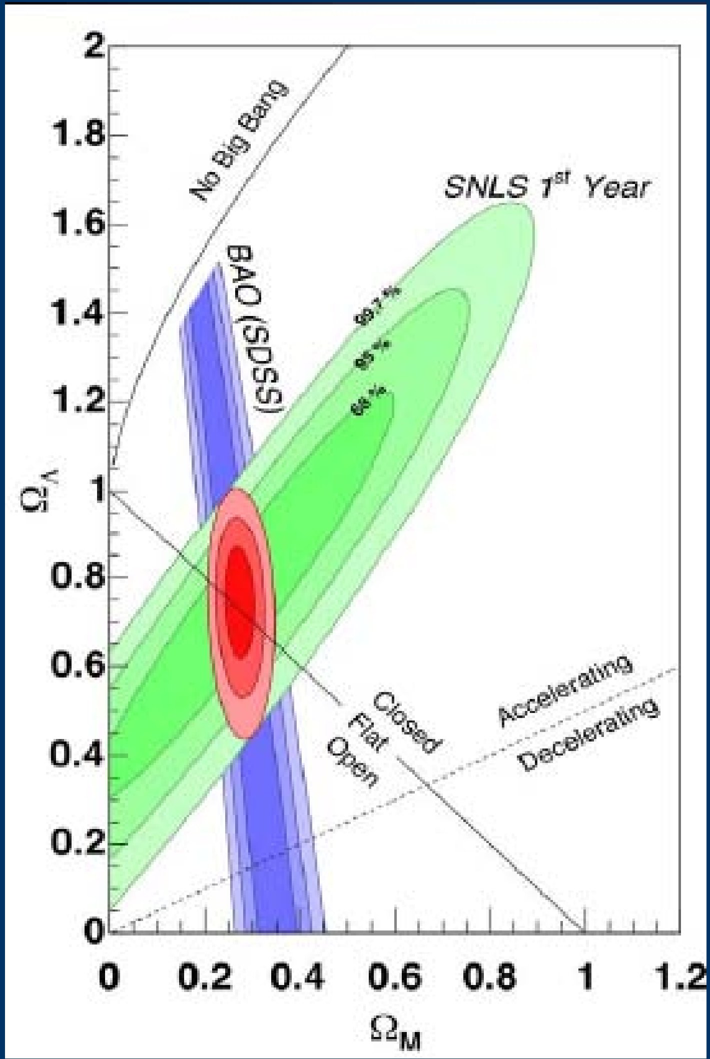
  Cosmological constant, quintessence, more complex equations of state, etc.

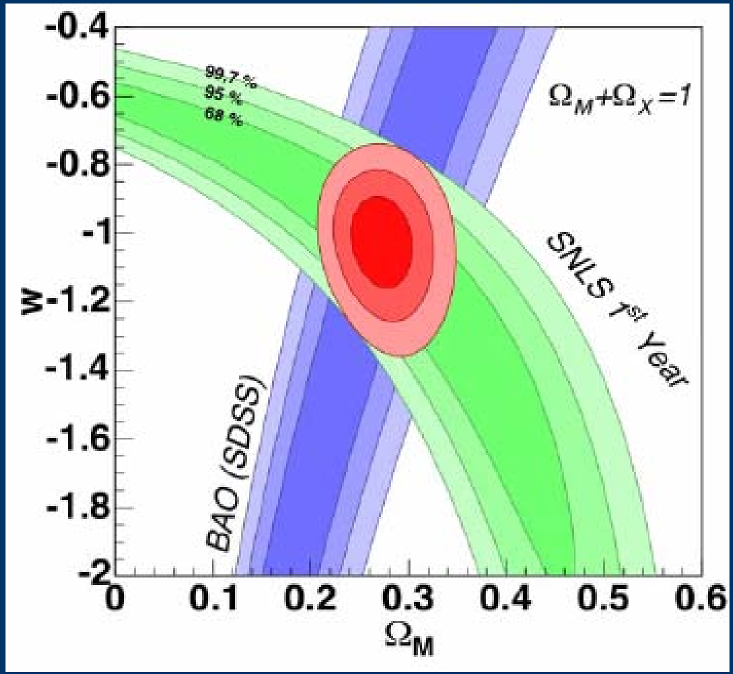
Supernova Legacy Survey (SNLS)

[Astier et al., astro-ph/0510447]



SNLS - SDSS





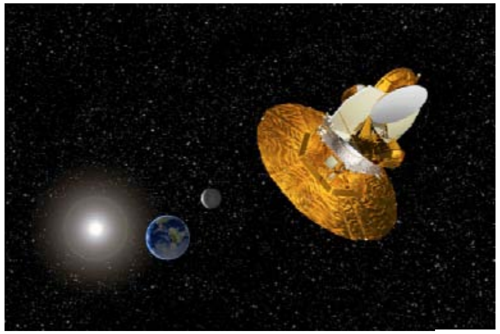
[Riess et al. 2004]

$$\omega = -1.02^{+0.13}_{-0.19}$$

[Astier et al. 2005]

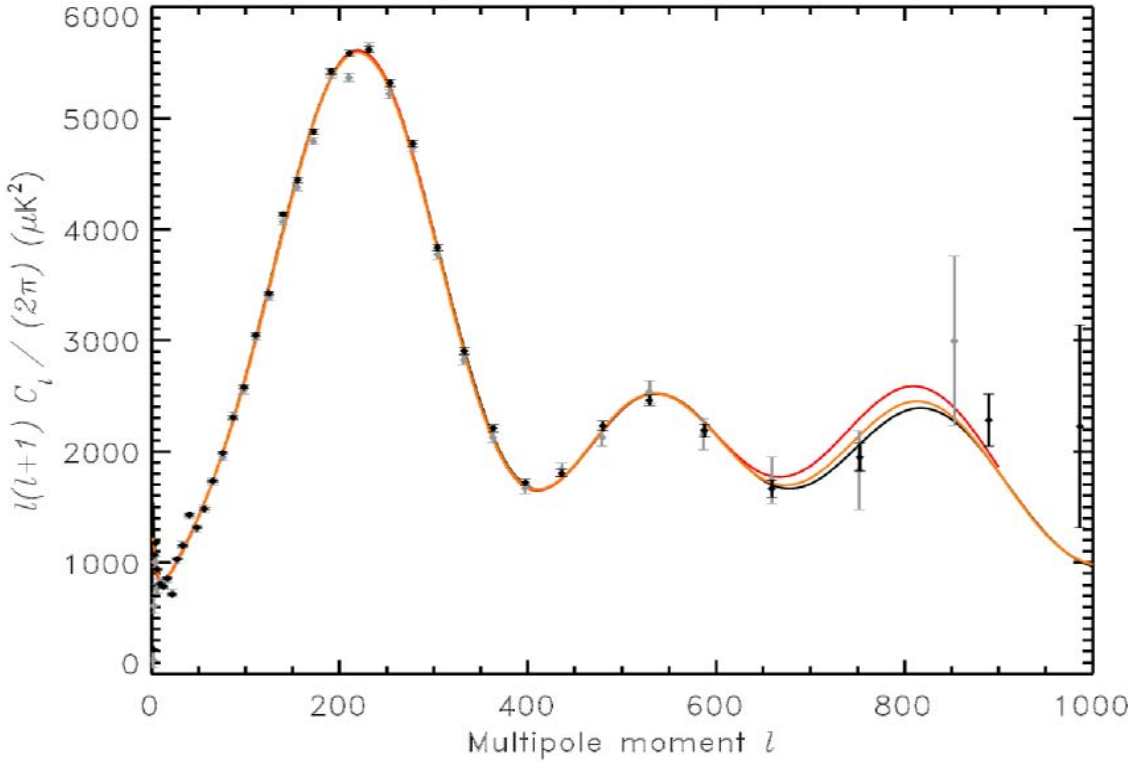
$$\omega = -1.023 \pm 0.090(stat) \pm 0.054(syst)$$

$$\Omega_m = 0.271 \pm 0.021(stat) \pm 0.007(syst)$$



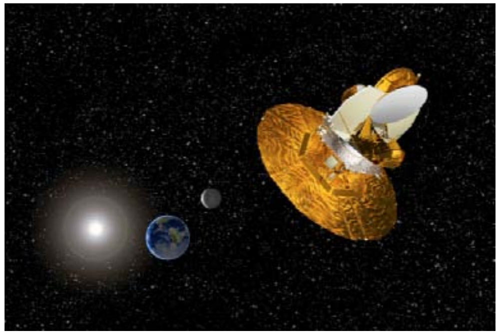
WMAP 3 Year Results

D.N. Spergel et al., astro-ph/0603449



$$(\Omega_m h^2, \Omega_b h^2, h, n_s, \tau, \sigma_8) \quad =$$

$$(0.127^{+0.007}_{-0.013}, 0.0223^{+0.0007}_{-0.0009}, 0.73^{+0.03}_{-0.03}, 0.951^{+0.015}_{-0.019}, 0.09^{+0.03}_{-0.03}, 0.74^{+0.05}_{-0.06})$$



WMAP 3 Year Results

D.N. Spergel et al., astro-ph/0603449

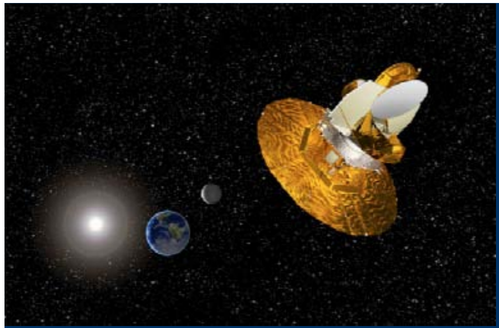
ΛCDM Model

Parameter	WMAP+ SDSS	WMAP+ LRG	WMAP+ SNLS	WMAP + SN Gold	WMAP+ CFHTLS
$100\Omega_b h^2$	$2.233^{+0.062}_{-0.086}$	$2.242^{+0.062}_{-0.084}$	$2.233^{+0.069}_{-0.088}$	$2.227^{+0.065}_{-0.082}$	$2.255^{+0.062}_{-0.083}$
$\Omega_m h^2$	$0.1329^{+0.0056}_{-0.0075}$	$0.1337^{+0.0044}_{-0.0061}$	$0.1295^{+0.0056}_{-0.0072}$	$0.1349^{+0.0056}_{-0.0071}$	$0.1408^{+0.0034}_{-0.0050}$
h	$0.709^{+0.024}_{-0.032}$	$0.709^{+0.016}_{-0.023}$	$0.723^{+0.021}_{-0.030}$	$0.701^{+0.020}_{-0.026}$	$0.687^{+0.016}_{-0.024}$
A	$0.813^{+0.042}_{-0.052}$	$0.816^{+0.042}_{-0.049}$	$0.808^{+0.044}_{-0.051}$	$0.827^{+0.045}_{-0.053}$	$0.846^{+0.037}_{-0.047}$
τ	$0.079^{+0.029}_{-0.032}$	$0.082^{+0.028}_{-0.033}$	$0.085^{+0.028}_{-0.032}$	$0.079^{+0.028}_{-0.034}$	$0.088^{+0.026}_{-0.032}$
n_s	$0.948^{+0.015}_{-0.018}$	$0.951^{+0.014}_{-0.018}$	$0.950^{+0.015}_{-0.019}$	$0.946^{+0.015}_{-0.019}$	$0.953^{+0.015}_{-0.019}$
σ_8	$0.772^{+0.036}_{-0.048}$	$0.781^{+0.032}_{-0.045}$	$0.758^{+0.038}_{-0.052}$	$0.784^{+0.035}_{-0.049}$	$0.826^{+0.022}_{-0.035}$
Ω_m	$0.266^{+0.026}_{-0.036}$	$0.267^{+0.018}_{-0.025}$	$0.249^{+0.024}_{-0.031}$	$0.276^{+0.023}_{-0.031}$	$0.299^{+0.019}_{-0.025}$

WMAP 3 + SNLS: $w = -0.97^{+0.07}_{-0.09}$

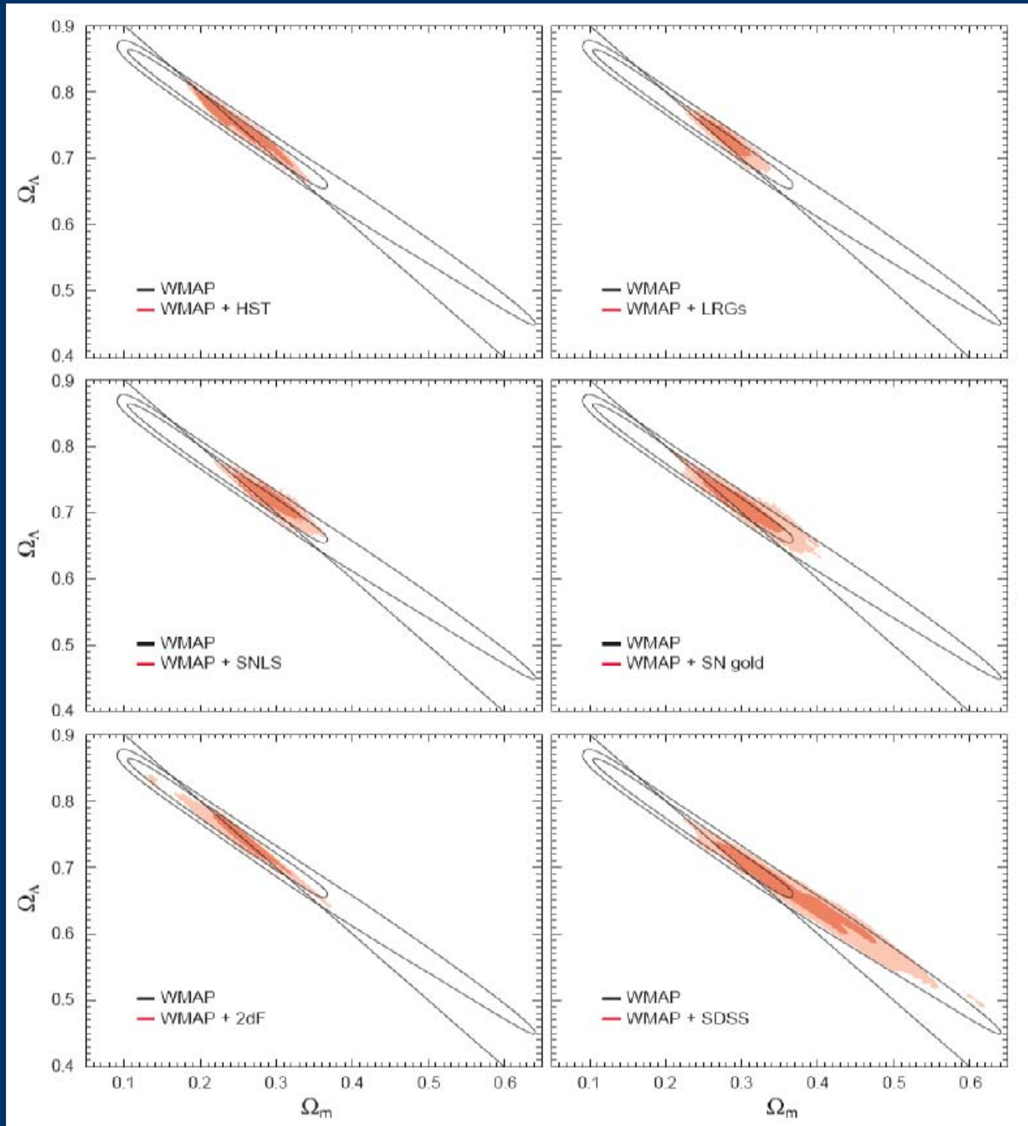
$$\omega = \frac{p}{\rho}$$

$$\Omega_k = -0.015^{+0.020}_{-0.016} \qquad \Omega_\Lambda = 0.72 \pm 0.04$$

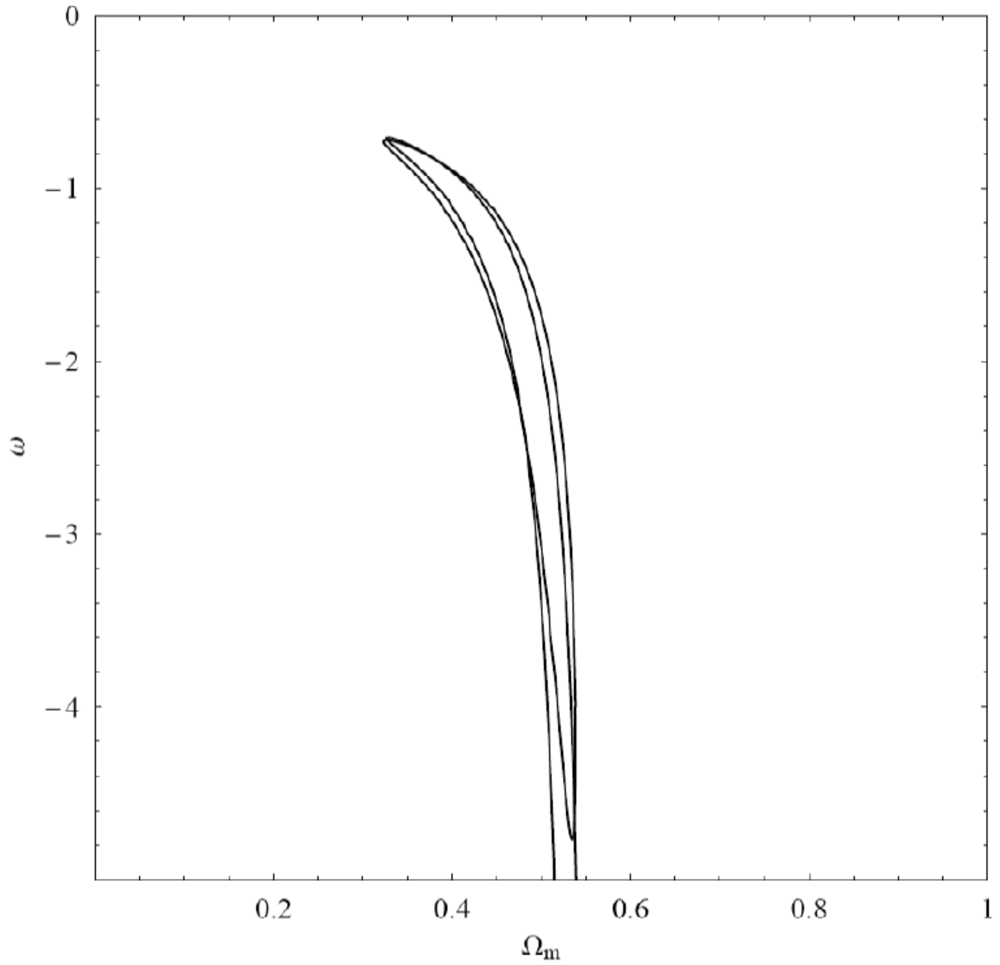


WMAP 3 Year Results

D.N. Spergel et al., astro-ph/0603449



Gamma-ray bursts and Dark Matter



Effect of the increase of high red shift GRBs (90, 500, 1000) for XCDM models

[O.B., Silva, Mon. Not. R. Ast .Soc. 365 (2006) 1149]

SWIFT

NASA
November 2004

Dark Matter Probe
O.B., P. Silva, MNRAS (2006)

Gamma-Ray Bursts Telescope

A Universe dominated by dark components

Cosmic Concordance
(ΛCDM)

$$\Omega_\Lambda \cong 0.72$$

$$\Omega_m \cong 0.28$$

$$\Omega_k \cong 0$$

Quintessence

Varying vacuum energy models

[Bronstein 1933; O.B. 1986; Ratra, Peebles 1988; Wetterich 1988; …]

- $V_0 \exp(-\lambda\phi)$ [Ratra, Peebles 1988; Wetterich 1988; Ferreira, Joyce 1998]
- $V_0\, \phi^{\alpha}$, $\alpha > 0$ [Ratra, Peebles 1988]
- $V_0\, \phi^{\alpha} \exp(\lambda\phi^2)$, $\alpha > 0$ [Brax, Martin 1999, 2000]
- $V_0\, [\exp(M_p / \phi) - 1]$ [Zlatev, Wang, Steinhardt 1999]
- $V_0\, (\cosh \lambda\phi - 1)^p$ [Sahni, Wang 2000]
- $V_0\, \sinh^{-\alpha}(\lambda\phi)$ [Sahni, Starobinsky 2000; Urena-López, Matos 2000]
- $V_0\, [\exp(\beta\phi) + \exp(\gamma\phi)]$ [Barreiro, Copeland, Nunes 2000]

- **Scalar-Tensor Theories of Gravity**

[Uzan 1999; Amendola 1999; O.B., Martins 2000; Fujii 2000; ...]

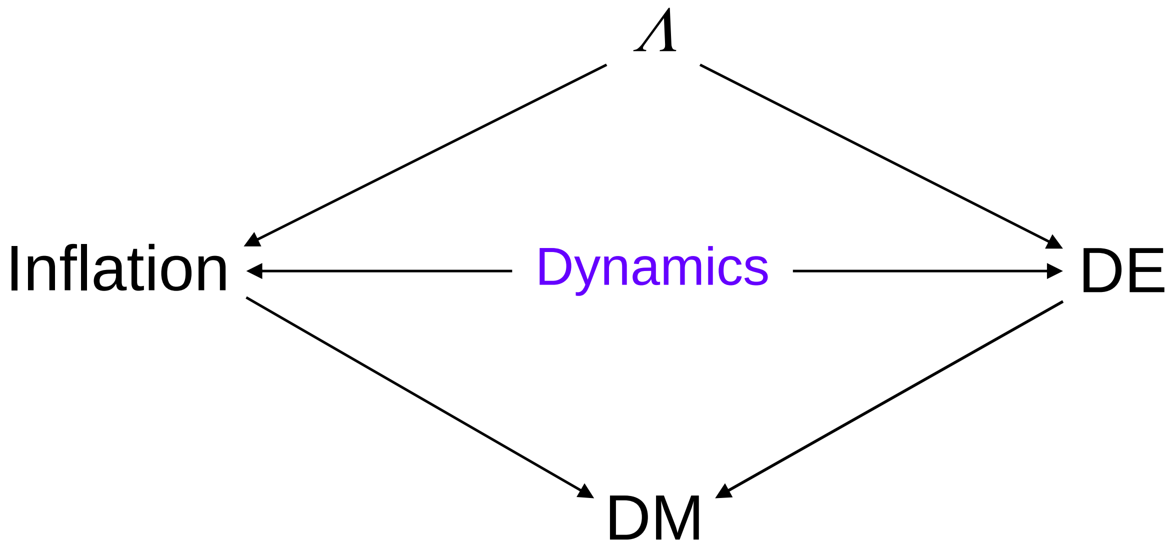
- $V_0 \exp(-\lambda\phi)\, [A + (\phi - B)^2]$ [Albrecht, Skordis 2000]
- $V_0 \exp(-\lambda\phi)\, [a + (\phi - \phi_0)^2 + b(\psi - \psi_0)^2 + c\,\phi(\psi - \psi_0)^2 + d\,\psi(\phi - \phi_0)^2]$

[Bento, O.B., Santos 2002]

Dark Energy and Dark Matter

"Quintessential Inflation"

[Peebles, Vilenkin 1999; Dimopoulos, Valle 2002; O.B., Duvvuri 2006, …]



Dark Energy – Dark Matter interaction

[Amendola 2000]

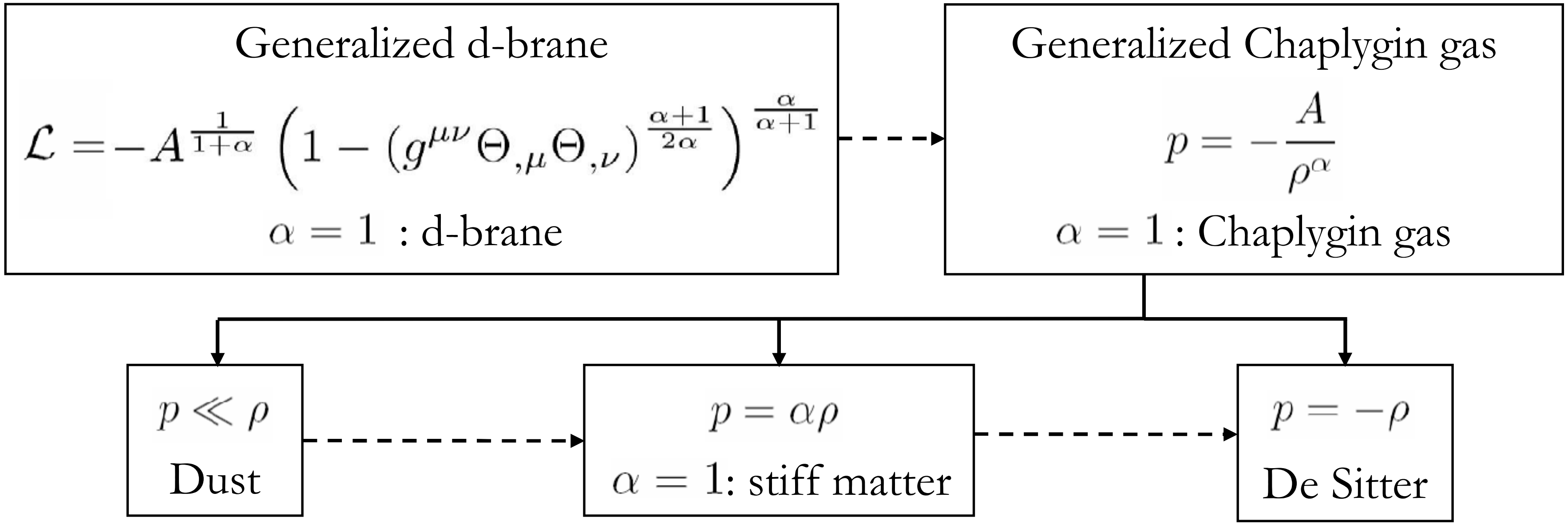
Dark Energy – Dark Matter Unification

[Kamenschik, Moschella, Pasquier 2001]

[Bilic, Tupper, Viollier 2002; Bento, O.B., Sen 2002]

Generalized Chaplygin gas model

- **Unified model for Dark Energy and Dark Matter**



[Bento, O.B., Sen 2002]

Dark Energy - Dark Matter Unification: Generalized Chaplygin Gas Model

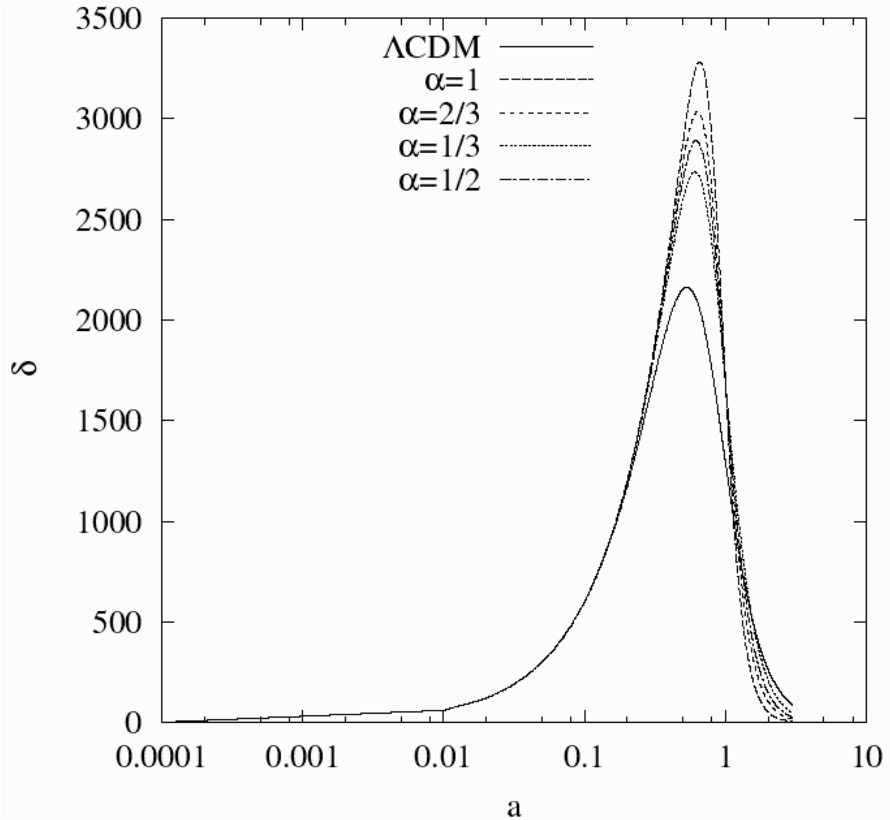
- CMBR Constraints [Bento, O. B., Sen 2003, 2004; Amendola et al. 2004]
- SNe Ia [O. B., Sen, Sen, Silva 2004; Bento, O.B., Santos, Sen 2005]
- Gravitational Lensing [Silva, O. B. 2003]
- Structure Formation *

  [Sandvik, Tegmark, Zaldarriaga, Waga 2004; Bento, O. B., Sen 2004; Bilic, Tupper, Viollier 2005; …]
- Gamma-ray bursts [O. B., Silva 2006]
- Cosmic topology [Bento, O. B., Rebouças, Silva 2006]
- Inflation [O.B., Duvvuri 2006]

Background tests:

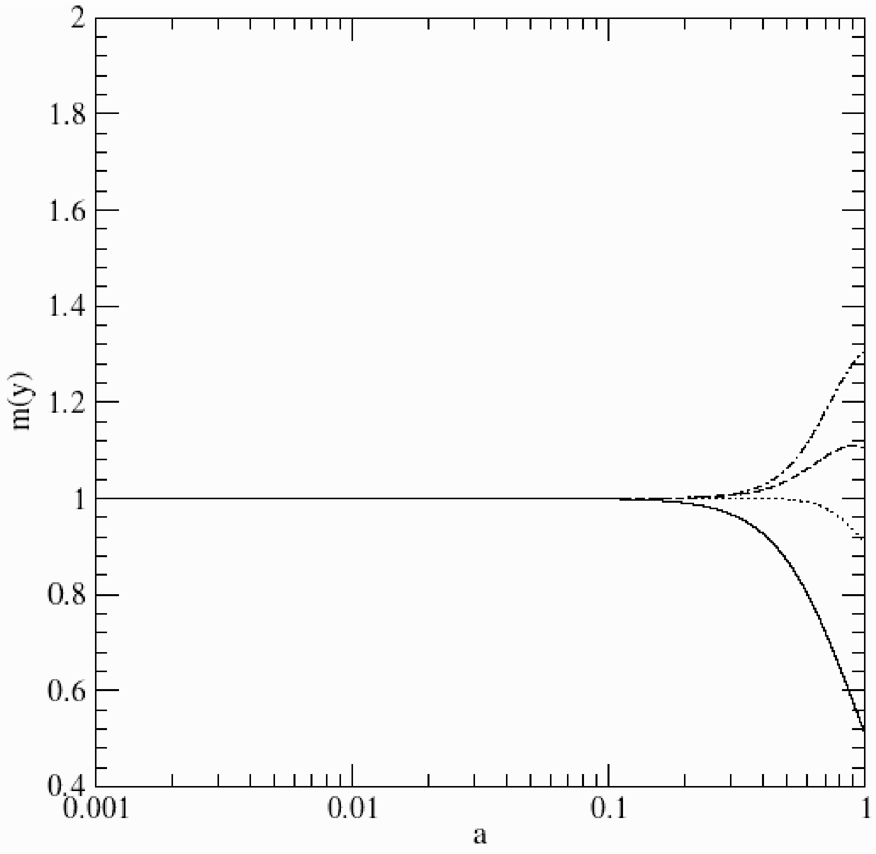
$$\alpha \le 0.6, \quad 0.65 \le A_s \le 0.85 \qquad A_s \equiv \frac{A}{\rho_{Ch0}^{1+\alpha}}$$

Structure formation: $\alpha \le 0.2$

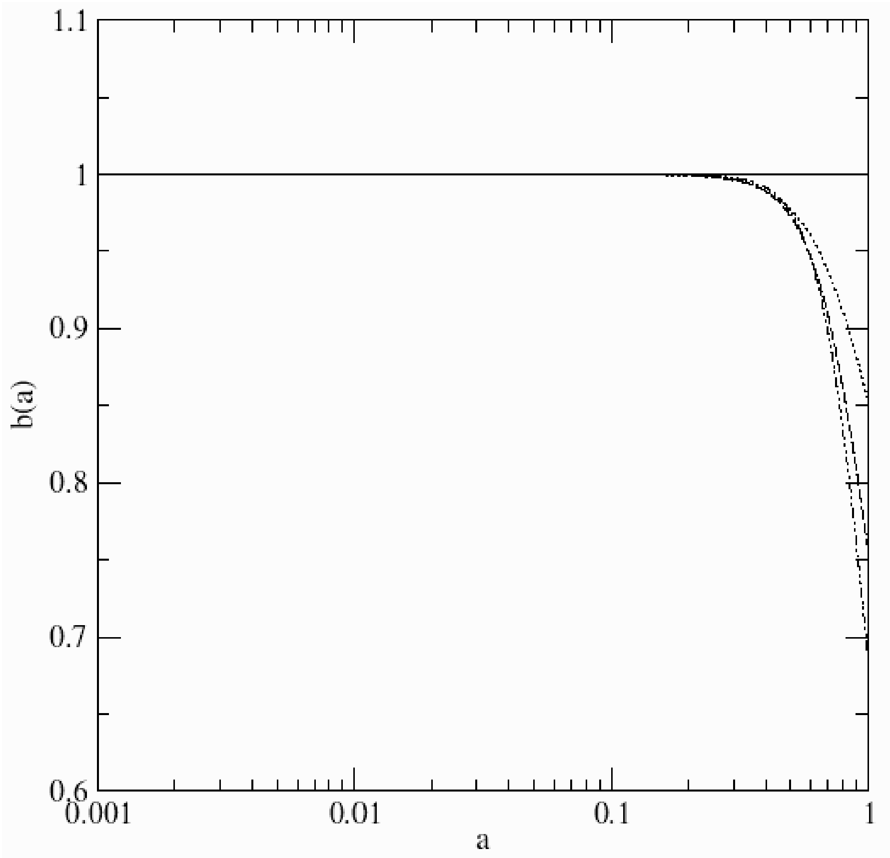


Density constrast $\delta(a_{eq})$ for different values of α, as compared with ΛCDM.

[Bento, O. B., Sen 2002]

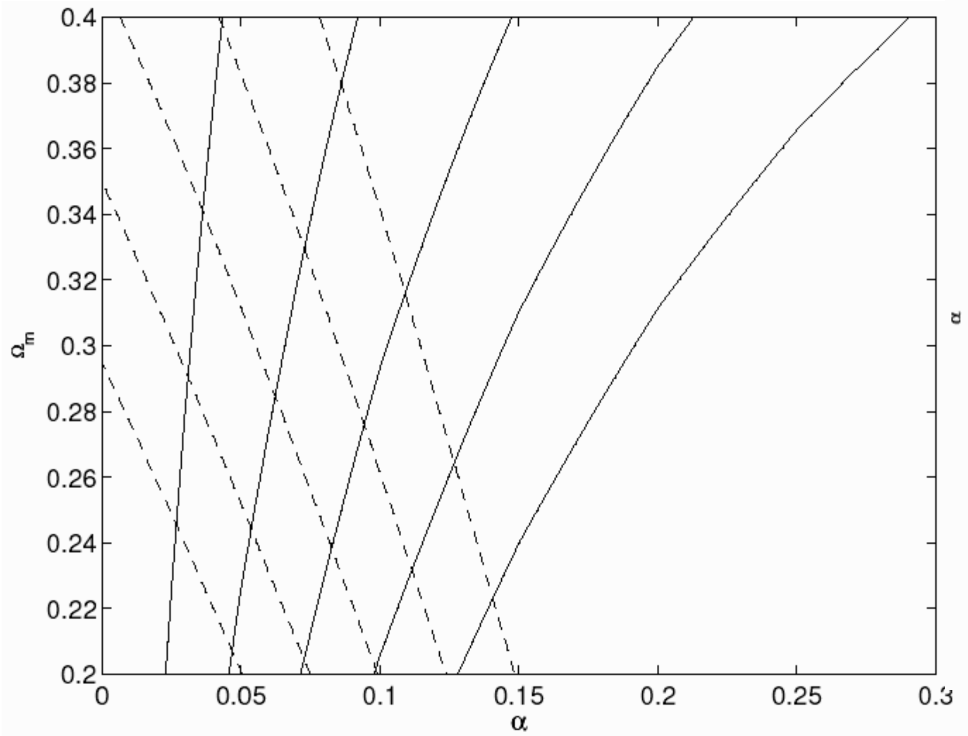


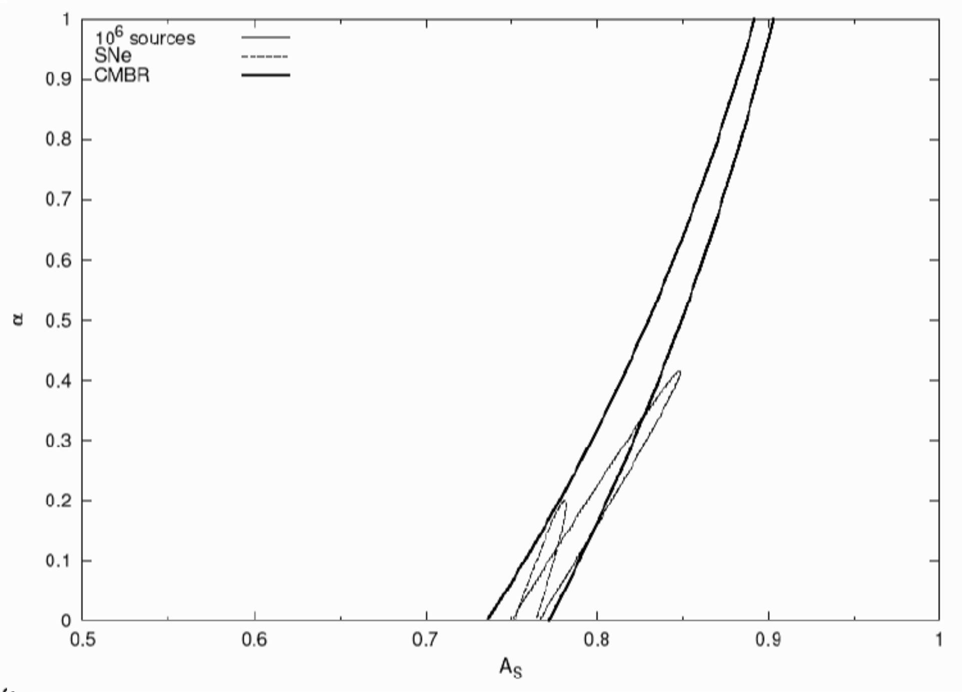
The growth factor $m(y)$ as a function of the scale factor a. The solid, dotted, dashed and dash-dot lines correspond to $\alpha = 0, 0.2, 0.4, 0.6$ respectively. It is assumed: $\Omega_{dm0} = 0.25$, $\Omega_{\Lambda 0} = 0.7$, $\Omega_{b0} = 0.05$ and $\alpha = 0.2$



The bias b as a function of the scale factor a. The solid, dotted, dashed and dash-dot lines correspond to $\alpha = 0, 0.2, 0.4, 0.6$ respectively. It is assumed: $\Omega_{dm0} = 0.25$, $\Omega_{\Lambda 0} = 0.7$, $\Omega_{b0} = 0.05$ and $\alpha = 0.2$

[Bento, O. B., Sen 2004]





Joint **68% CL** confidence regions for Model II using both SNe, gravitational lensing statistics and CMBR constraints.

Contours for parameters ***b*** and ***m*** in the $\Omega_m - \alpha$ plane. Solid lines are for ***b*** whereas dashed lines are for ***m***. For ***b***, contour values are ***0.98***, ***0.96***, ..., ***0.9*** from left to right. For ***m***, contour values are ***0.6***, ***0.65***, ..., ***0.8*** from left to right.

[Bento, O. B., Sen 2004]

[Silva, O. B. 2003]

Pioneer 10 anomalous deceleration

Pioneer 10/11 anomalous deceleration (20 AU – 70 AU):

$$a_{Pio} = (8.5 \pm 1.3) \times 10^{-10} m/s^2$$

[Anderson, Laing, Lau, Liu, Nieto, Turyshev 2002]

Cause:

Systematical effects ?
Thermal effects ? **[Scheffer 2003]**
Kuiper Belt gravity ? No ! **[Anderson et al. 2002, Nieto 2005, O.B., Vieira 2005]**
Scalar field ? **[O.B., Páramos 2004]**
Post-Newtonian model with running coupling consts. ? **[Jaekel, Reynaud 2005]**
…

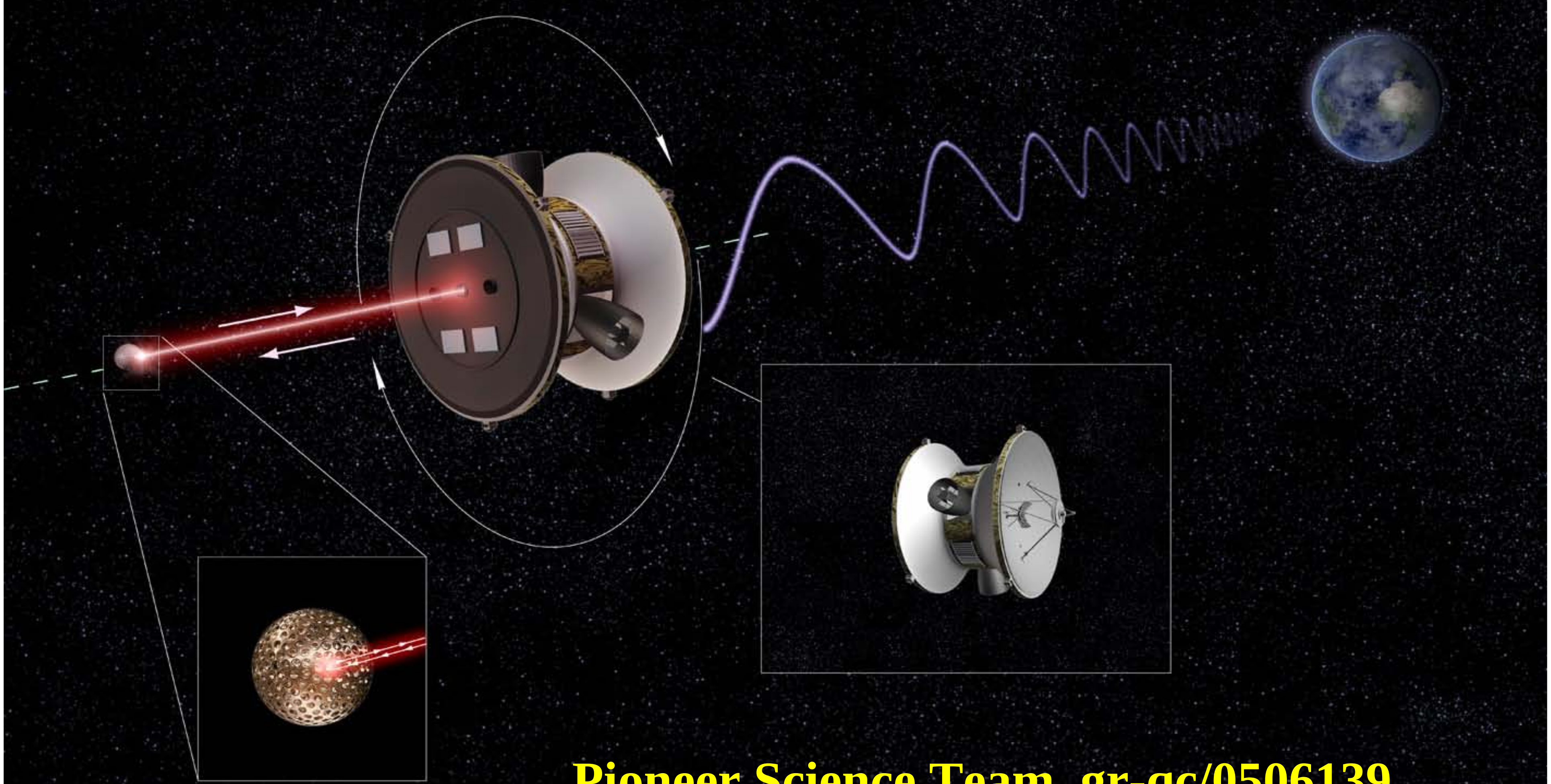
Deceleration due to dragging: $a_{Pio} = O(1) \frac{\rho_{Med.} v_{Pio}^2 A_{Pio}}{m_{Pio}}$

$$v_{Pio} = 11.6 - 12.2 km/s, A_{Pio} = 5.9 m^2, m_{Pio} = 241 kg \Rightarrow \rho_{Med.} = 3 \times 10^{-19} g/cm^3$$

DM $\rho_{DM} \cong \rho_{Halo} \cong 6 \times 10^{-24} g/cm^3 \Rightarrow a_{DM} \cong 2 \times 10^{-5} a_{Pio}$

DE $\rho_{DE} \cong 6 \times 10^{-30} g/cm^3 \Rightarrow a_{DE} \cong -2 \times 10^{-11} a_{Pio}$

A Mission to Test the Pioneer Anomaly



Pioneer Science Team, gr-qc/0506139

Dark Matter Detection

[Baudis 2005]

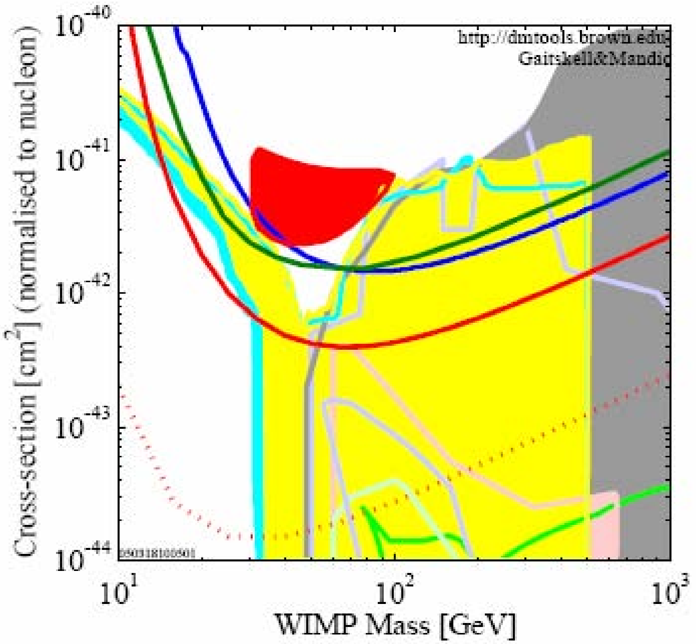


Figure 5: Experimental results and theoretical predictions for spin-independent WIMP nucleon cross sections versus WIMP mass. The data (from high to low cross sections) show the DAMA allowed region (red) [24], the latest EDELWEISS result (blue) [29], the ZEPLIN I preliminary results (green) [30] and the CDMS results from Tower 1 at Soudan (red) [26]. Also shown is the expectation for 5 CDMS towers at Soudan (red dashed). The SUSY theory regions are shown as filled regions or contour lines, and are taken from [31].

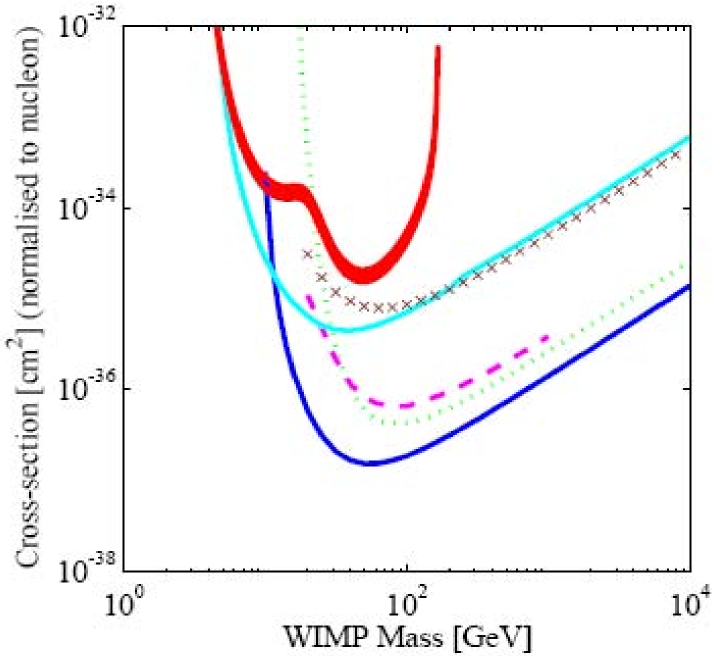
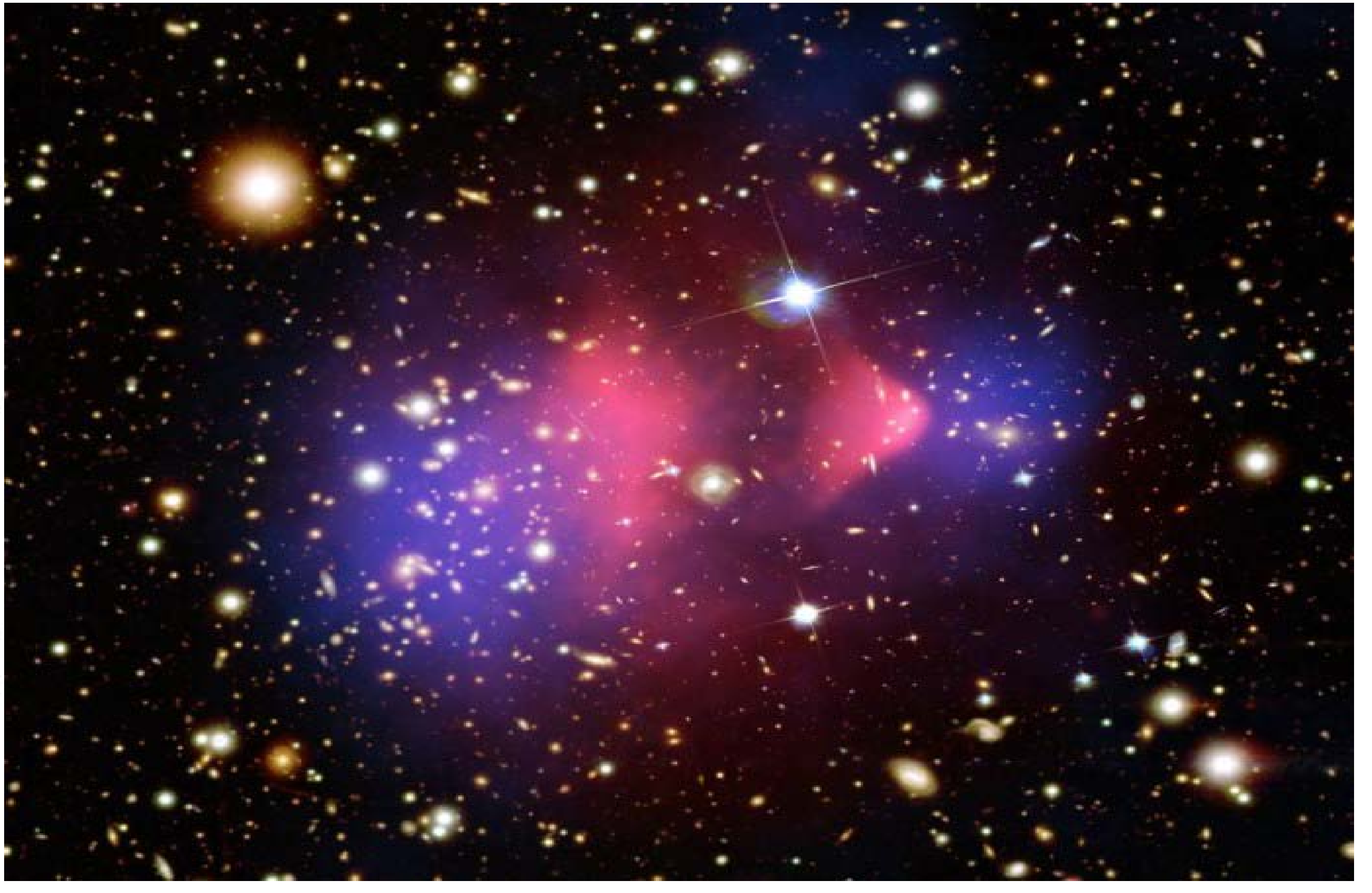


Figure 6: Experimental results for spin-dependent WIMP couplings (90% C.L. contours), for the case of a pure neutron coupling. The curves (from high to low cross sections) show the DAMA annual modulation signal (filled red region), the CDMS Soudan Si data (red crosses), the CDMS Stanford Si data (cyan), EDELWEISS (magenta dashed), DAMA/Xe (green dotted) and the CDMS Soudan Ge data (solid blue). For details and references see [38].

Merging Galaxy Cluster 1E 0657-56

[Clowe et al., astro-ph/0608407]



"Bullet" Cluster

Self-Interacting Dark Matter

[Spergel, Steinhardt 2000]

Motivation: **“cuspy core” problem**

Model:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2} m_\phi^2 \phi^2 - \frac{g}{4!}\phi^4 + g' v \phi^2 h$$

Higgs decay width

$$\Gamma(h \to \phi\phi) = 5.23 \left(\frac{m_h}{115 \text{ GeV}} \right)^{-1} g'^2 \text{ GeV}$$

[Bento, O.B., Rosenfeld, Teodoro 2000]
[Silveira, Zee 1988]
[Bento, O.B., Rosenfeld 2001]

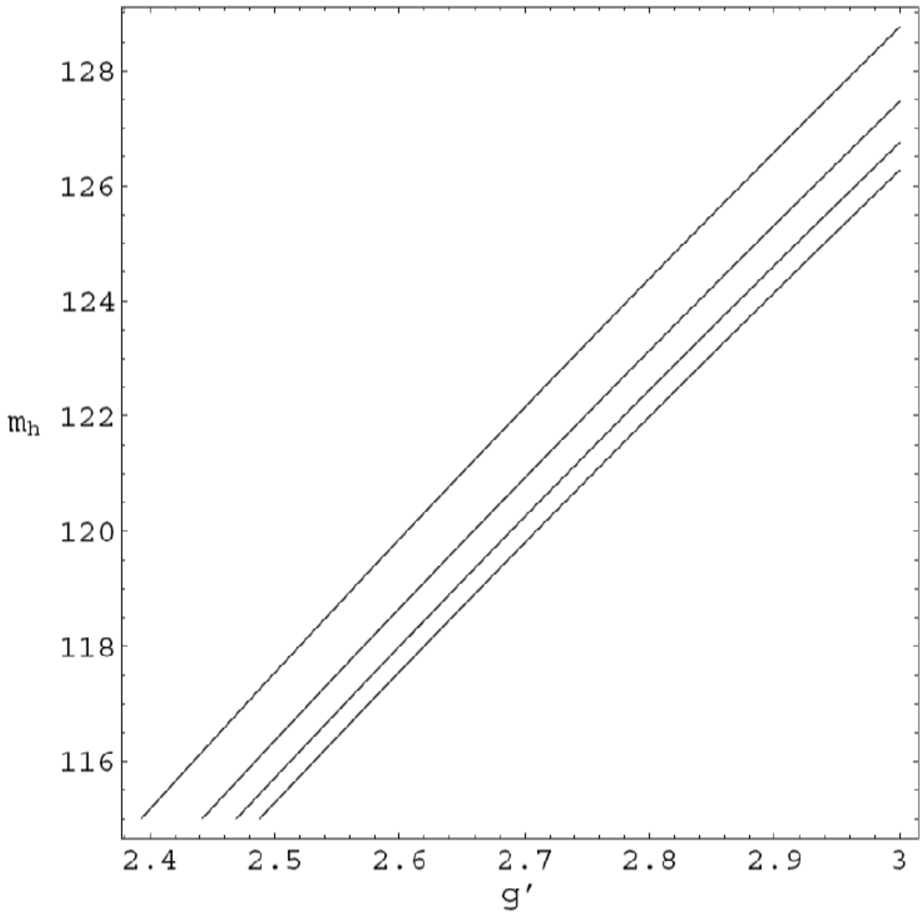


FIG. 2. Contour of $\Omega_\phi h^2 = 0.3$ as a function of m_h (in GeV) and g', for $m_\phi = 0.5$ GeV (top), 1.0, 1.5 and 2 GeV (bottom).

[Bento, O.B., Rosenfeld 2001]

Direct Dark Energy Detection ?

- **Spectrum noise in Josephson junctions** **[Beck, Mackey 2005]**

$$\frac{\pi\hbar}{c^3}\nu_c^4 \cong \rho_{DE} = (3.9 \pm 0.4)\frac{GeV}{m^3} \Rightarrow \nu_c \cong (1.69 \pm 0.05)\times 10^{12}\,Hz$$

- **No! Zero-energy fluctuations are not measurable …**

[Jetzer, Straumann 2005]

- **DE-gauge field coupling: variation of the “fine structure constant”**

[Olive, Pospelov 2002; Gardner 2003; …]

[O.B., Lehnert, Potting, Ribeiro 2003; Bento, O.B., Santos 2004]

Variation of the electromagnetic coupling via direct Q-electromagnetic interaction

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{2} R + \mathcal{L}_b + \mathcal{L}_Q + \mathcal{L}_{em} \right]$$

$$\mathcal{L}_Q = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi + \frac{1}{2} \partial^\mu \psi \partial_\mu \psi - V(\phi, \psi)$$

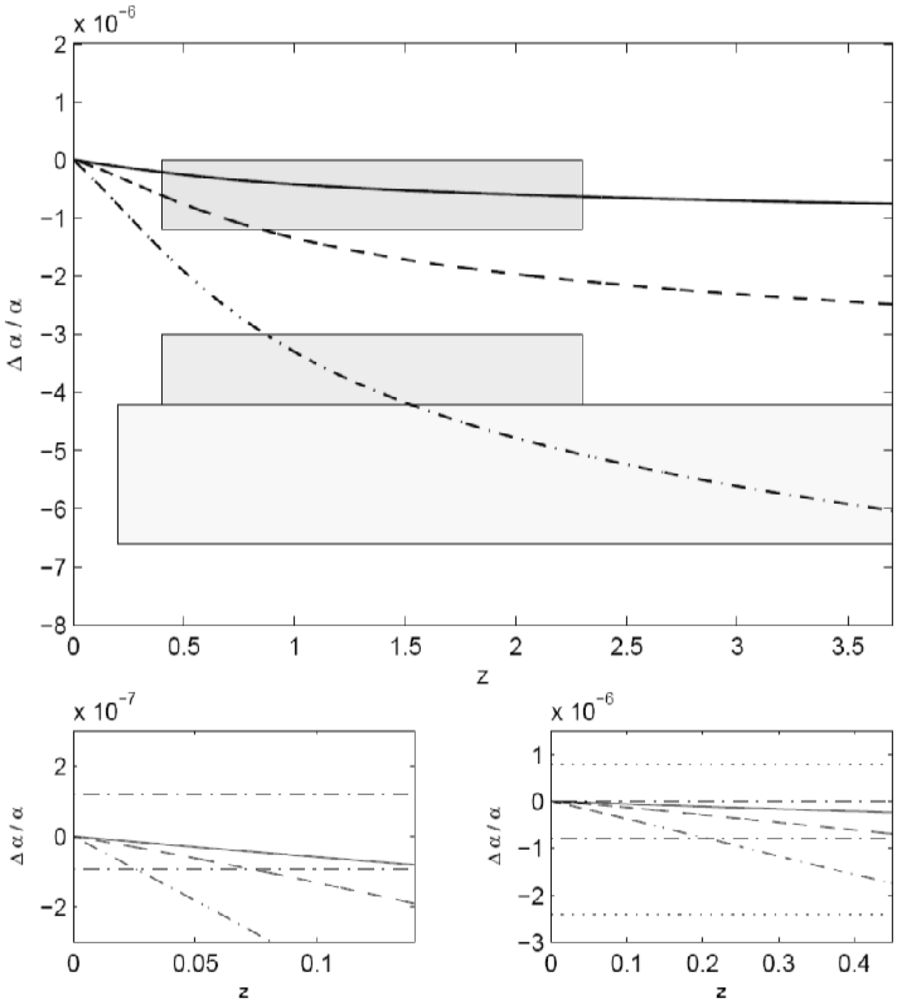
$$V(\phi, \psi) = e^{-\lambda \phi} P(\phi, \psi)$$

$$\begin{aligned} P(\phi, \psi) \ = \ & A \ + (\phi - \phi_*)^2 + B \ (\psi - \psi_*)^2 \\ & + C \ \phi(\psi - \psi_*)^2 + D \ \psi(\phi - \phi_*)^2 \end{aligned}$$

$$\mathcal{L}_{em} = -\frac{1}{4} B_F(\phi, \psi) F_{\mu\nu} F^{\mu\nu}$$

$$B_F(\phi, \psi) = 1 - \zeta_1(\phi - \phi_0) - \zeta_2(\psi - \psi_0)$$

[Bento, O.B., Santos 2004]



Oklo

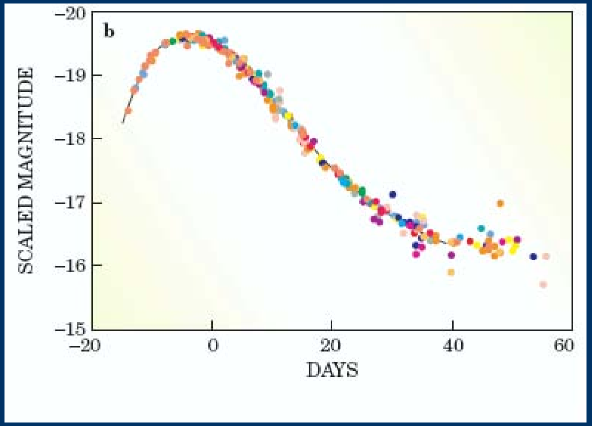
Meteorites

FIG. 4: Evolution of α for a transient acceleration model with 2×10^{-6} and $\zeta_2 = 8\times10^{-5}$ (full line), $\zeta_1 = 5.3\times10^{-6}$ and $\zeta_2 = 3\times10^{-5}$ (dashed line), $\zeta_1 = 1.4\times10^{-5}$ and $\zeta_2 = 7\times10^{-4}$ (dash-dotted line). Line and box conventions are those of

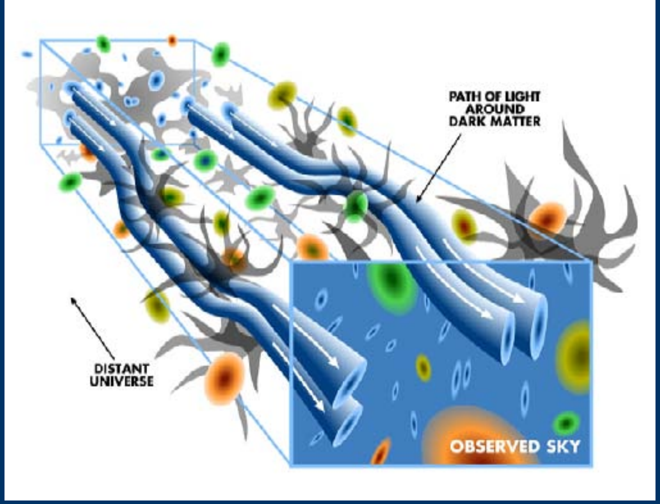
[Bento, O.B., Santos 2004]

Large Dark Energy Surveys

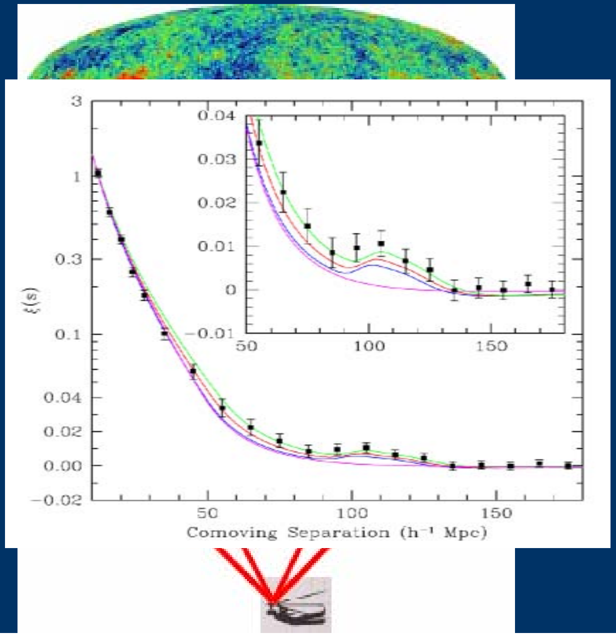
SNAP, DUNE…



Supernovae **Standard Candles**
Luminosity Distance



Cosmic Shear **Evolution of DM perts.**



Baryon Acoustic Oscillations
Standard ruler
Angular diameter distance

Dark Matter

Modified Newtonian Dynamics (MOND)

[Milgrom 1983, Bekenstein, Milgrom 1984, ..., Bekenstein 2004]

Motivation: Flatness Rotation Curve of Galaxies

$$\vec{a} = \mu\left(\frac{|\vec{g}|}{a_0}\right)\vec{g} = -\mu\left(\frac{|\vec{g}|}{a_0}\right)\nabla\phi$$

$$\mu(x) = \begin{cases} 1 \;\; if \;\; x \gg 1 \\ x \;\; if \;\; x \ll 1 \end{cases}$$

$a_0 \approx 1.2 \times 10^{-10}\ m/s^2$ **- universal acceleration**

Tully-Fisher Law: $L_H \propto v_c^4$ **as** $L_H \propto M = (Ga_0)^{-1} v_c^4$

TeVeS^2 version: F-function problem

$$S_s = -\frac{1}{2}\int\left[\sigma^2 h^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta} + \frac{1}{2}G\ell^{-2}\sigma^4 F(kG\sigma^2)\right](-g)^{1/2}d^4x$$

MOND

Tensor-Vector-Scalar field theory, $S = S_g + S_s + S_v + S_m$:

$$S_g = (16\pi G)^{-1} \int g^{\alpha\beta} R_{\alpha\beta} (-g)^{1/2} d^4x$$

$$S_s = -\frac{1}{2} \int \left[\sigma^2 h^{\alpha\beta} \phi_{,\alpha} \phi_{,\beta} + \frac{1}{2} G \ell^{-2} \sigma^4 F(kG\sigma^2) \right] (-g)^{1/2} d^4x$$

$$S_v = -\frac{K}{32\pi G} \int \left[g^{\alpha\beta} g^{\mu\nu} \mathfrak{U}_{[\alpha,\mu]} \mathfrak{U}_{[\beta,\nu]} - 2(\lambda/K)(g^{\mu\nu} \mathfrak{U}_\mu \mathfrak{U}_\nu + 1) \right] (-g)^{1/2} d^4x$$

$$S_m = \int \mathcal{L}(\tilde{g}_{\mu\nu}, f^\alpha, f^\alpha{}_{|\mu}, \cdots)(-\tilde{g})^{1/2} d^4x$$

Conformal transformation to the physical metric: $(-\tilde{g})^{1/2} = e^{-2\phi}(-g)^{1/2}$

MOND in Post-Newtonian regime

Scalar field: $\phi(r) = \phi_c - \frac{kGm}{4\pi r}$

Vector field:

$$\left(\mathfrak{U}^{[\alpha;\beta]}{}_{;\beta} + \mathfrak{U}^{\alpha}\mathfrak{U}_{\gamma}\mathfrak{U}^{[\gamma;\beta]}{}_{;\beta}\right) + 8\pi G\sigma^2\left[\mathfrak{U}^{\beta}\phi_{,\beta}\, g^{\alpha\gamma}\phi_{,\gamma} + \mathfrak{U}^{\alpha}(\mathfrak{U}^{\beta}\phi_{,\beta})^2\right]$$
$$= 8\pi G(1 - e^{-4\phi})\left[g^{\alpha\mu}\mathfrak{U}^{\beta}\tilde{T}_{\mu\beta} + \mathfrak{U}^{\alpha}\mathfrak{U}^{\beta}\mathfrak{U}^{\gamma}\tilde{T}_{\gamma\beta}\right]$$

- **Timelike vector tracks the metric** **[Bekenstein 2004]**

$\mathfrak{U}^{\alpha} = (\sqrt{-g^{00}}, 0, 0, 0)$ **consistent with eq. of motion**

- **Einstein eq.**

$$G_{\alpha\beta} = 8\pi G\left[\tilde{T}_{\alpha\beta} + (1 - e^{-4\phi})\mathfrak{U}^{\mu}\tilde{T}_{\mu(\alpha}\mathfrak{U}_{\beta)} + \tau_{\alpha\beta}\right] + \Theta_{\alpha\beta}$$

$$\tau_{\alpha\beta} \equiv \sigma^2\left[\phi_{,\alpha}\phi_{,\beta} - \frac{1}{2}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu}\, g_{\alpha\beta} - \mathfrak{U}^{\mu}\phi_{,\mu}\left(\mathfrak{U}_{(\alpha}\phi_{,\beta)} - \frac{1}{2}\mathfrak{U}^{\nu}\phi_{,\nu}\, g_{\alpha\beta}\right)\right]$$

$$\Theta_{\alpha\beta} \equiv K\left(g^{\mu\nu}\mathfrak{U}_{[\mu,\alpha]}\mathfrak{U}_{[\nu,\beta]} - \frac{1}{4}g^{\sigma\tau}g^{\mu\nu}\mathfrak{U}_{[\sigma,\mu]}\mathfrak{U}_{[\tau,\nu]}\, g_{\alpha\beta}\right) - \lambda\mathfrak{U}_{\alpha}\mathfrak{U}_{\beta}$$

- **Parametrization of the metric**

$$g_{\alpha\beta}\, dx^\alpha dx^\beta = -e^\nu dt^2 + e^\varsigma[d\varrho^2 + \varrho^2(d\theta^2 + \sin^2\theta d\varphi^2)]$$

$$-g_{00} = e^\nu = 1 - R/r + \alpha_2(R/r)^2 + ...$$

$$g_{rr} = e^\sigma = 1 + \beta_1 R/r + \beta_2(R/r)^2 + ...$$

- **Expansion of Einstein eq. up to order r^{-4}**

$$\lambda = \frac{K(2+\beta_1-4\alpha_2)}{4}\frac{R^2}{r^4} \qquad 8\pi G\tau_{00} = 8\pi G\tau_{rr} = \frac{kR^2}{16\pi r^4}$$

$$\theta_{00} = \frac{K(-2\beta_1-3+8\alpha_2)}{8}\frac{R^2}{r^4} \quad , \quad \theta_{rr} = -\frac{K}{8}\frac{R^2}{r^4}$$

- **Solution:** $\beta_1 = 1 \;,\; \alpha_2 = \frac{1}{2} \;,\; \beta_2 = \frac{3}{8} + \frac{1}{16}K - \frac{k}{32\pi}\left(\frac{R}{r}\right)^2$

- **Transformation into physical, isotropic PPN metric yields**

$$\beta = 1 \quad , \quad \gamma = 1$$

(like GR !)

Dynamic solution for the vector field

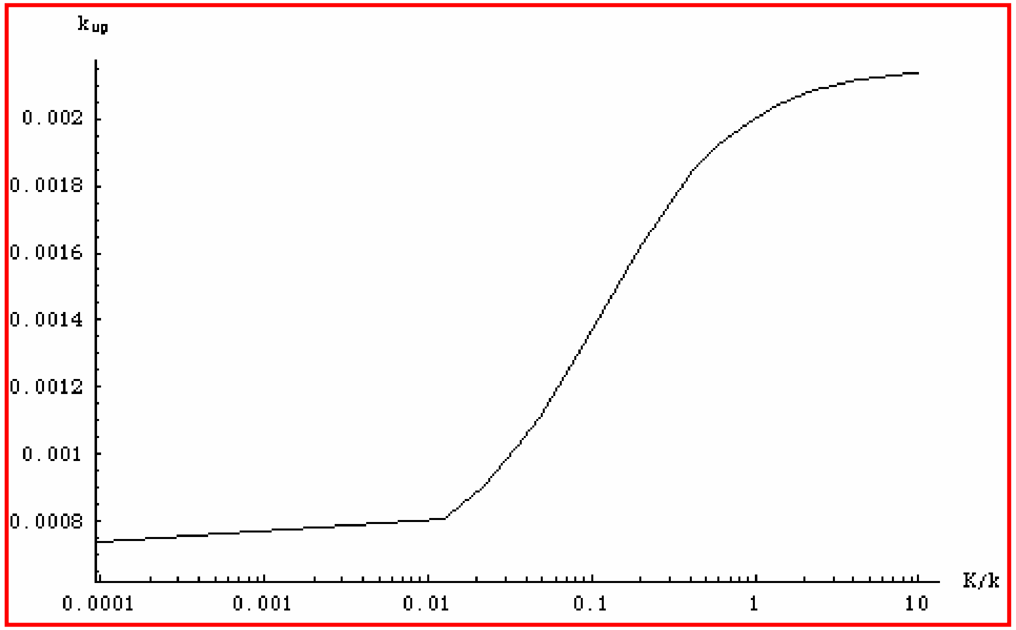
- **Assume** $\mathfrak{U}^\alpha = (\mathfrak{U}^0(r), \mathfrak{U}^r(r), 0, 0) \quad , \quad \mathfrak{U}^\alpha \mathfrak{U}_\alpha = -1$
- **Solution:**

$$\beta = 1 + \frac{K}{\left(1 + 9\frac{K\pi}{k}\right)^2} - \frac{k}{\pi}\left(\frac{7}{8} + \frac{2}{1 + 9\frac{K\pi}{k}}\right) \quad , \quad \gamma = 1$$

(different from GR!)

Constraint $|\beta - 1| < 6 \times 10\text{-}4$
allows for $k < k_{up}$

[O.B., Páramos, to appear]



Consistency with Cosmology

- **CMBR**

  i) (Potentially) compatible

  [Skordis, Mota, Ferreira, Boehm 2005]

  ii) Problem with the third peak

  $$\frac{P_{\Lambda CDM}}{P_{MOND}} \simeq 2\times 10^{2}$$

  [Slosar, Melchiorri, Silk 2005]

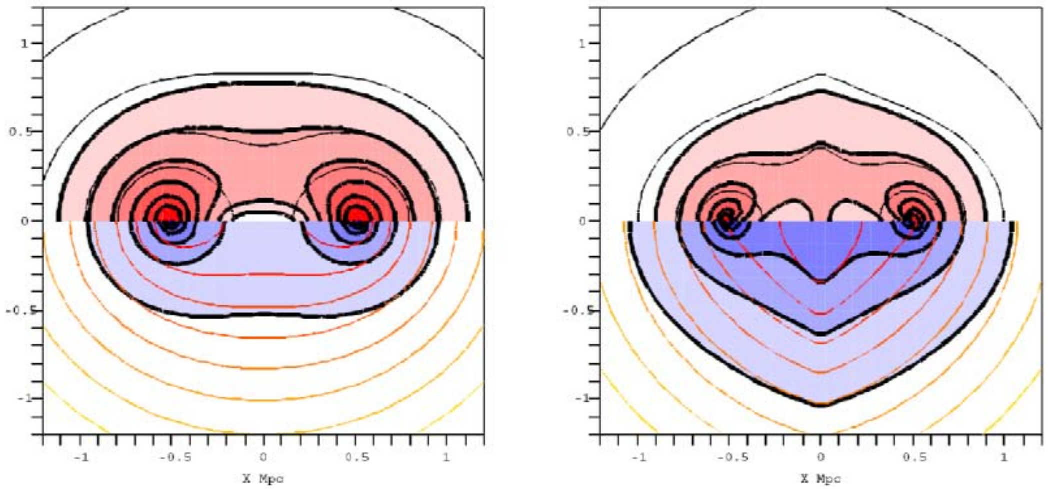
- **Gravitational lensing – great potential for testing**

  [Zhao, Bacon, Taylor, Horne 2005]

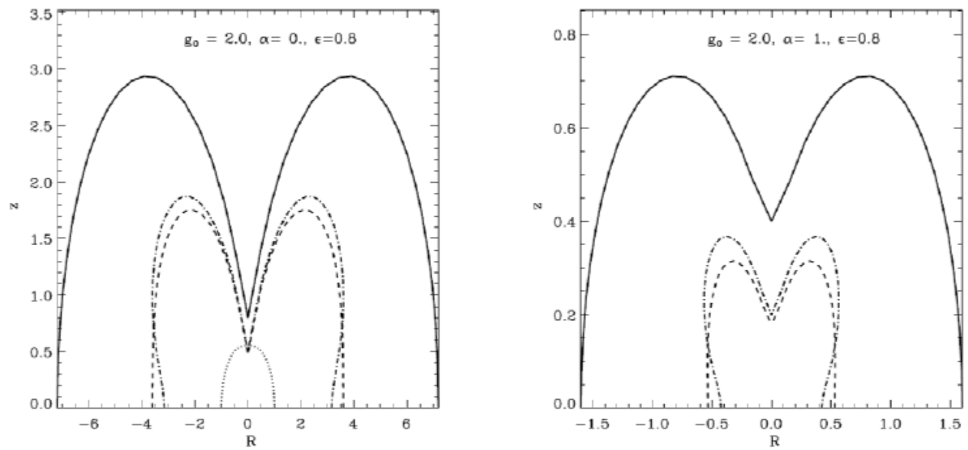
Can MOND take a bullet ?

[Angus, Famaey, Zhao 2006]

- **Doubled and tripled-centered baryonic systems**



- **Multi-field TeVeS gravity**



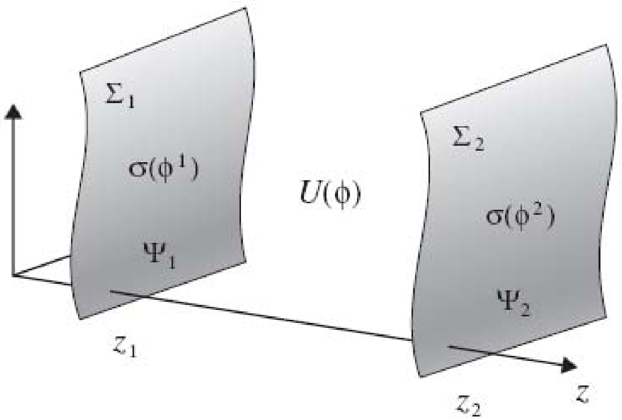
Newtonian (baryons + DM) (full)
MOND (dashed)
TeVeS (scalar field) (dot-dashed)

Dark Energy
Self-accelerating gravity models

[Dvali, Gabadadze, Porrati 2000; Deffayet 2001; Freese, Lewis 2002; …]

Motivation: 5D Braneworlds

E.g. BPS-branes (Randall-Sundrum, Dilatonic): bulk scalar field



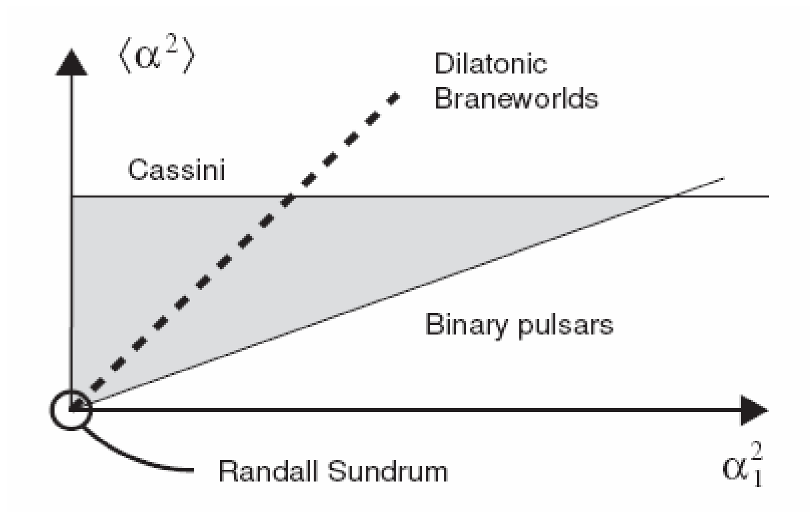
$$U = \partial_\phi \sigma - \sigma^2$$

$$\alpha \equiv \frac{1}{\sigma}\frac{\partial \sigma}{\partial \phi}$$

PPN:

$$\gamma - 1 = -\frac{4}{3}\left\langle \alpha^2 \right\rangle$$

$$\beta - 1 = \frac{1}{9}\left(\left\langle \alpha^2 \right\rangle - \alpha_1^2\right)$$



[Palma 2006]

Self-accelerating gravity models

- "Infrared" Modifications of Gravity (***r_c= 3 Gpc*** – crossover constant):
- PPN: $\beta = 1, \gamma = 1$
- Lense-Thirring effect unchanged [Iorio 2006]
- DGP $H^2 + \frac{k}{a^2} = \left(\sqrt{\frac{8\pi\,\rho}{3M_{Pl}^2} + \frac{1}{4r_c^2}} + \frac{1}{2r_c} \right)^2$ [Dvali, Gabadadze, Porrati 2000] [Deffayet 2001]

$$r_c = \frac{M_{Pl}^2}{2M_5^3}$$

- DT $H^2 + \frac{k}{a^2} = \frac{8\pi\,\rho}{3M_{Pl}^2} + \frac{1}{r_c^{2-\beta}} \left(H^2 + \frac{k}{a^2} \right)^{\beta/2}$ [Dvali, Turner 2003]
- Cardassian $H^2 = \frac{8\pi}{3M_{\rm Pl}^2} (\rho + b\rho^n) - \frac{k}{a^2}$ [Freese, Lewis 2002]

$$\ddot{a}(t \leq t_0) \rightarrow n < 2/3$$

Cosmological Constraints

Model	Parameters	SN	SN+SDSS	SN+SDSS+CMBR	SN+SDSS+CMBR+T
	Ω_m	0.46	0.28	0.28	0.29
ΛCDM	Ω_k	−0.44	0.033	−0.003	−0.020
	χ^2	181.24	183.76	183.93	184.44
	Ω_m	0.33	0.27	0.27	0.28
DGP	Ω_k	−0.56	−0.32	0.014	−0.021
	χ^2	181.36	182.04	190.53	192.34
	β	−10	1.0	0.26	0.23
DT	Ω_m	0.49	0.27	0.28	0.29
	Ω_k	0.032	−0.32	−0.002	−0.02
	χ^2	180.55	182.04	183.54	184.11
	n	−6.15	0.33	0.042	0.041
Card	Ω_m	0.33	0.27	0.28	0.29
	Ω_k	0.33	−0.76	−0.003	−0.020
	χ^2	178.77	182.08	183.72	184.23

TABLE I: Best fit parameters for the ΛCDM, DGP, DT and Cardassian models for different combinations of observational constraints (SN = SNe Ia gold sample, SDSS = SDSS baryon acoustic oscillations, CMBR = CMBR shift parameter and T =Poincaré dodecahedral space topology for $\gamma = 50° \pm 6°$).

[Bento, O.B., Rebouças, Santos 2006]

Cosmological Constraints

- **Baryon Acoustic Oscillations**

$$\mathcal{A} = \sqrt{\Omega_m} \left(\frac{H_0}{H(z_{lrg})} \right)^{1/3} \left[\frac{1}{z_{lrg}\sqrt{|\Omega_k|}} \, \mathcal{S}\left(y(z_{lrg})\right) \right]^{2/3}$$

$$S(x) \equiv (\sin(x), \sinh(x), x) \text{ for } \Omega_k < 0,\ \Omega_k > 0\ ,\ \Omega_k = 0$$

LRG (SDSS): $\mathcal{A}_0 = 0.469 \pm 0.017$ **[Eisenstein et al. 2005]**

$$z_{lrg} = 0.35$$

- **CMBR Shift Parameter ($\ell \rightarrow \mathcal{R}\ell$)** $\mathcal{R} = \sqrt{\frac{\Omega_m}{|\Omega_k|}}\, S(y(z_{lss}))$

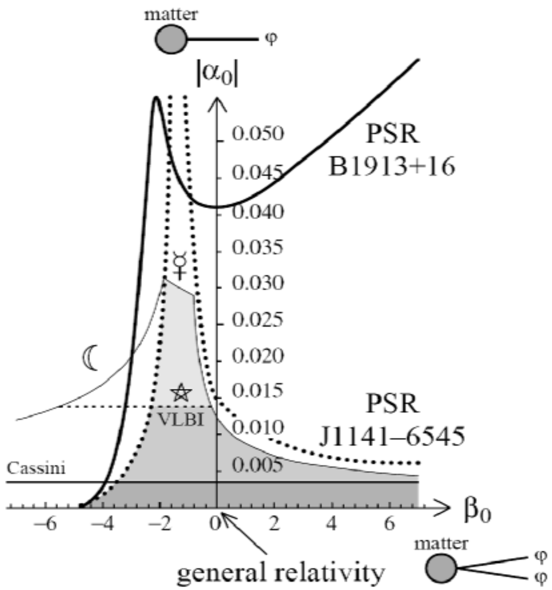
WMP 3: $\mathcal{R}_0 = 1.716 \pm 0.062$

Scalar-Tensor Theories of Gravity

$$S = \frac{c^3}{4\pi G}\int\sqrt{-g}\left\{\frac{R}{4} - \frac{1}{2}(\partial_\mu\varphi)^2 - V(\varphi)\right\} + S_{\rm matter}\left[\text{matter}; \tilde{g}_{\mu\nu} \equiv A^2(\varphi)g_{\mu\nu}\right]$$

$$\ln A(\varphi) \equiv \alpha_0(\varphi - \varphi_0) + \frac{1}{2}\beta_0(\varphi - \varphi_0)^2 + \mathcal{O}(\varphi - \varphi_0)^3$$

Binary Pulsars **(B1913+16; J1141-6545)**



$$\gamma - 1 = -\frac{2\alpha_0}{1+\alpha_0^2}$$

$$\beta - 1 = \frac{\alpha_0^2\beta_0}{(1+\alpha_0^2)^2}$$

$$\Rightarrow$$

$$\frac{\beta-1}{\gamma-1} < 1.1$$

$$\alpha_0 < 0.060, \beta_0 > -4.5$$

[Esposito-Farese 2004]

Conclusions

- Resolving the dichotomy DE - DM X Modified Gravity will require a concerted effort and a whole new programme of dedicated experiments in space:

- To observe SNe (SNe "factories"), gamma-ray bursts, gravitational lensing, cosmic shear, etc, so to characterize the properties of DE and DM, or alternatively, to find evidence for the inadequacy of General Relativity

- To test General Relativity and examine the implications of its contending theories or extensions (scalar-tensor theories, braneworld models, strings)

- For the search of evidence of new forces with ranges of about hundreds AU and for resolving the Pioneer anomaly problem