Istituto Nazionale di Fisica Nucleare | Consiglio Nazionale delle Ricerche | University of Trento | INFM | BEC BOSE EINSTEIN CONDENSATION

Thermal effects of the Casimir forces on ultra-cold gases

Mauro Antezza

University of Trento, INFN and
CNR-INFM BEC Center on Bose-Einstein Condensation, Trento, Italy

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The Team

Prof. Sandro Stringari

Prof. Lev P. Pitaevskii

Atom-Atom force

- ***Boyle and Gay-Lussac*** **ideal gas lows** $PV = nRT$ could be explained by the kinetic theory of non-interacting point atoms (Joule, Kroning, Clausius,..), but are **hardly exact**

- ***J.D. van der Waals*** (1873): eq. of state $\left(P + \frac{a}{v^2}\right)(V - b) = nRT$

- ***London*** (1930!): interaction potential between two atoms due to fluctuations of the atomic electric dipole moment **d** $V_{VL} \propto -\frac{1}{r^6}$

$\langle d_i \rangle = 0, \langle d_i^2 \rangle \neq 0, \vec{d} = \alpha \vec{E}$ ➡ ***dispersion forces*** (it is necessary only that $\alpha \neq 0$, the vacuum is a q.s. with **observable physical consequences!**)

\+ ***orientation forces*** (***Keesom***,T, perm. dipoles)
\+ ***induction forces*** (***Debye***,q-d) = 3 types vdW forces

- ***Casimir*** and ***Polder*** (1947): inclusion of retardation effect $c \neq \infty$ and at large distance $r >> \lambda_c$

$$V_{CP} \propto -\frac{1}{r^7}$$

Lifshitz Theory

- by adding the vdW force between the atoms of the **two plates** and assuming a pairwise potential V=-B/r^n but this was experimentally wrong!

- the vdW force is not additive: the force between two atoms depends of the presence of a third atom

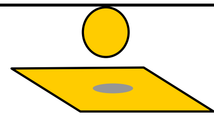
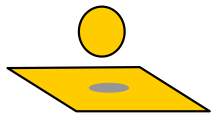
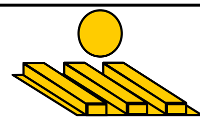
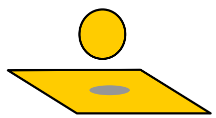
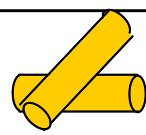
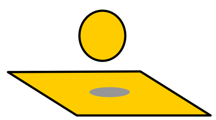
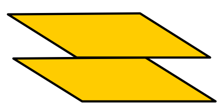
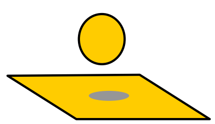
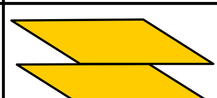
- ***Lifshitz*** (1955), ***Dzyaloshinskii*** and ***Pitaevskii*** (1961) developed a **Macroscopic General Theory of the vdW Forces** motivated by the experimental discrepancy with microscopic-additive theories

I.E. Dzyaloshinskii, E.M. Lifshitz and L.P. Pitaevskii, Advances in Physics 38, 165 (1961). Heroic Era!

- ***Lifshitz*** assumed the dielectrics characterized by **randomly fluctuating sources** as demanded by the **FDT** and solved the Maxwell equations using the Green function method

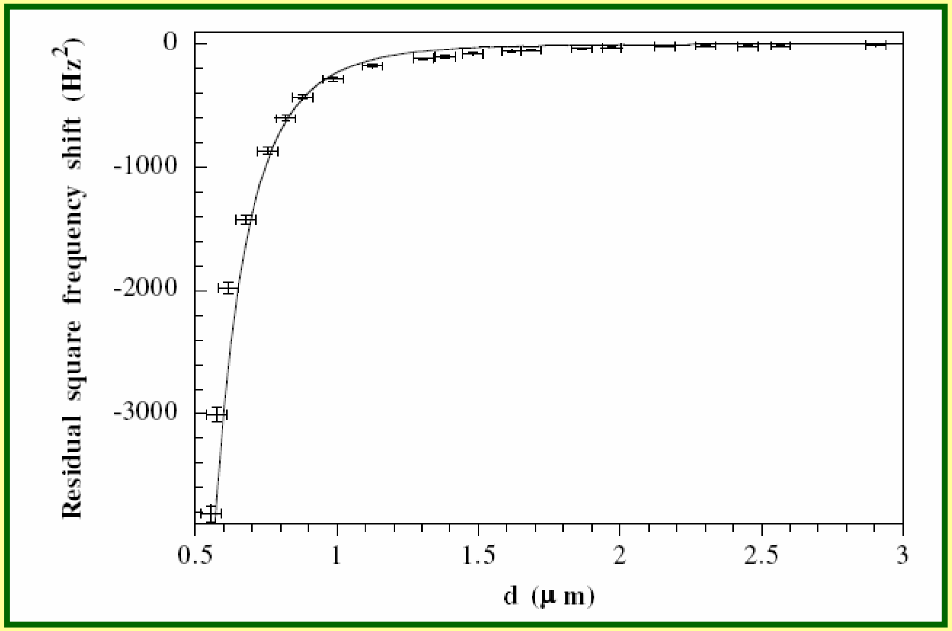
-***Ginzburg*** (1979): "the calculations are so cumbersome that they were not even reproduced in the relevant Landau and Lifshitz volume where, as a rule, all important calculations are given"

Recent Measurements of Casimir Force

Investigators	Year	Geometry	Method	Distance Scale (nm)	Materials	Pressure (mbar)	Temp (K)	Accuracy (%)
S. K. Lamoreaux	1997		Torsion pendulum	600 - 6000	Au(500nm)	10^{-4}	300	5
U. Mohideen & A. Roy	1998		AFM	100 - 900	Al (300nm) + AuPd (20nm)	$5x10^{-2}$	300	2
A. Roy and U. Mohideen	1999		AFM	100 - 900	Al (250nm)+ AuPd (8nm)	$5x10^{-2}$	300	2
G. L. Klimthitskaya, A. Roy, U. Mohideen and V. M. Mostepanenko	1999		AFM	100 - 900	Al (300nm) + AuPd (20nm)	$5x10^{-2}$	300	1
T. Ederth	2000		Piezo-tube manipulator	20 - 100	50µm Au wires coated in thiol SAM	1000	300	1
H. B. Chan, V. A. Aksyuk, R. N. Kleiman, D. J. Bishop & F. Capasso	2001		MEMS torsion bar capacitance	90 - 1000	Au (200nm) + Cr underlayer	1000	300	1
G. Bressi, G. Carugno, R. Onofrio & G. Ruoso	2002		Interfero-metry	500 - 3000	Cr (50nm) on Si	10^{-5}	300	15
R. S. Decca, D. Lopez, E. Fischbach & D. E. Krause	2003		MEMS torsion bar capacitance	200 - 2000	Cu/Au	10^{-4}	300	1
NANOCASE	2005 -		AFM, MEMS	10 - 1000	Si, Au	10^{-11}	20 - 1000	<1

Mesurement of Casimir-Polder and Lifschitz force

- Behaviour of **Casimir-Polder force** well **explored** experimentally at **short distances** (mainly forces between metallic bodies)



Bressi et al. PRL 2002
(plate-plate configuration)

- Behaviour at **larger distances** (few microns) **less explored**. In particular thermal effects of the force not yet measured

- **Cold atoms** are natural **candidates** to explore **thermal** effects of the force at moderately large distances (5-10 microns).

Surface-atom interaction has been the object of systematic experimental and theoretical studies in recent years.

Motivations :

- Open theoretical and experimental questions (ex: role of **e.m. thermal fluctuations, usually masked**)
- Perspectives for applications **(atom chips, ..)**
- New constraints on hypothetical **non-Newtonian forces** at short distances

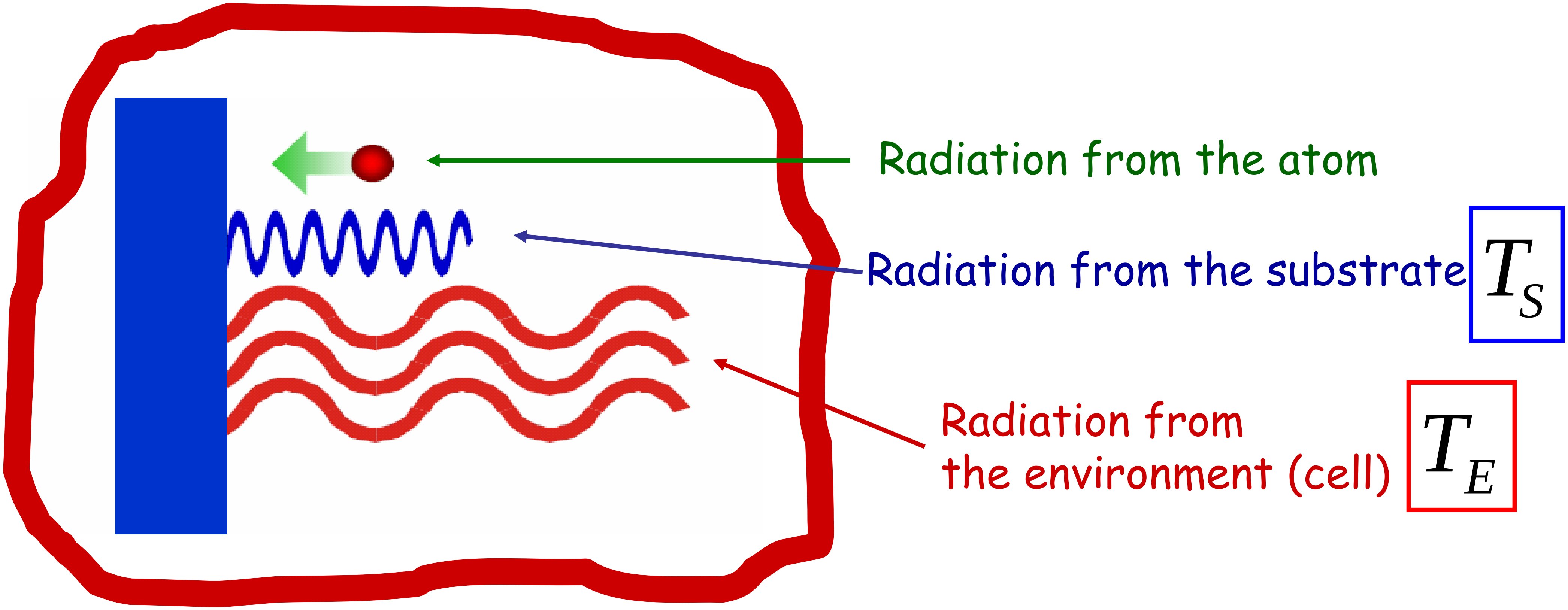
Experiments with **cold atoms**

- Shih and Parsegian (1975): deflection of atomic beam (VL)
- Anderson (1988): deflection of atomic beam (VL), Rydberg atoms
- Hinds (1993): deflection of atomic beam (CP)
- Aspect (1997): reflection from atomic mirror
- Shimizu (2001, 2005): reflection from solid surface
- Vuletic (2004): BEC stability near surfaces
- Ketterle (2004): BEC reflection from solid surface
- Cornell (2005): BEC center of mass oscillation (CP)
- Cornell (2006): BEC center of mass oscillation (Thermal)

Plan of the talk

- Surface-Atom force at thermal equilibrium
- Surface-Atom force out of thermal equilibrium
- Recent Experimental results
- Surface-Surface force out of thermal equilibrium

Surface-atom force



$$\vec{F}(\vec{r}) = \left\langle d_i^{tot}(t)\vec{\nabla}E_i^{tot}(\vec{r},t)\right\rangle \approx \left\langle d_i^{ind}(t)\vec{\nabla}E_i^{fl}(\vec{r},t)\right\rangle + \left\langle d_i^{fl}(t)\vec{\nabla}E_i^{ind}(\vec{r},t)\right\rangle$$

Force includes **zero-point** (**or vacuum**) fluctuations effects + **thermal** (**or radiation**) fluctuations effects (**crucial at large distance!**)

Electric Field

$$\vec{E}[\omega;\vec{r}] = \int_V \overline{G}[\omega;\vec{r},\vec{r}\,'] \bullet \vec{P}[\omega] \quad \mathrm{d}\vec{r}$$

Fluctuations Dissipation Theorem

$$\left\langle P_i^{fl}[\omega;\vec{r}]P_j^{fl+}[\omega',\vec{r}\,']\right\rangle_S = \frac{\hbar\varepsilon''(\omega)}{2} \coth\left(\frac{\hbar\omega}{2k_B T}\right) \delta(\omega-\omega')\delta(\vec{r}-\vec{r}\,')\delta_{ij}$$

Result at thermal equilibrium: L-D-P Theory

$$F^{eq}(T,z) = \frac{\hbar}{\pi}\int_0^\infty d\omega \coth\left(\frac{\hbar\omega}{2k_BT}\right)\mathrm{Im}\left[\alpha(\omega)\partial_{z_2} G_{ii}[\omega;\vec{r}_1,\vec{r}_2]\Big|_{\vec{r}_1=\vec{r}_2=\vec{r}}\right]$$

$$\coth\left(\frac{\hbar\omega}{2k_BT}\right) = 1 + \frac{2}{e^{\hbar\omega/k_BT}-1}$$

Vacuum fluctuations : T=0

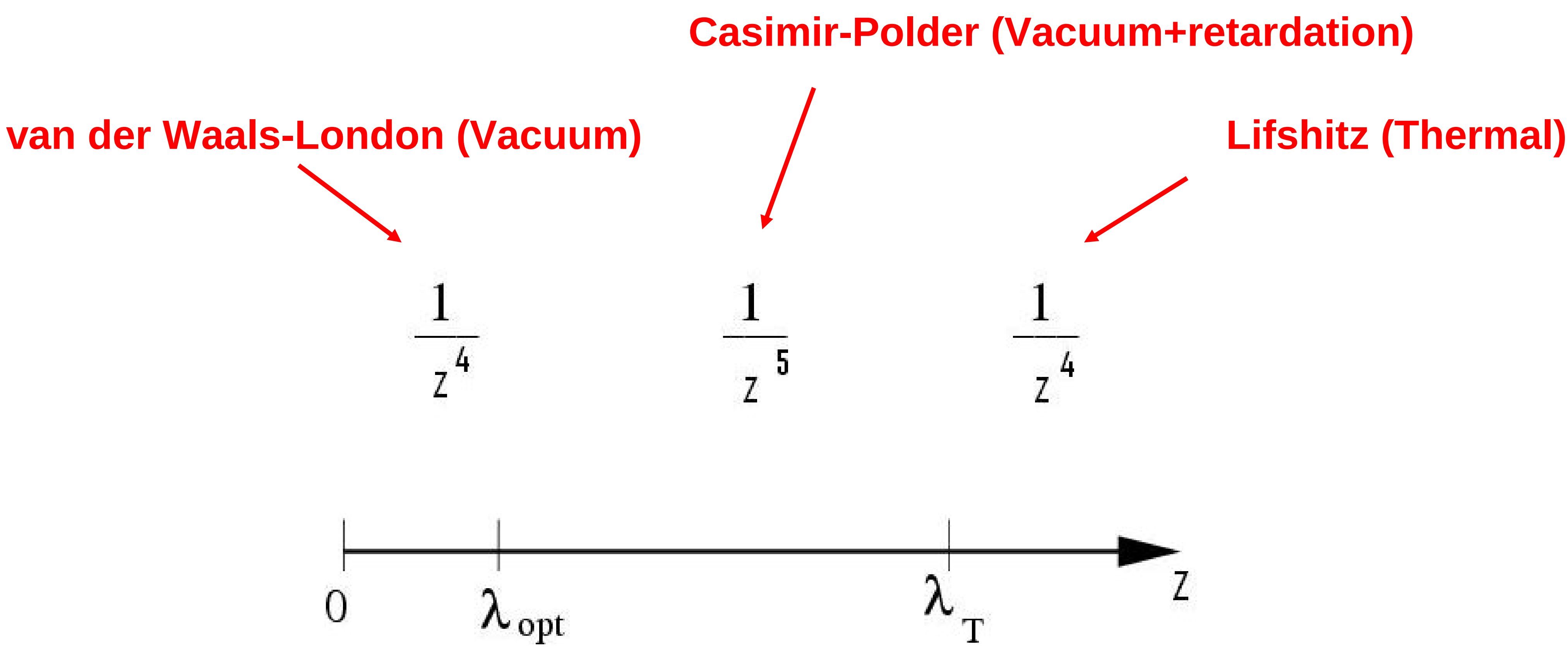
Thermal fluctuations

$$F^{eq}(T,z) = F_0(z) + F_{th}^{eq}(T,z)$$

Relevant length scales at Equilibrium

-Optical length λ_{opt} fixed by optical properties of the substrate (typically fractions of microns)

- Thermal photon wavelength ($\lambda_T = \hbar c / k_B T \approx 7.6\,\mu m$ at room temperature)



Asymptotic behaviour at thermal equilibrium

$$z << \lambda_{opt}$$

$$F_0(z) \to F_{VL}(z) = -\frac{3\hbar}{4\pi z^4}\int_0^\infty \alpha(i\xi)\frac{\varepsilon(i\xi)-1}{\varepsilon(i\xi)+1}d\xi$$

$$\varepsilon(i\xi) = 1 + \frac{2}{\pi}\int_0^\infty \omega\frac{\varepsilon''(\omega)}{\omega^2+\xi^2}d\omega$$

$$\alpha(i\xi) = \frac{2}{\pi}\int_0^\infty \omega\frac{\alpha''(\omega)}{\omega^2+\xi^2}d\omega$$

$$\lambda_{opt} << z << \lambda_T$$

$$F_0(z) \to F_{CP}(z) = -\frac{3}{2}\frac{\hbar c\alpha_0}{\pi z^5}\frac{(\varepsilon_0-1)}{(\varepsilon_0+1)}\phi(\varepsilon_0)$$

$$z >> \lambda_T$$

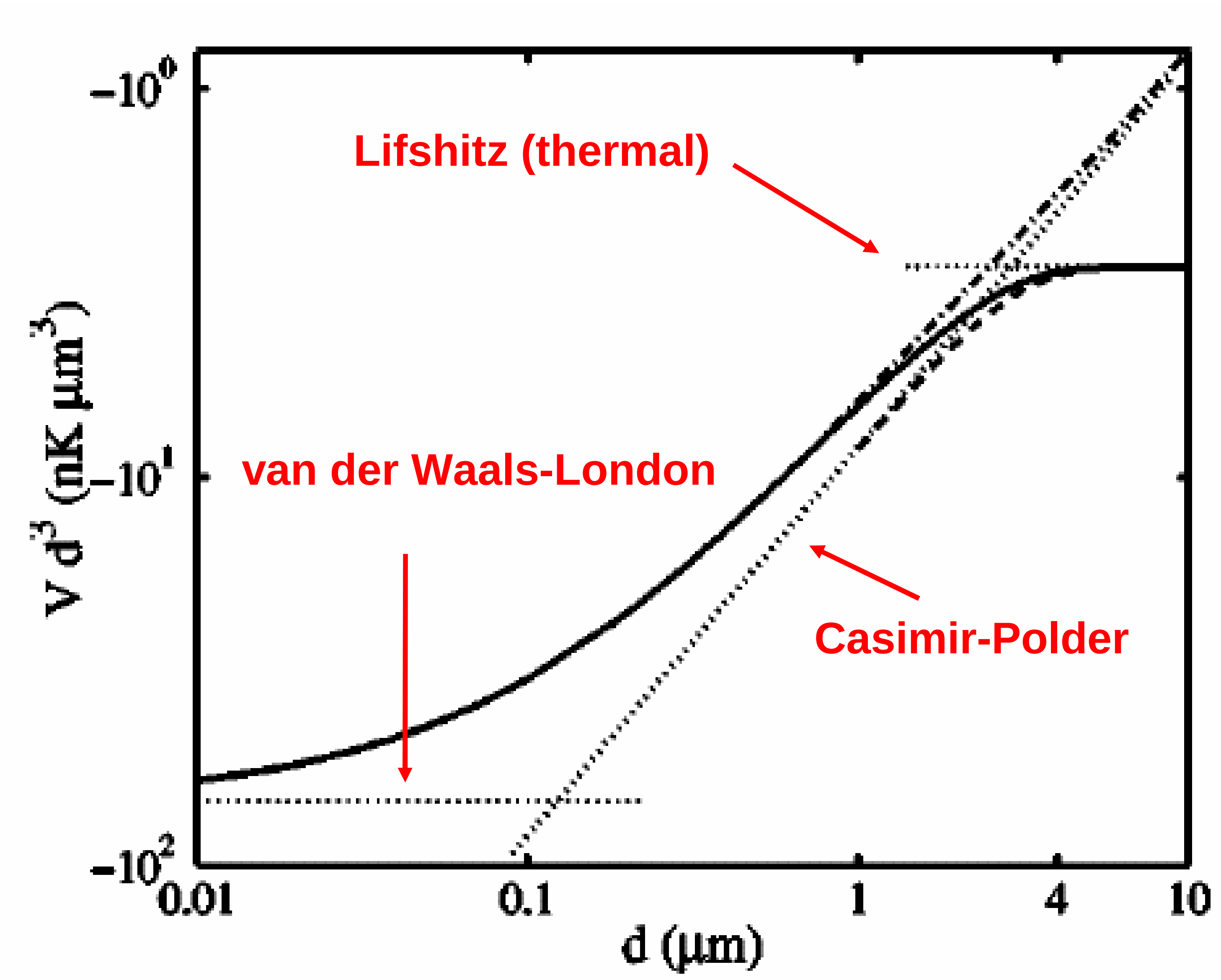
$$F^{eq}(T,z) \to F_{Lif}(T,z) = -\frac{3}{4}\frac{kT\alpha_0}{z^4}\frac{(\varepsilon_0-1)}{(\varepsilon_0+1)}$$

increases linealy with T

- Only **static optical properties** $(\alpha_0, \varepsilon_0)$ characterize the asymptotic behaviour of Casimir-Polder and thermal (Lifshitz) forces
- At smaller distances (van der Waals regime) dynamical optical properties $\varepsilon(\omega) = \varepsilon'(\omega) + i\varepsilon''(\omega)$ end $\alpha(\omega) = \alpha'(\omega) + i\alpha''(\omega)$ are needed

Surface (sapphire) atom (rubidium) interaction at T=300K

[M. Antezza, L.P. Pitaevskii, S.Stringari, Phys.Rev A70, 053619 (2004)]



- Casimir-Polder force already detected in various experiments
- How to detect thermal effects ?

-Surface-atom force extremely **weak at large distances**
(typically 10E-4 gravity at 4-5 microns)

- **At room temperature thermal effects** prevail only above 5-6 microns and are consequently **difficult to measure**

-***Possible strategies:***

- ***increase T***
  (thermal effect increases linearly with T, but vacuum in the chamber?)

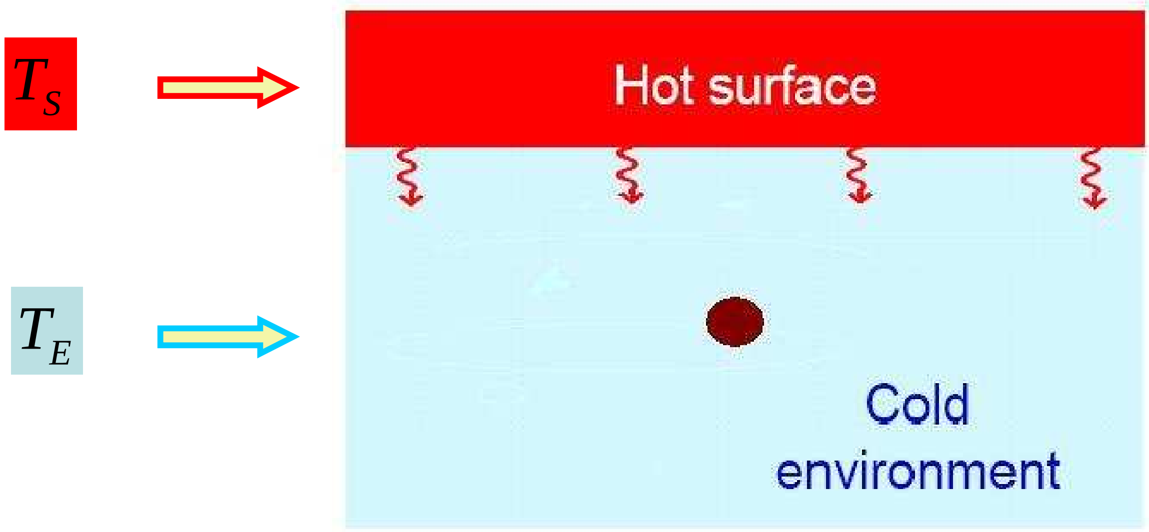
- ***out of thermal equilibrium*** configurations
  (if surface is hotter than environnment thermal effect increases **quadratically with surface temperature**)

Surface-atom force out of thermal equilibrium

- **Thermal effect** in surface-atom force can be **tunable** by varying substrate and environment temperatures.

- What happens if substrate and environment **temperatures are different** ?

- How to **describe environment** radiation and to calculate **field average values**?



Or viceversa: cold surface and hot environment

short distances

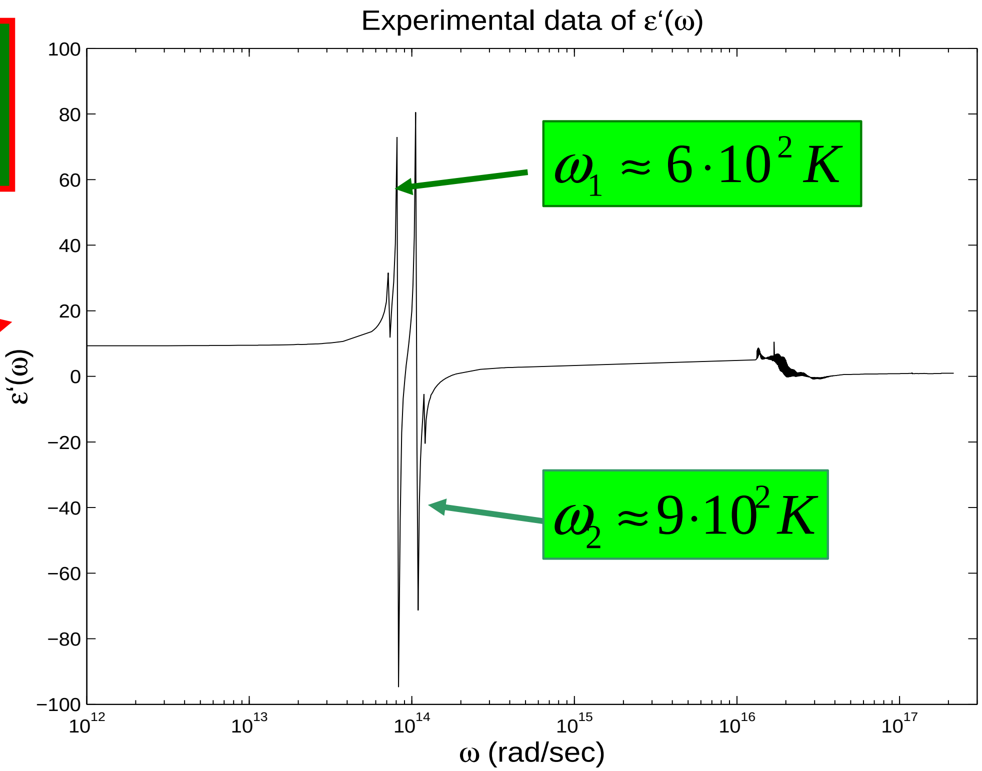
C. Henkel, K. Joulain, J.-P. Mulet and J.-J. Greffet, J. Opt. A **4**,S109 (2002)
M. Antezza, L.P. Pitaevskii and S. Stringari, PRL **95**, 113202 (2005)

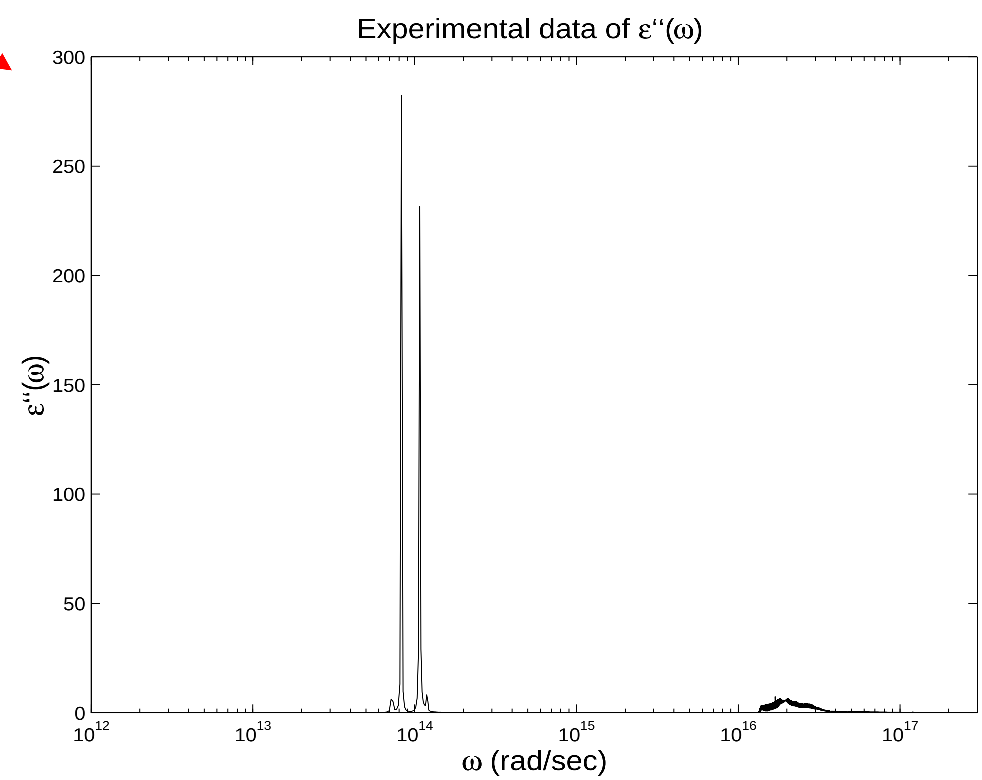
medium and long distance behaviour

Dynamic dielectric and polarizability functions

Sapphire (Al_2O_3) substrate

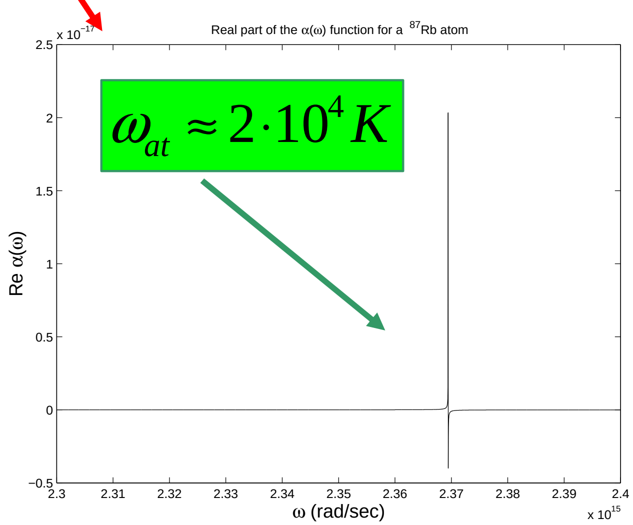
$$\varepsilon(\omega) = \varepsilon'(\omega) + i\varepsilon''(\omega)$$





Rubidium (^{87}Rb) atoms

$$\alpha(\omega) = \alpha'(\omega) + i\alpha''(\omega)$$



Thermal component of the force

$F_{th}^{eq}(T,z)$

$$F_{th}^{eq,ff}(T,z) = \frac{2\hbar}{\pi}\int_0^\infty d\omega \frac{\alpha'(\omega)\partial_{z_2} \operatorname{Im} G_{ii}[\omega;\vec{r}_1,\vec{r}_2]\big|_{\vec{r}_1=\vec{r}_2=\vec{r}}}{e^{\hbar\omega/k_B T}-1}$$

$$F_{th}^{eq,df}(T,z) = \frac{2\hbar}{\pi}\int_0^\infty d\omega \frac{\alpha''(\omega)\partial_{z_2} \operatorname{Re} G_{ii}[\omega;\vec{r}_1,\vec{r}_2]\big|_{\vec{r}_1=\vec{r}_2=\vec{r}}}{e^{\hbar\omega/k_B T}-1}$$

$$\alpha''(\omega) \approx \delta(\omega-\omega_{at})$$

$$k_B T \ll \hbar\omega_{at} \Rightarrow F_{th}^{eq}(T,z) \cong F_{th}^{eq,ff}(T,z)$$

$$k_B T_S, k_B T_E \ll \hbar\omega_{at}$$

Field fluctuations provide leading term also out of thermal equilibrium

- Atom does not contribute to thermal radiation!

- Thermal component of the force is determined by Stark effect

$$F_{th} = \frac{1}{2}\alpha_0 \partial_z < E^2 >_{th}$$

New asymptotic behaviour out of thermal equilibrium

$$F(T_S, T_E, z) = F^{eq}(T_E, z) + F_{th}(T_S, 0, z) - F_{th}(T_E, 0, z)$$

substrate → T_S ; **environment** → T_E

$$F^{neq} = -\frac{\pi}{6}\frac{\alpha_0 k_B^2 (T_S^2 - T_E^2)}{z^3 c\hbar}\frac{\varepsilon_0 + 1}{\sqrt{\varepsilon_0 - 1}}$$

holds at low temperature

M. Antezza, L.P.Pitaevskii and S.Stringari, Phys. Rev. Lett. **95**,093202 (2005)

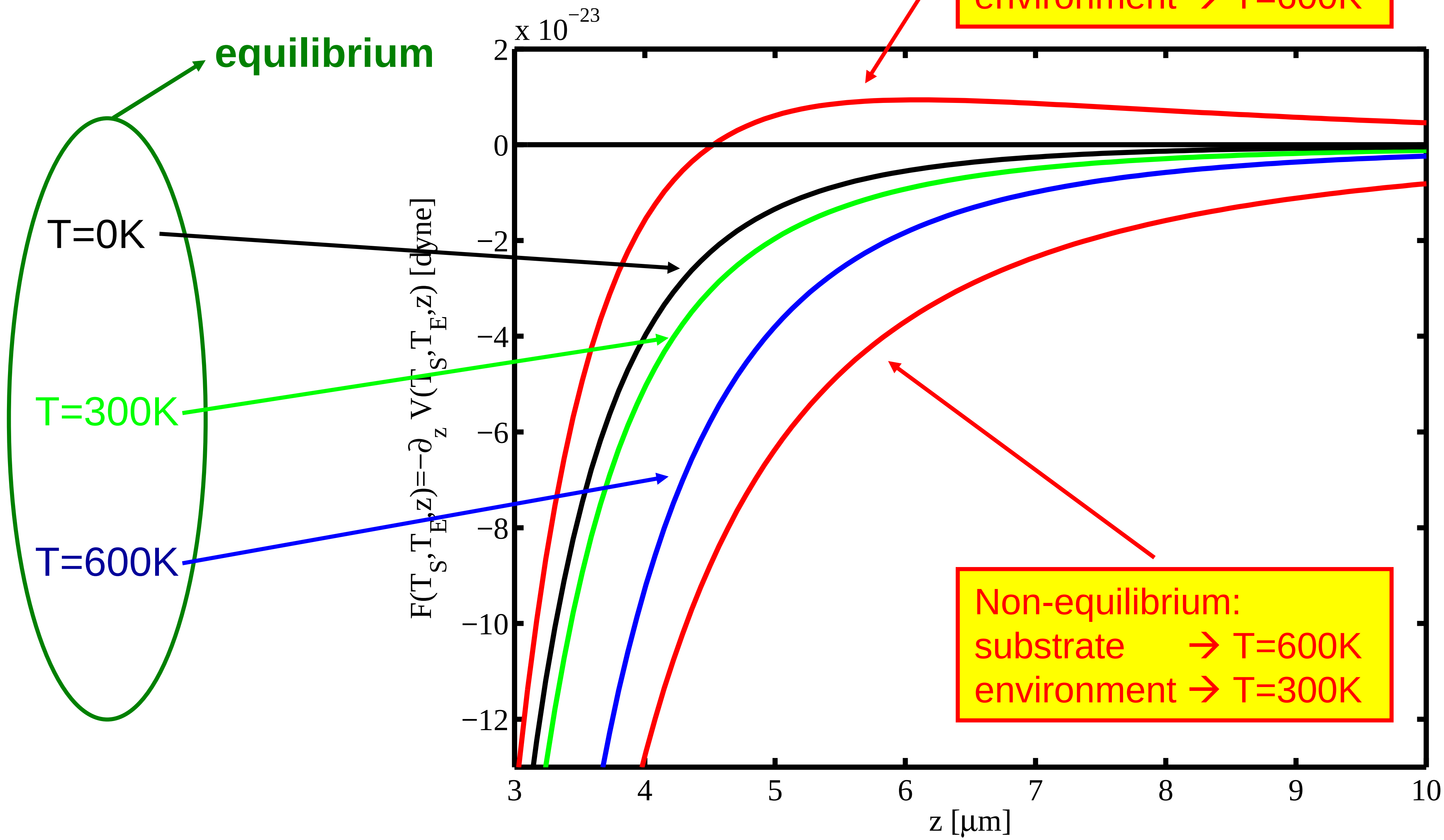
- force **decays slower than** at thermal **equilibrium**:

$$F^{eq} = -\frac{3k_B T \alpha_0 (\varepsilon_0 - 1)}{4z^4(\varepsilon_0 + 1)}$$

- force depends on **temperature** more **strongly** than at equilibrium
- force can be **attractive** or **repulsive** depending on relative temperatures of substrate and environment
- force has **quantum** nature
- simple extension to **metals** (Drude model $\varepsilon'' = 4\pi\sigma/\omega$)

Thermal effects on the surface-atom force

- Sapphire substrate
- Rubidium atoms



Measuring the Casimir-Polder forceusing ultracold atomic gases

Availability of Bose-Einstein condensates and degenerate Fermi gases yields new perspectives in the study of surface-atom forces

Possible experiments

- **Collective oscillations** with BEC's: **first experiment** at **JILA** (2005) (sensitive to the **gradient** of the force)

  oscillations

Bose-Einstein-condensed gases are dilute, ultracold samples characterized by unique properties of coherence and superfluidity. They give rise, among others, to a variety of collective oscillations (S. Stringari (1996))

- **Bloch oscillations** with ultracold degenerate gases (sensitive to the **force**)

  oscillations+interference

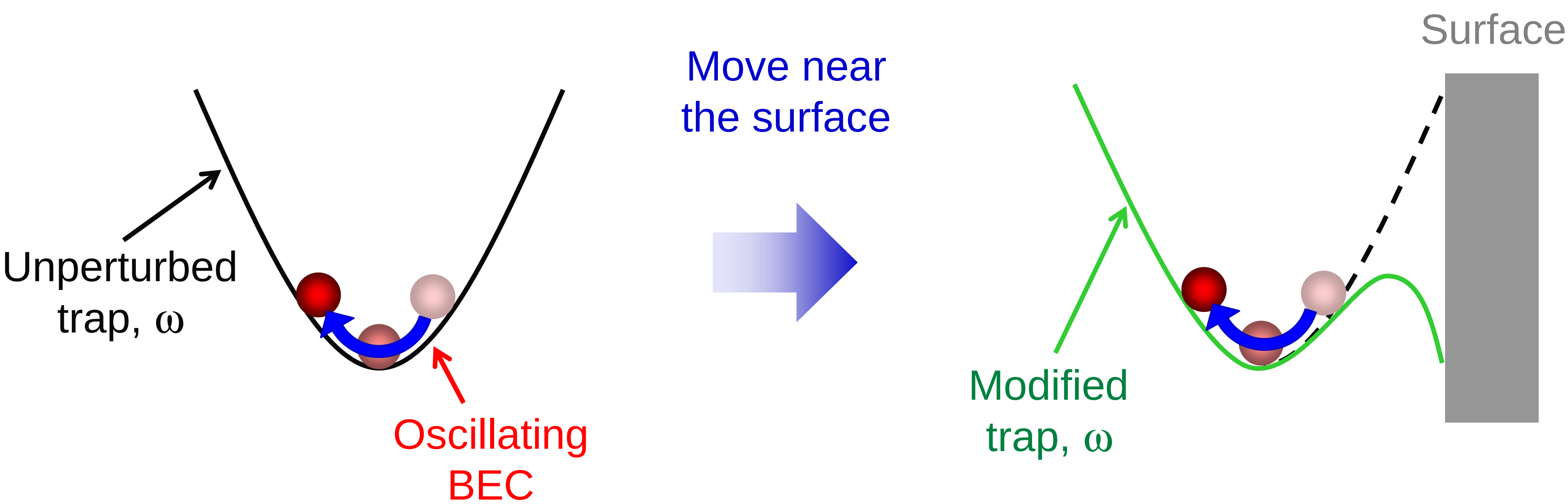
- Macroscopic **BEC** phase **interference** in **double well** potentials (sensitive to the **potential**)

  interference

Measuring atom-surface interactions: dipolar oscillations of a BEC

Use trapped BEC as a mechanical oscillator:
Measure changes in oscillation frequency

Attractive force -> Trap frequency decrease



Frequency shift of collective oscillations of a BEC

In M. Antezza, L.P. Pitaevskii and S. Stringari, PRA 70, 053619 (2004), the surface-atom force has been calculated and used to predict the frequency shift of the center of mass oscillation of a trapped Bose-Eisntein condensate, including:

-Effects of finite size of the condensate

-Non armonic effects due to the finite amplitude of the oscillations

-Dipole (center of mass) and quadrupole (long lived mode) frequency shifts

In the presence of harmonic potential + surface-atom force frequency of center of mass motion is given by

$$V_{ho}(\vec{r}) = \frac{m}{2}\omega_x^2 x^2 + \frac{m}{2}\omega_y^2 y^2 \frac{m}{2}\omega_z^2 z^2$$

$$\omega_{cm}^2 - \omega_z^2 = \frac{1}{m}\int n_0(\vec{r})\partial_z^2 V_{surf-at}(z)d\vec{r} + \frac{a^2}{8m}\int n_0(\vec{r})\partial_z^4 V_{surf-at}(z)d\vec{r}$$

Linear approximation

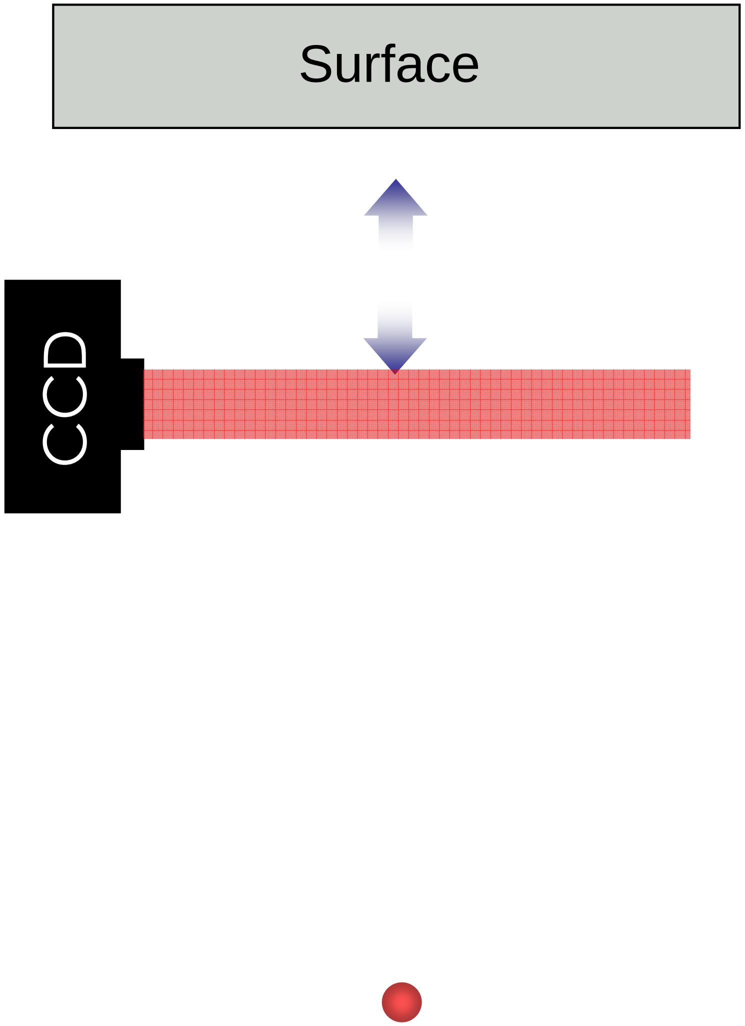
First non-linear correction

a= amplitude of c.m. oscillation

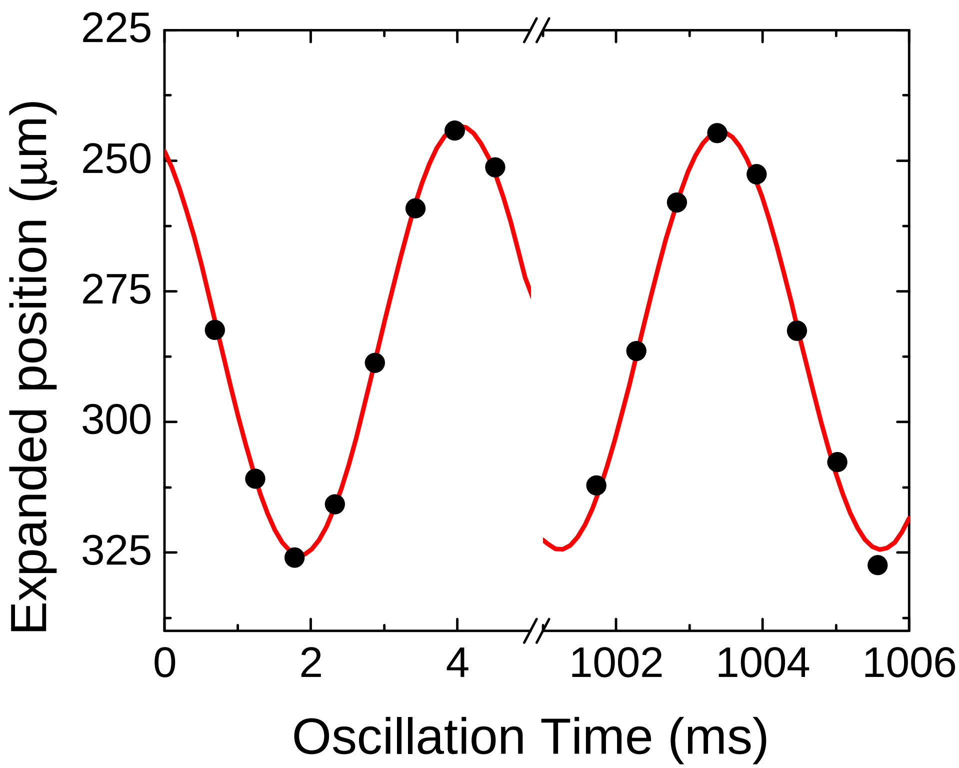
$$Z_{cm} = Z_0 + a\cos(\omega t)$$

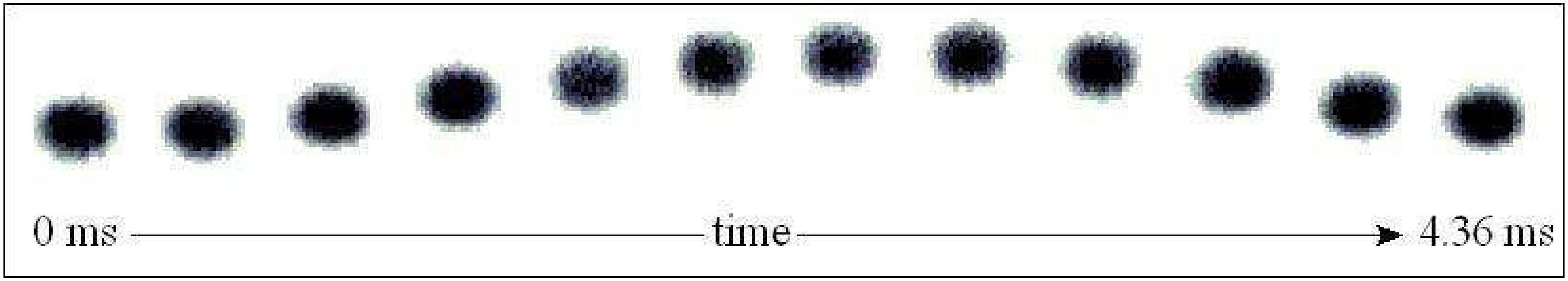
$n_0(r) \equiv$ Thomas-Fermi inverted parabola

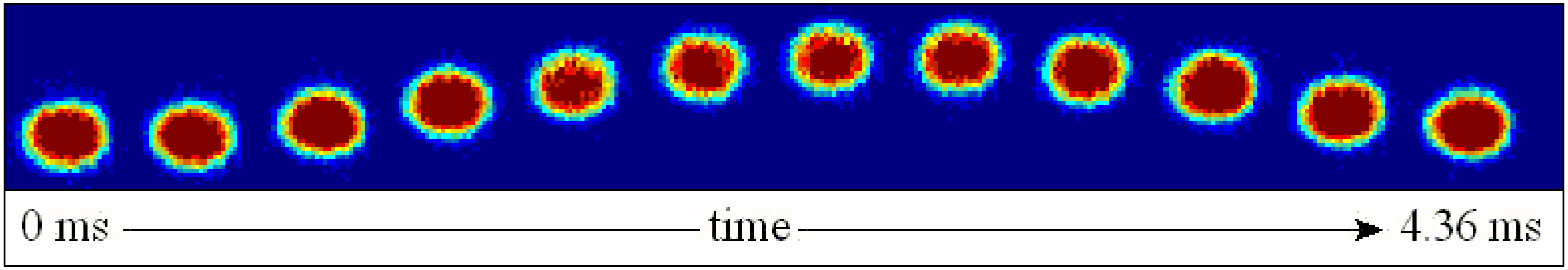
Measuring atom-surface interactions: dipolar oscillations of a BEC



1) Make BEC far from surface
2) Push BEC a few microns from surface
3) Excite oscillation vertically
4) Switch to anti-trapped state (atoms fall)
5) Image atoms on CCD camera







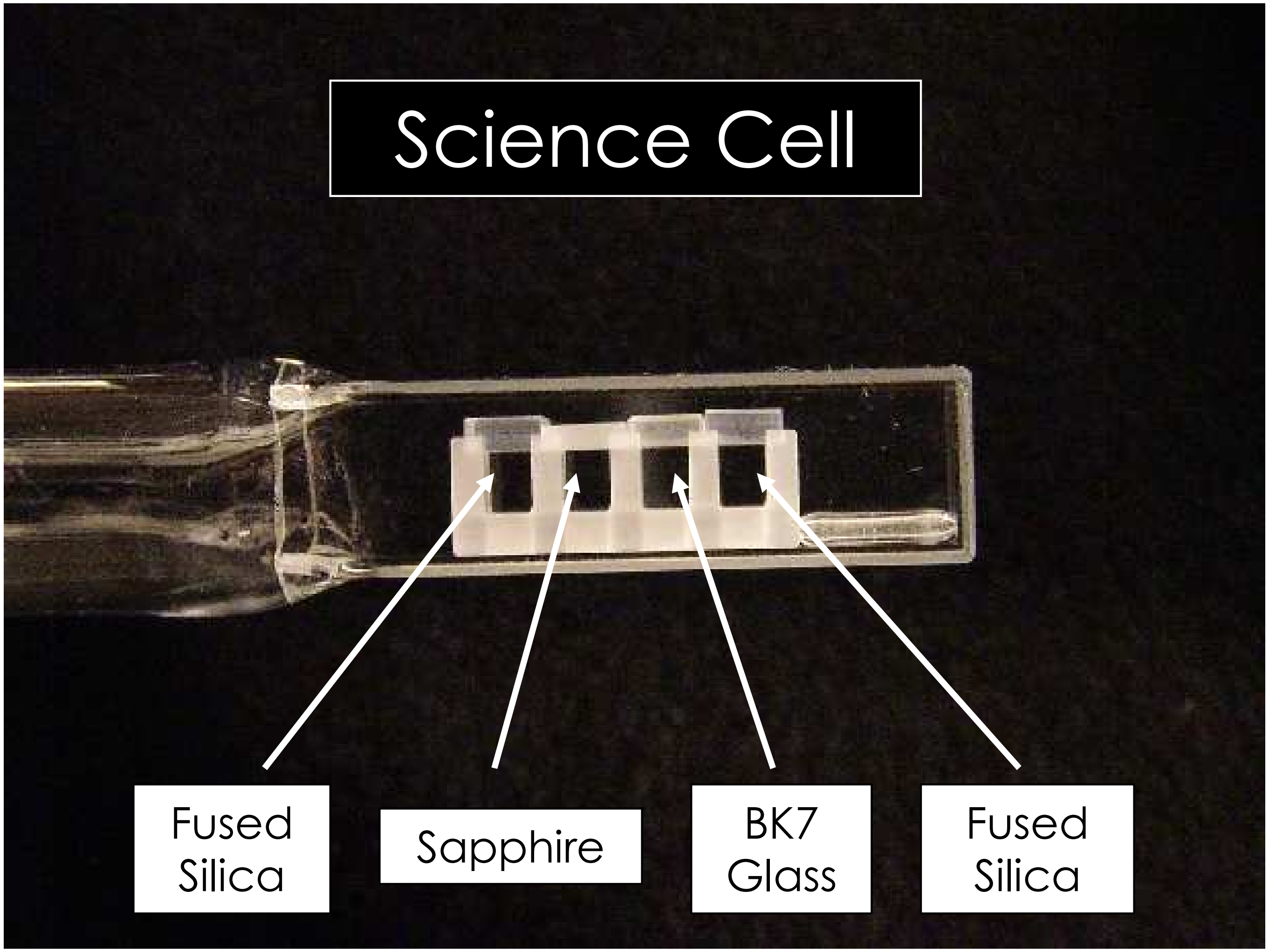
Dipole mode oscillation:

Damping time ~**10 seconds**

Frequency resolution ~**10 mHz**

→ *FFS* resolution ~**4 x 10^{-5}**

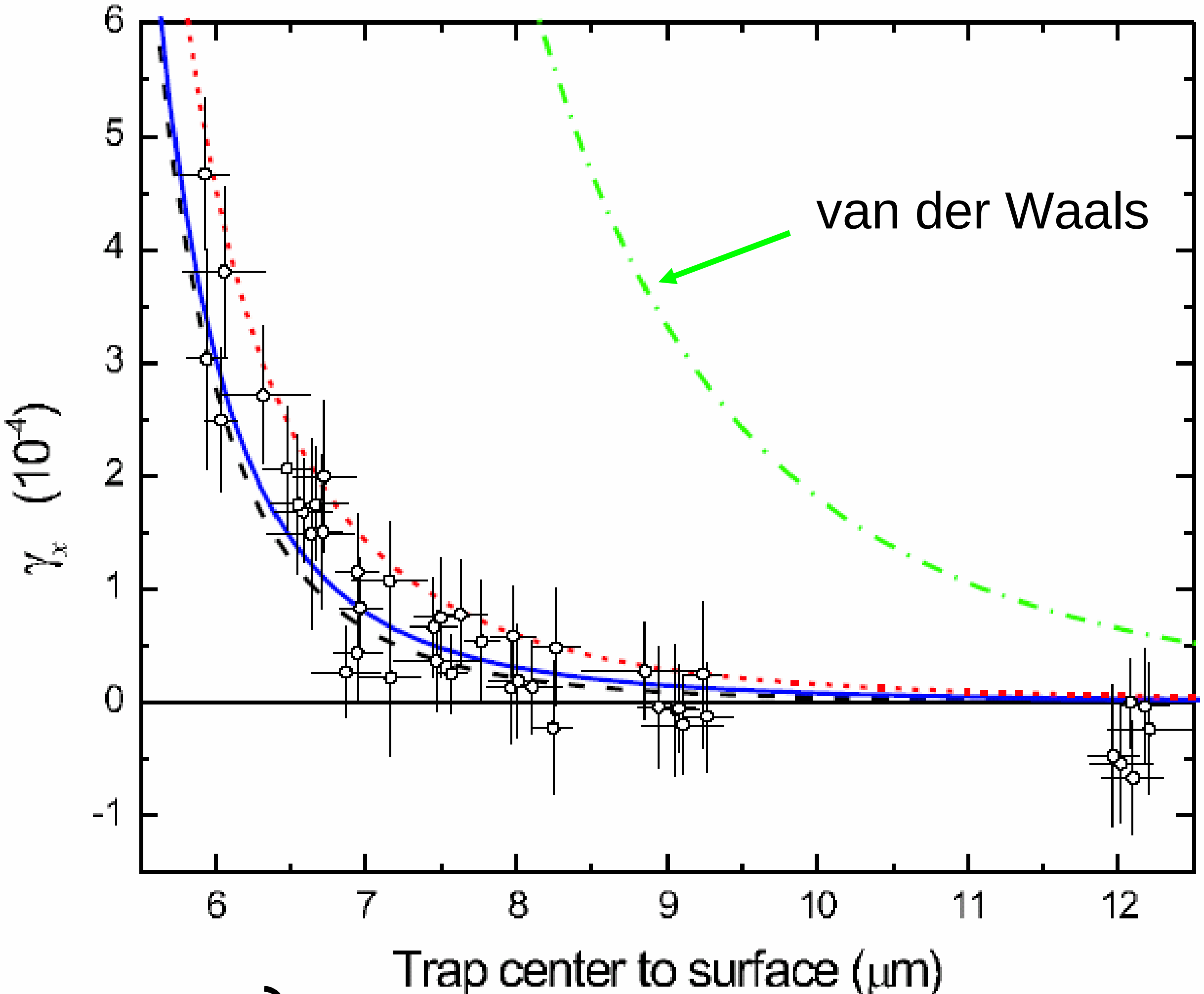
The experimental apparatus



- Multiple dielectric surfaces! Amorphous glass, crystalline sapphire.
- No conducting objects near atoms!
- Can sustain high temperatures and be compatible with UHV!)

Measurement of Casimir-Polder (+Lifshitz?) force with Bose-Einstein condensates

Exp: D.M. Harber, J.M. Obrecht, J.M. McGuirk ,and E.A. Cornell, PRA 72, 033610 (2005)



- Fused Silica substrate
- Rubidium atoms
- Experiment at room temperature

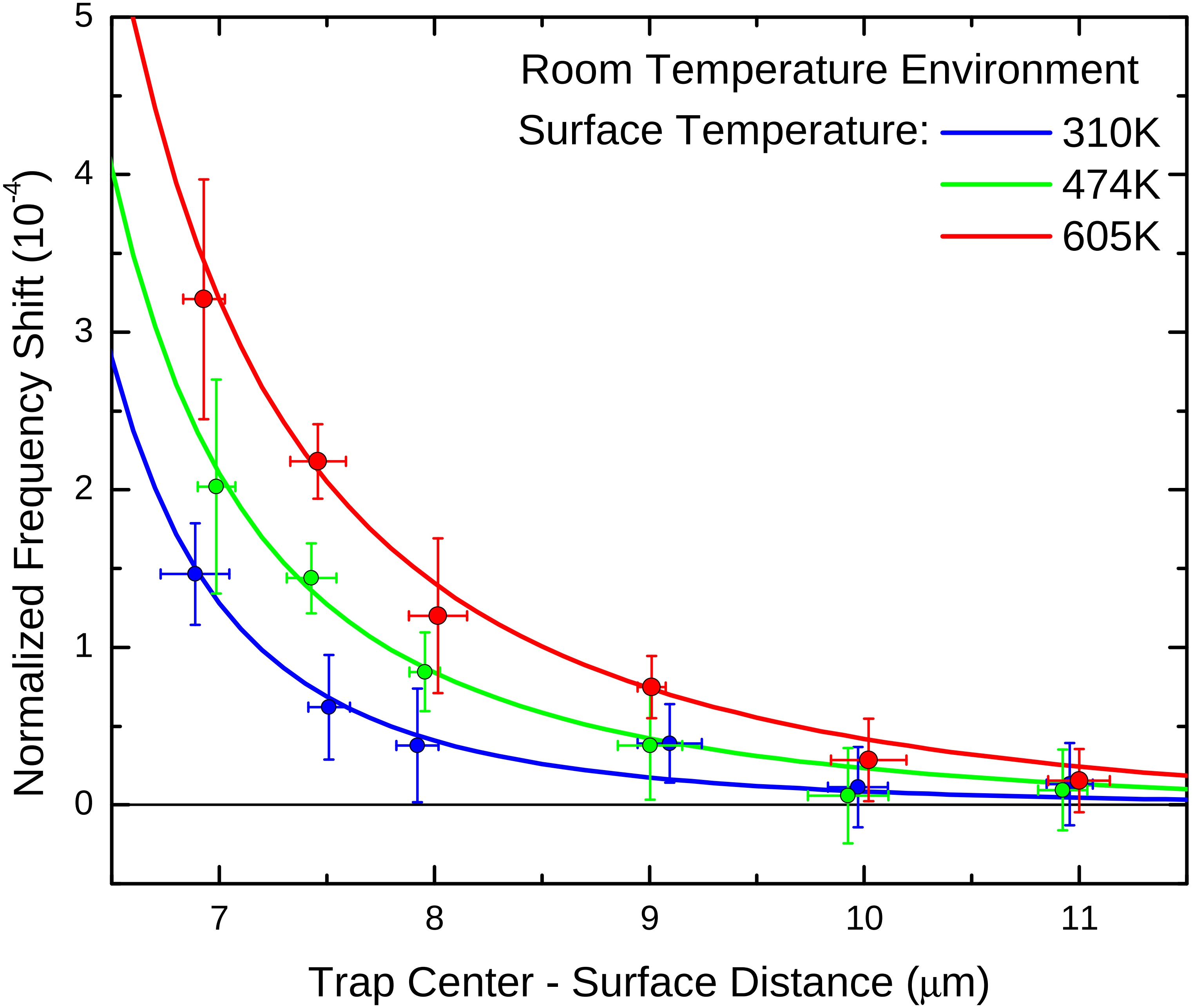
Thermal effects not yet evident some months ago!

Frequency shifts strongly enhanced by non equilibrium effects !?!

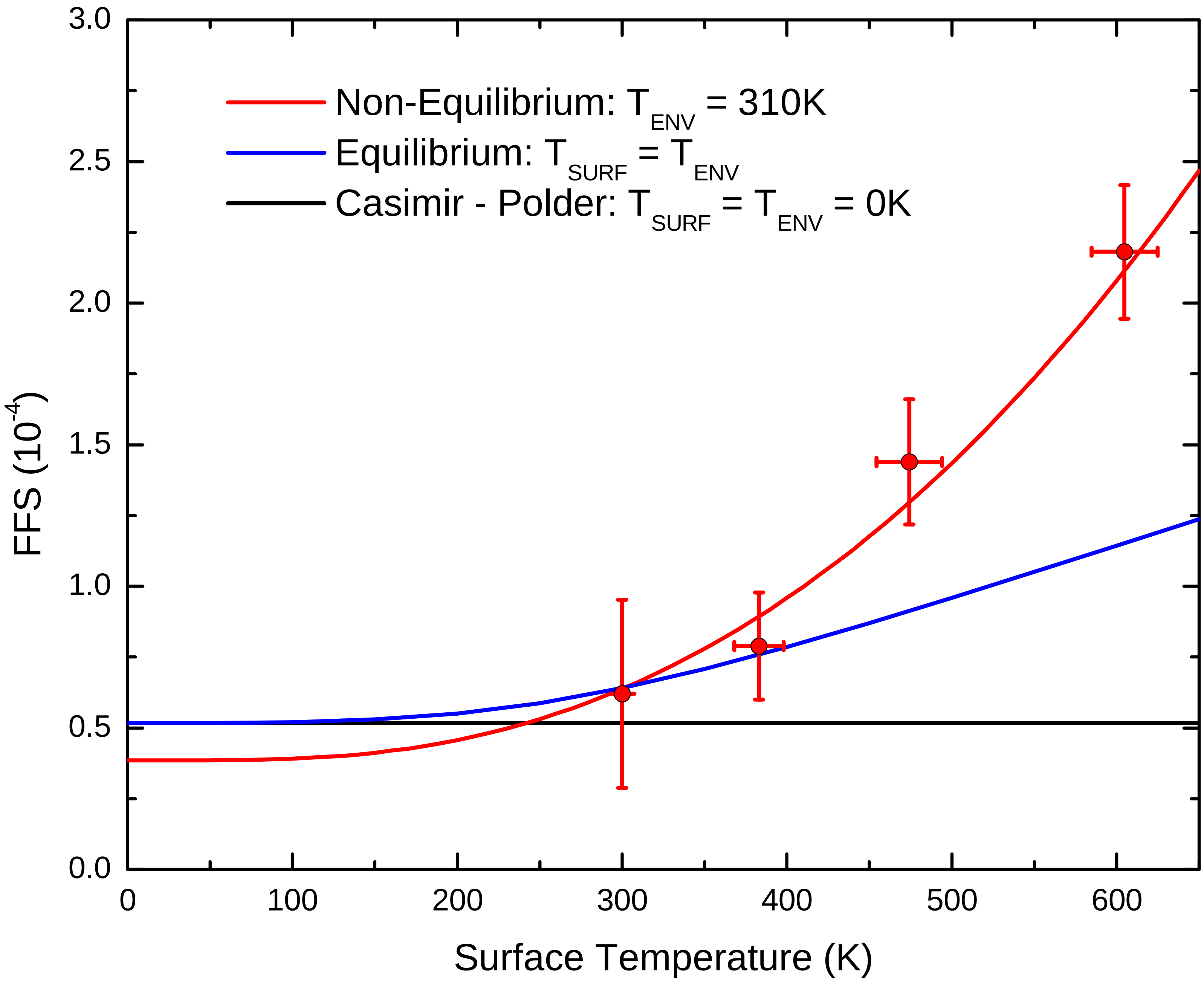
---- 500 K
---- 300 K
---- 0 K

Theory: Antezza, Pitaevskii, Stringari, Phys. Rev A **70**, 053619 (2004)

Recent Experimental results from JILA



J.M. Obrecht, R.J. Wild, M. Antezza, L.P. Pitaevskii, S. Stringari, and E.A. Cornell, submitted (2006), arXiv:physics/0608074



Non-Equilibrium: T_{ENV} = 310K
Equilibrium: T_{SURF} = T_{ENV}
Casimir - Polder: T_{SURF} = T_{ENV} = 0K
FFS (10^{-4})
Surface Temperature (K)
0.0
0.5
1.0
1.5
2.0
2.5
3.0
0
100
200
300
400
500
600

- Experiment on collective oscillation probes **gradient of the force**

- Due to finite size of condensate and amplitude of oscillation **experiment does not probe the effects locally** (average sensitive to shorter distances where thermal effects are weaker).

Bloch oscillations: new strategy for high precision measurements

Sensitive measurement of forces at micron scale using Bloch oscillations
I. Carusotto, L. Pitaevskii, S. Stringari, G. Modugno, and M. Inguscio,
Phys. Rev. Lett. **95**, 093202 (2005)

- **Center of mass** oscillation
  - measures **gradient** of the force
  - **mechanical** approach (oscillation in coordinate space)

- **Bloch** oscillation
  - measures directly the **force**
  - **interferometric** approach (oscillation in momentum space)

Bloch oscillations in atomic gases

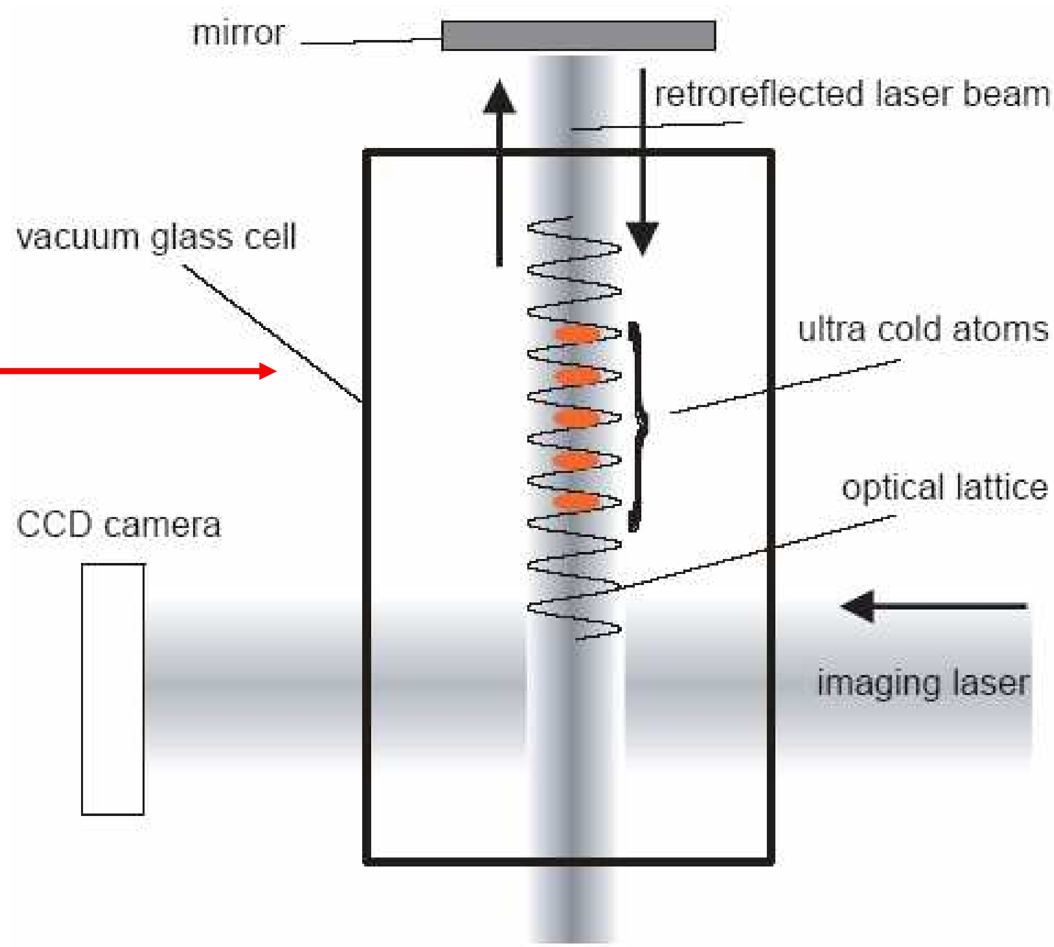
- atomic gas initially feels **3D harmonic** trap+ **gravity** + **periodic** confinement

- at t=0 one switches off harmonic trap
  System feels periodic potential + gravity and starts oscillating (Bloch oscillation).

- After given evolution time the periodic potential is switched off. Atomic gas falls down, expands and is hence imaged.

- For ideal gas imaged profiles are proportional to initial momentum distribution

$$\omega_{Bloch} = mg\lambda / 2\hbar$$



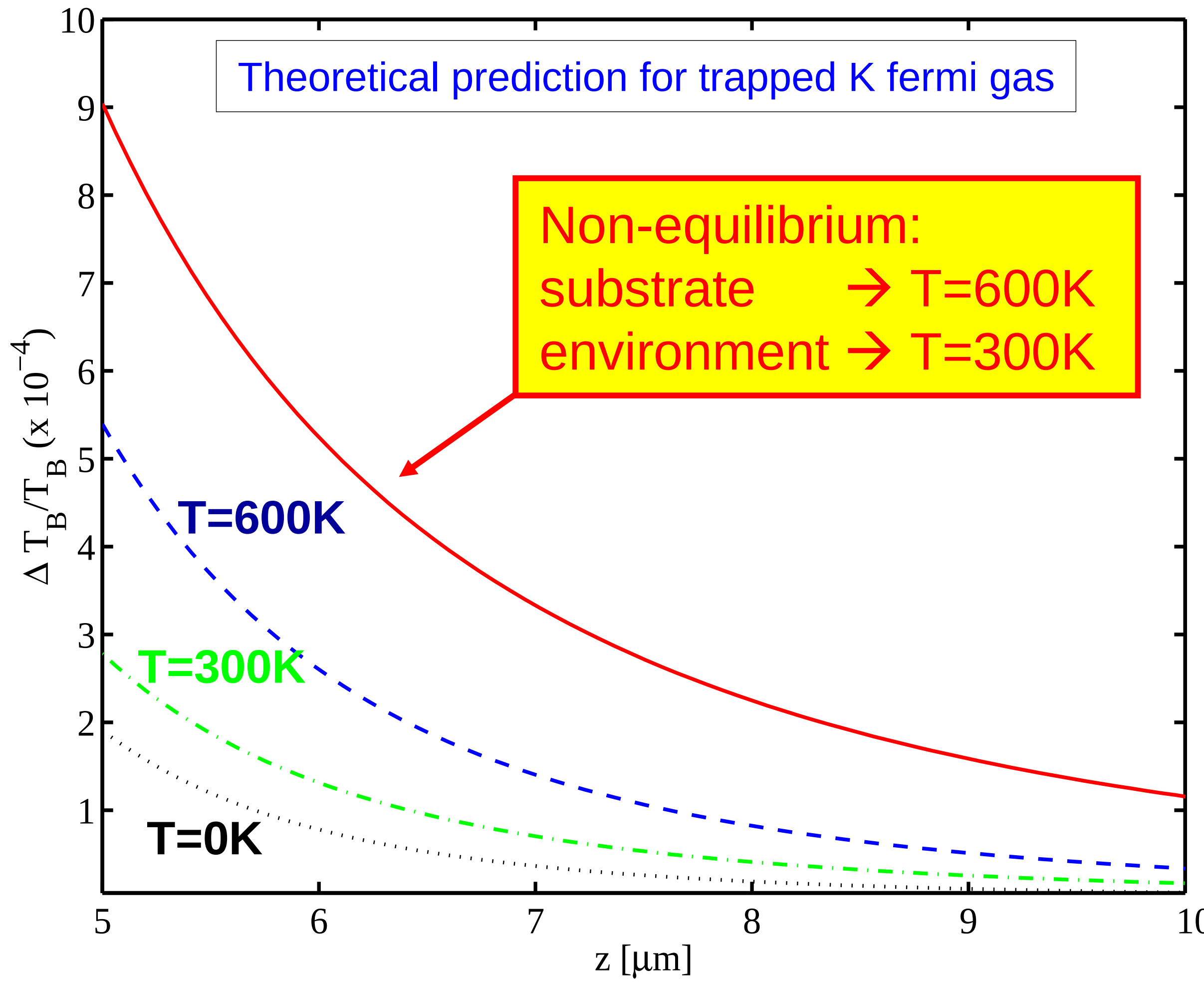
Atoms filling different wells evolve with different phase due to gravity !
(interferometric tool)

Surface-atom force effects on the Bloch frequency

Bloch oscillations of a trapped gas in an optical lattice in presence of gravity and surface-atom interactions: change in the ext. force → change in the Bloch frequency

$$\hbar \dot{q} = F = mg + F_{CP}$$

$$F_g / F_{CP} \approx 10^4$$



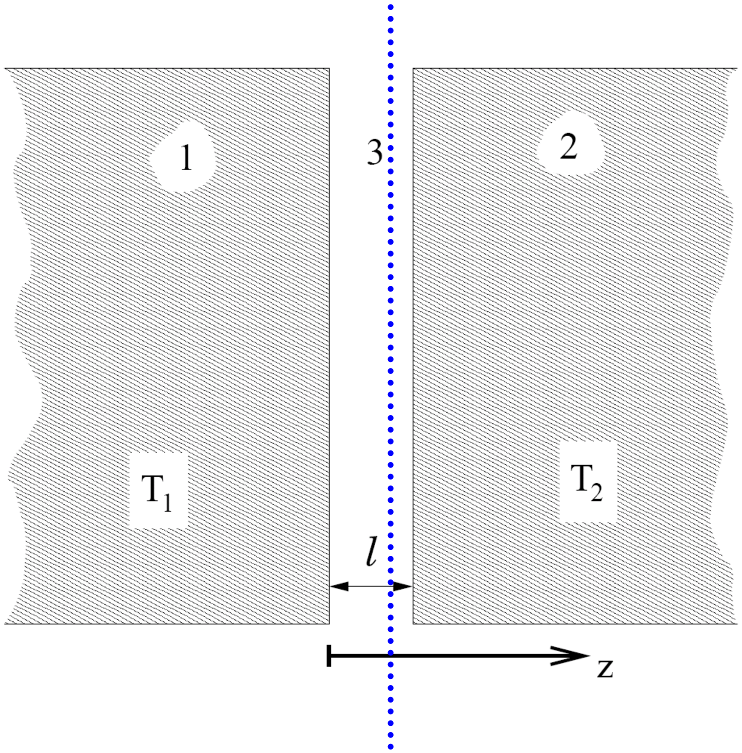
Sensitivity required: $\Delta T_B/T_B = 10^{-4}$ -10^{-5}

Carusotto, Antezza, Pitaevskii, Stringari

Some High Precision Experiments on Bloch Oscillations with Cold atoms

- M. Ben Dahan *et al.*, Phys. Rev. Lett. **76**, 4508 (1996)
- E. Peik *et al.*, Phys. Rev. A **55**, 2989 (1997)
- R. Battesti *et al.*, Phys. Rev. Lett. **92**, 253001 (2004)
- G. Roati *et al.*, Phys. Rev. Lett. **92**, 230402 (2004)
- P. Lemonde, and P. Wolf, Phys. Rev. A **72**, 033409 (2005)
- G. Ferrari *et al.*, Phys. Rev. Lett. **97**, 060402 (2006)

Surface-surface pressure out of thermal equilibrium



$$P^{neq}(T,l) = \langle T_{zz}(\vec{r},t) \rangle$$

- Role of thermal fluctuation?
- is it possible to recover the surface-atom force?
- what for asymptotics?

- Dorofeyev, J. Phys. A: Math. Gen. 31, 4369 (1998) – equal materials
- Dorofeyev et al., Phys. Rev. E 65, 026610 (2002) – different materials

⇓

$$P_{th}^{neq}(T_1,T_2,l) = \frac{P_{th}^{eq}(T_1,l)}{2} + \frac{P_{th}^{eq}(T_2,l)}{2}$$

Our Results

$$P_{th}^{neq}(T_1,T_2,l) = P_{th}^{neq}(T_1,0,l) + P_{th}^{neq}(0,T_2,l)$$

$$P_{th}^{neq}(T,0,l) = \frac{P_{th}^{eq}(T,l)}{2} + \Delta P_{th}(T,l)$$

$$P_{th}^{neq}(0,T,l) = \frac{P_{th}^{eq}(T,l)}{2} - \Delta P_{th}(T,l)$$

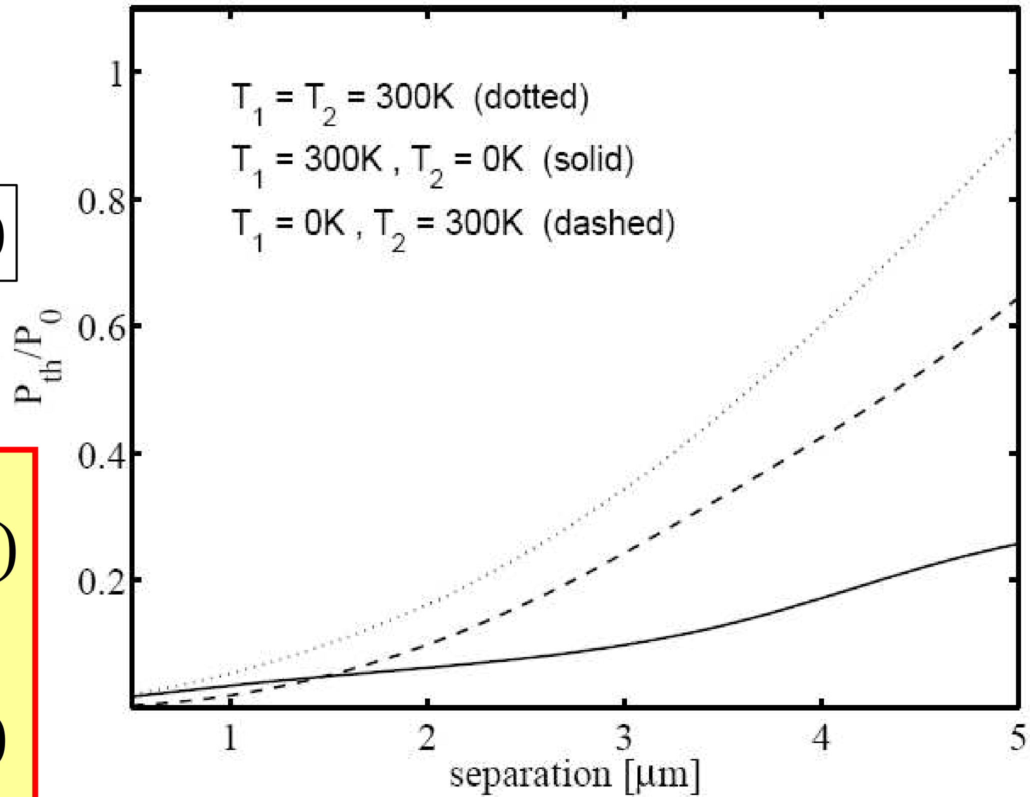


FIG. 1: Relative contribution of the thermal component with respect to the zero-temperature component of the pressure between two different materials: Fused Silica (SiO_2, body 1) and Silicon (Si, body 2).

$$\Delta P_{\rm th}^{\rm PW}(T,l) = -\frac{\hbar}{4\pi^2}\int_0^\infty {\rm d}\omega \frac{1}{e^{\hbar\omega/k_BT}-1}\int_0^k {\rm d}Q\, Q\, q_z \sum_{\mu=s,p}\left(|r_2^\mu|^2 - |r_1^\mu|^2\right)\left(\frac{1}{|D_\mu|^2} - \frac{1}{1-|r_1^\mu\, r_2^\mu|^2}\right),$$

$$\Delta P_{\rm th}^{\rm EW}(T,l) = \frac{\hbar}{2\pi^2}\int_0^\infty {\rm d}\omega \frac{1}{e^{\hbar\omega/k_BT}-1}\int_k^\infty {\rm d}Q\, Q\, {\rm Im}q_z\, e^{-2l{\rm Im}q_z}\sum_{\mu=s,p}\frac{{\rm Im}\,(r_1^\mu)\,{\rm Re}\,(r_2^\mu) - {\rm Im}\,(r_2^\mu)\,{\rm Re}\,(r_1^\mu)}{|D_\mu|^2},$$

M. Antezza, L.P. Pitaevskii, S. Stringari, V.B. Svetovoy, arXiv:cond-mat/0607205 (2006)

Asymptotic behaviours for the surface-surface pressure

Equilibrium

$$P^{\rm eq,PW}_{\rm th}(T,l) = \frac{k_B T\zeta(3)}{4\pi l^3},$$

$$P^{\rm eq,EW}_{\rm th}(T,l) = -\frac{k_B T\zeta(3)}{4\pi l^3} + \frac{k_B T}{16\pi l^3}\int_0^\infty \mathrm{d}x\, x^2 \left[\frac{\varepsilon_{10}+1}{\varepsilon_{10}-1}\frac{\varepsilon_{20}+1}{\varepsilon_{20}-1}e^x - 1\right]^{-1},$$

$$l \gg \lambda_T \max_{m=1,2}\left[\frac{\varepsilon_{m0}}{\sqrt{\varepsilon_{m0}-1}}\right].$$

$$l \gg \lambda_T$$

Non Equilibrium

$$P^{\rm neq,PW}_{\rm th}(T,0,l) \to \frac{k_B T}{l^3}\frac{\zeta(3)}{16\pi}\left[2 - \frac{\sqrt{\varepsilon_{10}-1}-\sqrt{\varepsilon_{20}-1}}{\sqrt{\varepsilon_{10}-1}+\sqrt{\varepsilon_{20}-1}} - \frac{\varepsilon_{20}\sqrt{\varepsilon_{10}-1}-\varepsilon_{10}\sqrt{\varepsilon_{20}-1}}{\varepsilon_{20}\sqrt{\varepsilon_{10}-1}+\varepsilon_{10}\sqrt{\varepsilon_{20}-1}}\right],$$

$$P^{\rm neq,EW}_{\rm th}(T,0,l) \to \frac{k_B T}{l^3}\frac{1}{8\pi^2}\int_0^\infty \mathrm{d}t\int_0^\infty \mathrm{d}x\,\frac{x^2\, e^{-x}}{t}\sum_{\mu=s,p}\frac{\mathrm{Im}\,[r_1^\mu(t)]\,\mathrm{Re}\,[r_2^\mu(t)]}{|1-r_1^\mu(t)r_2^\mu(t)\,e^{-x}|^2},$$

Asymptotic behaviours for the surface-rarefied body pressure

At equilibrium: for $\ell >> \lambda_T$

$$P_{th}^{eq}(T,\ell) = \frac{k_B T}{16\,\pi\,\ell^3}\,\frac{\varepsilon_{10}-1}{\varepsilon_{10}+1}\,(\varepsilon_{20}-1)$$

$$\varepsilon_{20} - 1 = 4\,\pi\,n\,\alpha$$

$$\lambda_T = \frac{\hbar c}{k_B T}$$

Out of equilibrium:
two different limiting procedures:

first $\ell \to \infty$ with fixed ε_{20}
and then $(\varepsilon_{20}-1) \to 0$:

$$P_{th}^{neq}(T,0,\ell) = \frac{k_B T \;\; C}{\ell^3}\,\frac{\varepsilon_{10}-1}{\sqrt{\varepsilon_{10}-1}}\,\sqrt{\varepsilon_{20}-1}$$

holding at $\ell >> \dfrac{\lambda_T}{\sqrt{\varepsilon_{20}-1}}$

The non additivity of the pressure (12) follows from the fact that for large l the main contribution in the force is produced by the *grazing waves* incident on the interface of the material 2 from the vacuum gap at small angles corresponding to small values of q_z/k, of order λ_T/l. If $q_z/k \leq \sqrt{\varepsilon_{20}-1}$, the reflection coefficients from the body 2 is not small even at small $\varepsilon_{20}-1$ and the body cannot be considered dilute from an electrodynamic point of view. This is a peculiarity of the non-equilibrium situation. In fact at equilibrium this anomalous contribution is canceled by the waves impinging the interface from the interior of the dielectric 2, close to the angle of total reflection (in a rarefied body such waves become grazing).

first $(\varepsilon_{20} - 1) \to 0$ with fixed ℓ

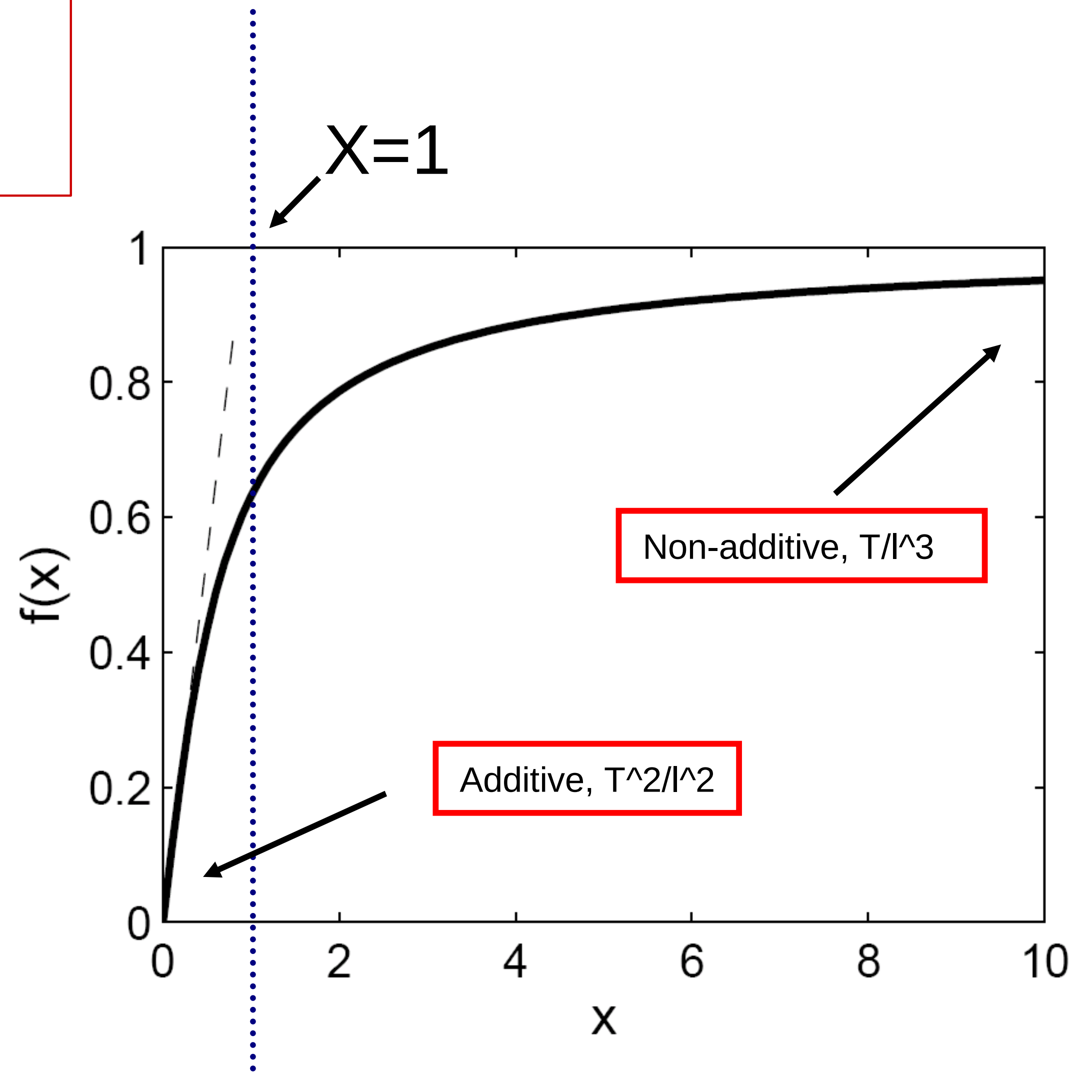
and then $\ell \to \infty$: ($n.b.$ PW and EW are equal!)

$$P_{th}^{neq}(T,0,\ell) = \frac{(k_B T)^2}{24 \ \ell^2 \ \hbar c} \frac{\varepsilon_{10} - 1}{\sqrt{\varepsilon_{10} - 1}} (\varepsilon_{20} - 1)$$

holding at $\lambda_T >> \ell >> \dfrac{\lambda_T}{\sqrt{\varepsilon_{20} - 1}}$

$$P_{\rm th}^{\rm neq}(T,0,l) = \frac{k_B T \, C}{l^3} \frac{\varepsilon_{10} + 1}{\sqrt{\varepsilon_{10} - 1}} \sqrt{\varepsilon_{20} - 1} \, f(x),$$

$$x = l\sqrt{\varepsilon_{20} - 1} / \lambda_T$$



1 - thermal dependence
2 - distance dependence
3 - non-additivity
4 - ranges of validity

What for the surface-atom force?

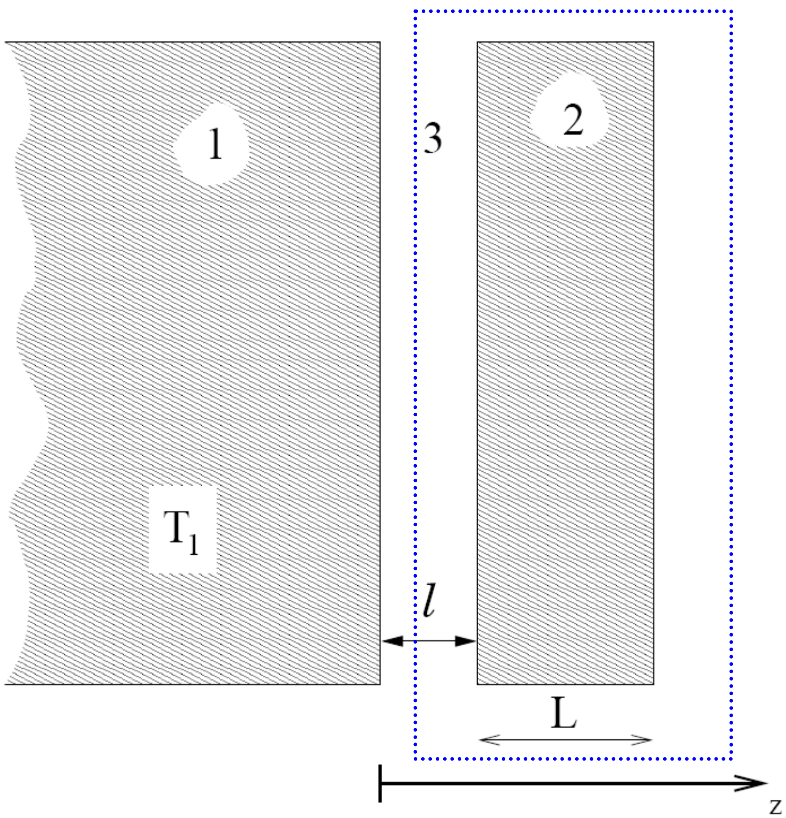
From the surface-rarefied body:

$$P_{th}^{neq}(T,0,\ell) = \frac{(k_B T)^2}{24\ \ell^2\ \hbar c}\ \frac{\varepsilon_{10}-1}{\sqrt{\varepsilon_{10}-1}}\ (\varepsilon_{20}-1)$$

But the surface-atom force is:

$$F_{th}^{neq}(T,0,\ell) = \frac{(k_B T)^2}{24\ \ell^3\ \hbar c}\ \frac{\varepsilon_{10}-1}{\sqrt{\varepsilon_{10}-1}}\ (\varepsilon_{20}-1)$$

What is the problem??



If the gas occupies a finite slab L and does not absorb the thermal radiation:

$$L \ll \lambda_T^2 / \ell \varepsilon_2''$$

the inclusion of the remote surface results in a PW contribution of the order

$$\propto (\varepsilon_{20}-1)^3$$

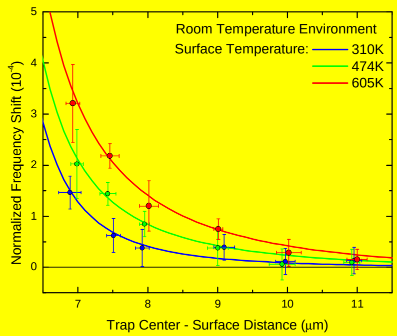
and hence should be omitted!

Conclusions

- Surface-atom force out of thermal equilibrium exhibits **new asymptotic (large distance) behaviour** and can provide **a new way to mesure thermal effects**

$$F(T_S, T_E, z) \to \frac{(T_S^2 - T_E^2)}{z^3}$$

-**Center of mass oscillation** of a trapped Bose-Einstein condensate provides powerful **mechanical tool** to detect surface-atom force at large distances, **agree** with theoretical **predictions for Casimir-Polder** force **(first measurement of any thermal effect)** (Trento-Boulder collaboration)



-Study of the surface-surface force out of thermal equilibrium and asymtotic non-additivity