General Relativistic Astrometry: the RAMOD project as a tool for highly accurate observations in space

Maria Teresa Crosta (P)[+],

D. Bini[*], B. Bucciarelli[+], F. de Felice[§], M.G. Lattanzi[+], A.Vecchiato [+]

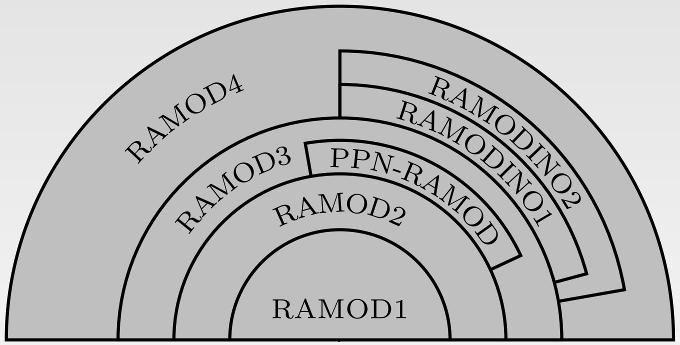
[+] INAF-Astronomical Observatory of Turin, Pino Torinese, Italy;

[*] ICRA International Center for Relativistic Astrophysics, University of Rome, Italy;

[§] Physics Departement G.Galilei , University of Padova, Italy

RAMOD&Relativistic Astrometry: from the observer to the star

Several Relativistic Astrometric MODels of increasing intrinsic accuracy (up to 0.1 μas) and adapted to many different satellite setting (including software engineering)



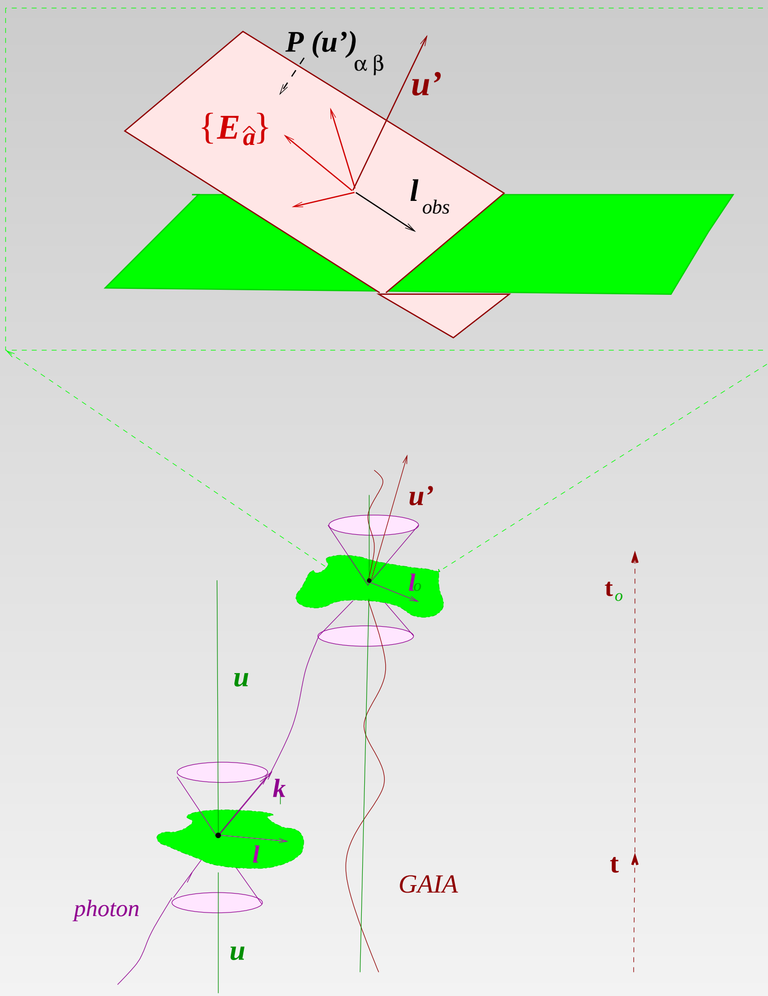
1. RAMOD1: a static non-perturbative model in the Schwarzschild metric of the Sun (de Felice at al., 1998,A&A, 332,1133)
2. RAMOD2: a dynamical extension of RAMOD1 (parallaxes and proper motions, de Felice at al., 2001,A&A, 373,336)
3. PPN-RAMOD: recasting RAMOD2 in the PPN Schwarzschild metric of the Sun (Vecchiato et al, 2003, A&A, 399,337)
4. RAMOD3: a perturbative model of the light propagations in the static field of the Solar System ($1/c^2$, de Felice et al., 2004, ApJ, 607,580)
5. RAMOD4: the extension of RAMOD3 to the $1/c^3$ level of accuracy ($1/c^3 \equiv 0.1\mu$as, de Felice et al., 2006, ApJ, in press)
6. RAMODINO1-2: satellite-observer model for Gaia (Bini et al., 2003, Class. Quantum Grav.,20,2251/4695)

The astrometric observable as a physical measurement

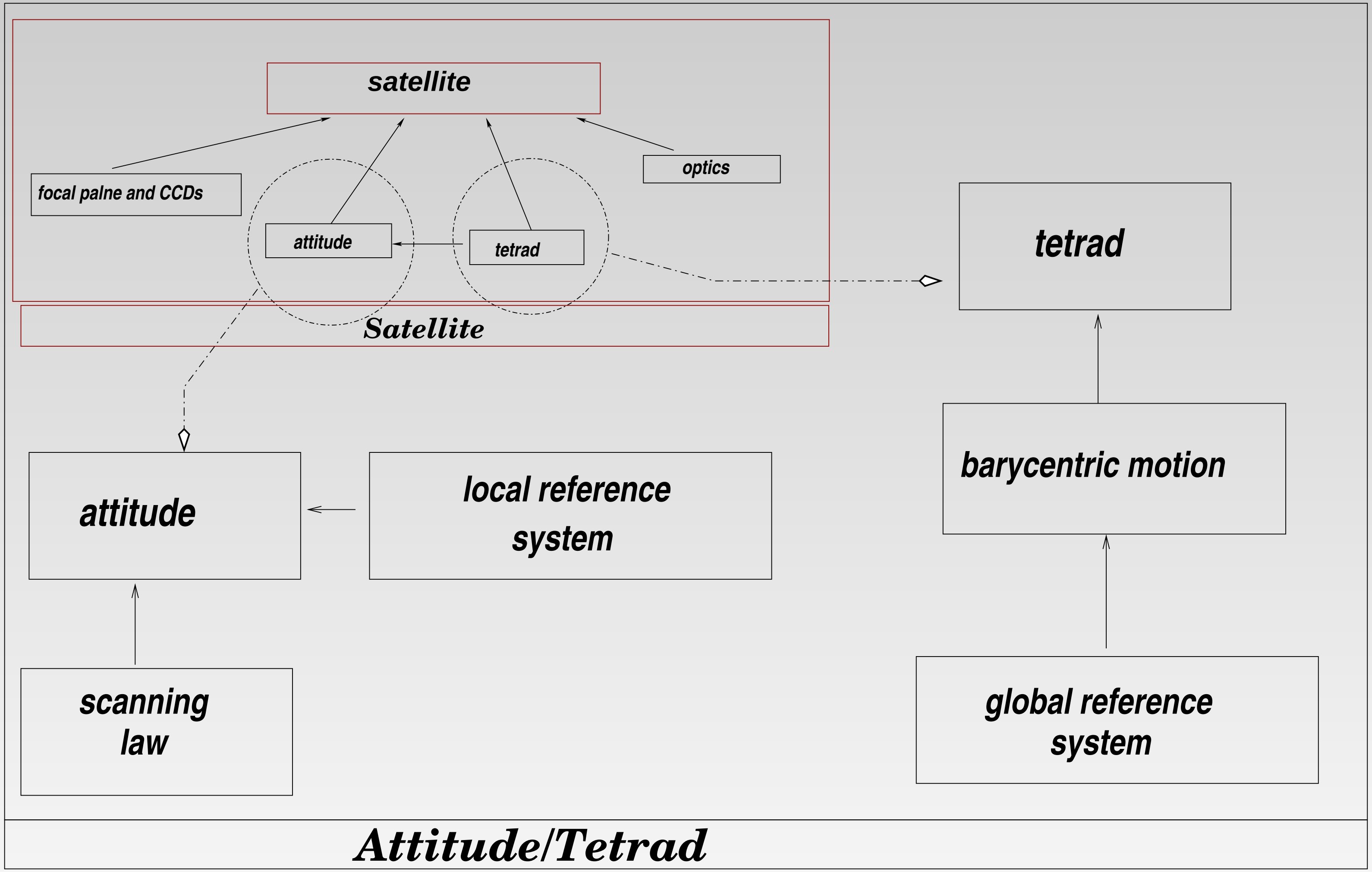
Modelling the Gaia observable requires to solve the inverse problem of light ray tracing, which connects the satellite to the emitting star. The *astrometric observable* ≡ angles that the incoming light ray forms with the axes of the spatial *attitude triad* $E_{\hat{a}}$ in the rest frame of the satellite:

$$\cos\psi_{(E_{\hat{a}},\ell_{obs})} \equiv \mathbf{e}_{\hat{a}} = \frac{P(u')_{\alpha\beta}k^{\alpha}\boldsymbol{E}^{\beta}_{\hat{a}}}{(P(u')_{\alpha\beta}k^{\alpha}k^{\beta})^{1/2}}$$

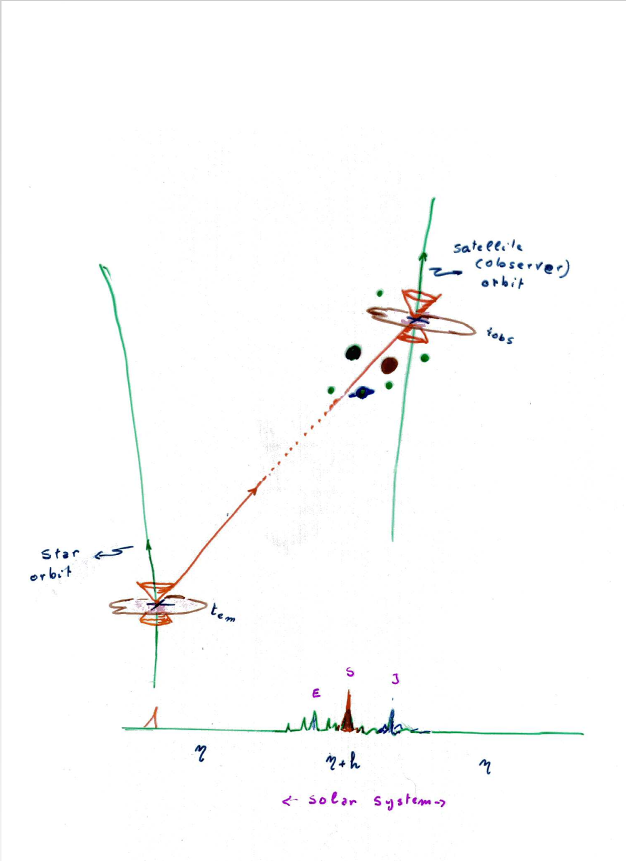
where $P(u')_{\alpha\beta} = g_{\alpha\beta} + u'_{\alpha}u'_{\beta}$. The incoming light ray k^{α} is the solution of the null geodesic considering the full gravitational field of the Solar System presented in RAMOD4 (de Felice et al., ApJ, in press, astro-ph/0609073).



The satellite attitude triad modelling

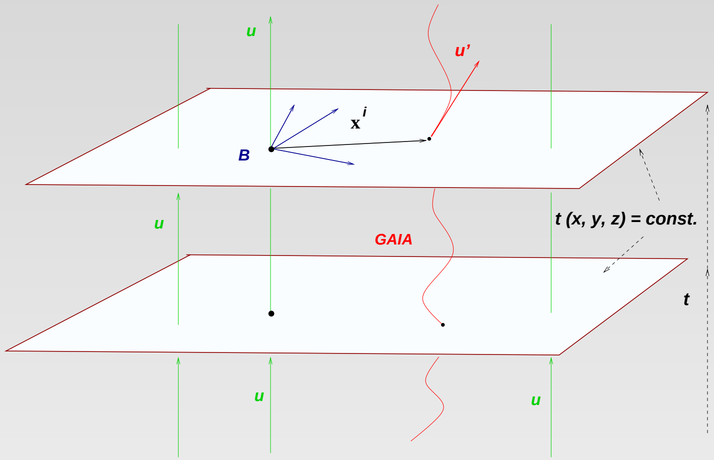


The astrometric set-up



- (1) The background geometry felt by the satellite:
  $g_{\alpha\beta} = (\eta_{\alpha\beta} + h_{\alpha\beta} + O(h^2)) \rightarrow$
  $g_{00} = -1 + \underset{2}{h}_{00} + O(4), \quad g_{0a} = \underset{3}{h}_{0a} + O(5),$
  $g_{ab} = 1 + \underset{2}{h}_{00}\delta_{ab} + O(4)$
  (compatible with retarded potential solutions and/or the IAU resolution B1.3, 2000)
- (2) satellite's trajectory: $\boldsymbol{u'} = T_s(\boldsymbol{\partial}_t + \beta_1\boldsymbol{\partial}_x + \beta_2\boldsymbol{\partial}_y + \beta_3\boldsymbol{\partial}_z)$
  time-like, unitary four-vector
  $\boldsymbol{\partial}_\alpha$'s $\equiv$ coordinate basis vectors relative to the barycentric coordinate system (BCRS)
  $\beta_i \equiv$ BCRS *coordinate* components of the satellite three-velocity

(3) *Global BCRS*: BCRS is identified by three spatial axes at the barycenter of the Solar System (**B**) and pointing to distant cosmic sources (kinematically non-rotanting);
the axes define a Carthesian-like coordinate system (x, y, z) and there exist space-like hypersurfaces with equation $t(x, y, z)$ =constant $\longrightarrow$ the function *t* is chosen as coordinate time.



(the satellite world line with respect to the Carthesian like coordinate system (x^i) and the space-like hypersurface)

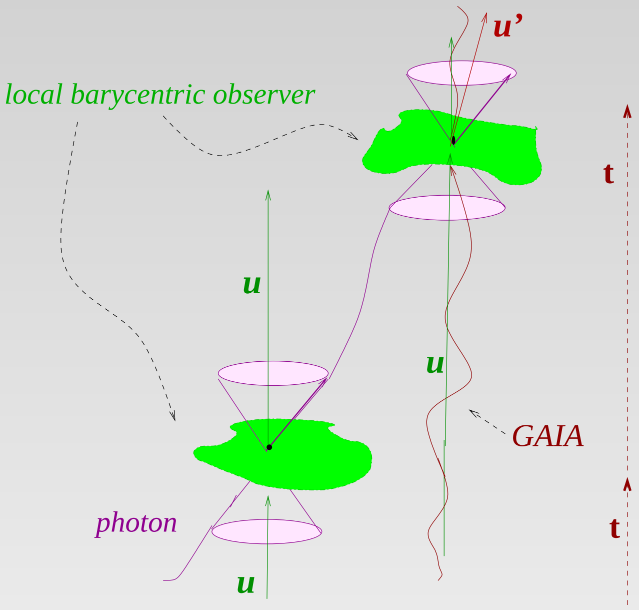
(4) *Local BCRS*: at any point in space-time there exists an observer at rest relative to the BCRS; world-line of **B**

$$\boldsymbol{u} = (g_{tt})^{-1/2}\boldsymbol{\partial}_t = (1+U)\boldsymbol{\partial}_t + O(4)$$

$\longrightarrow$ local triad of space-like vectors which point to the local coordinate directions (U gravitational potential);
the proper time of $\boldsymbol{u}$ is the barycentric proper-time t, since

$$u^\alpha = \frac{dx^\alpha}{d\sigma} = (-g_{00})^{1/2}\delta^\alpha_0$$

$\sigma(x^i, t)$ is the world line parameter of $\boldsymbol{u}$



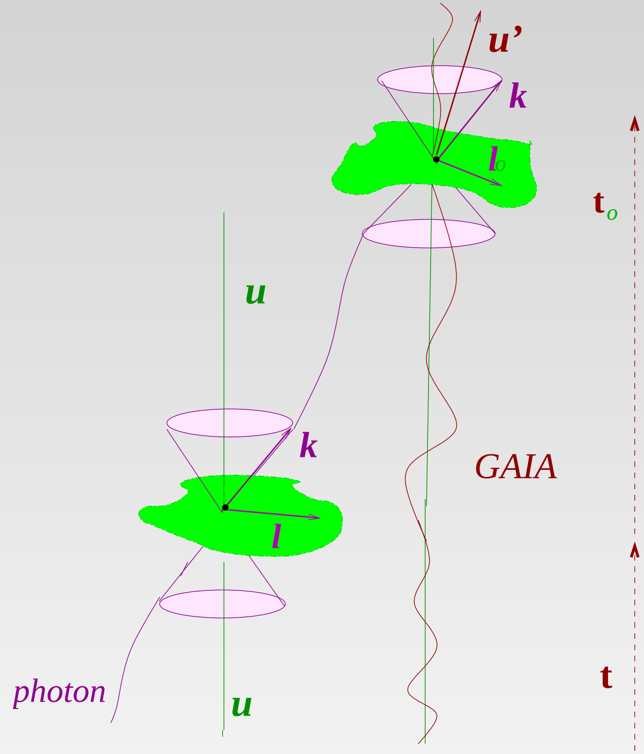
(5) *Light trajectory*: The light signal arriving at the local BCRS is $\boldsymbol{\ell} = P(u)^{\rho}{}_{\sigma} k^{\sigma}$ (*local line-of-sight*, $P(u)^{\rho}{}_{\sigma}$ operator which projects into the rest space of $\boldsymbol{u}$), which is the solution of

$$\frac{d\ell^{\alpha}}{d\sigma} = F^{\alpha}(\partial_{\beta} h(x, y, z, t), \ell^{i}(\sigma(x)))$$

A general solution is:

$$\ell^{i}(\sigma) = f^{i}(\sigma, \ell^{k}_{obs})$$

which links the parameters of the star to the physical measurements (*condition equation*)-> the mathematical characterization of Gaia's attitude triad is *essential* to solve the boundary value problem in the process of reconstructing the light trajectory



The mathematical rest frame: the tetrad

The rest-frame of an observer consists of a *clock* (satellite proper-time) + a *space* (triad of orthonormal axes).
The mathematical quantity which defines a rest-frame of a given observer is the *tetrad adapted to that observer*:

$$g_{\mu\nu}\lambda^{\mu}_{\hat{\alpha}}\lambda^{\nu}_{\hat{\beta}} = \eta_{\hat{\alpha}\hat{\beta}}$$

$\lambda_{\hat{0}} \equiv u'$ (space-time history of the observer in a given space-time)
$\lambda_{\hat{a}} \equiv$ spatial triad of space-like vectors
There are many possible *spaces* to be fixed within a satellite $\longrightarrow$

which is the actual attitude frame for Gaia?

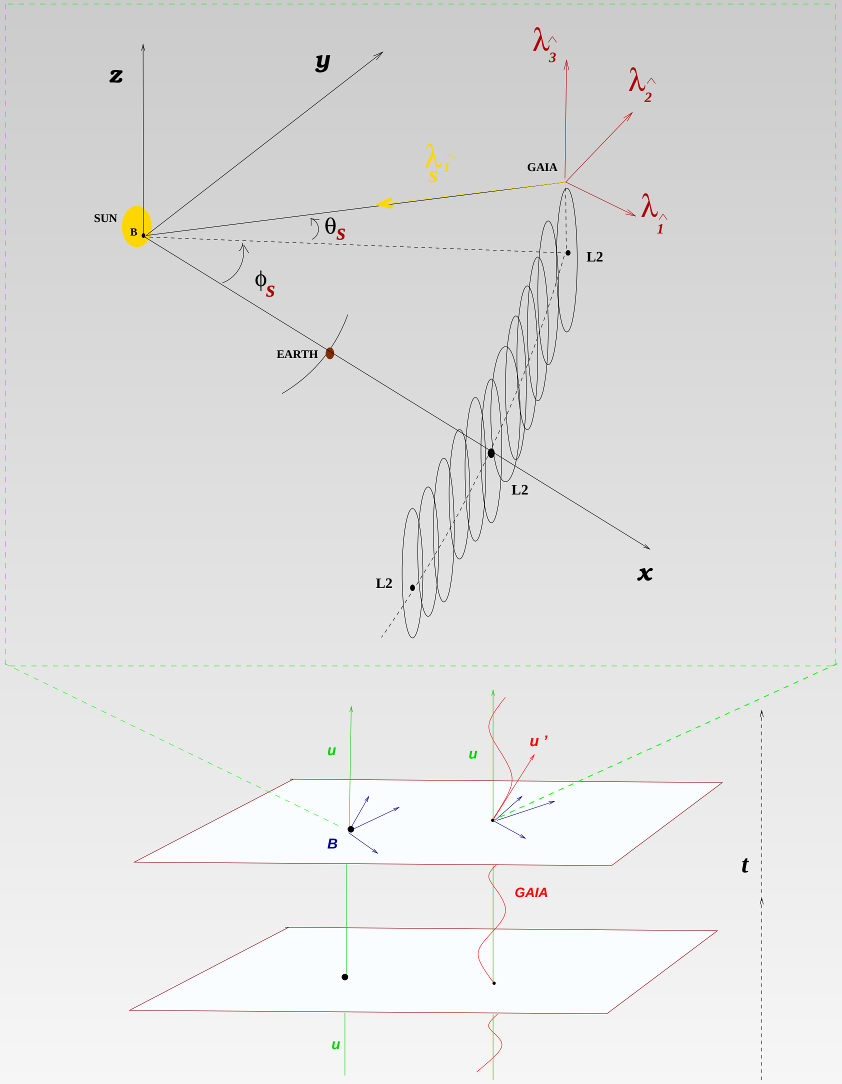
The attitude frame for Gaia

- First step: we need to identify the spatial direction to the geometrical center of the Sun as seen from within the satellite w.r.t. the local BCRS defined at each point of the satellite's trajectory;

$$\Downarrow$$

the new triad adapted to the observer $\boldsymbol{u}$ is

$$\underset{s}{\boldsymbol{\lambda}}_{\hat{a}} = \mathcal{R}_2(\theta_s)\mathcal{R}_3(\phi_s)\boldsymbol{\lambda}_{\hat{a}}$$



The boosted triad

- Second step: we boost the vectors of the triad $\underset{s}{\boldsymbol{\lambda}}_{\hat{1}}$ along the satellite relative motion

$$\Downarrow$$

$$\underset{bs}{\boldsymbol{\lambda}}{}^{\alpha}_{\hat{a}} = P(u')^{\alpha}{}_{\sigma} \left[\underset{s}{\lambda}{}^{\sigma}_{\hat{a}} - \frac{\gamma}{\gamma+1} \nu^{\sigma} \left(\nu^{\rho} \underset{s}{\lambda}{}_{\rho \hat{a}} \right) \right]_{\hat{a}=1,2,3}$$

(Jantzen, Carini and Bini, 1992, *Annals of Physics* **215** and references therein)
$\nu^{\alpha} = \frac{1}{\gamma}(u'^{\alpha} - \gamma u^{\alpha})$ relative spatial four-velocity of $\boldsymbol{u}'$ w.r.t. $\boldsymbol{u}$
$\gamma = -u'^{\alpha} u_{\alpha}$ relative Lorentz factor

The vector $\underset{bs}{\boldsymbol{\lambda}}_{\hat{1}}$ *identifies the direction to the Sun as seen by the satellite* as a Sun-locked frame

The Gaia attitude frame

Final steps in order to obtain the Gaia attitude frame:

- i) rotate the Sun-locked triad by an angle $\omega_p t$ about the vector $\underset{bs}{\boldsymbol{\lambda}}_{\hat{1}}$ which constantly points to the Sun; ω_p is the angular velocity of precession,
- ii) rotate the resulting triad by a fixed angle $\alpha = 50^\circ$ about the image of the vector $\underset{bs}{\boldsymbol{\lambda}}_{\hat{2}}$ under rotation i), and
- iii) rotate the triad obtained after step ii) by an angle $\omega_r t$ about the image of the vector $\underset{bs}{\boldsymbol{\lambda}}_{\hat{1}}$ under the previous two rotations; ω_r is the angular velocity of the satellite spin.

$$\Downarrow$$

$$\boldsymbol{E}_{\hat{a}} = \mathcal{R}_1(\omega_r t)\mathcal{R}_2(\alpha)\mathcal{R}_1(\omega_p t)\underset{bs}{\boldsymbol{\lambda}}_{\hat{a}} \qquad \hat{a} = 1,\ 2,\ 3$$

Gaia attitude triad

Explicit coordinate components of the Gaia attitude triad

$$E_{\hat{1}}^{\alpha} = \cos\alpha \underset{\text{bs}}{\lambda_{\hat{1}}^{\alpha}} - \sin\alpha\cos(\omega_p t)\underset{\text{bs}}{\lambda_{\hat{2}}^{\alpha}} - \sin\alpha\cos(\omega_p t)\underset{\text{bs}}{\lambda_{\hat{3}}^{\alpha}} \quad (1)$$

$$\begin{aligned} E_{\hat{2}}^{\alpha} = & -\sin\alpha\sin(\omega_r t)\underset{\text{bs}}{\lambda_{\hat{1}}^{\alpha}} + \\ & +[\cos(\omega_r t)\cos(\omega_p t) - \sin(\omega_r t)\sin(\omega_p t)\cos\alpha]\underset{\text{bs}}{\lambda_{\hat{2}}^{\alpha}} \\ & +[\cos(\omega_r t)\sin(\omega_p t) + \sin(\omega_r t)\cos(\omega_p t)\cos\alpha]\underset{\text{bs}}{\lambda_{\hat{3}}^{\alpha}} \end{aligned} \quad (2)$$

$$\begin{aligned} E_{\hat{3}}^{\alpha} = & -\sin\alpha\cos(\omega_r t)\underset{\text{bs}}{\lambda_{\hat{1}}^{\alpha}} \\ & -[\sin(\omega_r t)\cos(\omega_p t) + \cos(\omega_r t)\sin(\omega_p t)\cos\alpha]\underset{\text{bs}}{\lambda_{\hat{2}}^{\alpha}} \\ & +[-\sin(\omega_r t)\sin(\omega_p t) + \cos(\omega_r t)\cos(\omega_p t)\cos\alpha]\underset{\text{bs}}{\lambda_{\hat{3}}^{\alpha}} \end{aligned} \quad (3)$$

(Bini, Crosta and de Felice, 2003, *Class. Quantum Grav.* **20** 4695)

The clock on board of Gaia

Time interval between two events in space-time

$$dT = -\frac{1}{c} g_{\alpha\beta} u'^{\alpha} dx^{\beta}$$

interval of proper time of an observer on board of the satellite (Crosta et al., *Proper frames and time scan for Gaia-like satellites*, 2004, ESA livelink, tech.note);
if we adopt the IAU metric

$$\begin{aligned} dT \quad \approx \quad & dt - c^{-2}\left[\left(\frac{v^2}{2} + w(\mathbf{x},t)\right) + v^i dr^i\right] \\ & + c^{-4}\left[\left(\frac{w^2(\mathbf{x},t)}{2} - \frac{v^4}{8} - \frac{3v^2 w(\mathbf{x},t)}{2} + 4w^i(\mathbf{x},t)v^i\right) dt \right. \\ & \left. + 4w^i(\mathbf{x},t)dr^i - \left(3w(\mathbf{x},t) + \frac{v^2}{2}\right) v^i dr^i\right], \end{aligned}$$

(IAU resolution B1.5, Second Recommendation)

Summary

1. RAMOD is a well-established framework of general relativistic astrometric models which can be extended to whatever accuracy and physical requirements (*i.e.* the metric);
2. RAMOD is fully operational from the theoretical stand-point, and ready to be implemented in an end-to-end simulation of the Gaia Mission (*i.e.*, estimation of the astrometric parameters of celestial objects from a well-defined set of relativistically measured quantities)

Thanks all the presents and all the absents happy people...