**INTERNATIONAL WORKSHOP**

**Advances in Precision Tests and Experimental Gravitation in Space**

# Matter Waves and the Detection of Gravitational Waves


Pacôme Delva, Marie-Christine Angonin, Philippe Tourrenc

l'Observatoire de Paris
LERMA
Laboratoire d'Étude du Rayonnement et de la Matière en Astrophysique

Université Pierre & Marie Curie
La Science à Paris

## Outline

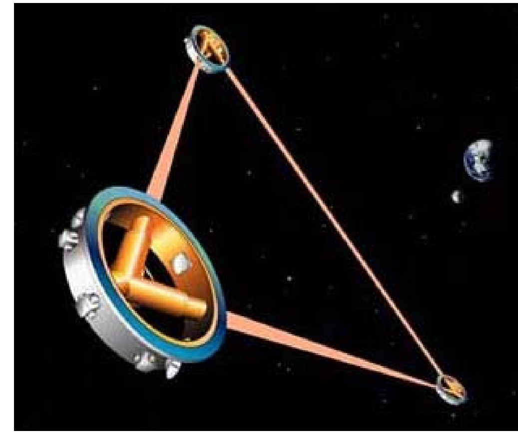
- Different experiments : free and « rigid » detectors
- The Michelson Morley interferometer
- The Ramsey-Bordé interferometer
- The matter wave interferometer characteristics
- Open Problems

# Different Experiments : Free and « Rigid » Detectors

Virgo

## Free Detectors

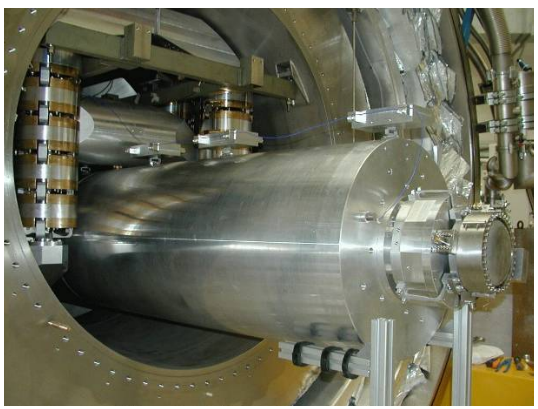
- Einstein coordinates (TT)
- $ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + h_{rs} dx^r dx^s$
- Direction of propagation $x^3$, r,s = 1,2
- Geodesics : $\frac{d^2 x^r}{dt^2} = -\dot{h}^r_s \frac{dx^s}{dt}$



LISA

(Ashby & Dreitlein 1975)

$$\begin{cases} x^r = X^r - \frac{1}{2} h^r_s X^s + O(\xi^4, h^2) \\ x^a = X^a - \frac{1}{4} \dot{h}_{rs} X^r X^s + O(\xi^4, h^2) \end{cases}$$

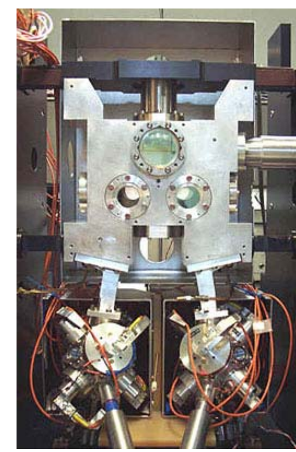


AURIGA

## « Rigid » Detectors

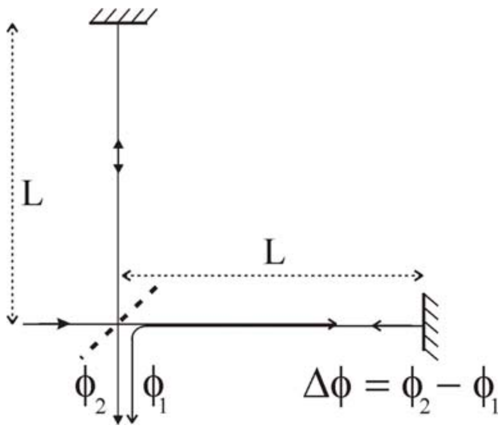
$$\begin{cases} Z \simeq z \simeq 0 \\ X^i \leq \xi \ll \Lambda \end{cases}$$

- The proper (or Fermi) reference frame
- $ds^2 = \eta_{\mu\nu} dX^\mu dX^\nu + \frac{1}{2} \ddot{h}_{rs} X^r X^s dT^2 + O(\xi^3, h^2)$
- Quasi newtonian interpretation : equation of elasticity with GWs interpreted as an extra force $\frac{\rho}{2} \ddot{h}^r_s x^s$
- Small experiments can be cooled to very low temperatures



LNE Syrte - Paris

# The Michelson Morley Interferometer




- Periodic gravitational wave (long wavelenght approximation) :

$$\begin{cases} h_{11}(z-ct) = -h_{22}(z-ct) = h_+ \cdot \sin(\Omega t) \\ h_{12}(z-ct) = h_{21}(z-ct) = h_\times \cdot \sin(\Omega t + \varphi_\times) \end{cases}$$

- Same method for light wave and matter wave (Linet & Tourrenc 76)

| | |
|---|---|
| V=v | for matter wave |
| V=c | for light wave |

**Free configuration**

$$\Delta\phi = -4\pi h_+ \cdot \frac{V}{\Omega\lambda} \cdot \sin\frac{\Omega L}{V} \cdot \sin\Omega t$$

$$L \ll \frac{V}{c} \cdot \Lambda$$

$$\widetilde{\Delta\phi} \sim 4\pi |h_+| \cdot \frac{L}{\lambda}$$

**« Rigid » configuration**

$$\Delta\phi = 4\pi h_+ \cdot \frac{L}{\lambda} \cdot \left(1 - \frac{V}{\Omega L}\sin\frac{\Omega L}{V}\right) \cdot \sin\Omega t$$

$$L \ll \frac{V}{c} \cdot \Lambda$$

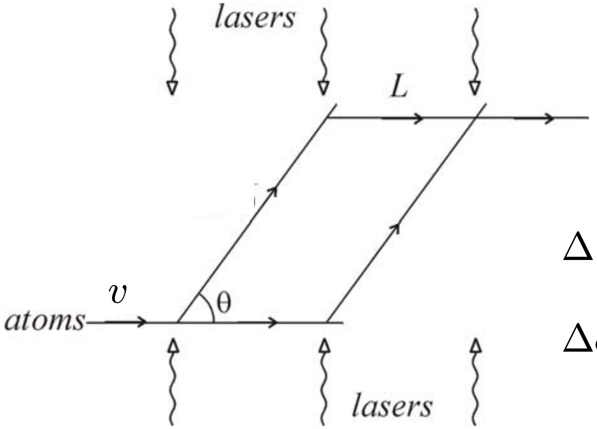
$$\Delta\phi \simeq 0$$

$$\frac{V}{c} \cdot \Lambda \ll L \ll \Lambda$$

$$\widetilde{\Delta\phi} \sim 4\pi |h_+| \cdot \frac{L}{\lambda}$$

- Two regimes for the rigid configuration for the matter wave interferometer (Roura et al. 2006)
- The second regime is not sensitive… ( $v \ll \Omega L$)

# The Ramsey-Bordé Interferometer (rigid configuration)




$$\Delta\phi = \Delta\phi_+ + \Delta\phi_\times$$

$$\Delta\phi_+ = -4\pi h_+ \frac{L}{\lambda} \sin\psi \tan^2\theta \left[\cos(\Omega t + \psi) + \frac{\sin\psi}{2\psi}\cos(\Omega t)\right] \qquad \psi = \frac{\Omega L}{2 v_0}$$

$$\Delta\phi_\times = -4\pi h_\times \frac{L}{\lambda} \cos\psi \tan\theta \left[\sin(\Omega t + \varphi_\times - \psi) - \frac{\sin\psi}{\psi} \tan\psi \cos(\Omega t + \varphi_\times)\right]$$

$$\tan\theta \simeq 1$$

$$L \ll \frac{V}{c} \cdot \Lambda$$

$$\frac{V}{c} \cdot \Lambda \ll L \ll \Lambda$$

$$\widetilde{\Delta\phi} \simeq 4\pi |h_\times| \cdot \frac{L}{\lambda}$$

$$\begin{aligned} \Delta\phi &\simeq -4\pi \cdot \tfrac{L}{\lambda} \cdot [h_+ \sin\psi \cos(\Omega t + \psi) \\ &+ h_\times \cos\psi \sin(\Omega t + \varphi_\times - \psi) \end{aligned}$$

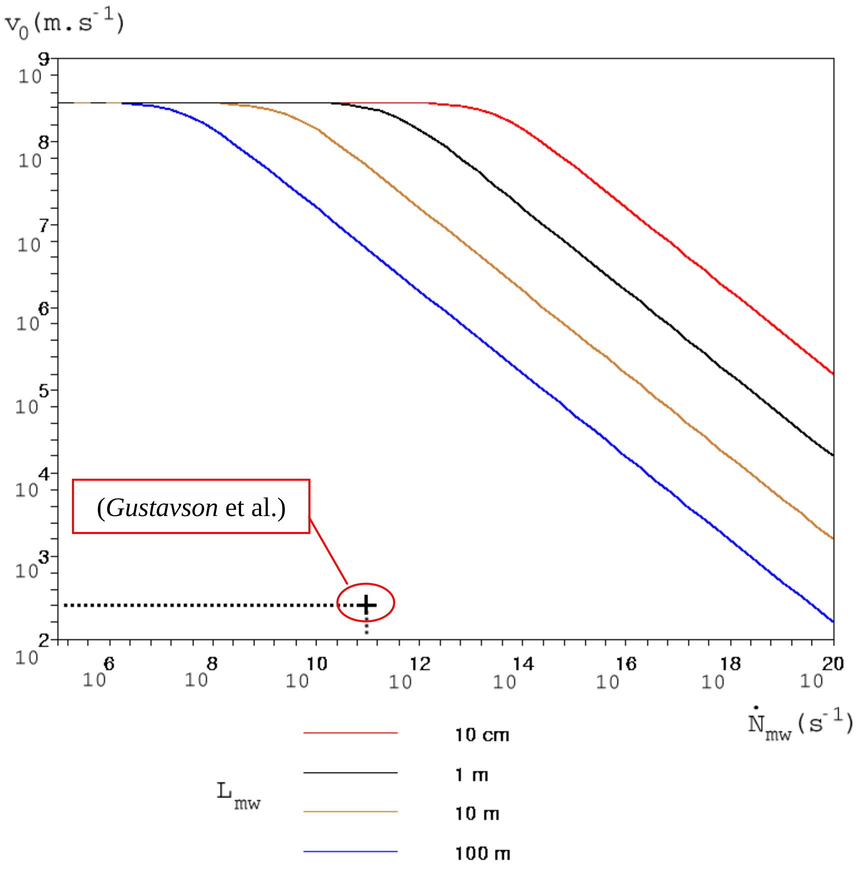
- The Ramsey-Bordé interferometer is sensitive in the first regime to the cross polarization
- The sensitivity is of the same order than the free Michelson configuration
- The second regime is sensitive to the two polarizations

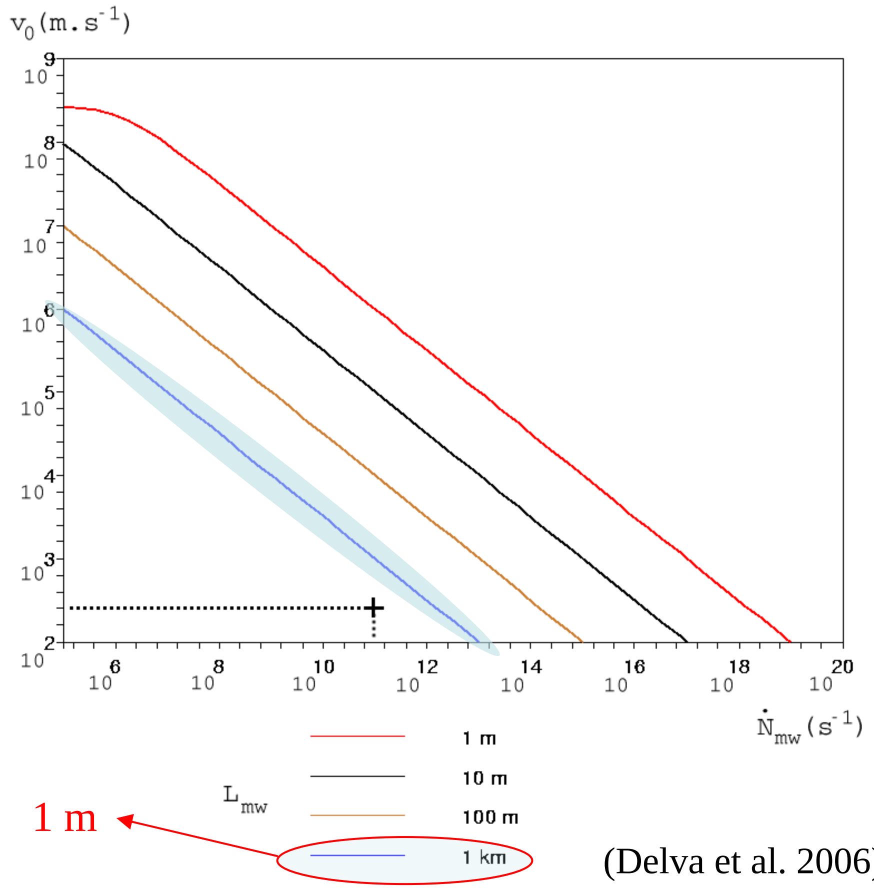
# The Matter Wave Interferometer Characteristics (if limited by shot noise)

Relativistic velocities needed to reach VIRGO sensitivities

Kilometric interferometer to reach the sensitivity of LISA with thermal atoms

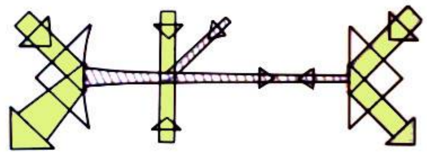
Matter wave cavity ?



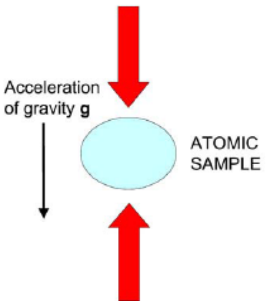




(Delva et al. 2006)

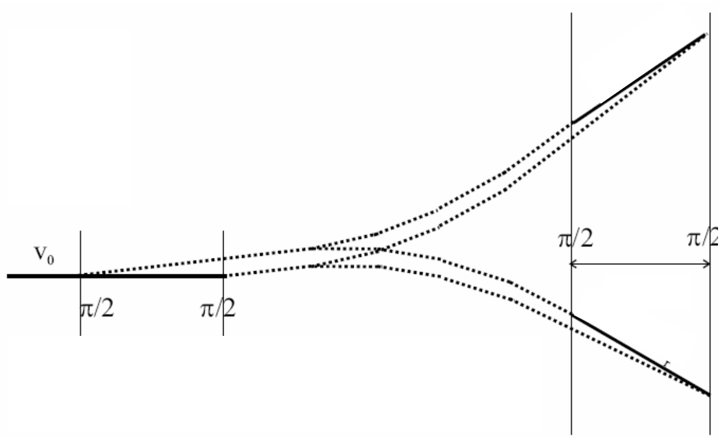
# Open Problems : Cavities and Deviations

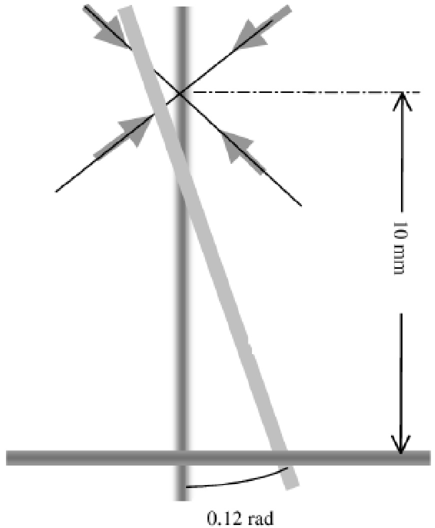


- Matter wave cavities
  - 1 m. cavities with 1000 bounces for a 1 meter interferometer to reach LISA sensitivity
  - Atomic cavity (Balykin & Letokhov 88, Balykin et al. 00)
  - Gravitational cavity with light induced mirrors (Wallis et al. 91)
  - Gravitational cavity with raman pulses (Impens et al. 06)
  - BEC cavity at the Institut d'Optique with atom guides




- High flux : collisions ?
- The high deviation of atoms
  - Many photon transfer (Peters et al 97, Cladé 05, Battesti et al 04)
  - Atom guides (Houde et al. 00)



- The thermal noise in the experiment bench

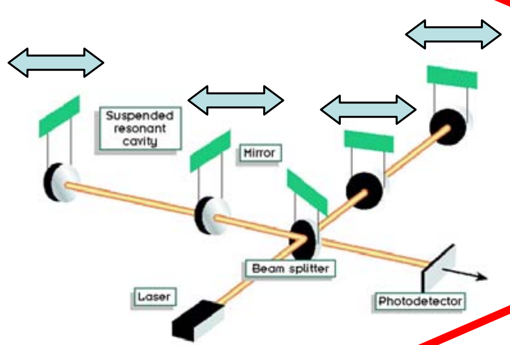
$$h_{TN} \sim \frac{1}{L}\left(\frac{4k_BT}{MQ\omega_0^2\Omega}\right)^{1/2}$$

$M \sim 500$ kg
$\omega_0 \sim 10^4\ \mathrm{s}^{-1}$
$Q \sim 10^7$
$T \sim 10^{-2}$ K
$\Omega \sim 10^{-2}\ \mathrm{s}^{-1}$

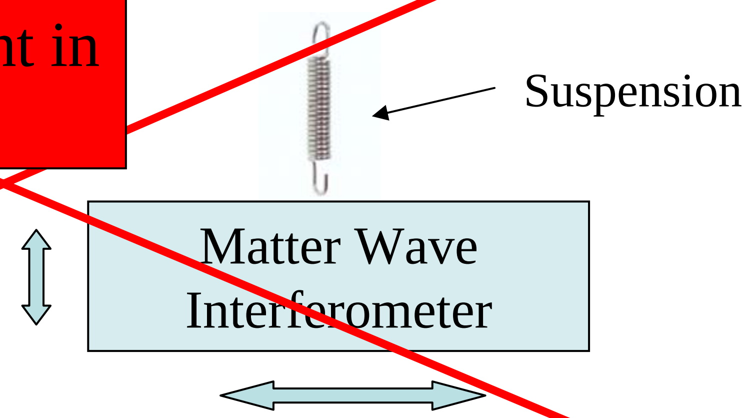
$$\Rightarrow \quad h_{TN} \sim 10^{-20} (/\sqrt{\mathrm{Hz}})$$

  - A small interferometer could be cooled to very low temperatures (L = 1 m)

- The thermal noise and the seismic noise in the suspension
  - The problem is different from the free interferometer : all the elements moves together at low frequency




Experiment in Space




- The newtonian noise ?

# Bibliography : Sensitivity of a MWI to Gravitational Waves


[1976] Linet, B., & Tourrenc, Ph., Can. J. Phys., 54, 1129

[1979] Stodolsky, L., 1979, Gen. Relativ. Gravit., 11, 391

[1989] Cai, Y. Q., & Papini, G., Class. Quantum Grav., 6, 407

[1994] Bordé, C. J., Karasiewicz, A., & Tourrenc, P., Intern. J. Mod. Phys. D, 3, 157

[1997] Bordé, C. J., in : P. Berman (Ed.), Atom Interferometry, Academic Press

[2001] Borde, C. J., Houard, J.-C., & Karasiewicz, A., LNP Vol. 562, 403

[2002] Lamine, B., Jaekel, M.-T., & Reynaud, S., European Physical Journal D, 20, 165

[2004] Vetrano, F., Tino, G., Bordé, C. J., Aspen Winter Conference

[2004] Bordé, C. J., Gen. Relativ. Gravit., 36, 475

[2004] Chiao, R. Y., & Speliotopoulos, A. D., J. Mod. Opt., 51, 861

[2006] Foffa, S., Gasparini, A., Papucci, M., & Sturani, R., Phys. Rev. D, 73(2)

[2006] Roura, A., Brill, D. R., Hu, B. L., Misner, C. W., & Phillips, W. D., Phys. Rev. D, 73(8)

[2006] Delva, P., Angonin, M.-C., & Tourrenc, Ph., Phys. Lett. A, 357, 249


THANK YOU FOR YOUR ATTENTION !

## Appendice : Sensibility of an atomic interferometer to metric effects

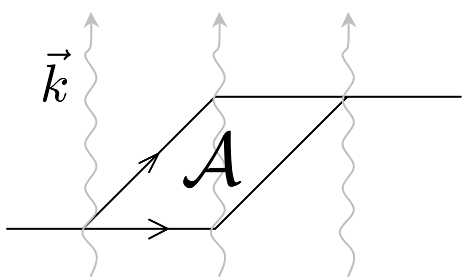
• Métrique : $\mathrm{d}s^2 = (\eta_{\mu\nu} + K_{\mu\nu})\,\mathrm{d}x^\mu \mathrm{d}x^\nu \;,\; K_{\mu\nu} \ll 1$

• Différence de phase dans un interféromètre :

$$\Delta\phi \sim \frac{c^2}{\hbar} \int K_{\mu\nu} p^\mu p^\nu \frac{\mathrm{d}t}{E}$$

(Formule de Linet-Tourrenc)

| Accélération a | Rotation Ω | Onde Gravitationnelle $h^{TT}$ |
|---|---|---|
| $K_{00} \sim \frac{aL}{c^2}$ | $K_{0i} \sim \frac{\Omega L}{c}$ | $K_{ij} \sim h^{\mathrm{TT}}$ |
| $\Delta\phi \sim \frac{ma}{\hbar} \cdot \frac{\mathcal{A}}{v}$ | $\Delta\phi \sim \frac{m\Omega}{\hbar} \cdot \mathcal{A}$ | $\Delta\phi \sim \frac{mh^{\mathrm{TT}}}{\hbar} \cdot \frac{\mathcal{A}}{T}$ |
| $\Delta\varphi \sim kaT^2$ | $\Delta\varphi \sim k\Omega vT^2$ | $\Delta\varphi \sim kh^{\mathrm{TT}} vT$ |




$$\mathcal{A} = \frac{\hbar k v T^2}{m}$$