Quantum-limited force measurement with an optomechanical device

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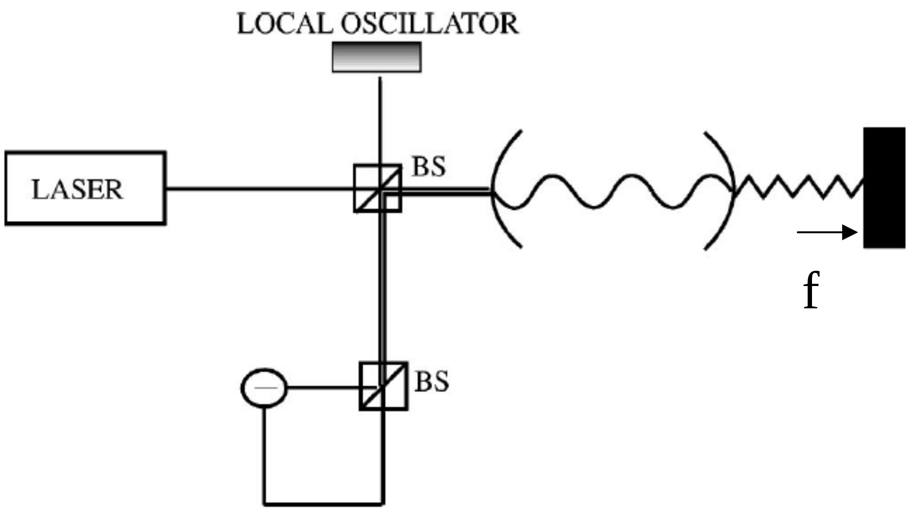
….Plus something on Heisenberg-limited interfer. in cavity-QED systems….

Optomechanical detection of a weak force

- Typical scheme: cavity with a movable mirror
- Coupled by **radiation pressure**

- Mirror = probe experiencing the force to be measured
- cavity field = meter reading out the mirror's position

- Mechanical force ⇒ momentum and position shift of a given vibrational mode of the mirror
- ⇒ phase shift of the reflected field
- Phase-sensitive measurement ⇒ detection of the force.

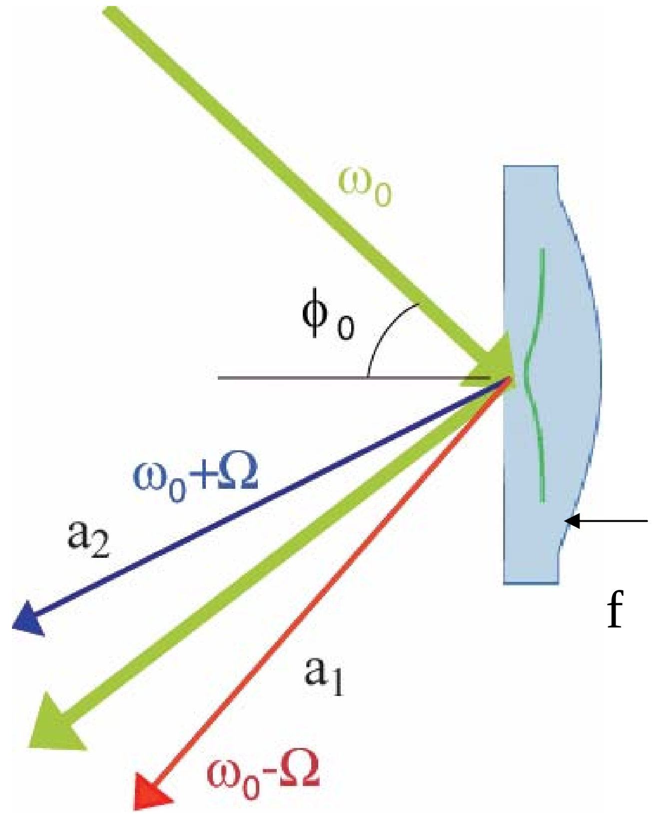


Crucial parameters:

- cavity finesse
- Input power (the one minimizing joint effect of shot noise and radiation pressure noise)

We propose a **new optomechanical scheme**, based on the detection of the **vibrational sidebands** of a strong, narrow-band laser field, incident on a **single mirror**

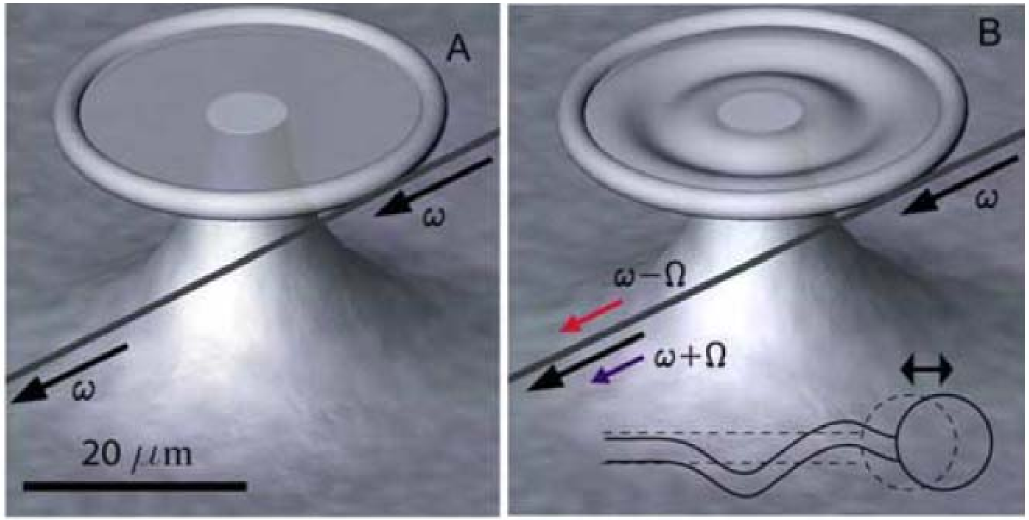
The intense driving mode @ ω_0 is reflected undisturbed, while the two sideband optical modes, initially in the vacuum state, can get photons scattered by the stationary vibrational mode

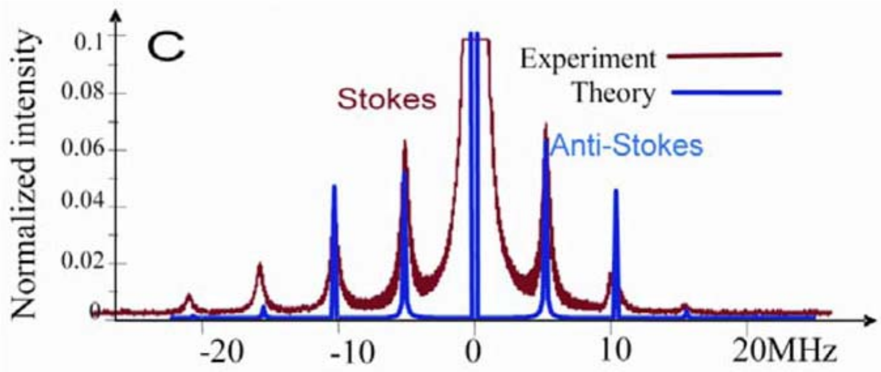


Similar to Brillouin scattering, induced however by radiation pressure and not by the modulation of the refractive index

a_1 @ $\omega_0 - \Omega$ = Stokes mode
a_2 @ $\omega_0 + \Omega$ = Anti-Stokes mode
b @ Ω = (quantized) mirror vibrational mode

Other possible implementation: **vibrating microtoroidal resonator** driven via an evanescent wave coupled laser (Vahala group, Caltech)





Observed transmitted spectrum, Carmon et al, PRL **94,** 223902 (2005)

General radiation pressure interaction Hamiltonian

for light impinging on a single (perfectly reflecting) mirror

$$\hat{H} = -\int_{\substack{mirror\\surface}} d^2\vec{r}\, \hat{P}(\vec{r},t)\, \hat{x}(\vec{r},t)$$

$\hat{x}(\vec{r},t)$ is the mirror surface deformation field and $\hat{P}(\vec{r},t)$ is the radiation pressure

We have a continuum of optical modes exciting many vibrational modes of the mirror which, in turn, scatter photons between these opt. modes.

However, we can drastically simplify the system and reduce it to an **effective three-mode problem** when we consider:

1. an intense, classical, quasi-monochromatic, incident field with frequency ω_0, small bandwidth $\Delta\nu_L$, and power P_L
2. a not too large detection bandwidth $\Delta\nu_{det}$ including only the first modulation sideband due to a **single** mirror vibrational mode (frequency Ω), at frequencies $\omega_0 \pm \Omega$.

Effective three mode interaction Hamiltonian

$$\hat{H}_{eff} = -i\hbar\chi(\hat{a}_1\hat{b} - \hat{a}_1^\dagger\hat{b}^\dagger) - i\hbar\theta(\hat{a}_2\hat{b}^\dagger - \hat{a}_2^\dagger\hat{b})$$

Analogous to optical parametric amplification leading to two-mode squeezing ⇔ generation of EPR-like entangled states between the Stokes and the vibrational mode

Beam-splitter like interaction between the anti-Stokes and the vibrational mode (analogous to optical frequency up-conversion)

Effective optomechanical coupling constants

$$\chi = \cos\phi_0 \sqrt{\frac{P_L \Delta\nu_{\det}^2(\omega_0 - \Omega)}{2M_{eff}\Omega c^2 \Delta\nu_L}}$$

$$\theta = \chi\sqrt{\frac{\omega_0 + \Omega}{\omega_0 - \Omega}}$$

Appreciable **quantum effects** expected for ***large* power P_L, and *small* M_{eff}** = effective mass of the vibrational mode, ∝ mode volume

In order to achieve a **quantum-limited detection sensitivity**, we consider a **micro-mechanical oscillator, with high resonance frequency**

The above interaction Hamiltonian is valid as long as

$$\Omega \gg \Delta\nu_{det} > \Delta\nu_L \approx 1/t_{int} \gg \gamma = \Omega/Q_M$$

Achievable parameter values could be

$$\Omega \approx 10^8 \text{ Hz} \gg \Delta\nu_{det} \approx 10^5 \text{ Hz} \gg \Delta\nu_L \approx 1/t_{int} \approx 10^3 \text{ Hz} \gg \gamma = \Omega/Q_M \approx 10 \text{ Hz}$$

If we neglect mechanical damping, time evolution is periodic in t_{int}, the duration of the driving laser pulse, with period

$$T = \frac{2\pi}{\Theta} = \frac{2\pi}{\sqrt{\theta^2 - \chi^2}}$$

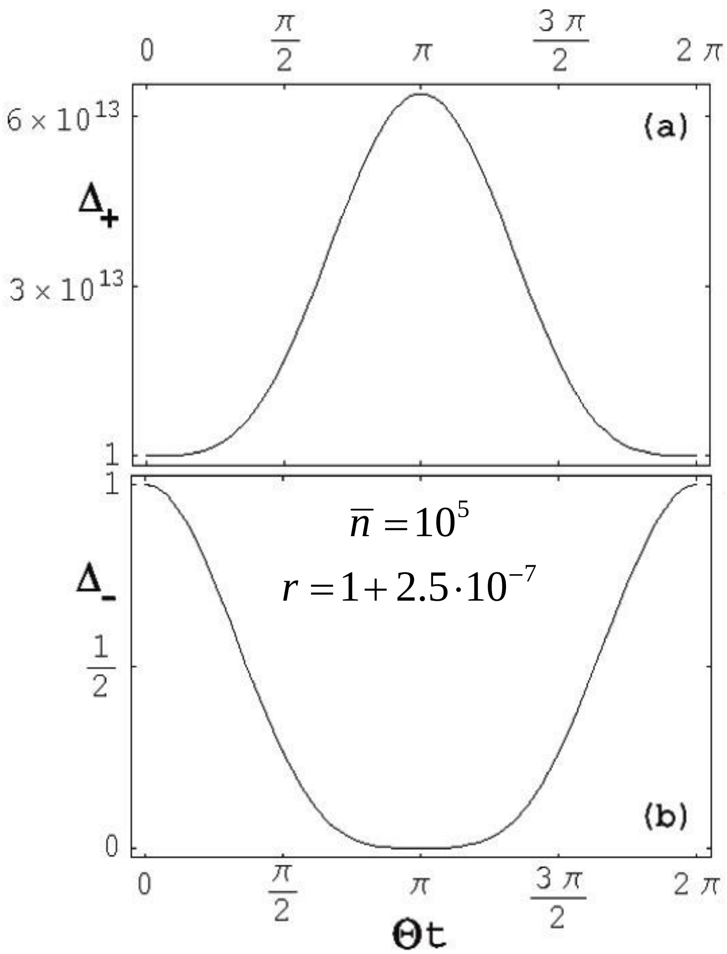
The dynamics depend upon three dimensionless parameters: the scaled dimensionless interaction time $\Theta\, t_{int}$, the mean vibrational thermal number n_T, (the mirror is assumed initially at thermal equilibrium), and the ratio r

$$r = \frac{\theta}{\chi} = \sqrt{\frac{\omega_0 + \Omega}{\omega_0 - \Omega}} \approx 1 + \frac{\Omega}{\omega_0} \approx 1 + 10^{-7}$$

(S. Pirandola et al., PRA **68**, 062317 (2003))

In particular, if $\Theta t_{int} = \pi$, thanks to radiation pressure, the two optical sidebands are in a **two-mode squeezed state,** independent of the mirror and its temperature

$$\left|\psi\right\rangle_{\pi} = \left(\frac{1-r^2}{1+r^2}\right)\sum_{n=0}^{\infty}\left(-\frac{2r}{1+r^2}\right)^n \left|n,n\right\rangle$$



Einstein-Podolski-Rosen correlations

For field quadratures X_i, P_j:

$$\Delta_{\mp} = \left\langle \left(X_1 \mp X_2\right)^2 \right\rangle = \left\langle \left(P_1 \pm P_2\right)^2 \right\rangle$$

In particular: $\Delta_{\mp}(\pi) = \left(\frac{r \mp 1}{r \pm 1}\right)^2$

$\Rightarrow$ Simultaneous eigenstate of "relative distance" and "total momentum" for $\Theta t_{int} = \pi$ and $r \to 1$

The difference between the two amplitude quadratures $\boldsymbol{X_1 - X_2}$, and the sum of the phase quadratures $\boldsymbol{P_1 + P_2}$ of the sideband modes, is highly squeezed

If we perform a **phase-sensitive detection of this combination of quadratures**, the reduced noise properties would allow to achieve **high-sensitive detection** of a force acting on the oscillator.

$$X_j = \frac{a_j + a_j^+}{\sqrt{2}}$$

$$P_j = \frac{a_j - a_j^+}{i\sqrt{2}}$$

We now explicitly include **mechanical damping** and **Brownian noise** ***$b_{in}(t)$*** and use **Heisenberg-Langevin equations** for the three-mode system

$$\dot{\hat{a}}_1(t) = \chi \hat{b}^\dagger(t)$$
$$\dot{\hat{b}}(t) = \chi \hat{a}_1^{\dagger}(t) - \theta \hat{a}_2(t) - 2\gamma \hat{b}(t) + 2\sqrt{\gamma}\hat{b}_{in}(t) + \Omega f(t)$$
$$\dot{\hat{a}}_2(t) = \theta \hat{b}(t)$$

with
$$\begin{aligned}\langle \tilde{b}_{in}(t)\tilde{b}_{in}^{\dagger}(t')\rangle &= (1+\overline{n})\delta(t-t'),\\ \langle \tilde{b}_{in}^{\dagger}(t)\tilde{b}_{in}(t')\rangle &= \overline{n}\delta(t-t'),\end{aligned}$$

We also consider the **possibility to have additional input two-mode squeezing** for the sidebands, and consider the following initial condition: $\rho_b^{th} \otimes |\psi_{12}\rangle\langle\psi_{12}|$

$$\rho_{th}^{b} = \sum_{n} \frac{\overline{n}^{n}}{(1+\overline{n})^{n+1}} |n\rangle\langle n|$$

$$|\Psi\rangle_{12} = \sqrt{1-\tanh^2 s}\sum_{n=0}^{\infty}(\tanh s)^n |n\rangle_1 |n\rangle_2$$

s = two-mode squeezing parameter

signal-to-noise ratio $SNR = \frac{S}{N} = \frac{\left|\left\langle P_1 + P_2 \right\rangle\right|}{\sqrt{\left\langle \left(P_1 + P_2\right)^2 \right\rangle - \left\langle P_1 + P_2 \right\rangle^2}}$

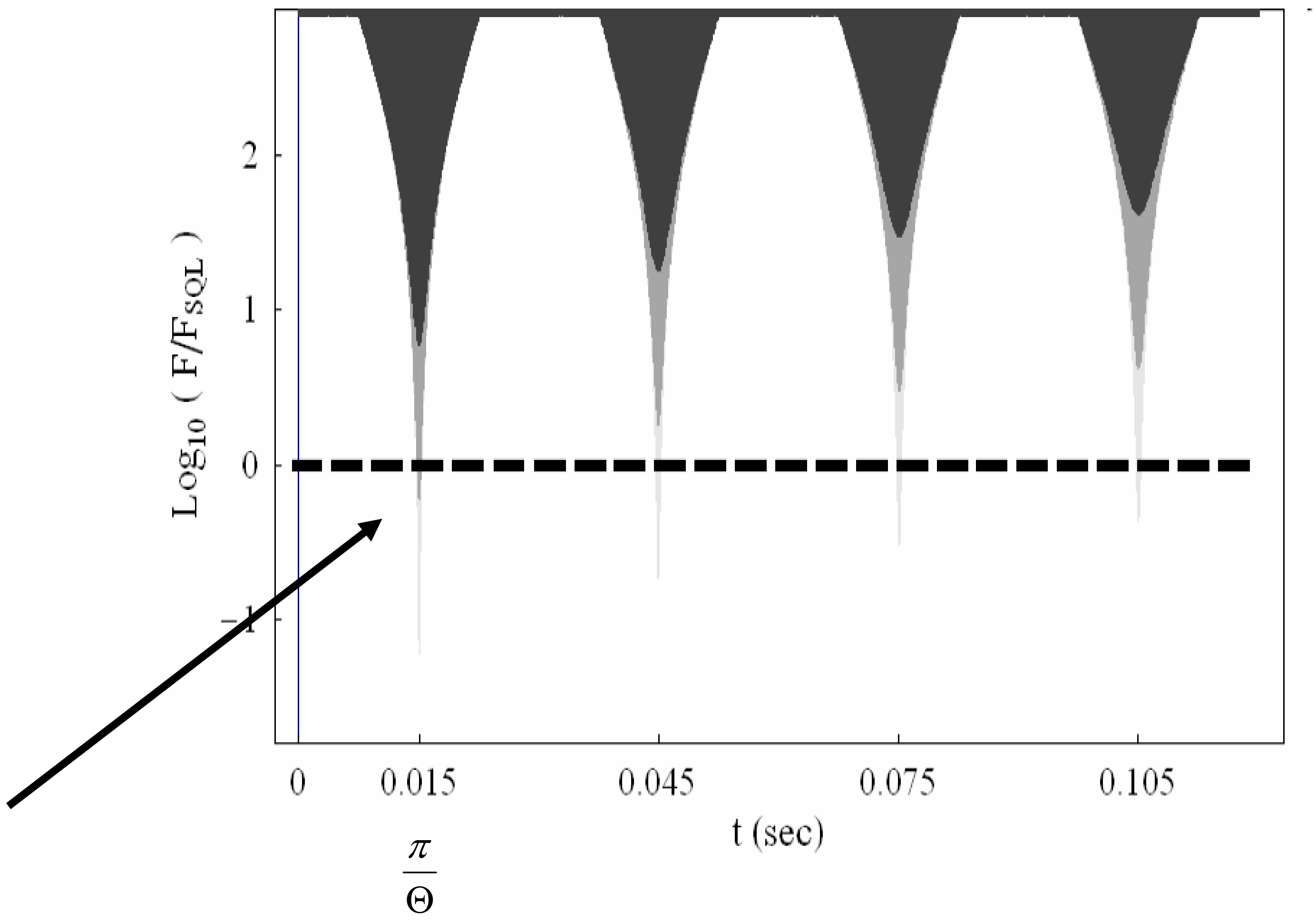
We characterize the force detection sensitivity through the **minimum detectable force**, i.e. the one realizing the condition ***SNR = 1***

$$F(t) = \frac{\sqrt{\left\langle \left(P_1(t) + P_2(t)\right)^2 \right\rangle - \left\langle P_1(t) + P_2(t) \right\rangle^2}}{\left|\left\langle P_1(t) + P_2(t) \right\rangle\right| / F(t)}$$

We compare it to the **standard quantum limit for the detection of a force**

$$F_{SQL} = \frac{\sqrt{\hbar \Omega M}}{\tau}$$

τ = observation time, $\tau \ll 1/\gamma$, $\tau \approx \Theta^{-1}$



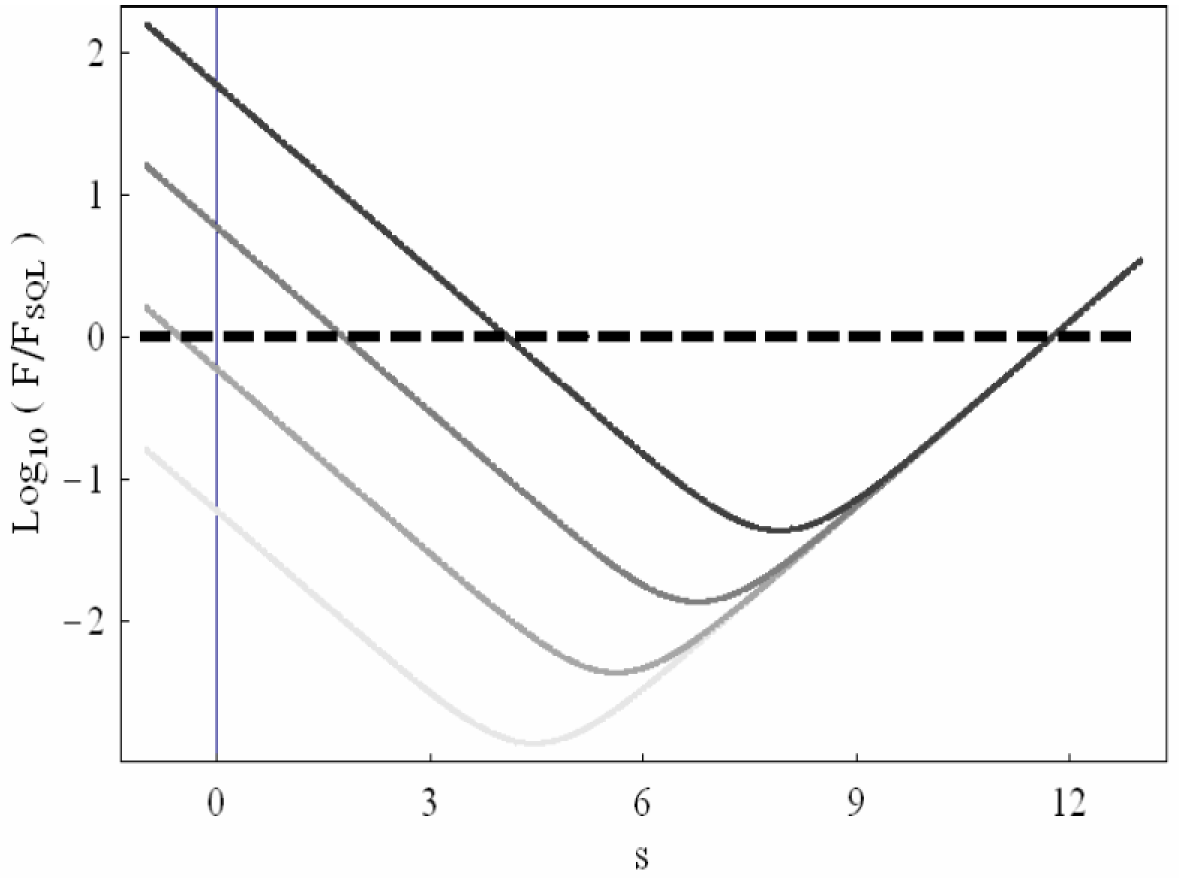
Envelope of the minimum detectable force F versus the interaction time, at three different values of damping, $\gamma = 0.01, 0.1, 1$ Hz, ($s = 0$), corresponding to increasingly darker grey curves. Only at low damping one goes below the SQL. **The best interaction time is $\tau = \pi/\Theta$, corresponding to the first peak**

Parameter	Value
$2\pi c/\omega_0$	600 nm
Ω	$2\pi \times 10^7$ Hz
P	50 mW
M	5×10^{-12} Kg
$\Delta\nu_{det}$	10^6 Hz
$\Delta\nu_{mode}$	10^2 Hz
γ	1 Hz

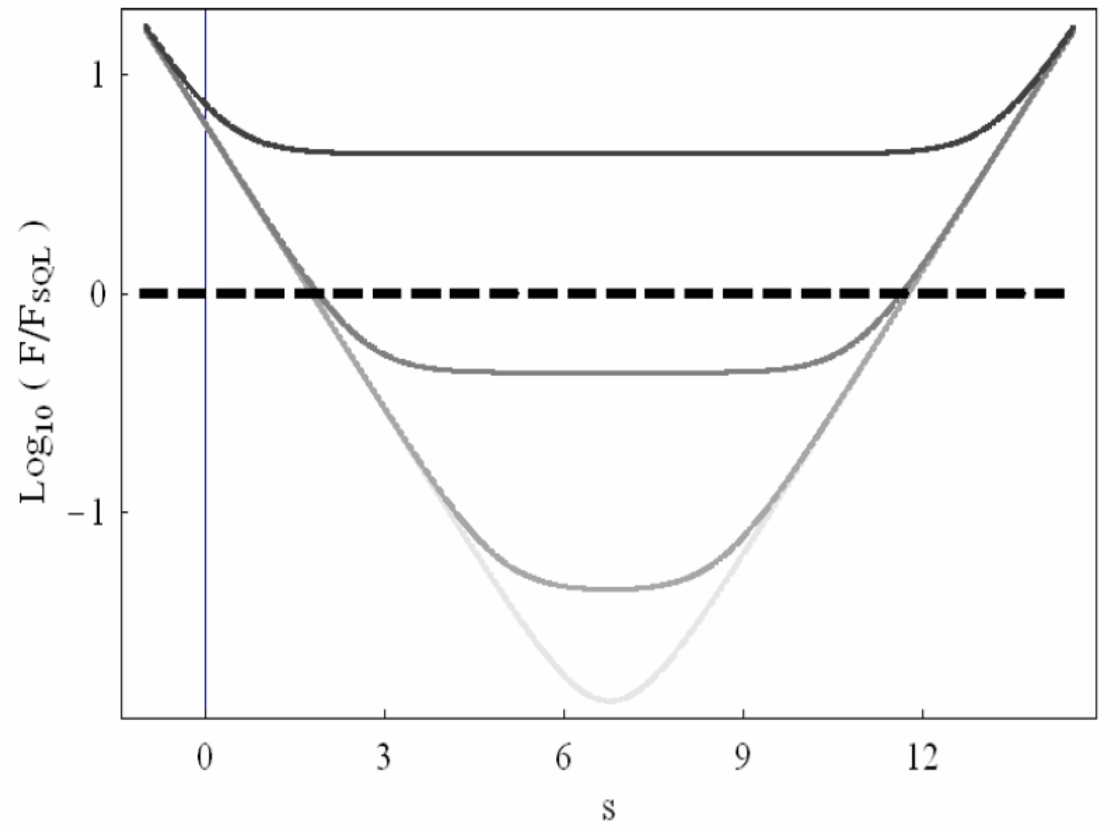
We fix $\tau = \pi/\Theta$ (≈15 msec with the values in the table), yielding

$$F_{SQL} = 12.2 \times 10^{-18}\ \text{N}$$

F versus squeezing s, at γ = 0.01, 0.1, 1, 10 Hz, and T = 0, corresponding to increasingly darker grey curves ⇒ **Input two-mode squeezing is able to compensate the effect of damping and one can go below the SQL.**



F versus squeezing s, at $\gamma = 1$ Hz, and T = 0, 0.03, 3, 300 K, corresponding to increasingly darker grey curves ⇒ **Input two-mode squeezing is able to compensate the effect of thermal noise at cryogenic temperatures and one can still go below the SQL.**



A cavity-QED scheme for Heisenberg-limited interferometry

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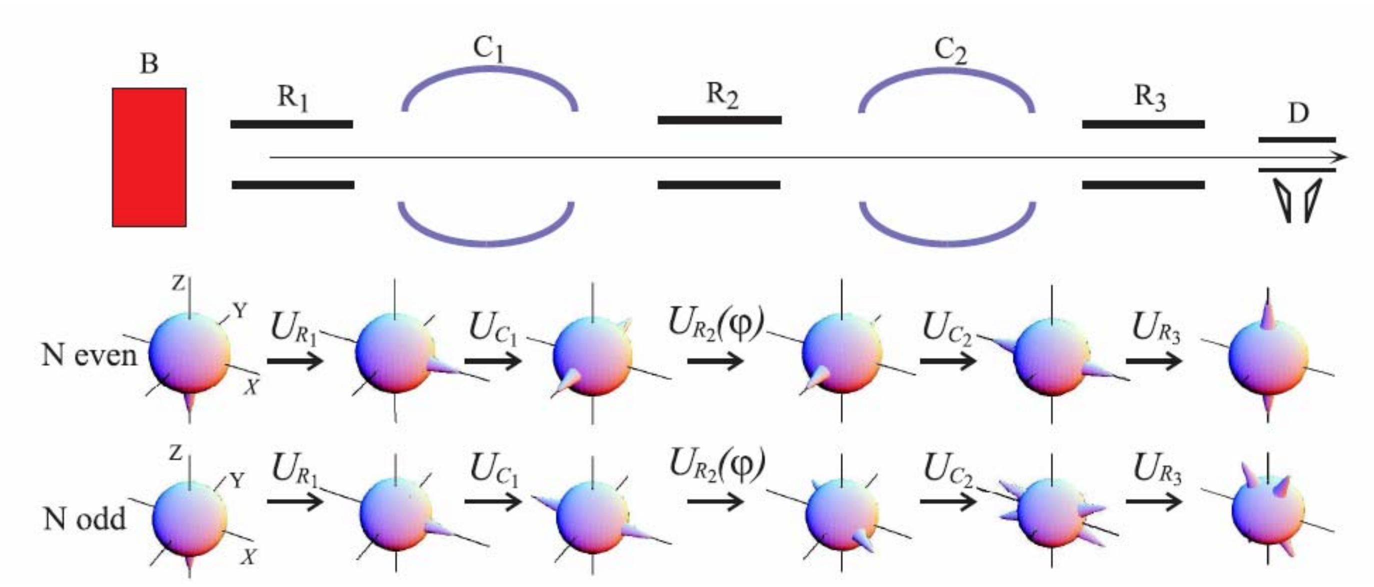
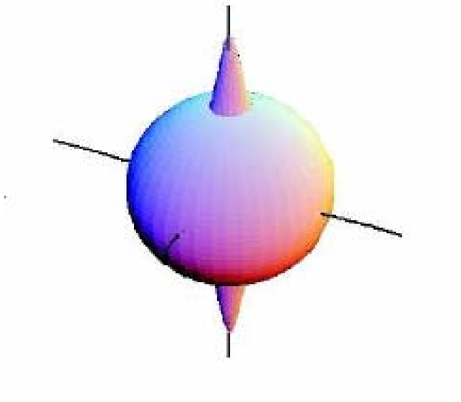


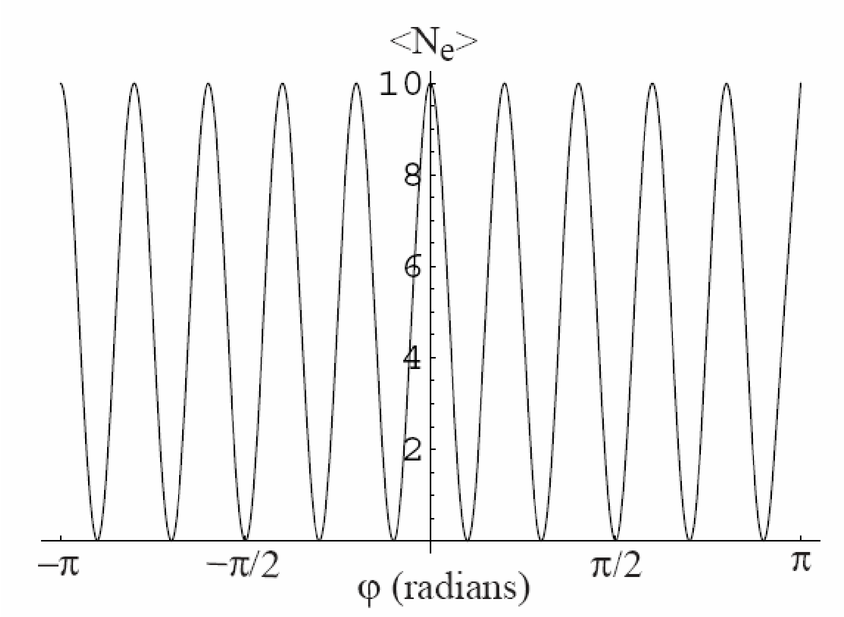
Two high-Q microwave cavities
Beam of circular Rydberg atoms

D. Vitali et al, in press on J. Mod. Optics



Maximally entangled states of N atoms: “atomic Schrodinger cat state”

$$|\psi_3\rangle = \frac{1}{\sqrt{2}}\left[\prod_{j=1}^{N}|e\rangle_j \cos\frac{N\varphi}{2} + \prod_{j=1}^{N}|g\rangle_j \sin\frac{N\varphi}{2}\right] = \frac{1}{\sqrt{2}}\left[\cos\frac{N\varphi}{2}|\theta=\pi\rangle + \sin\frac{N\varphi}{2}|\theta=0\rangle\right]$$



N even, ideal case

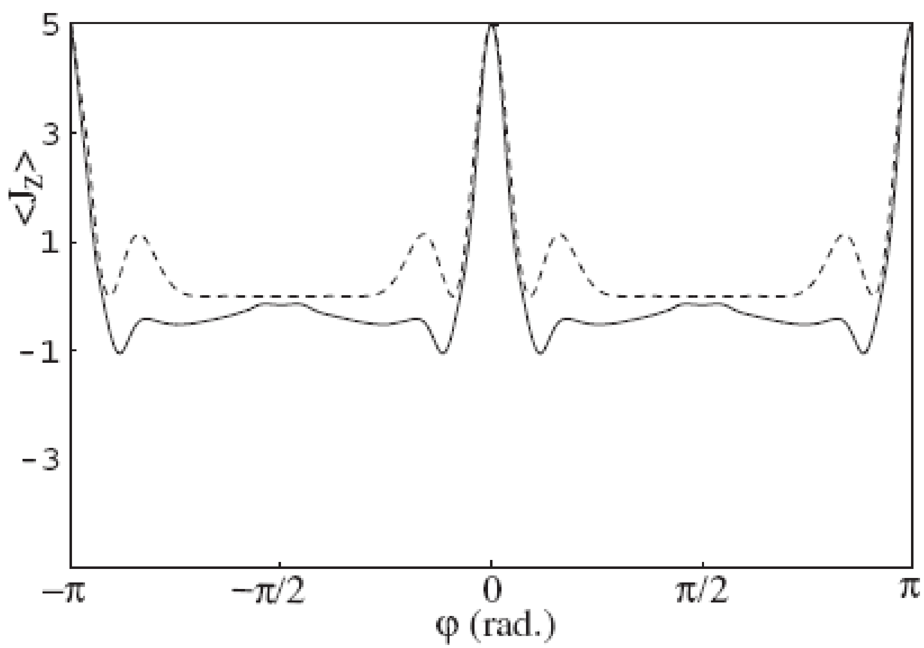
$$\langle N_e(\varphi)\rangle = N\cos^2\frac{N\varphi}{2} \qquad \Delta\varphi = \frac{1}{N}$$

Heisenberg-limited interferometry

Main problem: ***N* is not fixed but it fluctuates from run to run in a Poissonian way** ⇒ the fringes may be washed out by the average

Solution: conditioning on the # of detected atoms and optimizing the signal

$$\langle \hat{J}_z \rangle^w = \sum_{N_d=0}^{\infty} P(N_d) w(N_d) \langle \hat{J}_z(N_d) \rangle^{cond}$$



$$\Delta\varphi_{\min} \simeq 1.3/\eta\bar{N}$$

Close to Heisenberg limit

Conclusions

1. We have proposed a new scheme for the detection of weak forces, based on the **heterodyne measurement of a combination of two sideband modes** of an intense driving laser, scattered by a vibrational mode of a highly reflecting mirror.

2. The presence of **nonzero input two-mode squeezing *s* of the two sidebands improve the sensitivity** and one can go below the SQL, with damping and not too low temperatures (for example, with a mechanical quality factor $\Omega/\gamma \approx 10^7$ and at $T \approx 3$ K).

3. At fixed damping, there is an **optimal *s* maximizing the force detection sensitivity,** because the input entanglement non-trivially interfere with the dynamically generated one, so that the best sensitivity is achieved at finite and not arbitrarily large *s*.