unusual LISA

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1) Gravitational coronography

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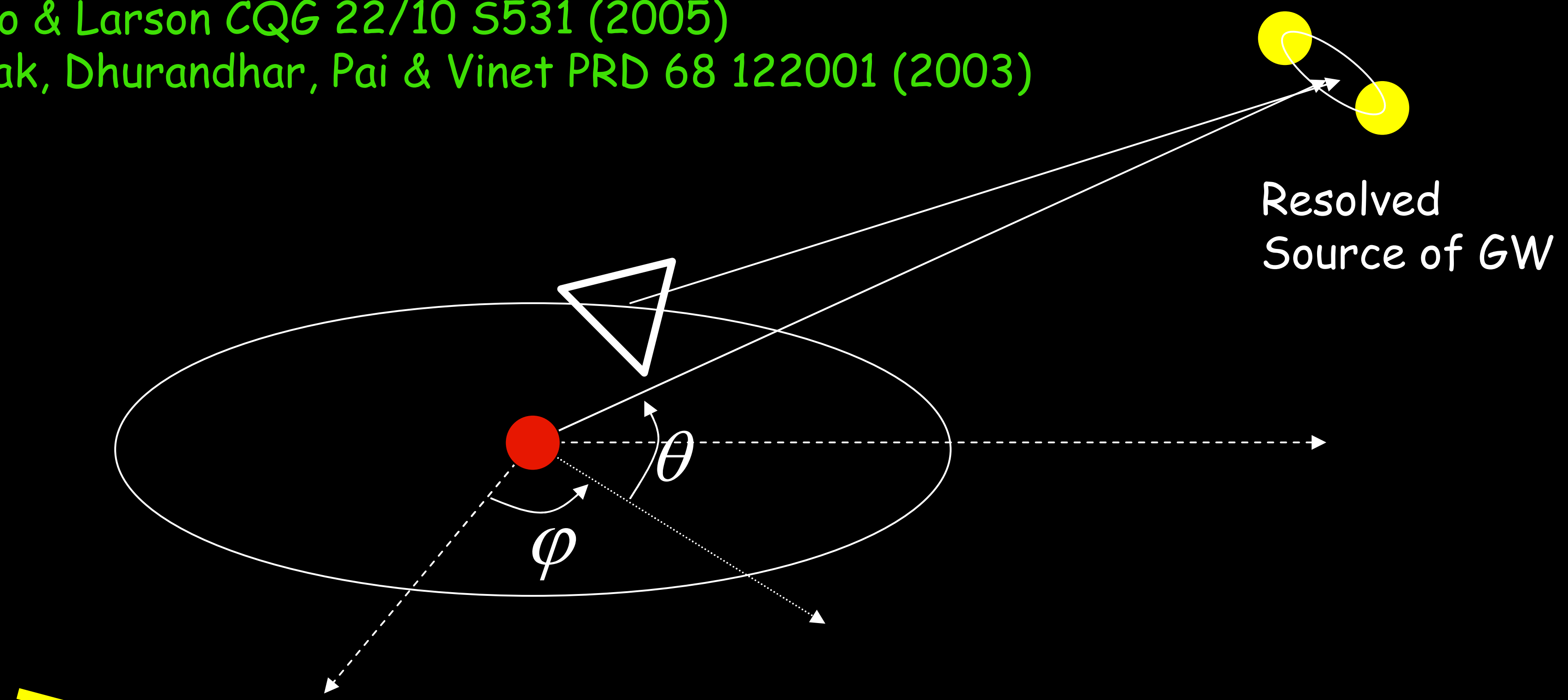
Gravitational coronography

Tinto & Larson CQG 22/10 S531 (2005)
Nayak, Dhurandhar, Pai & Vinet PRD 68 122001 (2003)



Data combination giving a zero response
For direction (θ,φ)

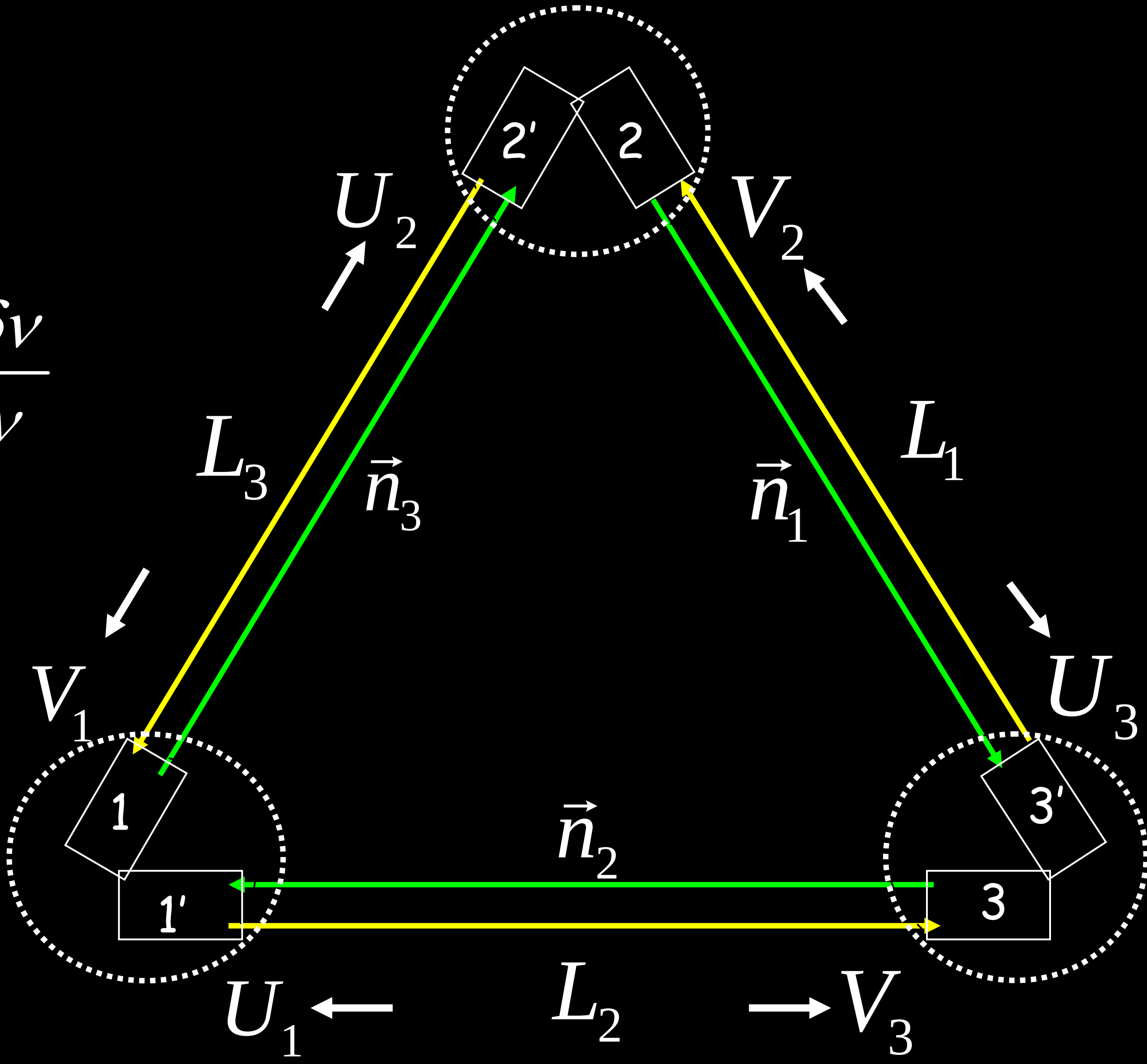
Benefits (conjecture)

- Occultation of a strong source for better analysis of its angular neighborhood

- Improving angular resolution

LISA

$$U,V \Box \frac{\delta\nu}{\nu}$$



Recall

- 6 main data channels (1 per phasemeter)
- There exist families of combinations of the 6 flows with properly chosen time delays that cancel dominant instrumental noises→TDI (Time Delay Interferometry)

  (Tinto, Armstrong, Estabrook, 99)
- They form a module and have generating parts (sets of generators).

  (Dhurandhar,Nayak,Vinet, 02)
- One may combine these generators for special purposes, keeping the noise cancelling property

Recall

- The generators have the form:

$$g = (p_1, p_2, p_3, q_1, q_2, q_3)$$

- Where (p_i, q_i) are formal polynomials in the 3 delay operators D_a

$$(D_a f)(t) \equiv f(t - L_a) \quad (a = 1, 2, 3)$$

Recall

6-uple of data:

$$\mathrm{U}(t) = [V_1(t), V_2(t), V_3(t), U_1(t), U_2(t), U_3(t)]$$

Generic noise-cancelling combination g:

$$\langle g \mid \mathrm{U} \rangle (t) = \sum_{i=1}^{3} [p_i V(t)_i + q_i U_i(t)]$$

= 0 when U,V represent laser phase fluctuations

Notation

Source oriented unit vector : $\vec{w}(\theta, \varphi)$

3 orthonormal vectors : $\vec{w}, \quad \vec{\theta} = \dfrac{\partial \vec{w}}{\partial \theta}, \quad \vec{\varphi} = \dfrac{1}{\sin\theta} \dfrac{\partial \vec{w}}{\partial \varphi}$

Unit vector along arm #a : $\vec{n}_a \quad (a = 1, 2, 3)$

Directional functions (spin 2 harmonics):

$$\xi_{a+} = (\vec{\theta} \cdot \vec{n}_a)^2 - (\vec{\varphi} \cdot \vec{n}_a)^2$$

$$\xi_{a\times} = 2(\vec{\theta} \cdot \vec{n}_a)(\vec{\varphi} \cdot \vec{n}_a)$$

Location of node #a : $\vec{r}_a$ notation $\mu_a = \vec{w} \cdot \vec{r}_a$

Notation

$h_+, h_\times$: the 2 polarization components of the GW

Data flow at node # 1 :

$$U_{1+,\times}(t) = -\frac{h_{+,\times}(t-\mu_1) - h_{+,\times}(t-\mu_3-L_2)}{2(1+\vec{w}\cdot\vec{n}_2)}\xi_{2+,\times}$$

$$V_{1+,\times}(t) = \frac{h_{+,\times}(t-\mu_1) - h_{+,\times}(t-\mu_2-L_3)}{2(1-\vec{w}\cdot\vec{n}_3)}\xi_{3+,\times}$$

Others are obtained by circular permutation of indices

Notation

Fourier space : transfer functions:

$$U_a = F_{U_a +}\tilde{h}_+ + F_{U_a \times}\tilde{h}_\times \quad , \quad V_a = F_{V_a +}\tilde{h}_+ + F_{V_a \times}\tilde{h}_\times$$

$$F_{V_{1+,\times}} = \frac{e^{i\omega\mu_1} - e^{i\omega(\mu_2 + L_3)}}{2(1 - \vec{w}\cdot\vec{n}_3)}\xi_{3+,\times} \, , \quad F_{U_{1+,\times}} = -\frac{e^{i\omega\mu_1} - e^{i\omega(\mu_3 + L_2)}}{2(1 + \vec{w}\cdot\vec{n}_2)}\xi_{2+,\times}$$

+ circular permutations

6-uple transfer:

$$\mathrm{F}_{+,\times} = (F_{V_1 +,\times}, F_{V_2 +,\times}, F_{V_3 +,\times}, F_{U_1 +,\times}, F_{U_2 +,\times}, F_{U_3 +,\times})$$

Notation

- Example of generators (Tinto et al.)

$$\alpha = (1, D_3, D_1 D_3, 1, D_1 D_2, D_2)$$
$$\beta = (D_1 D_2, 1, D_1, D_3, 1, D_2 D_3)$$
$$\gamma = (D_2, D_2 D_3, 1, D_1 D_3, D_1, 1)$$

and ς

- Vector generator: $\vec{\mathrm{Y}} = \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}$

Algebraic solution

Generic combination C :

$$C = C_1\alpha + C_2\beta + C_3\gamma = \vec{\mathrm{C}} \cdot \vec{\mathrm{Y}}$$

Transfer function:

$$\tilde{h}_C(f) = \langle C \,|\, \mathrm{F}_+ \rangle \tilde{h}_+(f) + \langle C \,|\, \mathrm{F}_\times \rangle \tilde{h}(f)_\times =$$

$$\vec{\mathrm{C}} \cdot \langle \vec{\mathrm{Y}} | \mathrm{F}_+ \rangle \tilde{h}_+(f) + \vec{\mathrm{C}} \cdot \langle \vec{\mathrm{Y}} | \mathrm{F}_\times \rangle \tilde{h}_\times(f)$$

Algebraic formal solution

For cancelling any signal, we must have simultaneously:

$$\vec{C} \cdot \left\langle \vec{Y} | F_{+} \right\rangle = 0, \quad \vec{C} \cdot \left\langle \vec{Y} | F_{\times} \right\rangle = 0$$

Thus:

$$\boxed{\vec{C} = \left\langle \vec{Y} | F_{+} \right\rangle \times \left\langle \vec{Y} | F_{\times} \right\rangle}$$

Explicit solution

1) The transfer functions $F_{\alpha+,\times} = \left\langle \alpha \,|\, \mathrm{F}_{+,\times} \right\rangle$

May be considered as scalar products with
The directional functions ξ :

$$\vec{\xi}_{+,\times} = (\xi_{+,\times 1}, \xi_{2+,\times}, \xi_{3+,\times})$$

then $\exists \vec{\alpha}, \vec{\beta}, \vec{\gamma} :\quad F_{\alpha,\beta,\gamma+,\times} = (\vec{\alpha}, \vec{\beta}, \vec{\gamma}) \cdot \vec{\xi}_{+,\times}$

And

$$\vec{\mathrm{C}} = \begin{bmatrix} (\vec{\beta} \times \vec{\gamma}) \cdot (\vec{\xi}_{+} \times \vec{\xi}_{\times}) \\ (\vec{\gamma} \times \vec{\alpha}) \cdot (\vec{\xi}_{+} \times \vec{\xi}_{\times}) \\ (\vec{\alpha} \times \vec{\beta}) \cdot (\vec{\xi}_{+} \times \vec{\xi}_{\times}) \end{bmatrix}$$

Explicit solution

Recall:

$$F_{V_{1+,\times}} = \frac{e^{i\omega\mu_1} - e^{i\omega(\mu_2+L_3)}}{2(1-\vec{w}\cdot\vec{n}_3)}\xi_{3+,\times}\,, \quad F_{U_{1+,\times}} = -\frac{e^{i\omega\mu_1} - e^{i\omega(\mu_3+L_2)}}{2(1+\vec{w}\cdot\vec{n}_2)}\xi_{2+,\times}$$

Notation:

$$g_a = e^{i\omega\mu_a}, \quad e_a = e^{i\omega L_a}$$

$$v_a = 1-\vec{n}_a\cdot\vec{w}, \quad u_a = 1+\vec{n}_a\cdot\vec{w}$$

$$G_3 = \frac{g_1 - g_2 e_3}{v_3}, \quad H_2 = \frac{g_1 - g_3 e_2}{u_2}$$

Explicit Solution

then:

$$\vec{\alpha} = \frac{1}{2}\begin{bmatrix} e_3 G_1 - e_2 H_1 \\ e_1 e_3 G_2 - H_2 \\ G_3 - e_1 e_2 H_3 \end{bmatrix}, \vec{\beta} = \frac{1}{2}\begin{bmatrix} G_1 - e_2 e_3 H_1 \\ e_1 G_2 - e_3 H_2 \\ e_1 e_2 G_3 - H_3 \end{bmatrix}, \vec{\gamma} = \frac{1}{2}\begin{bmatrix} e_2 e_3 G_1 - H_1 \\ G_2 - e_1 e_3 H_2 \\ e_2 G_3 - e_1 H_3 \end{bmatrix}$$

Invariance under simultaneous circular permutation of:
- Vectors
- Components
- Indices

Explicit Solution

The direction $\vec{w}_B$ of the source is constant
In the barycentric frame

There exists a linear mapping to the LISA frame :

$$\vec{w}(t_o) = \mathrm{R}(t_o) \cdot \vec{w}_B$$

Orbital time parameter, very slowly varying with respect to
The « signal time »

Explicit Solution

All functions may be expressed in terms of $\vec{w}$

$$\begin{cases} (u,v)_1 = 1 \mp w_2, \quad \mu_1 = L w_1 / \sqrt{3} \\ (u,v)_2 = 1 \pm \frac{1}{2}(w_2 - \sqrt{3} w_1), \quad \mu_2 = -L(w_1 + \sqrt{3} w_2) / 2\sqrt{3} \\ (u,v)_3 = 1 \pm \frac{1}{2}(w_2 + \sqrt{3} w_1), \quad \mu_3 = -L(w_1 - \sqrt{3} w_2) / 2\sqrt{3} \end{cases}$$

$$\vec{\eta} \equiv \vec{\xi}_+ \times \vec{\xi}_\times = \frac{\sqrt{3}}{4} w_3 \begin{bmatrix} 1 + w_3^2 - 2w_1^2 + 2w_2^2 \\ 1 + w_3^2 + w_1^2 - w_2^2 - 2\sqrt{3} w_1 w_2 \\ 1 + w_3^2 + w_1^2 - w_2^2 + 2\sqrt{3} w_1 w_2 \end{bmatrix}$$

Explicit Solution

$$C_1 = \frac{\eta_1}{4u_2v_2u_3v_3}(1-e_1^2)\left[u_2v_3(\bar{g}_1 - e_3\bar{g}_2 - e_2\bar{g}_3 + e_2e_3g_1^2) - u_3v_2e_2e_3(g_1^2 - e_2\bar{g}_2 - e_3\bar{g}_3 + e_2e_3\bar{g}_1)\right]$$

$$+\frac{\eta_2}{4u_1v_1u_3v_3}\Big\{(1-e_1e_2e_3)\left[v_1v_3(\bar{g}_1 - e_3\bar{g}_2 - e_1g_2^2 + e_1e_3\bar{g}_3)\right.$$

$$\left. -u_1u_3e_2(\bar{g}_3 - e_1\bar{g}_2 - e_3g_2^2 + e_1e_3\bar{g}_1)\right]$$

$$+(e_1 - e_2e_3)\left[u_1v_3(g_2^2 - e_3\bar{g}_3 - e_1\bar{g}_1 + e_1e_3\bar{g}_2)\right.$$

$$\left. -u_3v_1e_2(\bar{g}_2 - e_1\bar{g}_3 - e_3\bar{g}_1 + e_1e_3g_2^2)\right]\Big\}$$

$$+\frac{\eta_3}{4u_1v_1u_2v_2}\Big\{(1-e_1e_2e_3)\left[u_1u_2(\bar{g}_1 - e_3\bar{g}_3 - e_1g_3^2 + e_1e_2\bar{g}_2)\right.$$

$$\left. -v_1v_2e_3(\bar{g}_2 - e_1\bar{g}_3 - e_2g_3^2 + e_1e_2\bar{g}_1)\right]$$

$$+(e_1 - e_2e_3)\left[v_1u_2(g_3^2 - e_2\bar{g}_2 - e_1\bar{g}_1 + e_1e_2\bar{g}_3)\right.$$

$$\left. -u_1v_2e_2(\bar{g}_3 - e_2\bar{g}_1 - e_1\bar{g}_2 + e_1e_2g_3^2)\right]\Big\}$$

33 different delays

Explicit Solution

For retrieving the time domain, simply replace the Phase factors

$$e_a = e^{i\omega L_a}, \, g_a = e^{i\omega \mu_a}$$

By delay operators

$$(D_a f)(t) = f(t - L_a), \quad (\Gamma_a f)(t) = f(t - \mu_a)$$

the delays L_a et μ_a are slowly varying

L_a Due to the orbital deformation of the triangle (flexing)

μ_a Due to the apparent motion of the source viewed from LISA

Implementation

- Generators (α, β, γ) explicited above are valid for A static LISA (1st generation TDI)
- For actually cancel the intrumental noises one must use The 2d generation TDI generators (more complex)
- For studying the gravitational response, the 1st generation is relevant
- For the gravitational response, the 2d generation amounts to an extra delay

Actual Coronographic Combination

$$C = C_1(t_o)\alpha^{(2)} + C_2(t_o)\beta^{(2)} + C_3(t_o)\gamma^{(2)}$$

Our coefficients (found above)

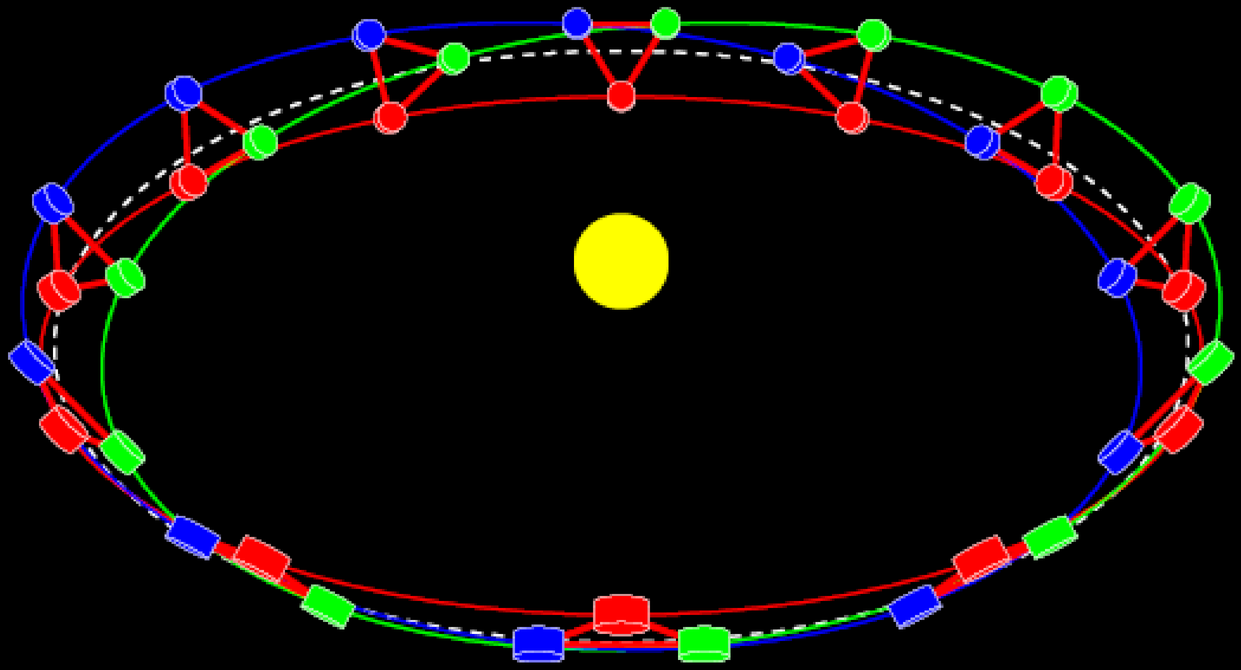
2d generation TDI generators

Programme :

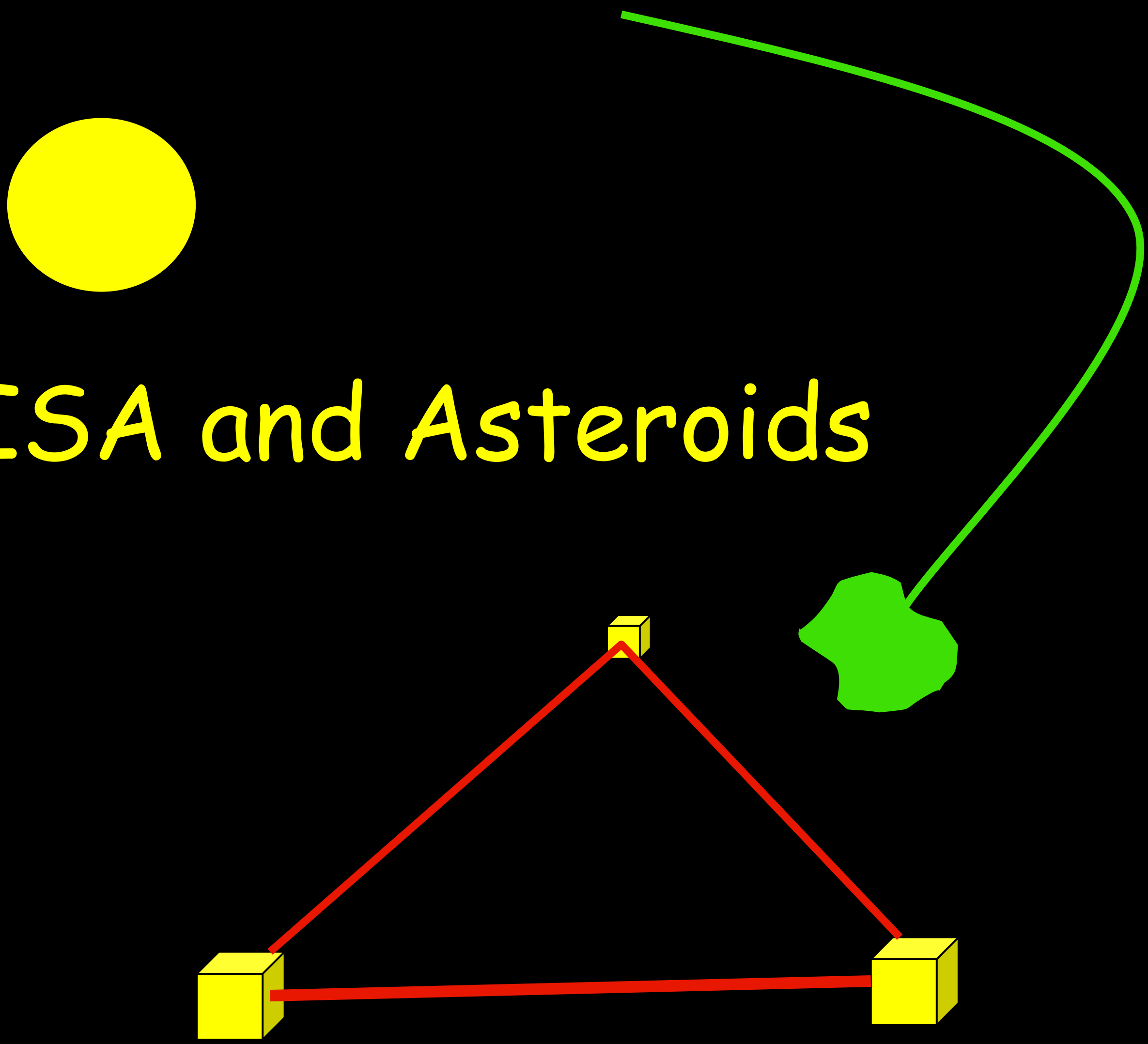
Testing the algorithm on mock data

Simulator : **LISACode** (APC-Paris+ARTEMIS-Nice)

http://www.apc.univ-paris7.fr/SPIP/article.php3?id_article=164



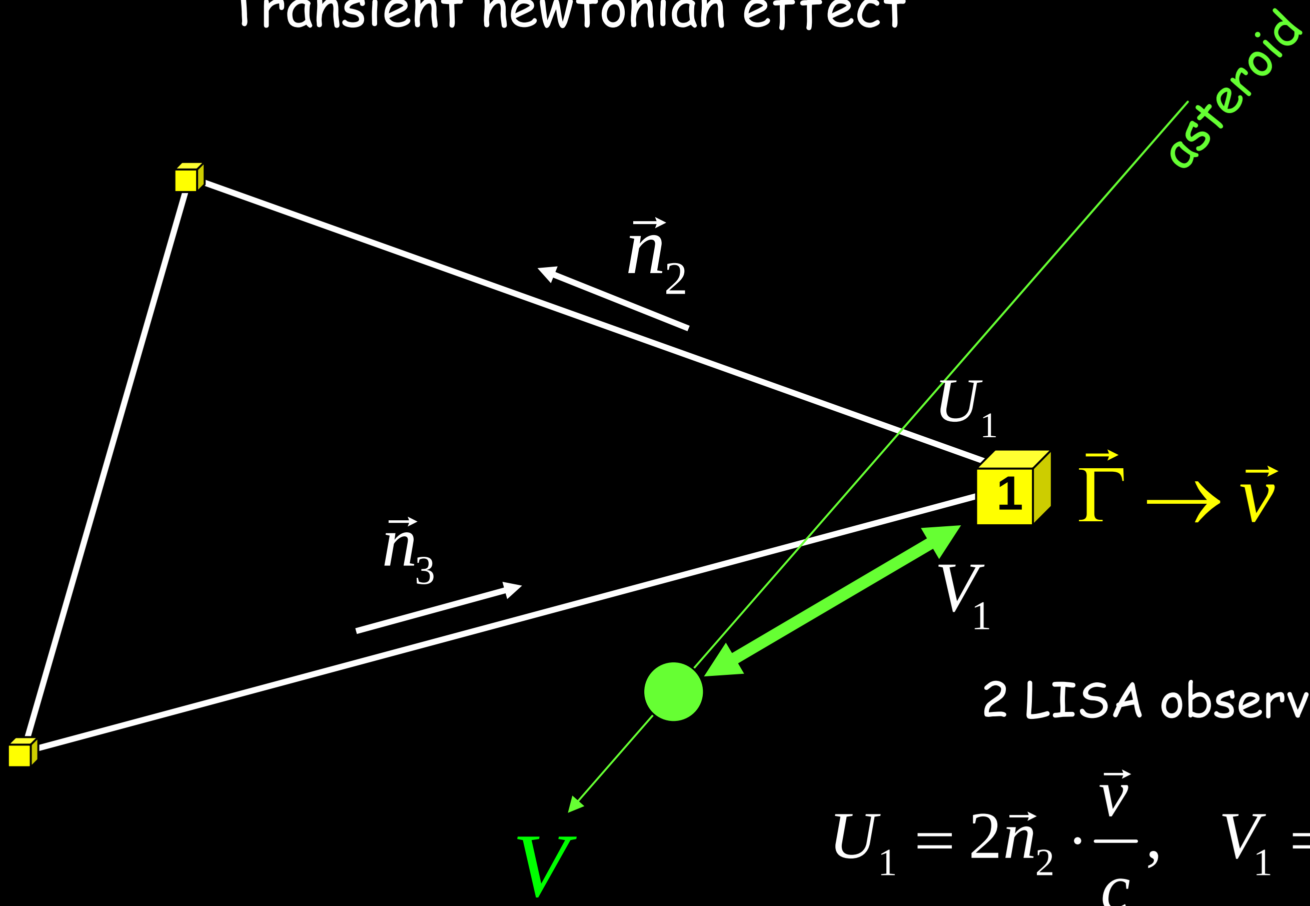
LISA and Asteroids



Vinet, Class. and Quantum Grav. 23 (2006) 4939-4944

An asteroid encounter

Transient newtonian effect



2 LISA observables

$$U_1 = 2\vec{n}_2 \cdot \frac{\vec{v}}{c}, \quad V_1 = 2\vec{n}_3 \cdot \frac{\vec{v}}{c}$$

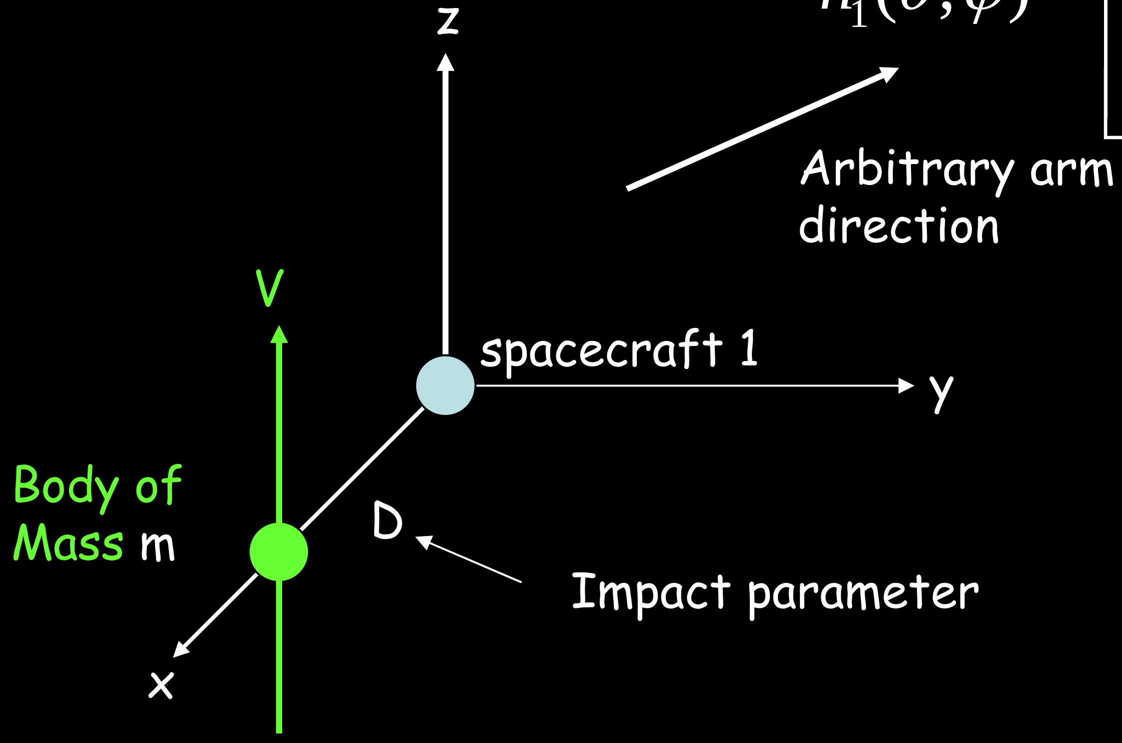
LISA response

Combinaisons TDI (X, α, ς) :

$$X = V_1 + U_1 = 2(\vec{n}_2 + \vec{n}_3) \cdot \frac{\vec{v}}{c} = -2\vec{n}_1 \cdot \frac{\vec{v}}{c}$$

natural frame

$$\vec{n}_1(\theta,\varphi) = \begin{bmatrix} \sin\theta\cos\varphi \\ \sin\theta\sin\varphi \\ \cos\theta \end{bmatrix}$$



Motion of spacecraft

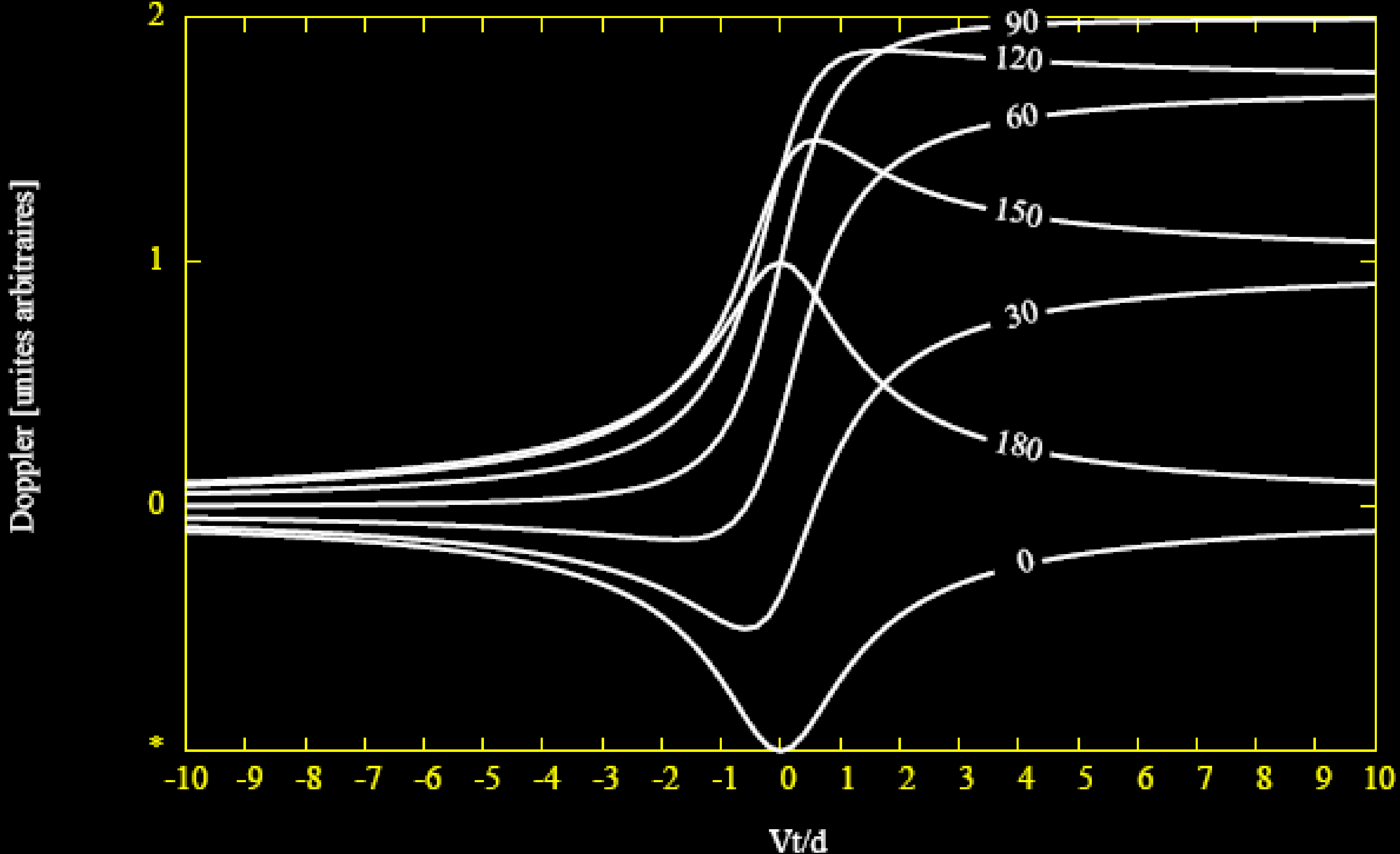
$$\vec{\Gamma}(t) = \frac{Gm}{D^2} \frac{1}{\left(1+V^2t^2/D^2\right)^{3/2}} \begin{bmatrix} 1 \\ 0 \\ Vt/D \end{bmatrix}$$

$$\vec{v}(t) = \frac{Gm}{DV} \begin{bmatrix} 1+\dfrac{Vt/D}{\left(1+V^2t^2/D^2\right)^{1/2}} \\ 0 \\ -\dfrac{1}{\left(1+V^2t^2/D^2\right)^{1/2}} \end{bmatrix}$$

Response filtered by TDI

$$X(t) = -\frac{2Gm}{DVc}\left\{ \sin\theta\cos\varphi\left(1 + \frac{Vt/D}{\sqrt{1+V^2t^2/D^2}} \right) - \cos\theta\frac{1}{\sqrt{1+V^2t^2/D^2}} \right\}$$

X(t) for various orientations (degrees) $\theta \quad (\varphi = 0)$



Fourier space

$$\tilde{\vec{\Gamma}}(\omega) = \frac{2Gm\omega}{V^2}\begin{bmatrix} K_1(\omega D/V) \\ 0 \\ -iK_0(\omega D/V) \end{bmatrix}$$

$$\frac{\tilde{\vec{v}}}{c}(\omega) = \frac{2Gm}{cV^2}\begin{bmatrix} iK_1(\omega D/V) \\ 0 \\ K_0(\omega D/V) \end{bmatrix}$$

(K_n: 2d kind modified Bessel f.)

$$\tilde{X}(\omega) = -\frac{4Gm}{cV^2}\left[iK_1(\omega D/V)\sin\theta\cos\varphi + K_0(\omega D/V)\cos\theta\right]$$

Spectral density of noise and SNR

$$S_X(f) = \left[8\sin^2(4\pi fL/c) + 32\sin^2(2\pi fL/c)\right] S_{acc}(f) +$$

$$16\sin^2(2\pi fL/c) S_{bq}(f)$$

Acceleration noise

Optical path noise

Angular average:

$$\left\langle \left|\tilde{X}(f)\right|^2 \right\rangle = \frac{1}{3}\left(\frac{4Gm}{cV^2}\right)^2 \left[K_1^2(2\pi fD/V) + K_0^2(2\pi fD/V)\right]$$

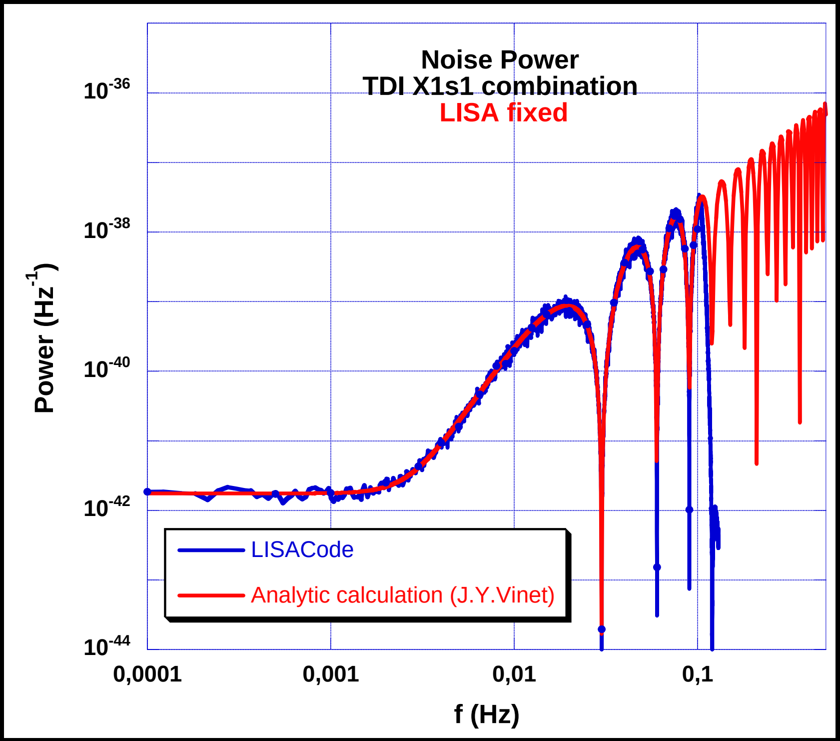
SNR power
Spectral density

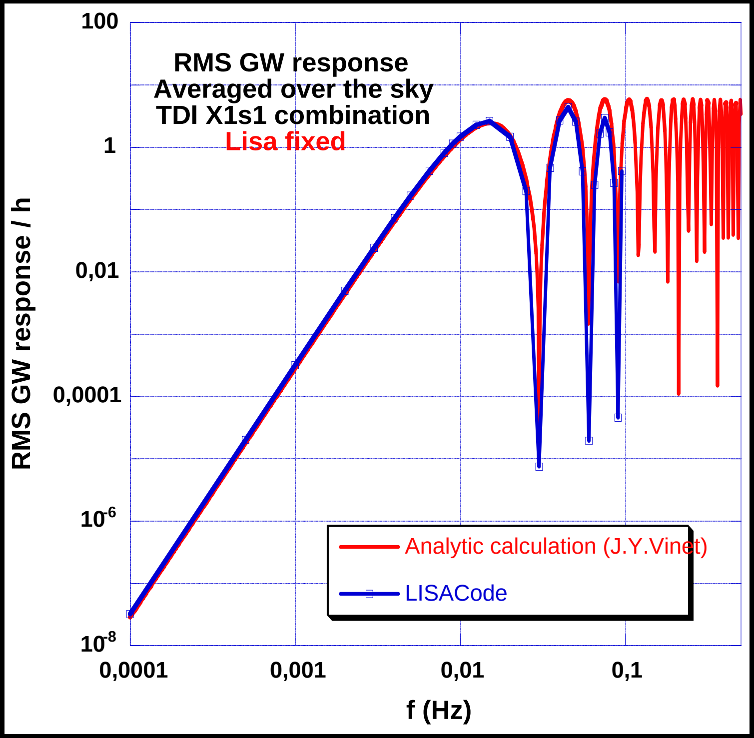
$$\rho(f) = 4\frac{\left\langle \left|\tilde{X}(f)\right|^2 \right\rangle}{S_X(f)}$$

Linear Spectral Density
Of residual noise

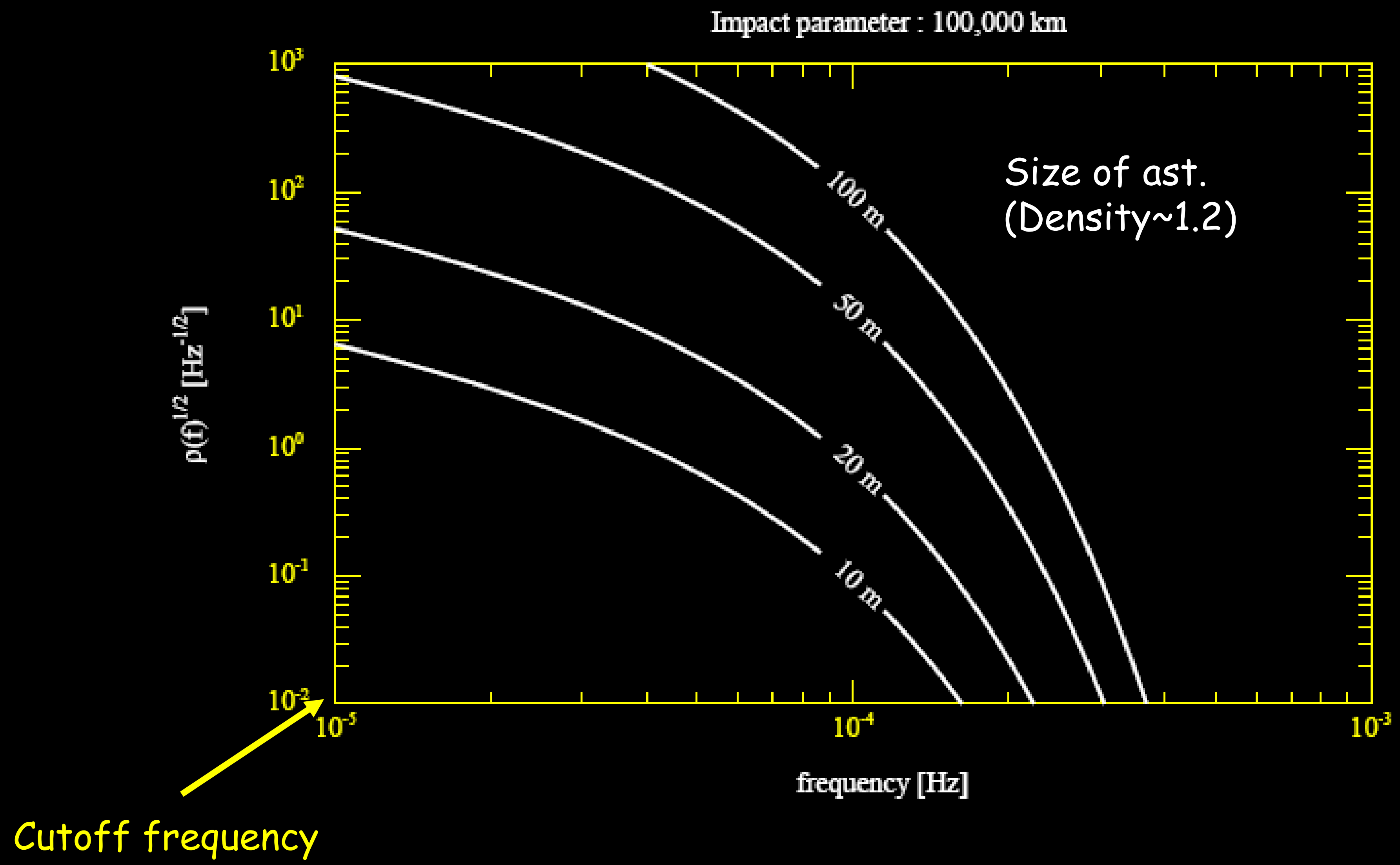
$$S_X^{1/2}(f)$$

Response to GW signal

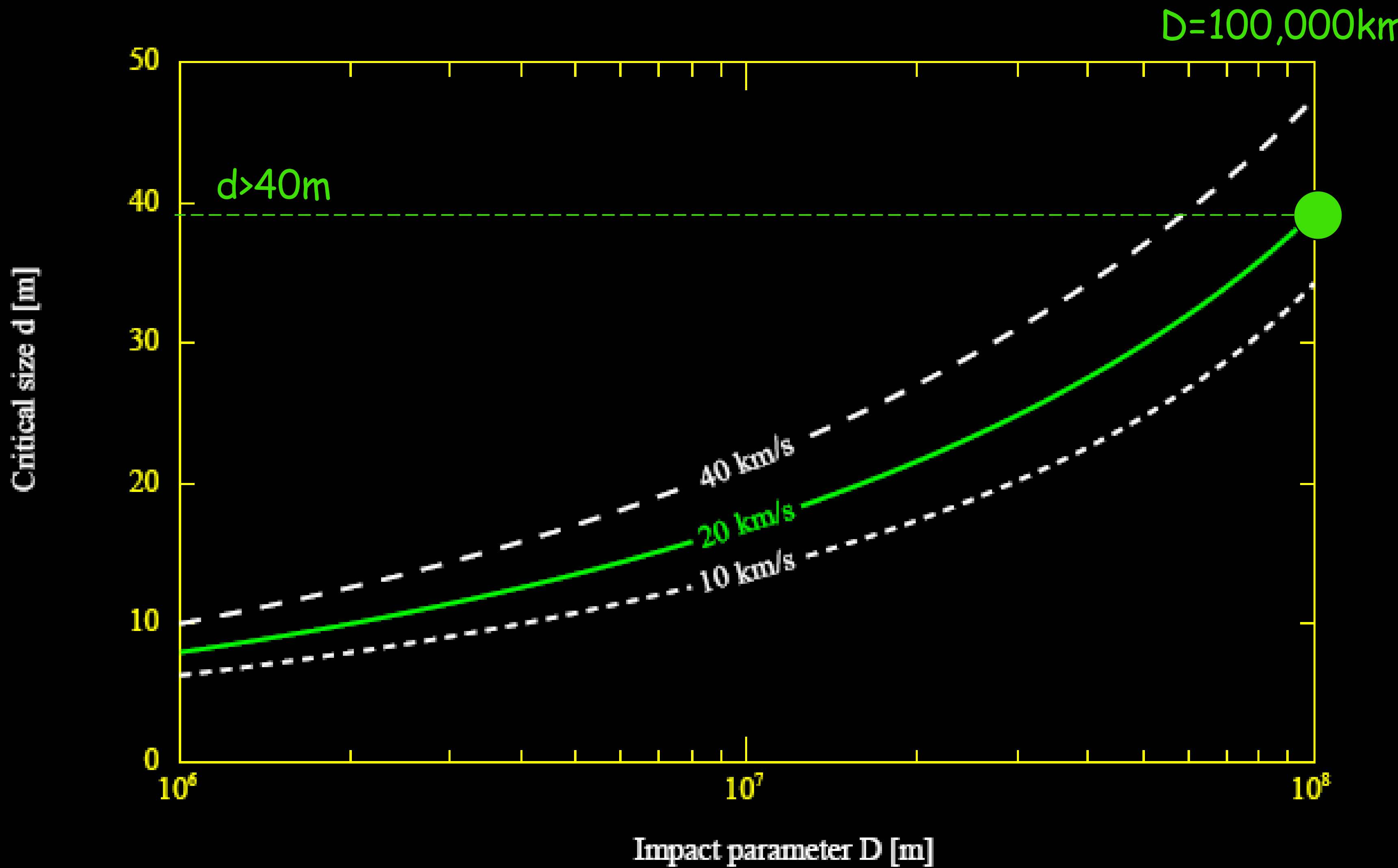




Linear spectral density of SNR for V=20 km/s



Detection condition (SNR>1)



Programme:

Assesment of a rate of detection

Monte Carlo simulation using
Realistic distributions of
asteroids masses and velocities