A new determination of α with cold rubidium atoms

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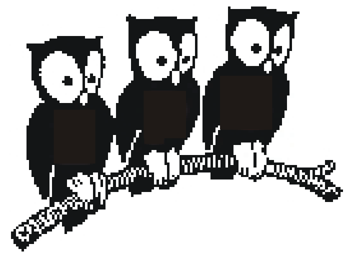

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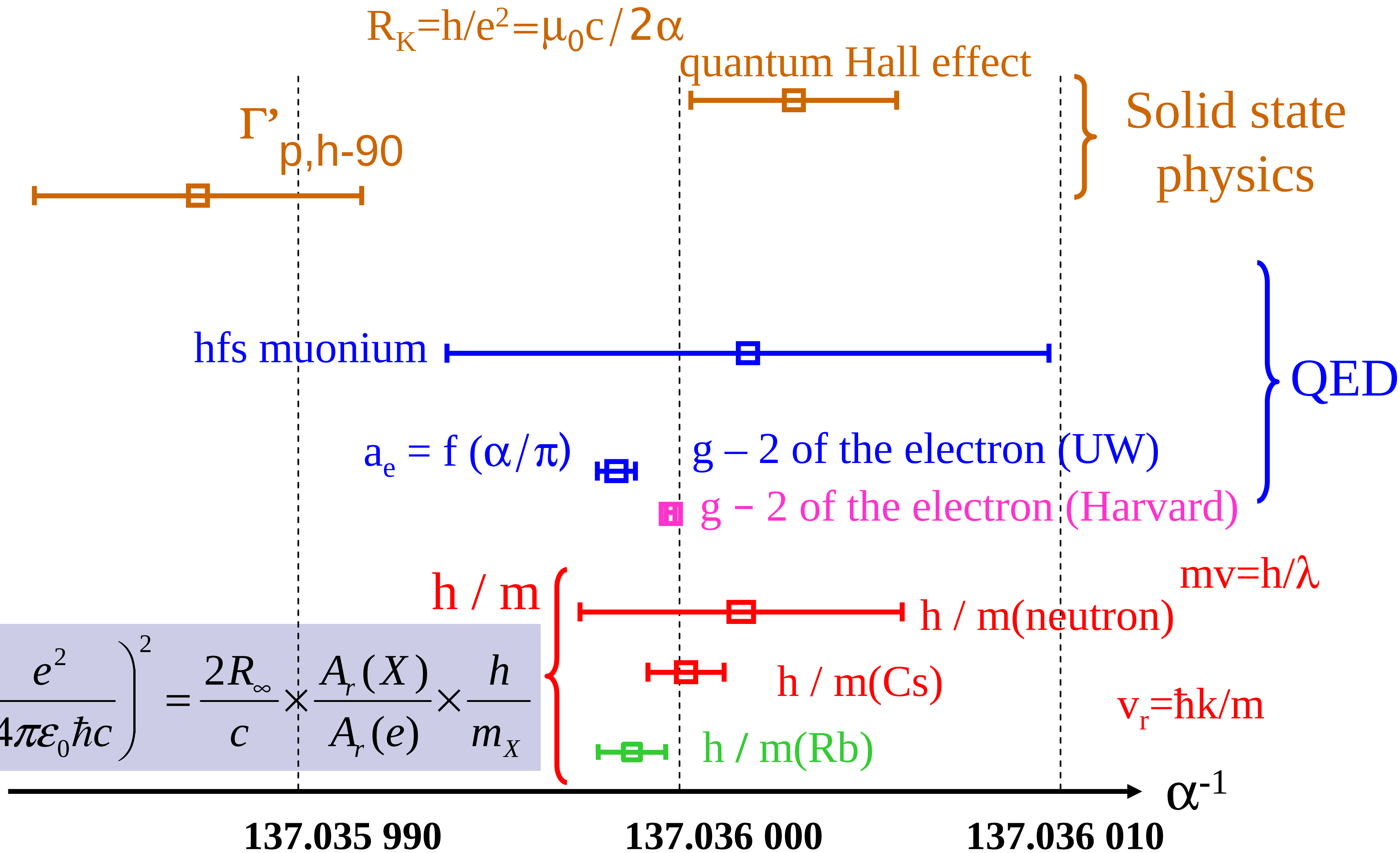
Determination of the fine structure constant

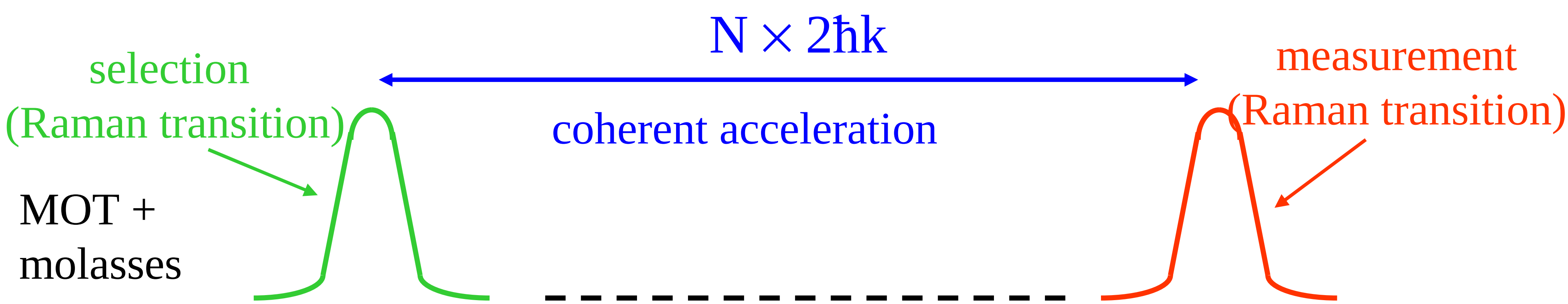


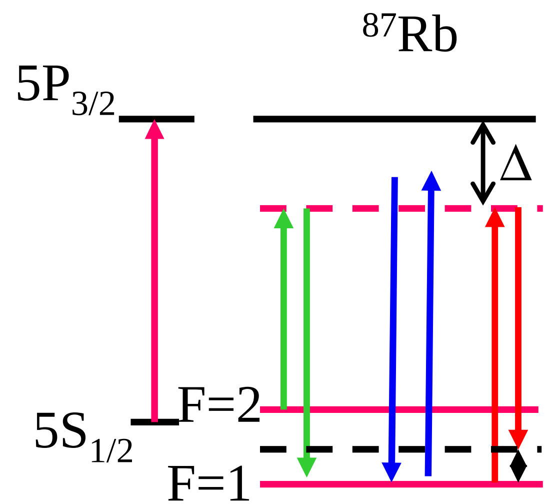
$$\alpha^2 = \left(\frac{e^2}{4\pi\varepsilon_0 \hbar c}\right)^2 = \frac{2R_\infty}{c} \times \frac{A_r(X)}{A_r(e)} \times \frac{h}{m_X}$$

*CODATA 2002 P. Mohr and B. Taylor, RMP, **77**, n°1, p. 1, january 2005*

*G. Gabrielse et al, PRL, **97**, 030802, 2006*

Principle of our experiment : measurement of the recoil velocity





- selection of an initial sub-recoil velocity class
- coherent acceleration : *N* Bloch oscillations, momentum transfer *2Nħk*
- measurement of the final velocity class

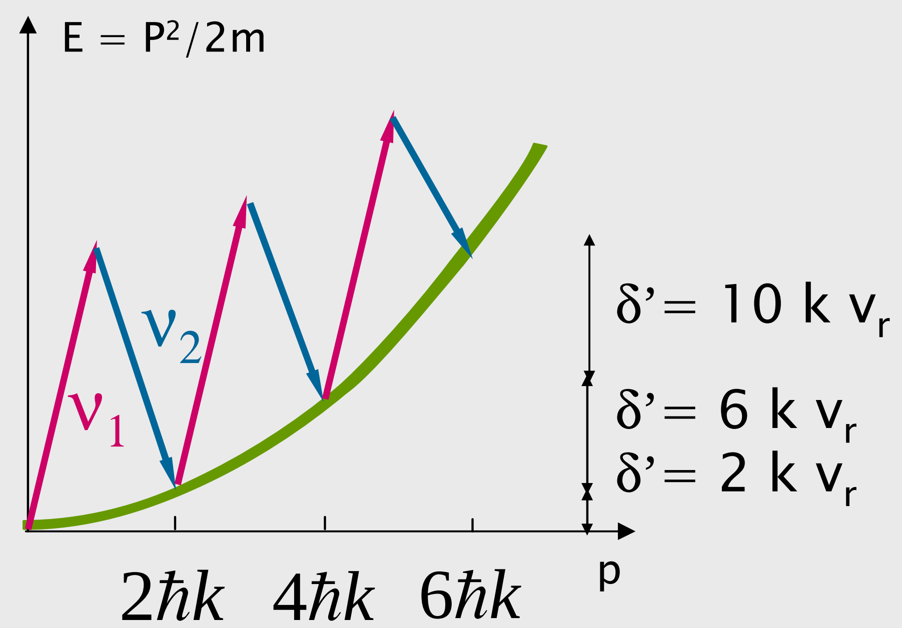
*J.L. Hall, Ch.J. Bordé, K. Uehara, PRL **37** (1976) 1339*

$$\sigma_{vr} = \sigma_v \,/\, (2N)$$

Bloch oscillations

Laboratory frame

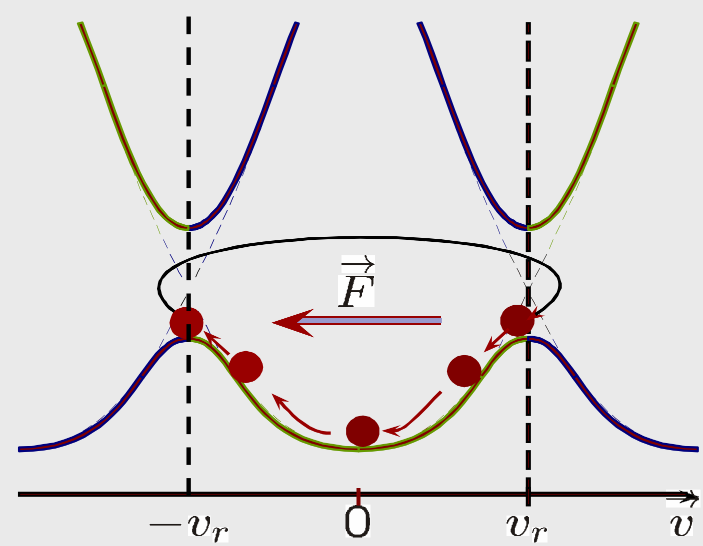
$$2\pi\delta' = 2\pi(\nu_1 - \nu_2) = 2kat$$



Accelerated frame

$$\delta' = 0$$

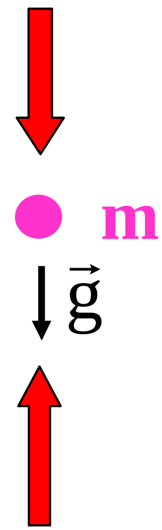
$$\vec{F} = -m\vec{a}$$



- Only one hyperfin level involved : coherent acceleration, $2\hbar k$ per cycle
- Acceleration ⇔ Bloch oscillations in the fundamental energy band

M. Ben Dahan et al , PRL, 76 (1996) 4508.

Two possibilities with vertical beams



Acceleration

$$m\Delta \mathrm{v} = mgt - N \times \frac{2h\nu}{c}$$

⇓

up and down accelerations

+

differential measurement

⇓

Measurement of h/m independent of *g*

Vertical standing wave

The atoms oscillate at the same place with the frequency

$$\nu_B = \frac{mg}{2\hbar k}$$

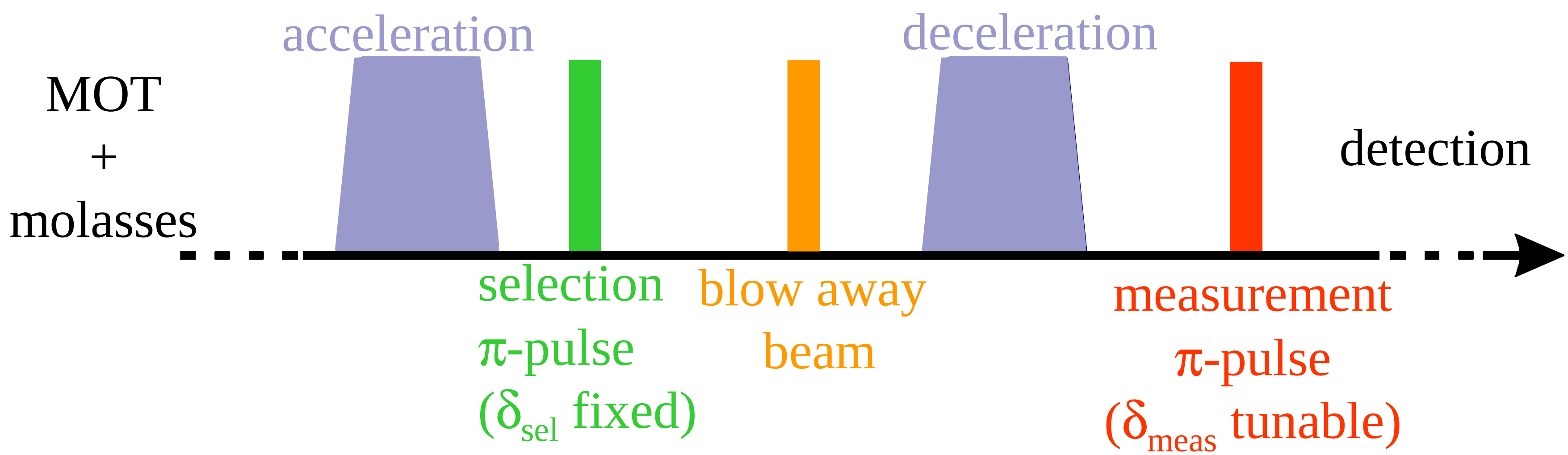
⇙ ⇘

measurement de *h/m* **gravimeter**

G. Ferrari et al , PRL, 97 (2006) 060402.

4000 oscillations in 7 s!

Experimental sequence



We measure (Doppler effect) : $\Delta V = \dfrac{\hbar(\delta_{sel} - \delta_{meas})}{(k_1 + k_2)}$

Acceleration in both opposite directions : $v_r = \dfrac{\Delta V^{up} - \Delta V^{down}}{2(N^{up} + N^{down})}$

$$v_r = \frac{\hbar k_B}{m}$$

$$\frac{\hbar}{m} = \frac{(\delta_{sel} - \delta_{meas})^{up} - (\delta_{sel} - \delta_{meas})^{down}}{2(N^{up} + N^{down})(k_1 + k_2)k_B}$$

Results

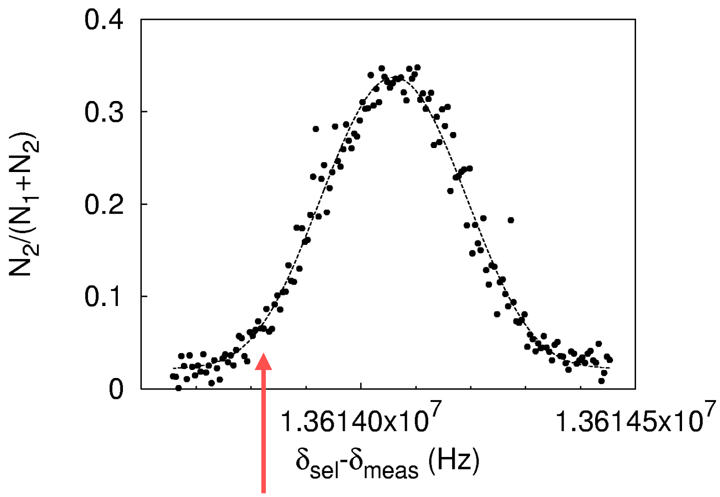
Transfer efficiency > 99.95% per oscillation (2 recoils)

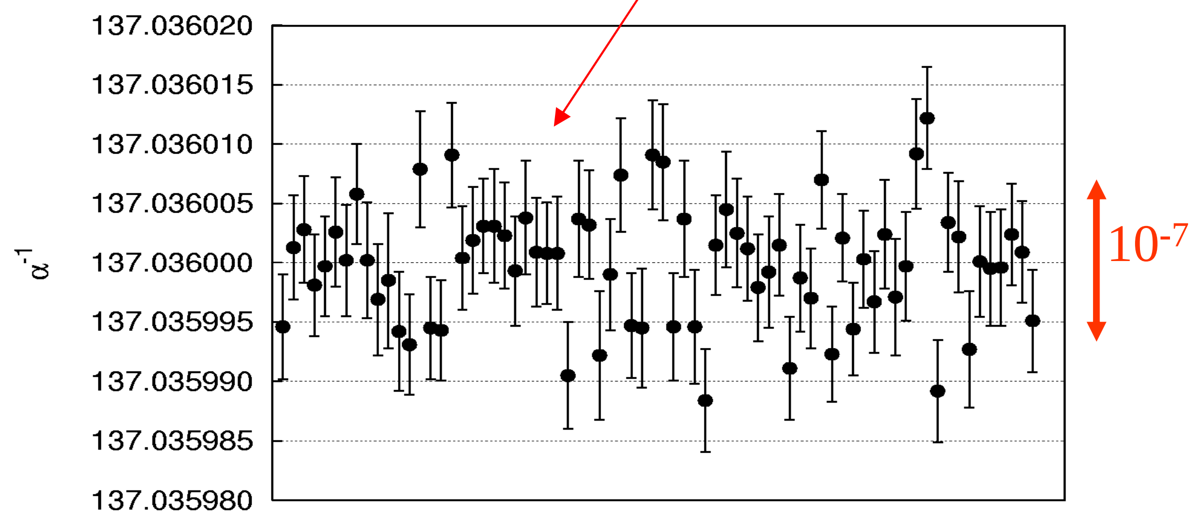
about 450 Bloch oscillations up and down → 1800 recoils

measurements performed in April 2005

1 point = 4 spectra

$\delta_{center} = 13614059.9 \pm 1.6 Hz$





1 point = 1 sequence

statistical uncertainty on α = 4.4×10^{-9}

total uncertainty on α = 6.7 ×10^{-9}

α^{-1} = 137.035 998 84 (91)

*Cladé et al, PRL, **96** (2006) 033001*

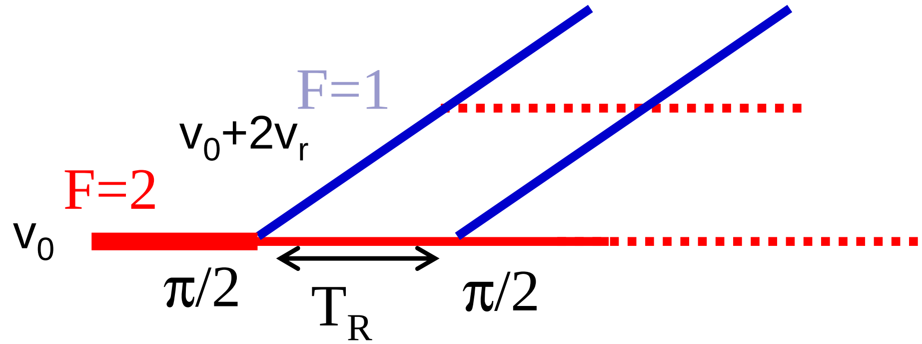
Error budget

Source	Correction (α^{-1})(ppb)	Uncertainty (α^{-1})(ppb)
✓Laser frequencies	0	0.8
✓Beams alignment	- 2	2
✓Wave front curvature and Gouy phase	- 8.2	4
✓2nd order Zeeman effect	6.6	2
✓Quadratic magnetic force	- 1.3	0.4
✓Gravity gradient	- 0.18	0.02
✓Light shift (one photon transition)	0	0.2
✓Light shift (two photon transition)	- 0.5	0.2
✓Light shift (Bloch oscillations)	0.46	0.4
✓Index of refraction (cold atomic cloud)	<0.1	0.3
✓Index of refraction (background vapor)	- 0.37	0.3
Global systematic effects	- 5.49	5.0
Statistical uncertainty		4.4
TOTAL		6.7

Cladé et al (submitted to PRA) α^{-1} = 137.035 998 84 (91)

Interferometric measurement of the recoil velocity

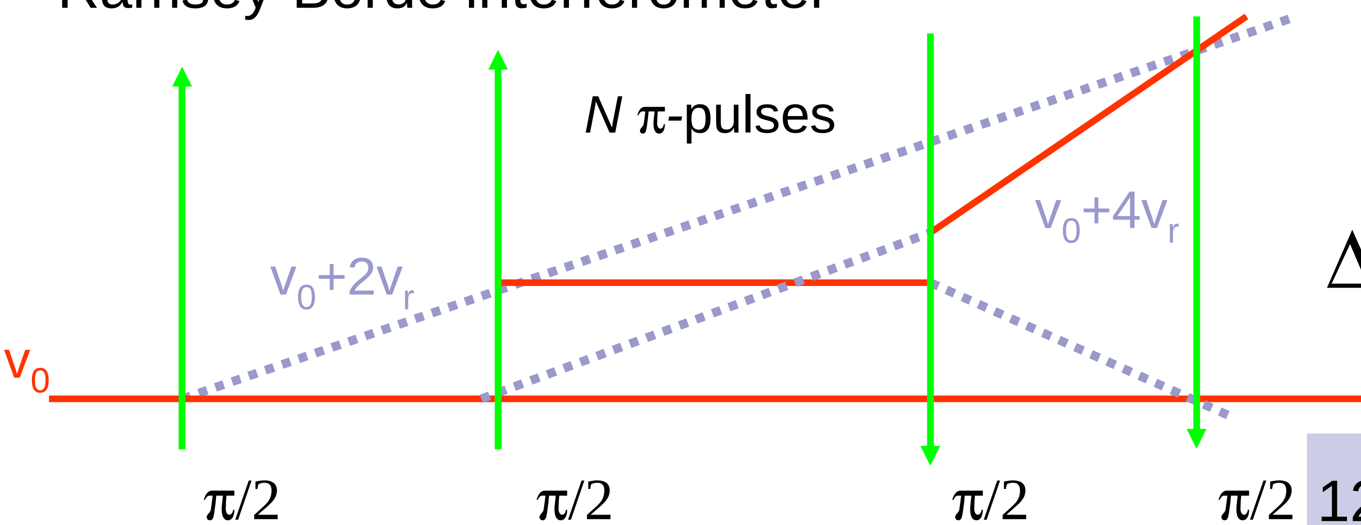
Ramsey interferometer



$$\Delta\phi_C = \frac{T_R}{\hbar}(E_C^b - E_C^a) = 2kT_R(\mathrm{v}_0 + \mathrm{v}_r)$$

$$\Delta\phi_{laser} = \delta T_R$$

Ramsey-Bordé interferometer



$$\Delta\phi_C = -4kT_R\mathrm{v}_r$$

independent of v_0

measure $2\mathrm{v}_r$

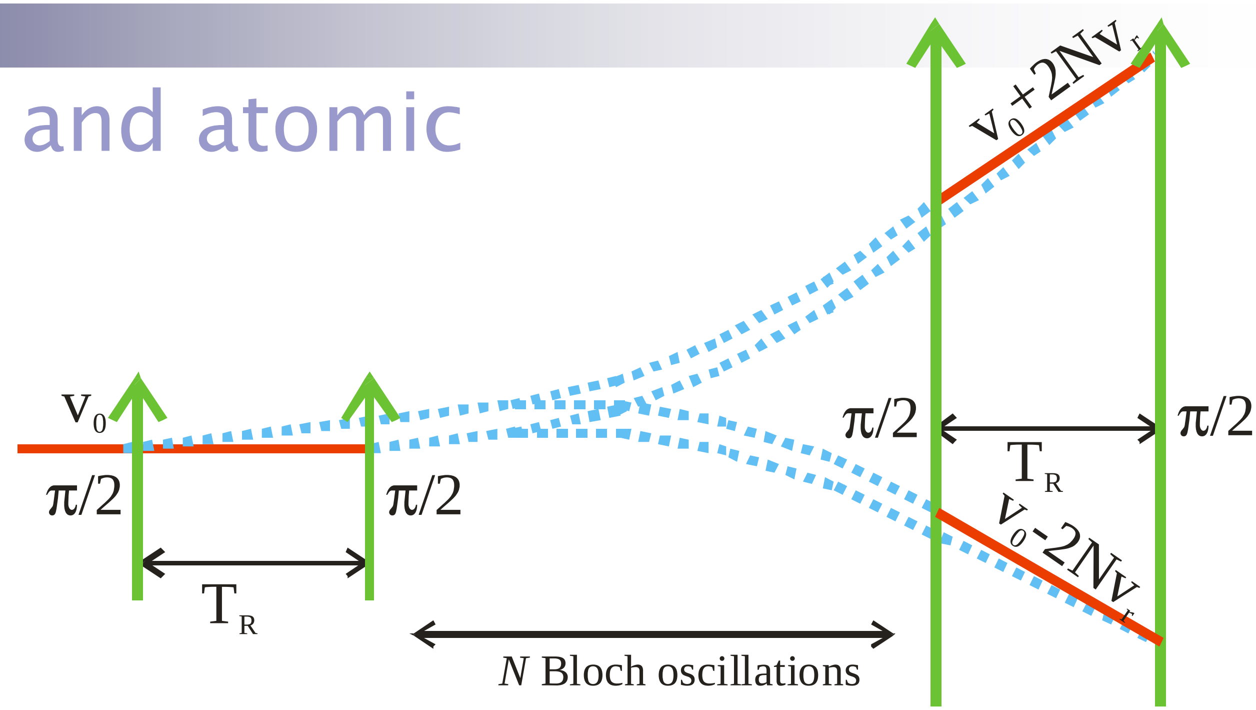
$$\Delta\phi_C = -4k(N+1)T_R\mathrm{v}_r$$

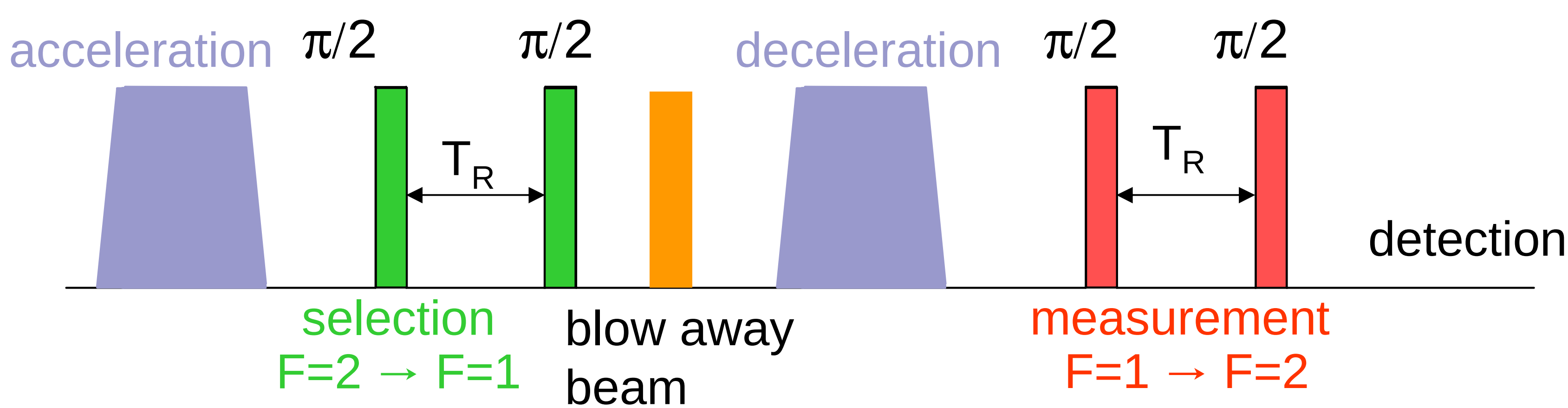
measure $2N\,\mathrm{v}_r$

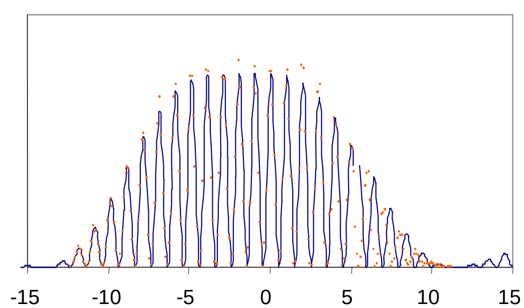
120 recoils transferred
uncertainty on α = 7.4×10^{-9}

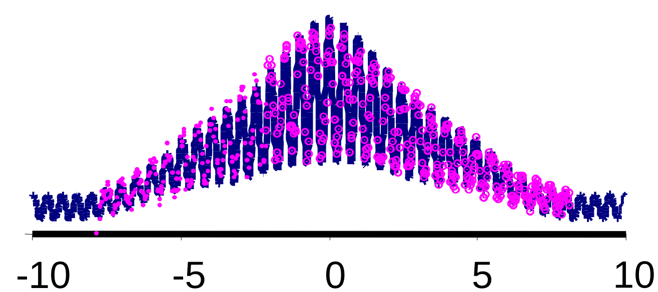
A. Wicht, J.M. Hensley, E. Sarajlic and S.Chu, Phys. Scr. ***T102****, 82 (2002)*

Bloch oscillations and atomic interferometry









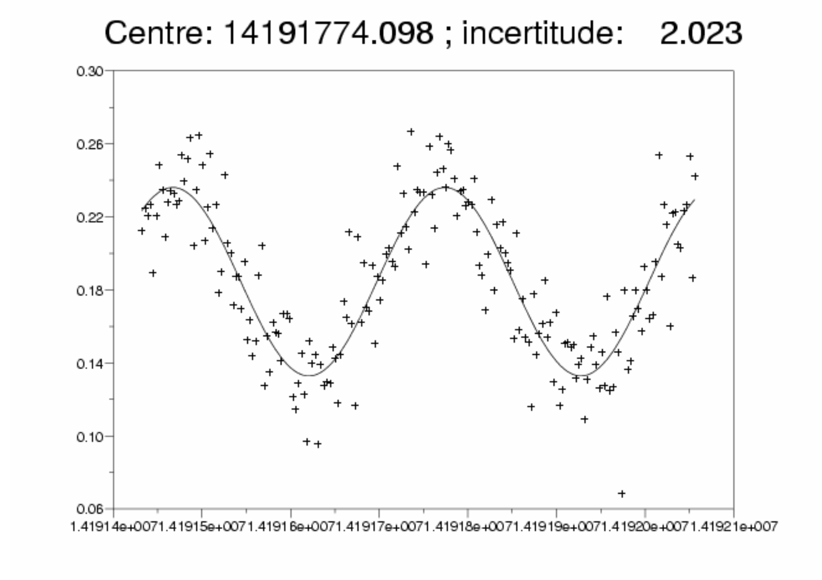
Preliminary tests

T_R=3.4 ms = π-pulse duration

$\pi/2$-pulse duration = 0.3 ms

Δ_{Raman} = 250 GHz and Δ_{Bloch}= 40 GHz

Up to 480 oscillations !



typically : 350 oscillations

statistical uncertainty for 5 determinations of $\alpha = 7.5\times10^{-9}$

4 spectra in « Rabi » configuration ⇒ h/m_{Rb} at 6.6×10^{-8}

4 spectra in « Ramsey » configuration ⇒ h/m_{Rb} at 2.9×10^{-8}

promising!

Further improvements

Statistical uncertainty

$$\sigma_{v_r} = \frac{\sigma_v}{2N}$$

Oscillations de Bloch (at the present time $N \sim 480$)

The number of Bloch oscillations is limited by the atomic longitudinal motion (500 oscillations & 12 ms , 6 cm).

Velocity measurement (at the present time $\sigma_v \sim 10^{-4} v_r$ in 10 minutes)

- a new vacuum cell and a 2D-MOT to increase the initial number of atoms.
- an actively stabilized anti-vibration plateforme to reduce vibrations.

Systematic effects

- a µ-metal shielding to reduce residual magnetic fields
- a Shack-Hartmann wave front analyser to control the beams curvature

→ $\sim 10^{-9}$

Towards a redefinition of the kilogram

The kilogram is the only SI base unit defined in terms of a material artefact
It is not invariable at a level of 10^{-8}

« Redefinition of kilogram : a decision whose time has come »
I. M. Mills et al., Metrologia **42**, 71-80 (2005)

One possible way :

- Fix the Planck constant h and relate mass and time units $E = h\nu = mc^2$

Realization of the kg using the watt balance which allows to compare :
- a mechanical power (displacement of a mass in the gravity field) $Mg\,\mathrm{v}$
- to an electrical power $UI \propto R_K K_J^2 = \frac{4}{h}$

This realization is based on the validity of the relations :

$$R_K = \frac{h}{e^2} = \frac{\mu_0 c}{2\alpha}$$ and $$K_J = \frac{2e}{h}$$

Need to be tested !

Von Klitzing constant   Josephson constant

Another possibility

- Fix the Avogadro constant (or the atomic mass unit)

  Mills et al. (2005)

At the present time, N_A is measured through the molar volume of a Si sphere

Morever

The watt balance gives h/M_{macro}

Recoil measurements give h/M_{atom}

both together can give a competitive value of N_A

Recent proposal

- Fix both h and N_A !

  « Redefinition of kilogram, ampere, kelvin, mole : … »
  Mills et al. Metrologia **43**, 227-246 (2006)
  (on going debate in the community of metrologists)

Conclusion

Highly precise frequency measurements allow very accurate determinations of fundamental constants leading to a lot of rich developments…

Refractive index

Recoil transmitted by one Bloch oscillation : $2\hbar k$ or $2n\hbar k$?
Doppler effect for the Raman transitions : 2kv or 2nkv ?

$$(n-1)=\pi\rho\frac{\Gamma}{\Delta}\left(\frac{\lambda}{2\pi}\right)^3$$

ρ: density
Γ: natural width
Δ: detuning

$$\Delta k=\frac{n\sigma}{2}\frac{\Gamma/2}{\Delta}$$

For the cold atoms

Initial atomic density : 10^{11} atoms/cm^3

Raman beams : Δ= 1050 GHz :

(n-1)= 4.10^{-10} (selection)

(n-1)<10^{-12} (measure)

Bloch beams : Δ = 40 GHz:

(n-1)=2.10^{-10} (selected atoms)

For the background vapor

density: 8.10^{8} atoms/cm^3 (n-1) ~ 4.10^{-10}

Index of refraction

*PRL **94** 170403 (2005) (MIT): Photon Recoil Momentum in Dispersive Media*

Observation : modification of recoil energy in a dispersive medium (BEC).

	Dispersive medium	Atoms
N_{tot}	N_0	N_1
n : index of refraction $N_1 << N_{tot}$	$2(1-n)N_1/N_0\hbar k$	$2n\hbar k$

Bloch oscillations :

$$p_{final} = 2n\hbar k + 2(n-1)\hbar k\,\frac{N_1}{N_0} = 2\hbar k$$

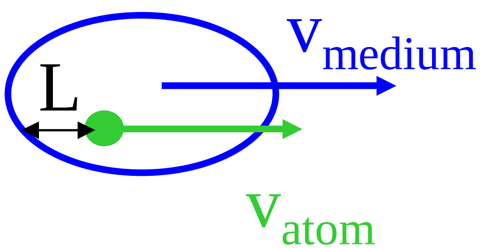
if η= 100%

Accelerated atoms ⇔ dispersive medium

otherwise ~(1-η)(n-1)

Raman transition :

Atomic cloud



$$\omega' = \omega - nkv_{atom} + (n-1)kv_{medium}$$

$$\omega' = \omega - kv_{atom} + (n-1)k(v_{medium} - v_{atom})$$

$dL/dt = 0 \Leftrightarrow v_{medium} = v_{atom}$ no effect

Refractive index

- Phase of the light (1) at the position of the atom i (x_i) : $\Phi_1(x_i)$
- Two photon transition : $\Phi = \Phi_1 - \Phi_2$
- Assum:
  - without dispersive media : $\Phi(x) = 2kx$
  - inside the medium : $d\Phi(x)/dx = 2nk$
  - uniform medium (N atoms), x_m of the center of the medium : $x_m = \Sigma_i x_i/N$
  - at the position x_m of the center of the medium effect of refractive index cancel from 1st and 2nd beam

$$\Phi(x) = 2(n-1)k(x - x_m) + 2kx$$

One Bloch oscillation :

- atom

$$\hbar \frac{d\Phi(x_i)}{dx_i} = 2n\hbar k + 2(1-n)\frac{\hbar k}{N} \approx 2n\hbar k$$

- medium

$$\hbar \frac{d\Phi(x_i)}{dx_j} = 2(1-n)\frac{\hbar k}{N}$$

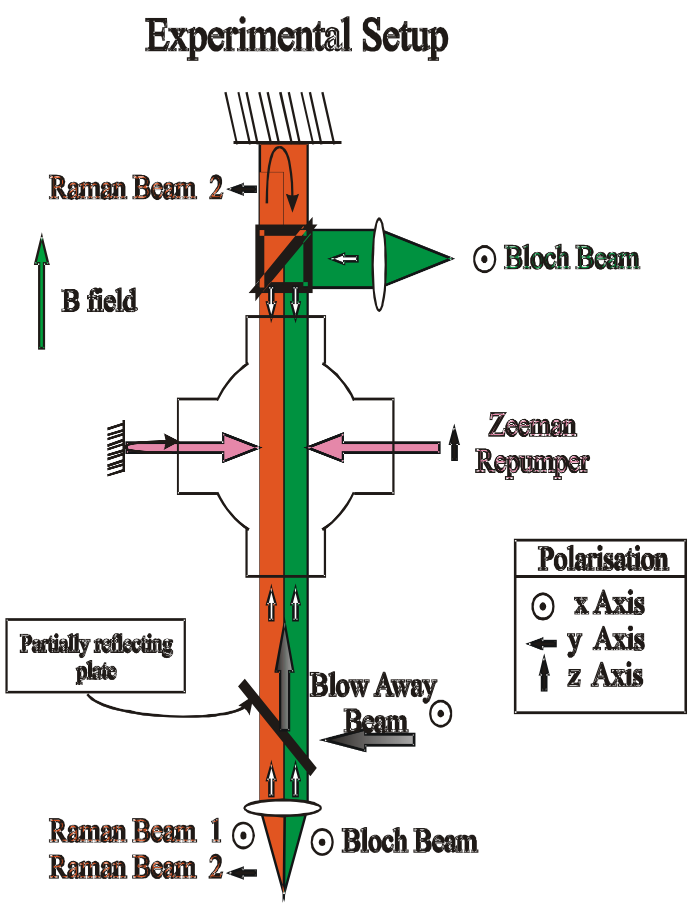
Raman transition : Doppler effect

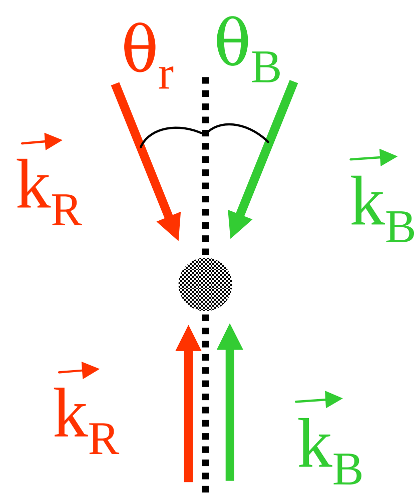
$$\frac{d\Phi(x(t),t)}{dt} \quad \rightarrow \quad \omega' = \omega - 2kv + 2(n-1)k(v - v_0)$$

Systematic effects

- **Lasers frequencies** : FP cavity → uncertainty 300kHz → $u_r(\alpha) = 8 \times 10^{-10}$

- **Beams misalignment** : Optical fibers to couple Raman/Bloch beams into the cell





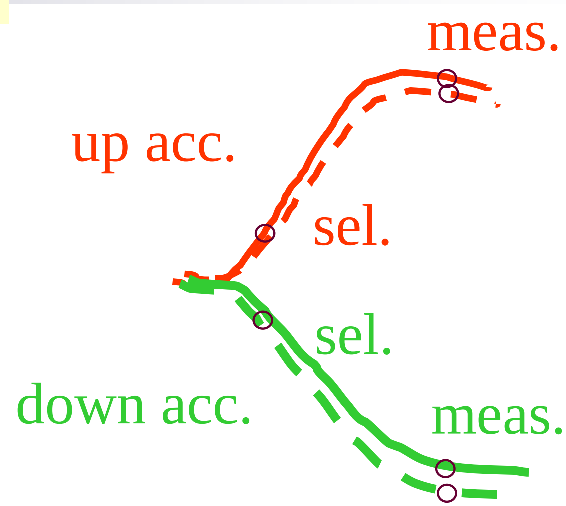
maximum misalignment :

$$\theta_r = 3\times10^{-5} \text{ rad}$$
$$\theta_B = 1.6\times10^{-4} \text{ rad}$$

Correction on α^{-1}: $-(2 \pm 2)\times10^{-9}$

Systematic effects

➢Gravity gradient :



R : Earth radius

t : spacing time / sel-meas = 12 ms

$$\text{Correction on } \alpha^{-1} \sim -\frac{g\,t^2}{R} \sim 10^{-10}$$

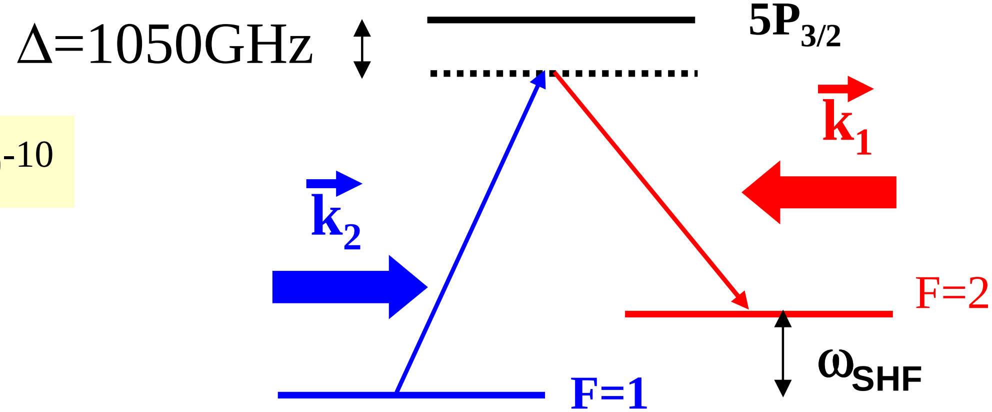
➢Level shifts :

- Light shift

Expansion of the cloud

$\Delta I = 10\%$ when $k^2_R \leftrightarrow k^1_R$

$$u_r(\alpha) = 3 \times 10^{-10}$$



-Magnetic field gradient = trajectory effect

$\Delta z = 0.3\text{mm}$ when $k^2_R \leftrightarrow k^1_R$

$$\text{Correction on } \alpha^{-1} \sim (6.6 \pm 2) \times 10^{-9}$$

➢Quadratic magnetic force :

$$\text{Correction on } \alpha^{-1} \sim \frac{(F/M)t}{2 N v_r} \sim (-1.3 \pm 0.4) \times 10^{-9}$$

Gouy phase and wave front curvature

- What is the momentum transferred to the atoms by laser beams ?

Gaussian beam : Plane waves superposition : $k_{//}^2 = \frac{\omega^2}{c^2} - k_\perp^2 < \frac{\omega^2}{c^2}$

Momentum transferred = gradient of the phase

$$E(r,z) = E(r)e^{i\phi(r,z)} \qquad p \rightarrow p + \hbar k_{eff} \qquad \text{avec } k_{eff}(r,z) = \partial_z \phi(r,z)$$

- Gaussian beam:

$$\phi(r,z) = k\,z - \phi_G(z) + k\frac{r^2}{2R}$$

Gouy phase

Curvature radius

$$k_{eff} = \frac{d\phi}{dz} = k - \frac{2}{k\,w^2(z)} - \frac{r^2}{2R^2} \times \frac{dR}{dz} \times k$$

k_{eff} can be measured with a wave front analyzer (R, w)

Possible realization of Avogadro constant (ccsd-00084607)

$$N_A = \frac{M_u}{m_u} = \frac{1}{h}\frac{h}{A_r(X)m_u}A_r(X)M_u = \frac{1}{h}\frac{h}{m(X)}A_r(X)M_u$$

Cold atom experiment and Watt balance $\longrightarrow$ realization of N_A

(g or h/m) (h if $R_K = h/e^2$ and $K_J = 2e/h$ are exact)

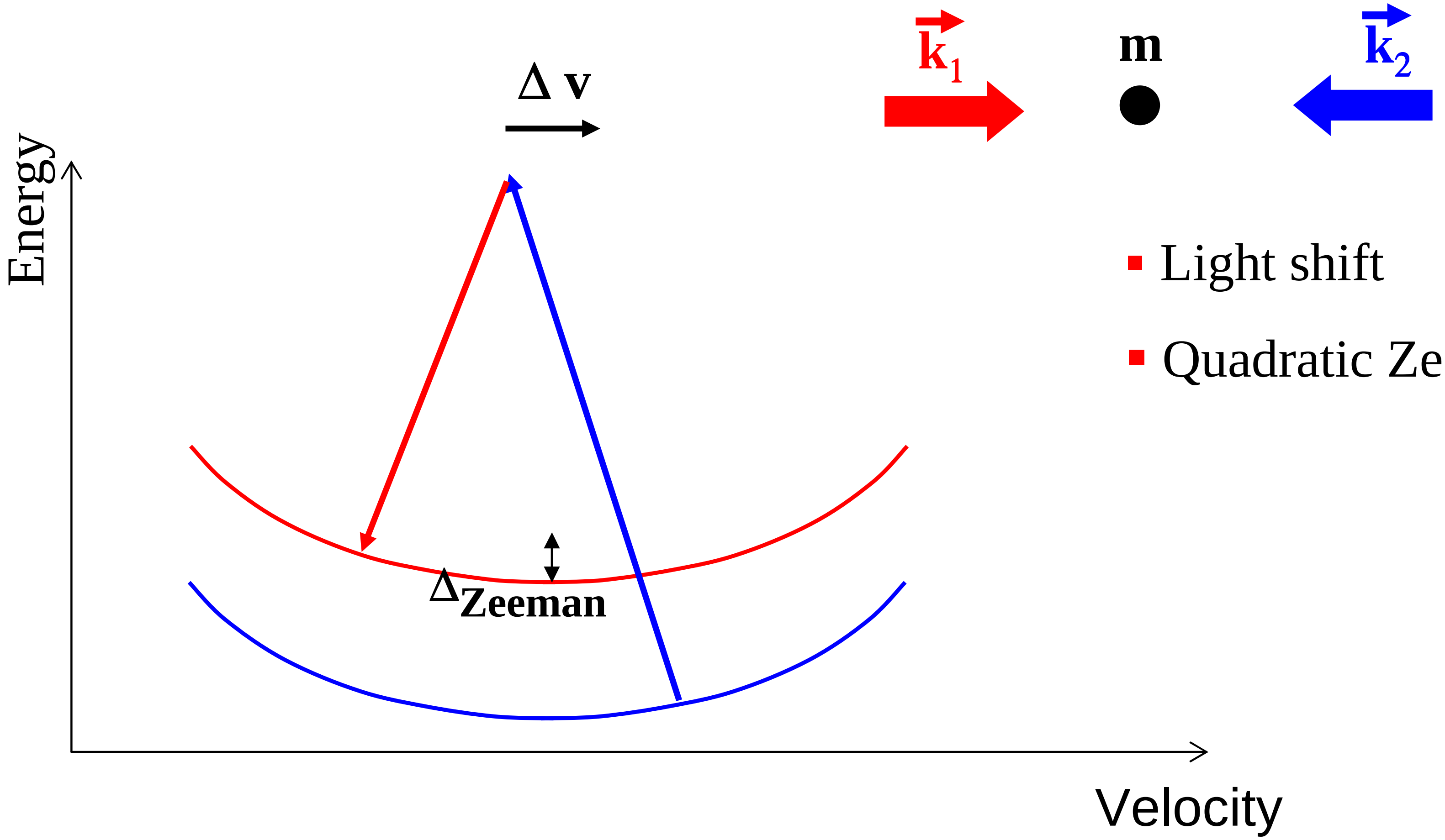
$$N_A^{(1)} = \left\{\frac{K_J^2 R_K g^{(w)}}{4}\right\}\left\{\frac{h}{m(^{87}Rb)g^{(a)}}\right\}\left\{\frac{g^{(a)}}{g^{(w)}}\right\}A_r(^{87}Rb)M_u$$

Watt balance — Bloch oscillations (stationary) — Relative gravimeters

$$N_A^{(2)} = \left\{\frac{K_J^2 R_K}{4}\right\}\left\{\frac{h}{m(^{87}Rb)}\right\}A_r(^{87}Rb)M_u$$

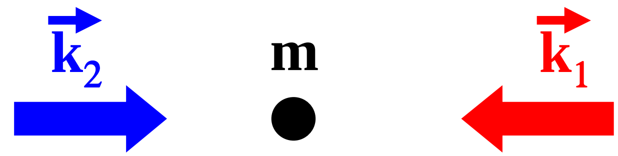
Watt balance — h/m experiments

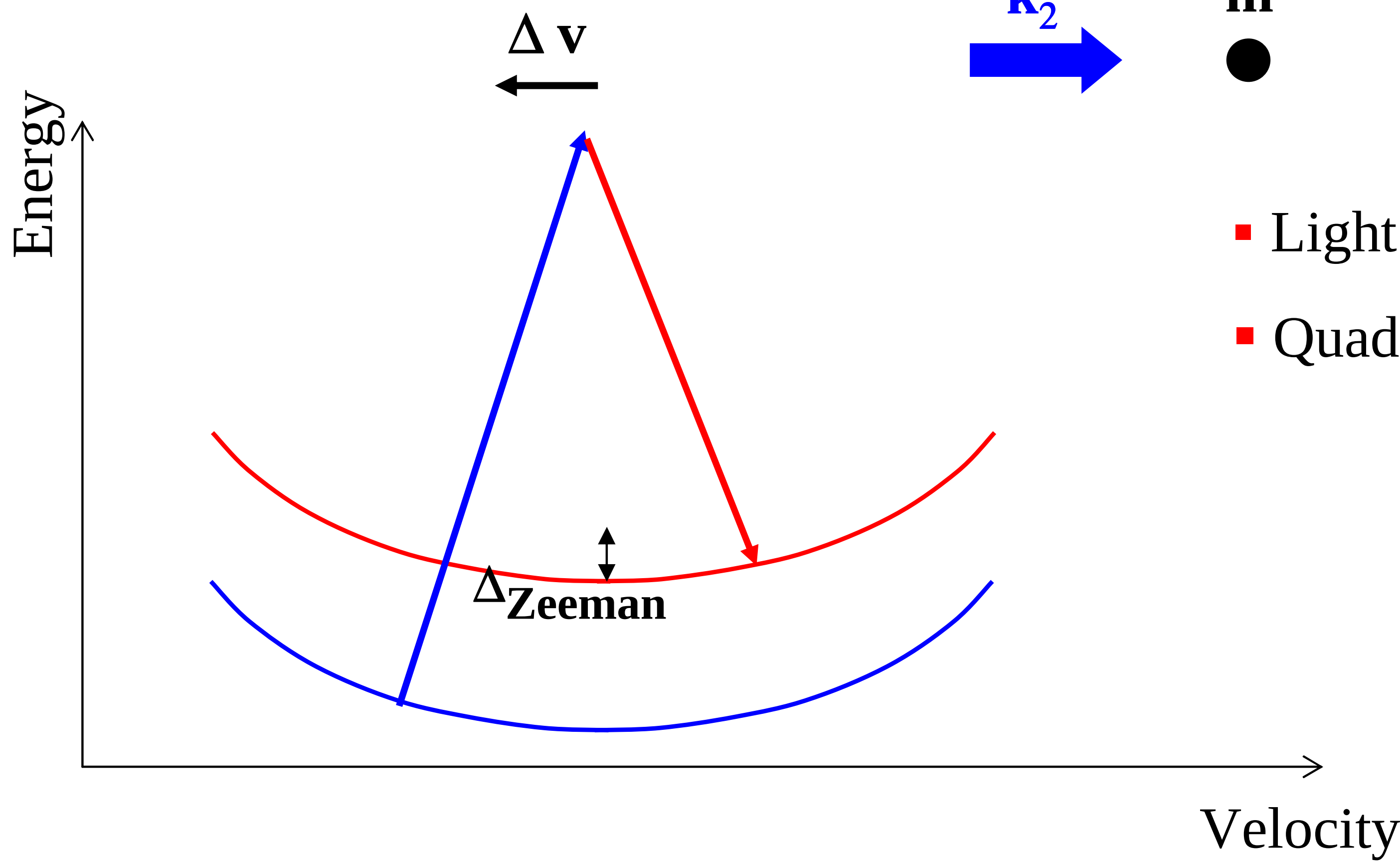
Reduction of constant systematic shifts



- Light shift
- Quadratic Zeeman shift

Reduction of systematic shifts





- Light shift
- Quadratic Zeeman shift

Compensation of energy shifts by inverting the direction of Raman beams

Two spectra →
one velocity measurement