FRAME-DRAGGING (GRAVITOMAGNETISM) AND ITS MEASUREMENT

INTRODUCTION

Frame-Dragging and Gravitomagnetism

EXPERIMENTS

- **Past, present and future experimental efforts to measure frame-dragging**
- **Measurements using satellite laser ranging**
- **The 2004-2006 measurements of the Lense-Thirring effect using the GRACE Earth's gravity models**

Ignazio Ciufolini (Univ. Lecce): Firenze 30-9-2006

DRAGGING OF INERTIAL FRAMES
(*FRAME-DRAGGING* as Einstein named it in 1913)

- **The local inertial frames are dragged by mass-energy currents: $\varepsilon\, u^{\alpha}$**

$$G^{\alpha\beta} = \chi\, T^{\alpha\beta} =$$
$$= \chi\, [(\varepsilon + p)\, u^{\alpha} u^{\beta} + p\, g^{\alpha\beta}]$$

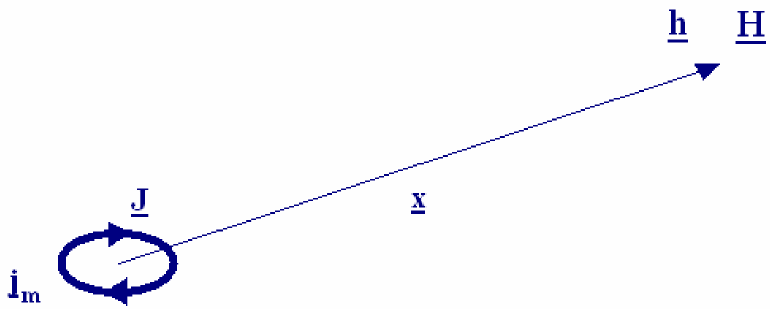
- It plays a key role in high energy astrophysics

  (Kerr metric)

Thirring 1918
Braginsky, Caves and Thorne 1977
Thorne 1986
Mashhoon 1993, 2001
Jantsen et al. 1992-97, 2001
I.C. 1994-2001

THE WEAK-FIELD AND SLOW MOTION ANALOGY WITH ELECTRODYNAMICS

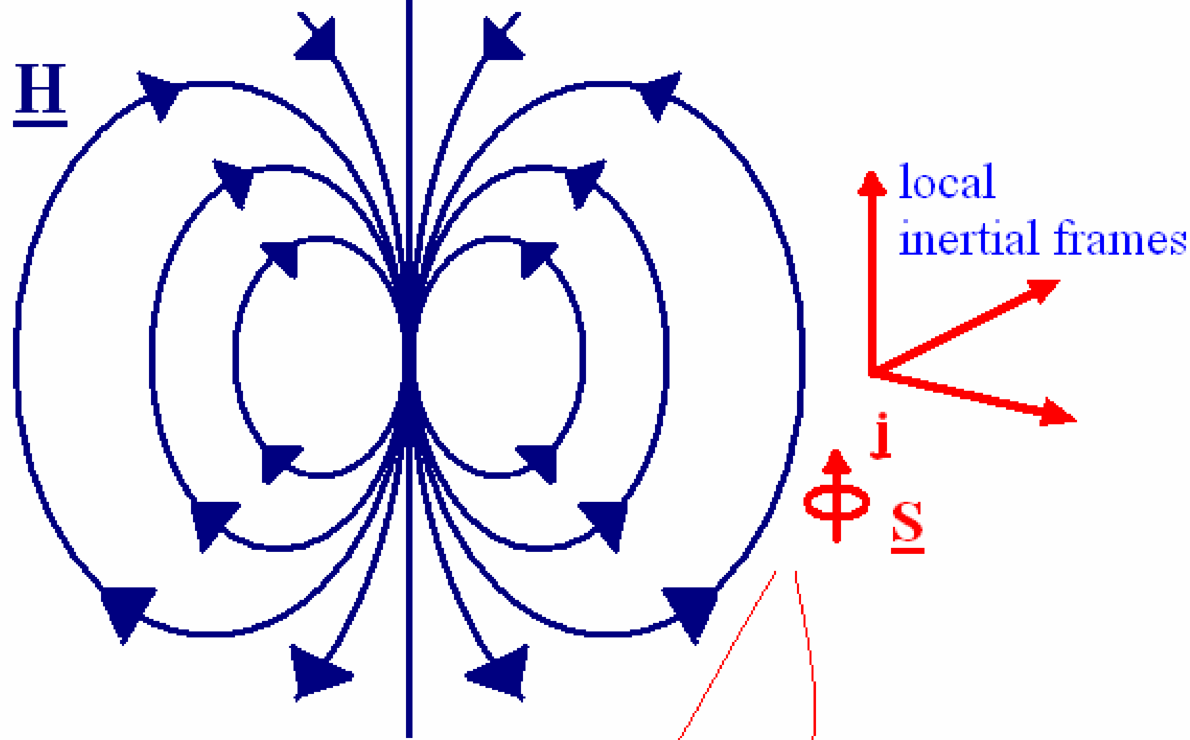
Gravitomagnetic Field in General Relativity



From weak field and slow motion limit of G= χ T:	**Electromagnetism**
$$\Delta\, h_{0i} \cong 16\,\pi\,\rho\, v^i$$ Lorentz gauge	$$\Delta\, \mathbf{A} = -\frac{4\pi}{c}\,\mathbf{j}$$

where $\mathbf{h} \equiv (h_{01},\ h_{02}\ , h_{03})$ is the gravitomagnetic potential

$$h_{0i}\,(\mathbf{x}) \cong -\,4\int \frac{\rho(\mathbf{x}')v^i(\mathbf{x}')}{\lvert\mathbf{x}-\mathbf{x}'\rvert}\, d^3x'$$	$$\mathbf{A}\,(\mathbf{x}) = \frac{1}{c}\int \frac{\mathbf{j}(\mathbf{x}')}{\lvert\mathbf{x}-\mathbf{x}'\rvert}\, d^3x'$$
$$\mathbf{h}\,(\mathbf{x}) \cong -\,2\,\frac{\mathbf{J}\times\mathbf{x}}{\lvert\mathbf{x}\rvert^3}$$	$$\mathbf{A}\,(\mathbf{x}) \cong \frac{\mathbf{m}\times\mathbf{x}}{\lvert\mathbf{x}\rvert^3}$$
The gravitomagnetic field is: $$\mathbf{H} = \nabla\times\mathbf{h} \cong 2\left[\frac{\mathbf{J}-3(\mathbf{J}\cdot\hat{\mathbf{x}})\,\hat{\mathbf{x}}}{\lvert\mathbf{x}\rvert^3}\right]$$	$$\mathbf{B} = \nabla\times\mathbf{A} \cong$$ $$\cong \frac{3\,\hat{\mathbf{x}}\,(\hat{\mathbf{x}}\cdot\mathbf{m}) - \mathbf{m}}{\lvert\mathbf{x}\rvert^3}$$
From weak field and slow motion limit of D u=0:	
$$m\,\frac{d^2\mathbf{x}}{dt^2} \cong m\,(\mathbf{G} + \frac{d\mathbf{x}}{dt}\times\mathbf{H})$$	$$m\,\frac{d^2\mathbf{x}}{dt^2} = q\,(\mathbf{E} + \frac{d\mathbf{x}}{dt}\times\mathbf{B})$$



$$\mathbf{F} = \left(\frac{1}{2}\mathbf{S}\cdot\nabla\right)\mathbf{H} \qquad \tau \cong \tfrac{1}{2}\mathbf{S}\times\mathbf{H} = \frac{d\mathbf{S}}{dt} \equiv \dot{\mathbf{\Omega}}\times\mathbf{S}$$

$$\dot{\mathbf{\Omega}} = -\tfrac{1}{2}\mathbf{H} = \frac{-\mathbf{J} + 3\,(\mathbf{J}\cdot\hat{\mathbf{x}})\,\hat{\mathbf{x}}}{|\mathbf{x}|^{3}}$$

Dragging of inertial frames:

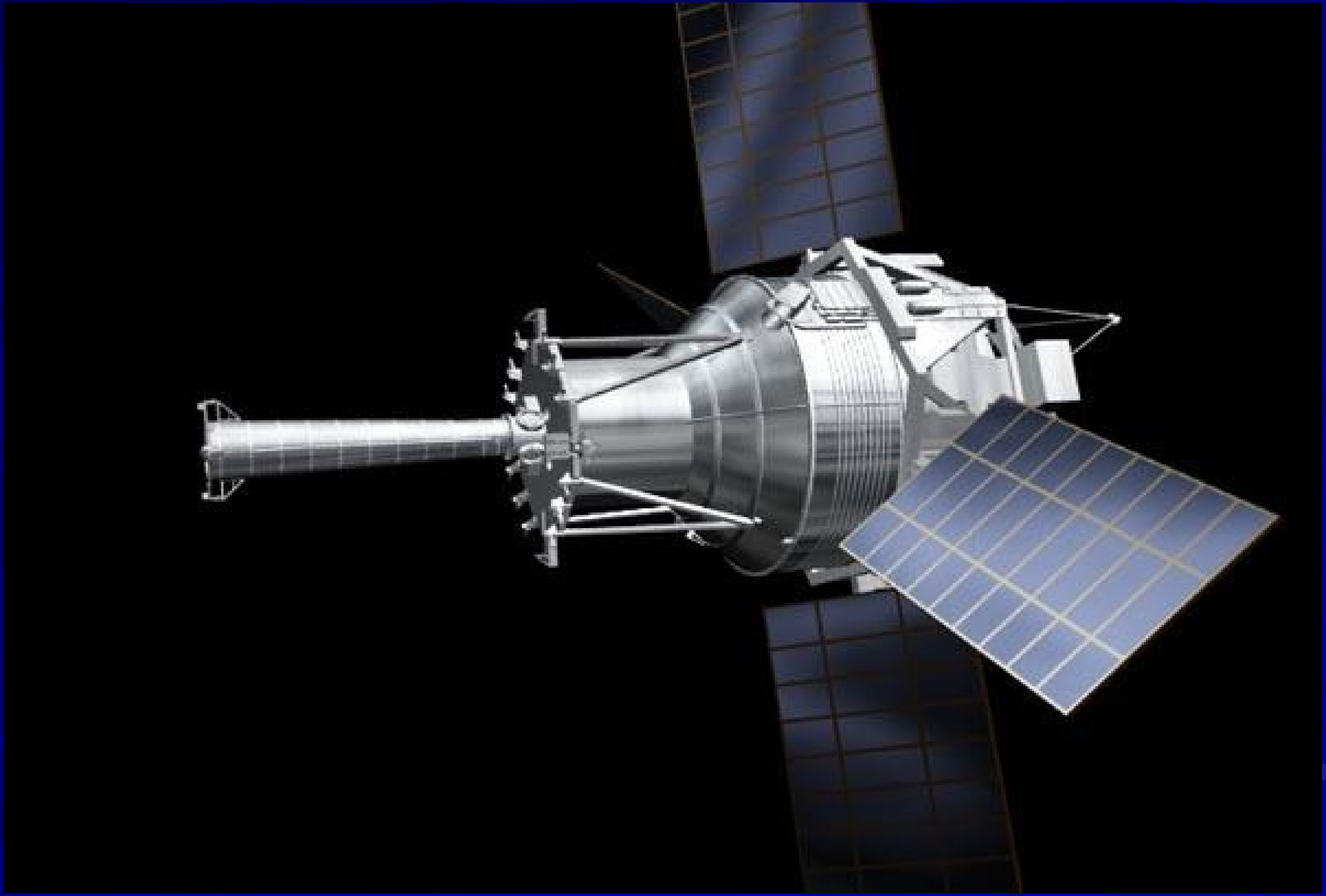
Mach principle in general relativity

GRAVITATION AND INERTIA

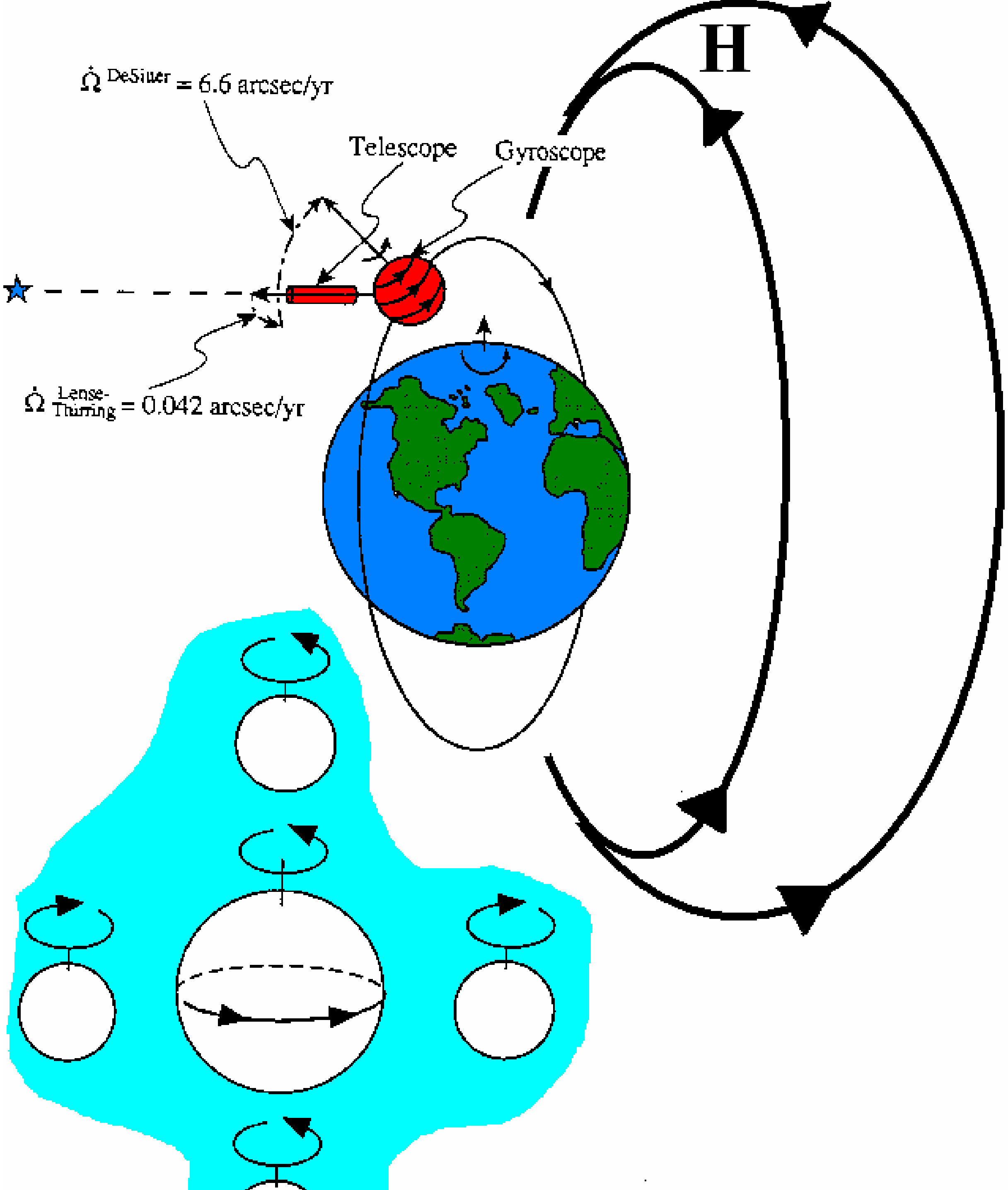
I.C. and J.A. Wheleer -1995

SOME EXPERIMENTAL ATTEMPTS TO MEASURE FRAME-DRAGGING AND GRAVITOMAGNETISM

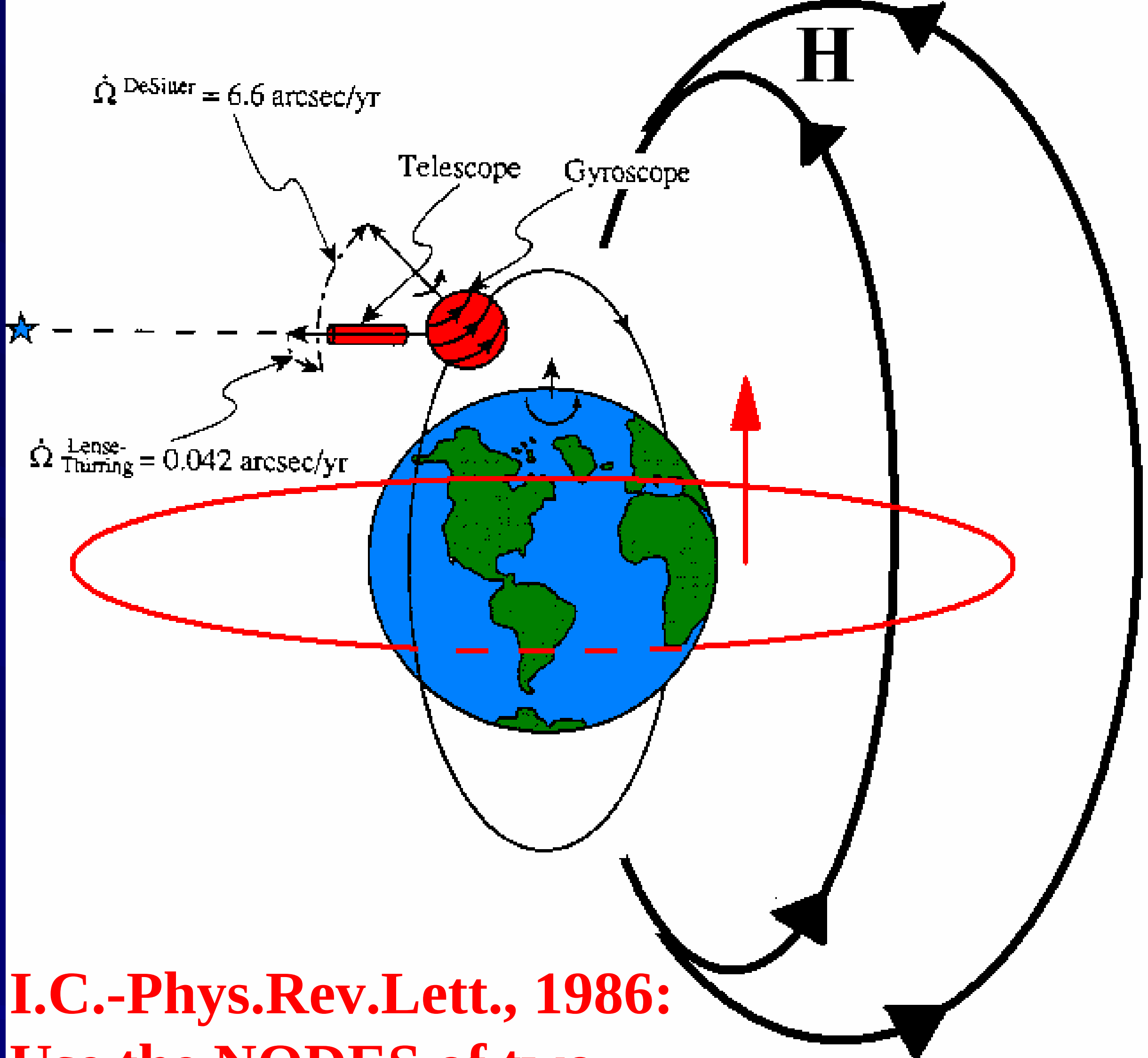
- **1896: Benedict and Immanuel FRIEDLANDER (torsion balance near a heavy flying-wheel)**
- **1904: August FOPPL (Earth-rotation effect on a gyroscope)**
- **1916: DE SITTER (shift of perihelion of Mercury due to Sun rotation)**
- **1918: LENSE AND THIRRING (perturbations of the Moons of solar system planets by the planet angular momentum)**
- **1959: Yilmaz (satellites in polar orbit)**
- **1976: Van Patten-Everitt (two non-passive counter-rotating satellites in polar orbit)**
- **1960: Schiff-Fairbank-Everitt (Earth orbiting gyroscopes)**
- **1986: I.C.: USE THE NODES OF TWO LAGEOS SATELLITES (two supplementary inclination, passive, laser ranged satellites)**
- **1988 : Nordtvedt (Astrophysical evidence from periastron rate of binary pulsar)**
- **1995-2006: I.C. et al. (measurements using LAGEOS and LAGEOS-II)**
- **1998: Some astrophysical evidence from accretion disks of black holes and neutron stars**



GRAVITY PROBE B



$\dot{\Omega}^{\text{DeSitter}} = 6.6$ arcsec/yr
Telescope
Gyroscope
H
$\dot{\Omega}_{\text{Lense-Thirring}} = 0.042$ arcsec/yr



I.C.-Phys.Rev.Lett., 1986:
Use the NODES of two LAGEOS satellites.

Measurement of the Lense-Thirring Drag on High-Altitude, Laser-Ranged Artificial Satellites

Ignazio Ciufolini

Center for Theoretical Physics, Center for Relativity, and Physics Department, University of Texas, Austin, Texas 78712

(Received 16 October 1984; revised manuscript received 19 April 1985)

We describe a new method of measuring the Lense-Thirring relativistic nodal drag using LAGEOS together with another similar high-altitude, laser-ranged satellite with appropriately chosen orbital parameters. We propose, for this purpose, that a future satellite such as LAGEOS II have an inclination supplementary to that of LAGEOS. The experiment proposed here would provide a method for experimental verification of the general relativistic formulation of Mach's principle and measurement of the gravitomagnetic field.

PACS numbers: 04.80.+z

In special and general relativity there are several precession phenomena associated with the angular momentum vector of a body. If a test particle is orbiting a rotating central body, the plane of the orbit of the particle is dragged by the intrinsic angular momentum J of the central body, in agreement with the general relativistic formulation of Mach's principle.[1]

In the weak-field and slow-motion limit the nodal lines are dragged in the sense of rotation, at a rate given by[2]

$$\dot{\Omega} = [2/a^3(1-e^2)^{3/2}]J, \tag{1}$$

where a is the semimajor axis of the orbit, e is the eccentricity of the orbit, and geometrized units are used, i.e., $G=c=1$. This phenomenon is the Lense-Thirring effect, from the names of its discoverers in 1918.[2]

In addition to this there are other precession phenomena associated with the intrinsic angular momentum or spin $\mathbf{S}$ of an orbiting particle. In the weak-field and slow-motion limit the vector $\mathbf{S}$ precesses at a rate given by[1] $d\mathbf{S}/d\tau = \dot{\mathbf{\Omega}} \times \mathbf{S}$ where

$$\dot{\mathbf{\Omega}} \equiv -\tfrac{1}{2}\mathbf{v}\times\mathbf{a} + \tfrac{3}{2}\mathbf{v}\times\nabla U + \frac{1}{r^3}\left[-\mathbf{J} + \frac{3(\mathbf{J}\cdot\mathbf{r})\mathbf{r}}{r^2}\right], \tag{2}$$

where $\mathbf{v}$ is the particle velocity, $\mathbf{a} \equiv d\mathbf{v}/d\tau - \nabla U$ is its nongravitational acceleration, $\mathbf{r}$ is its position vector, τ is its proper time, and U is the Newtonian potential.

The first term of this equation is the Thomas precession.[3] It is a special relativistic effect due to the noncommutativity of nonaligned Lorentz transformations. It may also be viewed as a coupling between the particle velocity $\mathbf{v}$ and the nongravitational forces acting on it.

The second (de Sitter[4]–Fokker[5]) term is general relativistic, arising even for a nonrotating source, from the parallel transport of a direction defined by $\mathbf{S}$; it may be viewed as spin precession due to the coupling between the particle velocity $\mathbf{v}$ and the static —$g_{\alpha\beta,0}=0$ and $g_{i0}=0$—part of the space-time geometry.

The third (Schiff[6]) term gives the general relativistic precession of the particle spin $\mathbf{S}$ caused by the intrinsic angular momentum $\mathbf{J}$ of the central body—$g_{i0}\neq 0$.

We also mention the precession of the periapsis of an orbiting test particle due to the angular momentum of the central body. This tiny shift of the perihelion of Mercury due to the rotation of the Sun was calculated by de Sitter in 1916.[7]

All these effects are quite small for an artificial satellite orbiting the Earth.

We propose here to measure the Lense-Thirring dragging by measuring the nodal precession of laser-ranged Earth satellites. We shall show that two satellites would be required; we propose that LAGEOS[8–10] together with a second satellite LAGEOS X with opposite inclination (i.e., with $I^X = 180° - I$, where $I \simeq 109.94°$ is the orbital inclination of LAGEOS) would provide the needed accuracy.

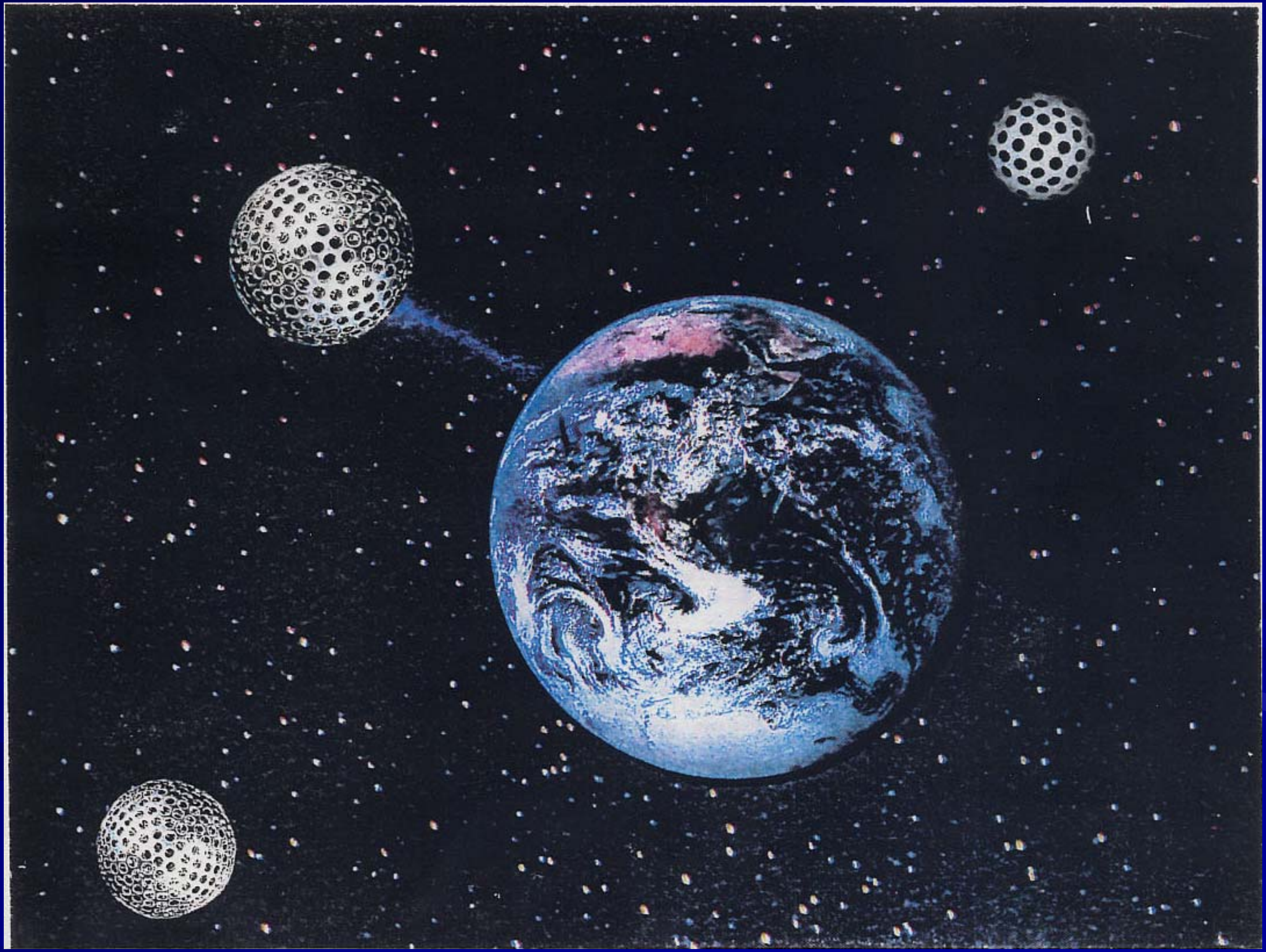
The major part of the nodal precession of an Earth satellite is a classical effect due to deviations from spherical symmetry of the Earth's gravity field —quadrupole and higher mass moments.[11] These deviations from sphericity are measured by the expansion of the potential $U(r)$ in spherical harmonics. From this expansion of $U(r)$ follows[11] the formula for the classical precession of the nodal lines of an Earth satellite:

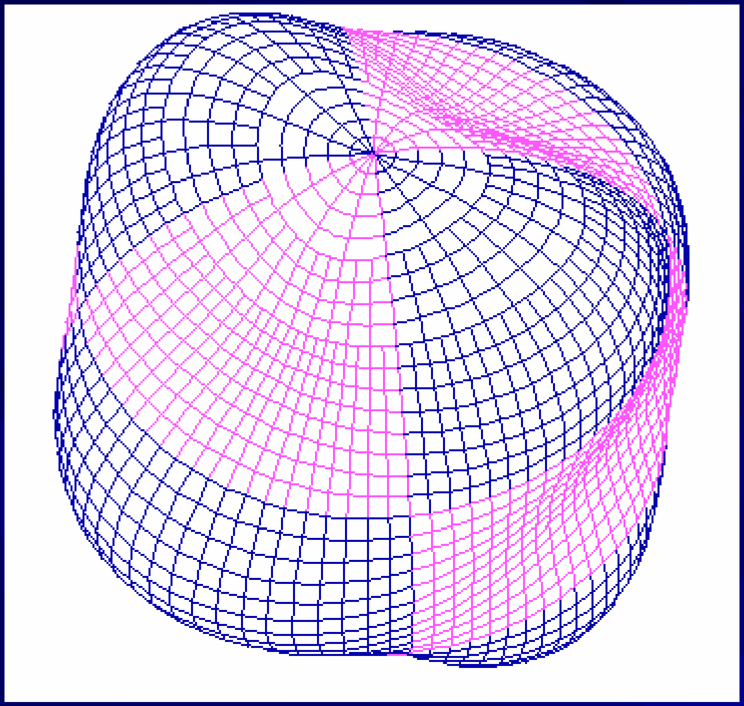
$$\dot{\Omega}_{\text{class}} \simeq -\frac{3}{2} n \left(\frac{R_\oplus}{a}\right)^2 \frac{\cos I}{(1-e^2)^2}\left\{J_2 + J_4\left[\frac{5}{8}\left(\frac{R_\oplus}{a}\right)^2 (7\sin^2 I - 4)\frac{1+\frac{3}{2}e^2}{(1-e^2)^2}\right] + \dots\right\}, \tag{3}$$

IC, PRL 1986: Use of the nodes of two laser-ranged satellites to measure the Lense-Thirring effect

John’s office, Univ. Texas at Austin, nearly 20 years ago

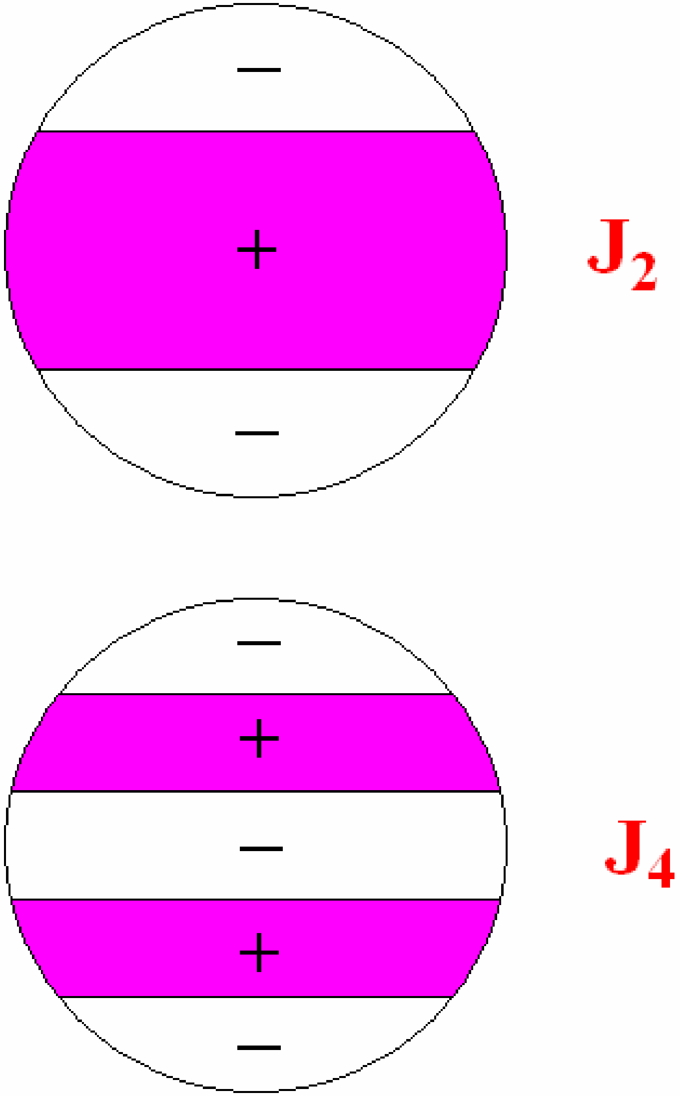
Satellite Laser Ranging



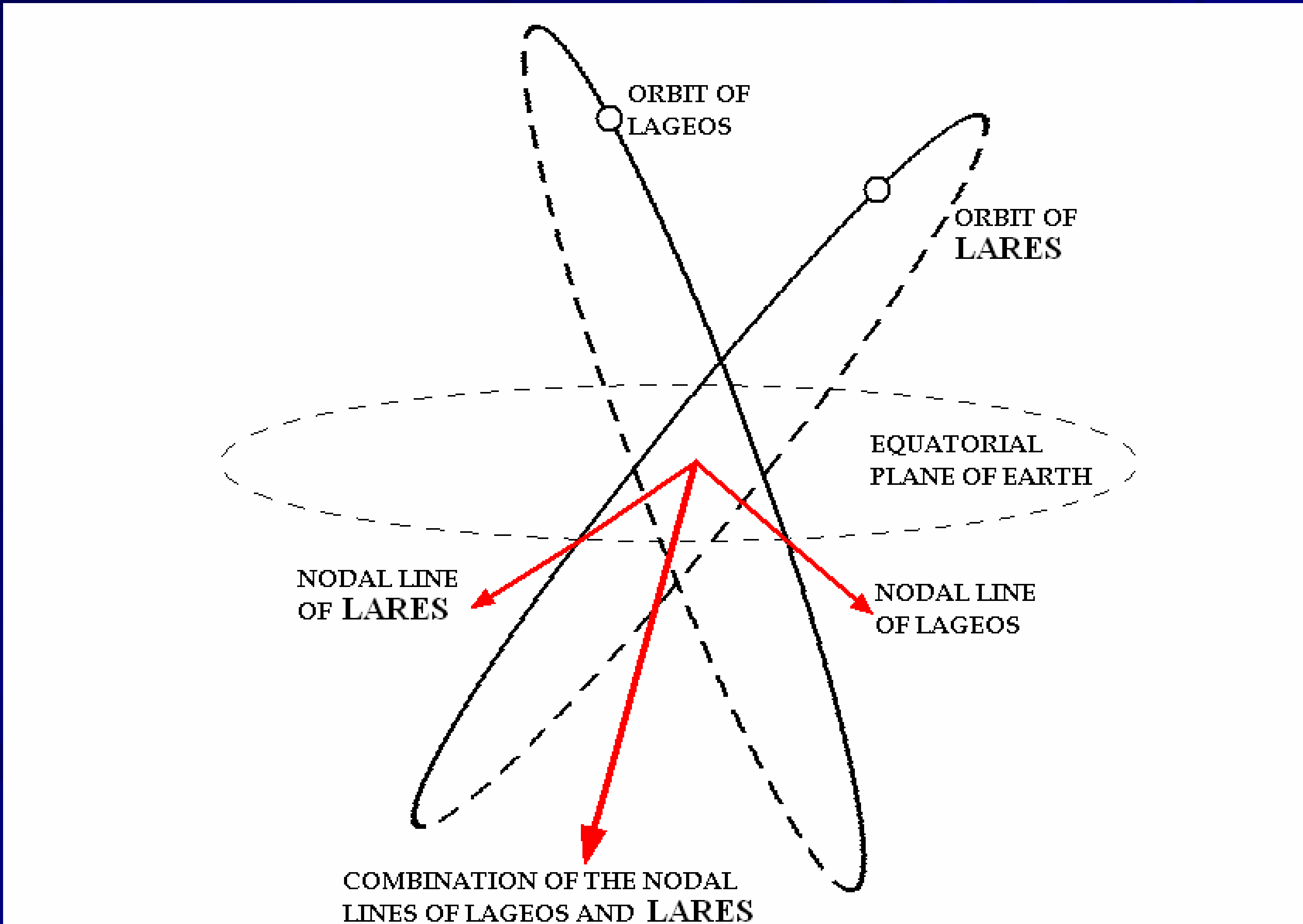


l=3, m=1

EVEN ZONAL HARMONICS



CONCEPT OF THE LAGEOS III / LARES EXPERIMENT



LARES

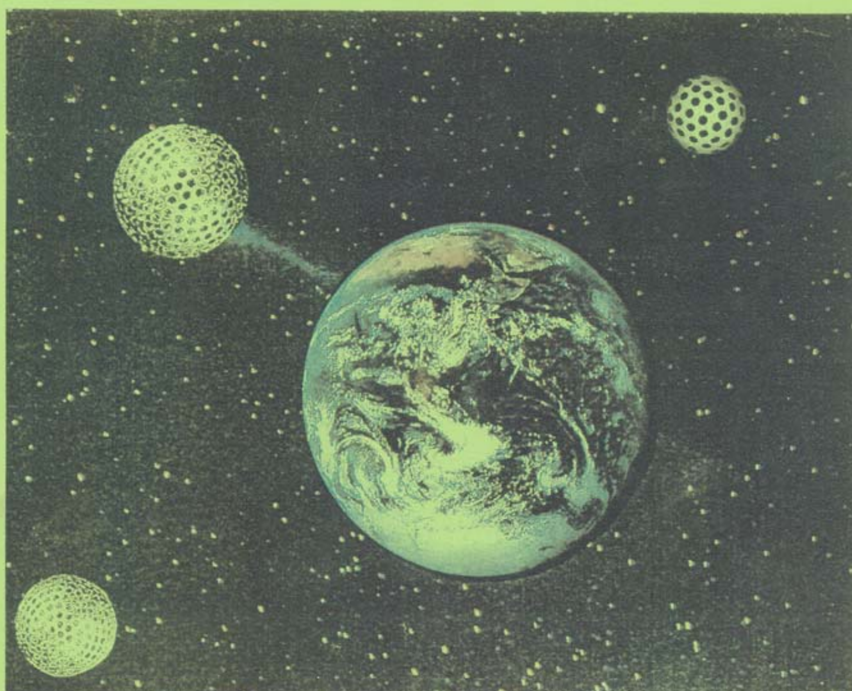
WEBER-SAT

A NEW SATELLITE FOR THE LARES EXPERIMENT

LAser RElativity experimentS

for

Testing General Relativity and Studying the Earth Gravitational Field



January 2003

MAIN COLLABORATION

University of Lecce

I.C.

University of Roma "La Sapienza"

A. Paolozzi

INFN of Italy

S. Dell'Agnello

University of Maryland

E. Pavlis

D. Currie

NASA-Goddard

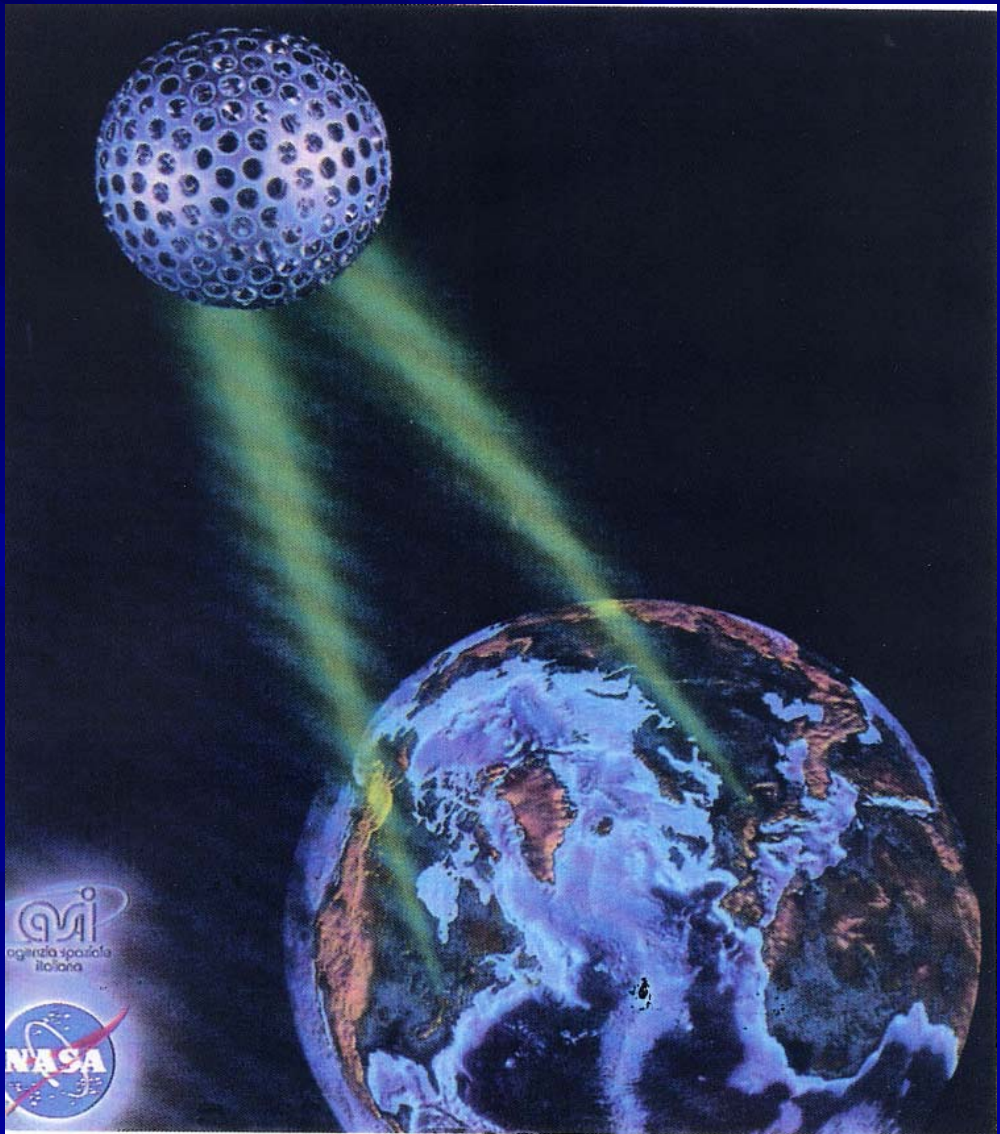
D. Rubincam

University of Texas at Austin

R. Matzner

Lageos II: 1992

However, NO LAGEOS satellite with supplementary inclination to LAGEOS has ever been launched. Nevertheless, LAGEOS II was launched in 1992.



International Journal of Modern Physics A, Vol. 4, No. 13 (1989) 3083–3145

A COMPREHENSIVE INTRODUCTION TO THE LAGEOS GRAVITOMAGNETIC EXPERIMENT: FROM THE IMPORTANCE OF THE GRAVITOMAGNETIC FIELD IN PHYSICS TO PRELIMINARY ERROR ANALYSIS AND ERROR BUDGET

IGNAZIO CIUFOLINI

CNR-Istituto di Fisica dello Spazio Interplanetario, Via G. Galilei-CP 27-00044 Frascati, Italy

and

Center for Space Research, The University of Texas at Austin, Austin, Texas 78712, USA

Received 3 May 1988
Revised 7 October 1988

The existence of the gravitomagnetic field, generated by mass currents according to Einstein geometrodynamics, has never been proved. The author of this paper, after a discussion of the importance of the gravitomagnetic field in physics, describes the experiment that he proposed in 1984 to measure this field using LAGEOS (Laser geodynamics satellite) together with another non-polar, laser-ranged satellite with the same orbital parameters as LAGEOS but a supplementary inclination.

The author then studies the main perturbations and measurement uncertainties that may affect the measurement of the Lense-Thirring drag. He concludes that, over the period of the node of ~ 3 years, the maximum error, using two nonpolar laser ranged satellites with supplementary inclinations, should not be larger than ~ 10% of the gravitomagnetic effect to be measured.

Table of Contents

IC IJMPA 1989: Analysis of the orbital perturbations affecting the nodes of LAGEOS-type satellites

(1) Use two LAGEOS satellites with supplementary inclinations

OR:

Use n satellites of LAGEOS-type to measure the first n-1 even zonal harmonics: J_2, J_4, … and the Lense-Thirring effect

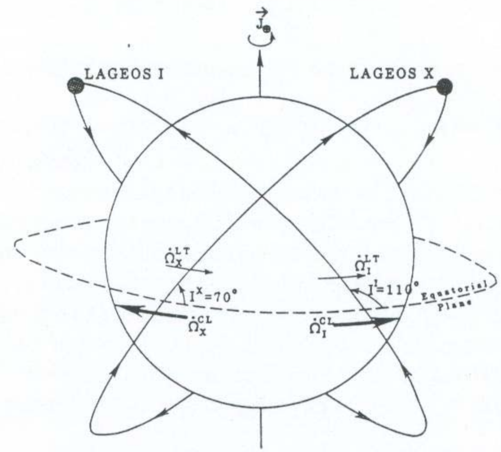


Fig. 5. The LAGEOS and LAGEOS *X* orbits and their classical and gravitomagnetic nodal precessions. A new[17] configuration to measure the Lense-Thirring effect.

For J_2, this corresponds, from formula (3.2), to an uncertainty in the nodal precession of 450 milliarcsec/year, and similarly for higher J_{2n} coefficients. Therefore, the uncertainty in $\dot{\Omega}^{\mathrm{Class}}_{\mathrm{Lageos}}$ is more than ten times larger than the Lense-Thirring precession.

A solution would be to orbit several high-altitude, laser-ranged satellites, similar to LAGEOS, to measure J_2, J_4, J_6, etc., and one satellite to measure $\dot{\Omega}^{\text{Lense-Thirring}}$.

Another solution would be to orbit polar satellites; in fact, from formula (3.2), for polar satellites, since $I = 90°$, $\dot{\Omega}^{\mathrm{Class}}$ is equal to zero. As mentioned before, Yilmaz proposed the use of polar satellites in 1959.[40,41] In 1976, Van Patten and Everitt[46,47] proposed an experiment with two drag-free, guided, counter-rotating, polar satellites to avoid inclination measurement errors.

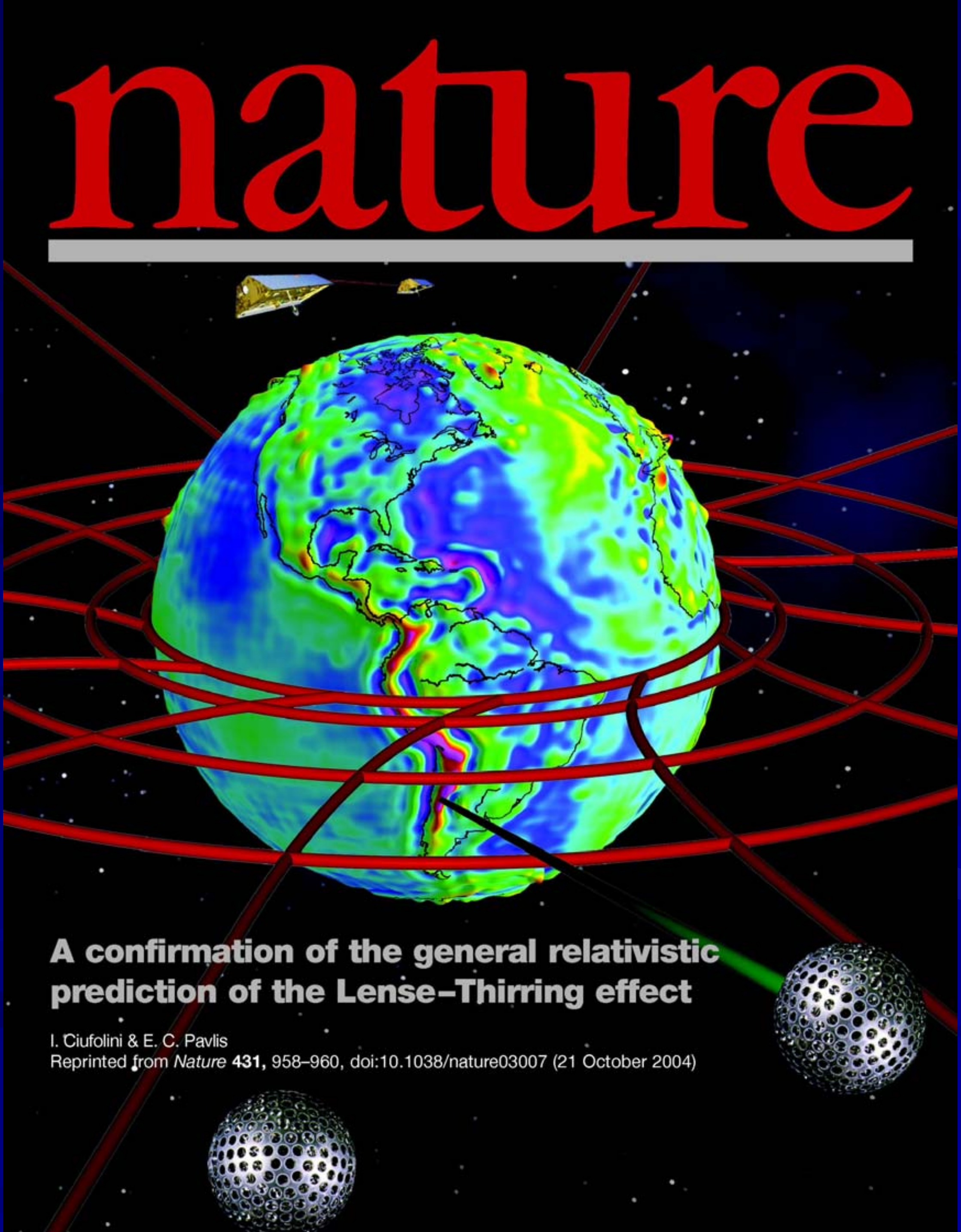
A new solution[15,16,17,21,22,23] would be to orbit a second satellite, of LAGEOS type, with the same semimajor axis, the same eccentricity, but the inclination supplementary to that of LAGEOS (see Fig. 5). Therefore, "LAGEOS X" should have the following orbital parameters:

$$I^X \cong \pi - I^I \cong 70°\,, \qquad a^X \cong a^I\,, \qquad e^X \cong e^I\,. \tag{3.3}$$

With this choice, since the classical precession $\dot{\Omega}^{\mathrm{Class}}$ is linearly proportional to $\cos I$, $\dot{\Omega}^{\mathrm{Class}}$ would be equal and opposite for the two satellites:

$$\dot{\Omega}^{\mathrm{Class}}_X = -\dot{\Omega}^{\mathrm{Class}}_I\,. \tag{3.4}$$

By contrast, since the Lense-Thirring precession $\dot{\Omega}^{\text{Lense-Thirring}}$ is independent of the inclination (Eq. (3.1)), $\dot{\Omega}^{\text{Lense-Thirring}}$ will be the same in magnitude and sign for both satellites:


nature
A confirmation of the general relativistic prediction of the Lense–Thirring effect
I. Ciufolini & E. C. Pavlis
Reprinted from Nature 431, 958–960, doi:10.1038/nature03007 (21 October 2004)

IL NUOVO CIMENTO Vol. 109 A, N. 12 Dicembre 1996

On a new method to measure the gravitomagnetic field using two orbiting satellites

I. Ciufolini
IFSI-CNR - Frascati, Italy
Dipartimento Aerospaziale, Università di Roma «La Sapienza» - Roma, Italy

(ricevuto il 20 Settembre 1996; approvato il 15 Novembre 1996)

Summary. — We describe a new method to obtain the first direct measurement of the Lense-Thirring effect, or dragging of inertial frames, and the first direct detection of the gravitomagnetic field. This method is based on the observations of the orbits of the laser-ranged satellites LAGEOS and LAGEOS II. By this new approach one achieves a measurement of the gravitomagnetic field with accuracy of about 25%, or less, of the Lense-Thirring effect in general relativity.

PACS 11.90 – Other topics in general field and particle theory.
PACS 04.80.Cc – Experimental test of gravitational theories.

1. – The gravitomagnetic field, its invariant characterization and past attempts to measure it

Einstein's theory of general relativity [1, 2] predicts the occurrence of a «new» field generated by mass-energy currents, not present in classical Galilei-Newton mechanics. This field is called the gravitomagnetic field for its analogies with the magnetic field in electrodynamics.

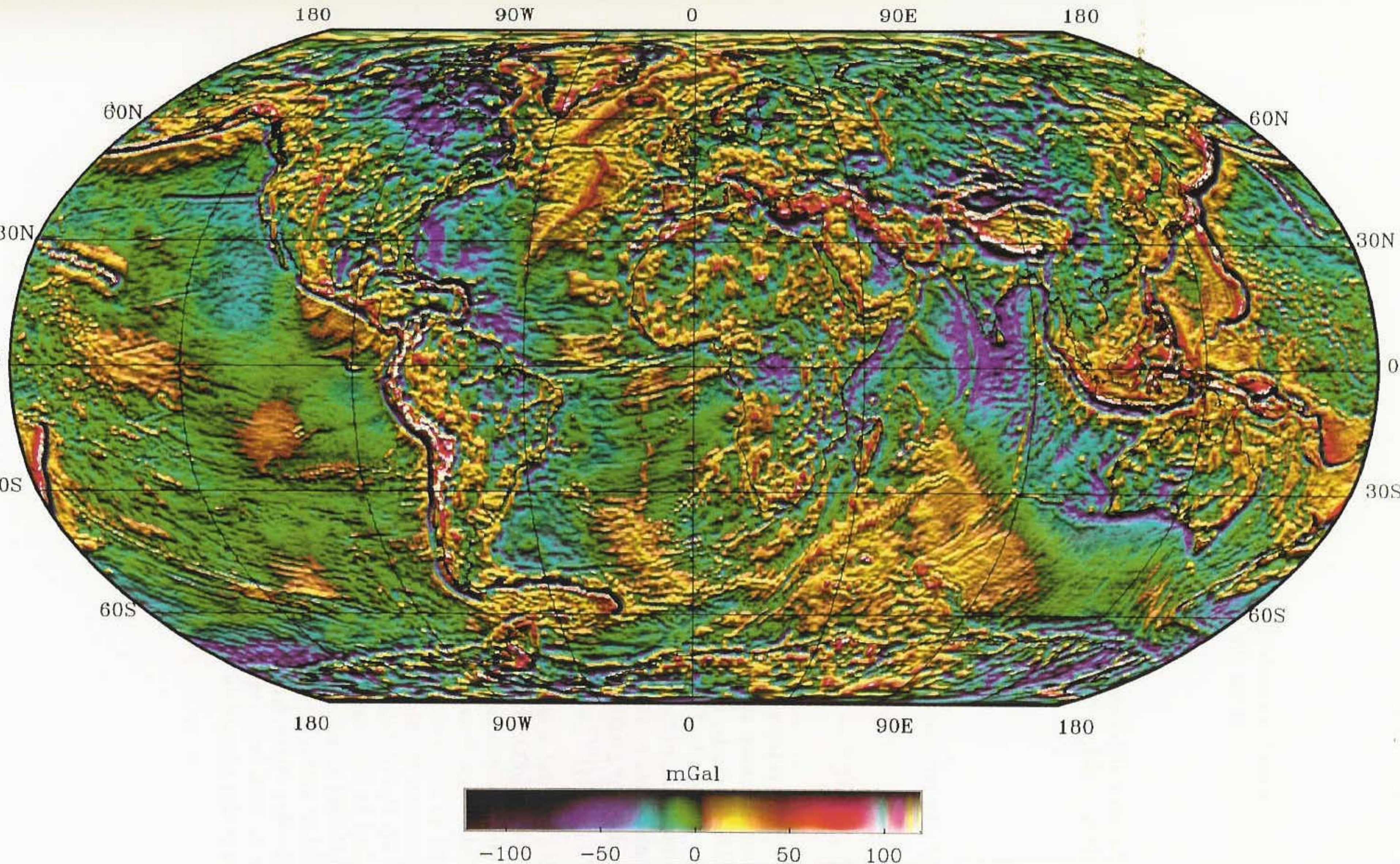
In general relativity, for a stationary mass-energy current distribution $\varrho_m \boldsymbol{v}$, in the weak-field and slow-motion limit, one can write [2] the Einstein equation in the Lorentz gauge: $\Delta \boldsymbol{h} \cong 16\pi\varrho_m \boldsymbol{v}$, where $\boldsymbol{h} \equiv (h_{01}, h_{02}, h_{03})$ are the $(0i)$-components of the metric tensor; $\boldsymbol{h}$ is called the gravitomagnetic potential. For a localized, stationary mass-energy distribution, in the weak-field and slow-motion limit, we can then write: $\boldsymbol{h} \cong -2((\boldsymbol{J} \times \boldsymbol{x})/r^3)$, where $\boldsymbol{J}$ is the angular momentum of the central body. In general relativity, one can also define [2] a gravitomagnetic field $\boldsymbol{H}$ given by $\boldsymbol{H} = \nabla \times \boldsymbol{h}$.

The Lense-Thirring effect is a consequence of the gravitomagnetic field and consists of a tiny perturbation of the orbital elements of a test particle due to the angular momentum of the central body. To characterize the gravitomagnetic field generated by the angular momentum of a body, and the Lense-Thirring effect, and distinguish it from other relativistic phenomena, such as the de Sitter effect, due to the

IC NCA 1996:
use the node of LAGEOS and the node of LAGEOS II to measure the Lense-Thirring effect

However, are the two nodes enough to measure the Lense-Thirring effect ??

EGM-96 GRAVITY MODEL



EGM96 Model and its uncertainties

Even zonals **l m**	value	Uncer-tainty in value	Uncer-tainty on node I	Uncer-tainty on node II	
20	**-0.484165 37 x 10-03**	**0.36x10-10**	**1 Ω_{LT}**	**2 Ω_{LT}**	
40	**0.5398738 6 x 10-06**	**0.1 x 10-09**	**1.5 Ω_{LT}**	**0.5 Ω_{LT}**	
60	**-0.149957 99 x 10-06**	**0.15x10-09**	**0.6 Ω_{LT}**	**0.9 Ω_{LT}**	
80	**0.4967116 7 x 10-07**	**0.23x10-09**	**0.07 Ω_{LT}**	**0.32 Ω_{LT}**	
10,0	**0.5262224 9 x 10-07**	**0.31x10-09**	**0.06 Ω_{LT}**	**0.11 Ω_{LT}**	

3 main unknowns: δC_{20} ,δC_{40} and LT

Needed 3 observables

we only have 2: $\delta\Omega_{I}$,$\delta\Omega_{II}$

(orbital angular momentum vector)

EGM96 Model and its uncertainties

Even zonals l m	value	Uncer-tainty in value	Uncer-tainty on node I	Uncer-tainty on node II	Uncer-tainty on Perigee II
20	-0.484165 37 x 10^{-03}	0.36×10^{-10}	1 Ω_{LT}	2 Ω_{LT}	0.8 ω_{LT}
40	0.5398738 6 x 10^{-06}	0.1×10^{-09}	1.5 Ω_{LT}	0.5 Ω_{LT}	2.1 ω_{LT}
60	-0.149957 99 x 10^{-06}	0.15×10^{-09}	0.6 Ω_{LT}	0.9 Ω_{LT}	0.31 ω_{LT}
80	0.4967116 7 x 10^{-07}	0.23×10^{-09}	0.07 Ω_{LT}	0.32 Ω_{LT}	0.78 ω_{LT}
10,0	0.5262224 9 x 10^{-07}	0.31×10^{-09}	0.06 Ω_{LT}	0.11 Ω_{LT}	0.34 ω_{LT}

3 main unknowns: $\delta C_{20}, \delta C_{40}$ and LT

Needed 3 observables:

2: $\delta\Omega_{I}, \delta\Omega_{II}$ (orbital angular momentum vector) plus

1: $\delta\omega_{II}$ **(Runge-Lenz vector)**

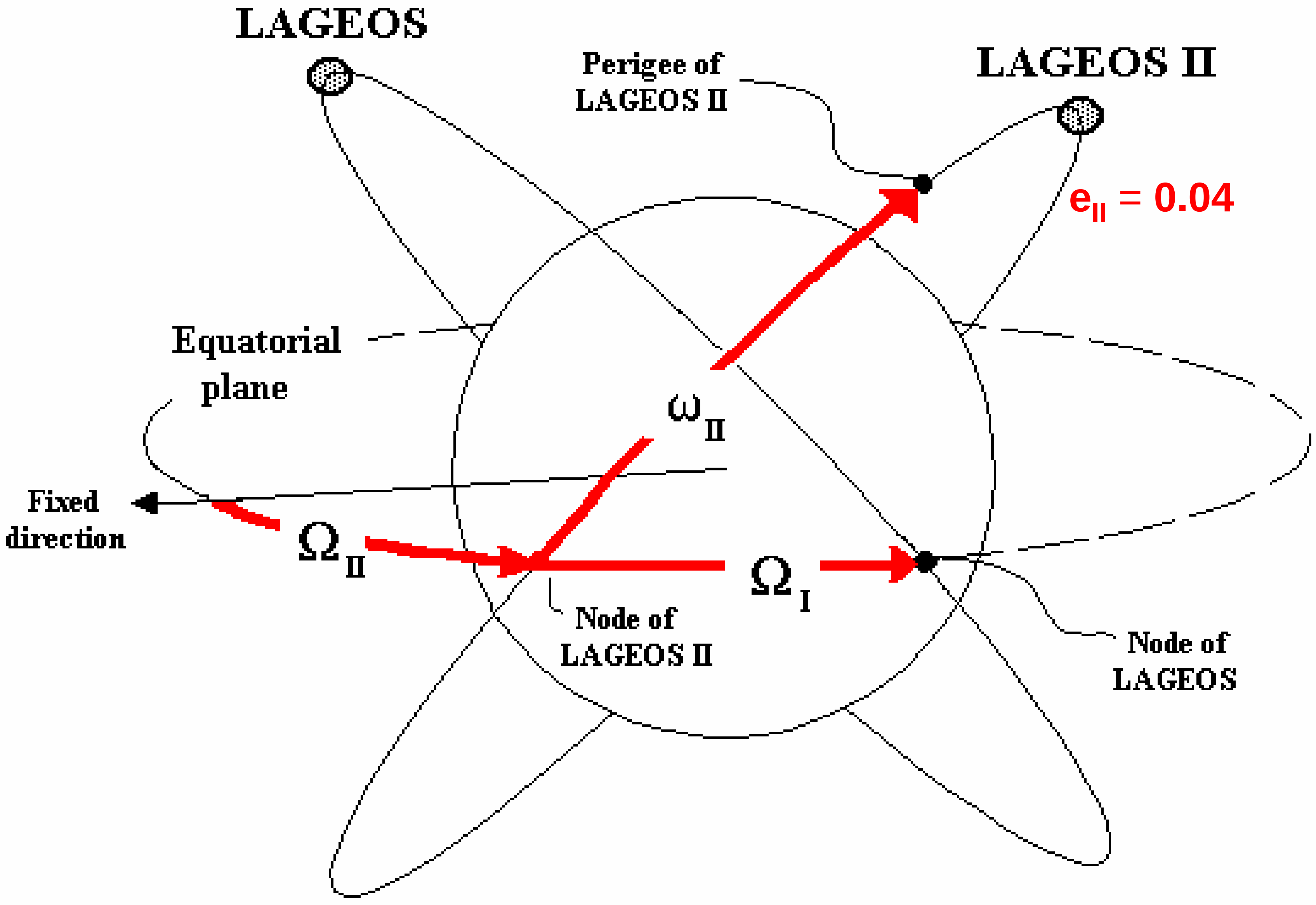
- $\delta\Omega_{I} = K_{2} \times \delta C_{20} + K_{4} \times \delta C_{40} + K_{2n} \times \delta C_{2n,0} + \mu$ **(31 mas/yr)**
- $\delta\Omega_{II} = K'_{2} \times \delta C_{20} + K'_{4} \times \delta C_{40} + K'_{2n} \times \delta C_{2n,0} + \mu$ **(31.5 mas/yr)**
- $\delta\omega_{II} = K''_{2} \times \delta C_{20} + K''_{4} \times \delta C_{40} + K''_{2n} \times \delta C_{2n,0} - \mu$ **(57 mas/yr)**

$$\mu = \delta\Omega_{I} + c_{1}\, \delta\Omega_{II} + c_{2}\, \delta\omega_{II} :$$

not dependent on δC_{20} and δC_{40} $(\mu = 1 \quad \text{in GR})$

TOTAL ERROR FROM EVEN ZONALS ≥ C60 = 13% Lense-Thirring

I.C., PRL 1986; I.C., IJMP-A 1989; I.C., NC-A 1996.


LAGEOS
Perigee of LAGEOS II
LAGEOS II
e$_{II}$ = 0.04
Equatorial plane
ω$_{II}$
Fixed direction
Ω$_{II}$
Ω$_{I}$
Node of LAGEOS II
Node of LAGEOS

I.C., NC A, 1996

IC
Nuovo Cimento A
1996

for LAGEOS II: $\dot{\omega}_{\text{LAGEOS II}} \cong 160°/\text{year}$, and the classical perigee precession is:

$$(11)\quad \dot{\omega}^{\text{Class}} = -\frac{3}{4}\, n \left(\frac{R_\oplus}{a}\right)^2 \frac{1-5\cos^2 I}{(1-e^2)^2} J_2 - \\ -[[15nR_\oplus^4(108+135e^2+208\cos(2I)+252e^2\cos(2I)+196\cos(4I)+ \\ +189e^2\cos(4I))]/(1024a^4(1-e^2)^4)]\,J_4 + \Sigma P_{2n} \times J_{2n},$$

where the P_{2n} are the coefficients (in the equation for the perigee rate) of the nonnormalized even zonal harmonics $J_{2n} \equiv -\sqrt{4n+1}\, C_{2n0}$. Thus, for the perigee of LAGEOS II, one has (in units of $\dot{\omega}_{\text{II}}^{\text{Lense-Thirring}}$):

	$\delta\dot{\omega}_{\text{II}}/\dot{\omega}_{\text{II}}^{\text{L-T}}$ due to JGM3 estimated errors	$\delta\dot{\omega}_{\text{II}}/\dot{\omega}_{\text{II}}^{\text{L-T}}$ due to difference (JGM3 − GEMT3)
δC_{20}	~ 1.1	~ 5.9
δC_{40}	~ 2.1	~ 5.3
δC_{60}	~ 0.41	~ 0.32
δC_{80}	~ 0.68	~ 0.8
$\delta C_{10,0}$	~ 0.22	~ 0.07

From these uncertainties in the perigee rate of LAGEOS II, similarly to what inferred for the nodal rates, it is manifest that the dominating error sources are due to the uncertainties in C_{20} and C_{40}.

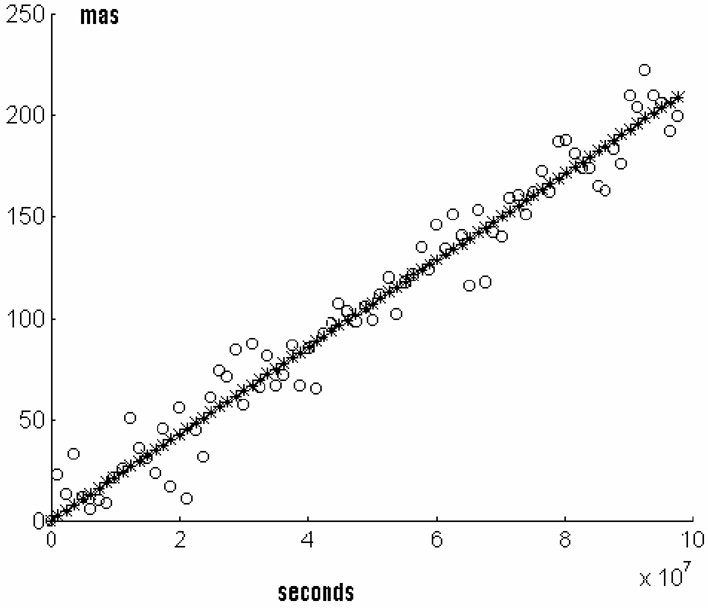
Thus, summarizing, we have now the three unknowns δC_{20}, δC_{40} and Lense-Thirring effect, and the three observable quantities $\dot{\Omega}_{\text{LAGEOS}}$, $\dot{\Omega}_{\text{LAGEOS II}}$, and $\dot{\omega}_{\text{LAGEOS II}}$.

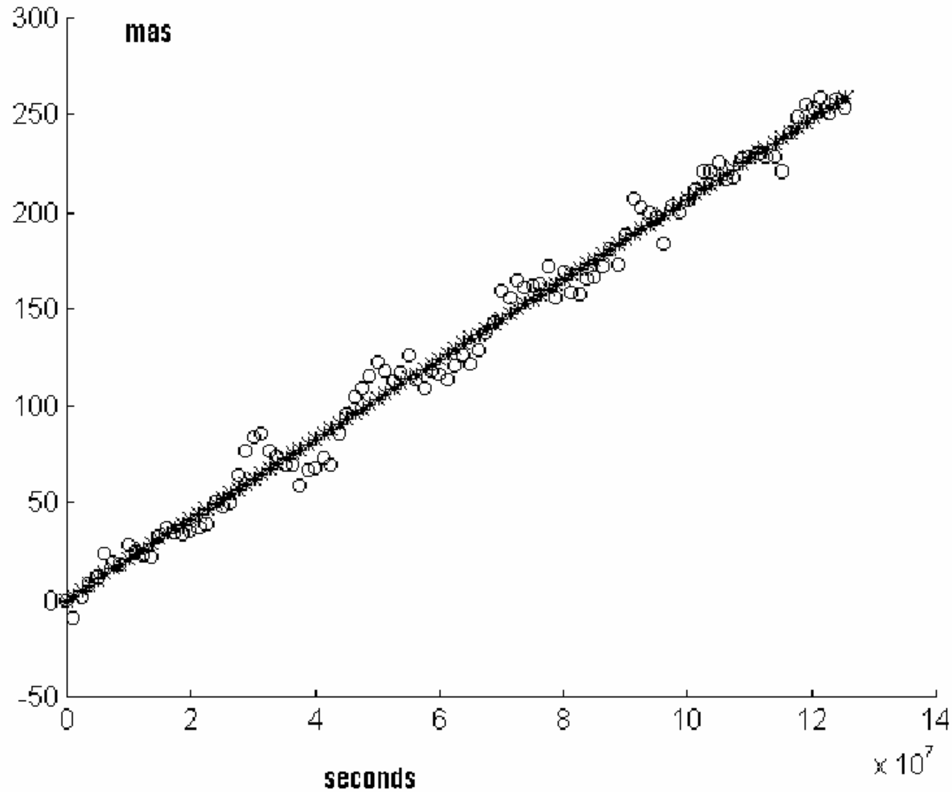
The main unmodeled part of the LAGEOS I nodal rate, due to the uncertainties in the even zonal harmonics, to the errors in the value of the orbital parameters (mainly the inclination), and including the Lense-Thirring effect (to be determined), is:

$$(12)\quad \delta\dot{\Omega}_I = (-9.3\cdot 10^{11}) \times \delta C_{20} - (4.62\cdot 10^{11}) \times \delta C_{40} + \Sigma N_{2n} \times \delta C_{2n0} + 6 \times \delta I_I + 31\mu,$$

where $\delta\dot{\Omega}$ is in units of milliarcsec/year, and δI in milliarcsec. This formula shows the main error sources in the calculated nodal rate (apart from the errors due to tides and to nongravitational perturbations; see below). In this formula the first two contributions are due to the uncertainties δC_{20} and δC_{40}, we then have the error due to the uncertainties in the higher even zonal harmonics δC_{2n0} (with $2n \geq 6$), and the error due to the uncertainties in the determination of the inclination δI_I. In this formula we have also included the Lense-Thirring [2] parameter μ, by definition 1 in general relativity: $\mu^{\text{GR}} \equiv 1$, that, if not incorporated in the modeling of the orbital perturbations, will affect the orbital residuals. One can write a similar expression for the node of LAGEOS II:

$$(13)\quad \delta\dot{\Omega}_{\text{II}} = (17.17\cdot 10^{11}) \times \delta C_{20} + \\ +(1.68\cdot 10^{11}) \times \delta C_{40} + \Sigma N''_{2n} \times \delta C_{2n0} + 5.3 \times \delta I_{\text{II}} + 31.5\mu$$

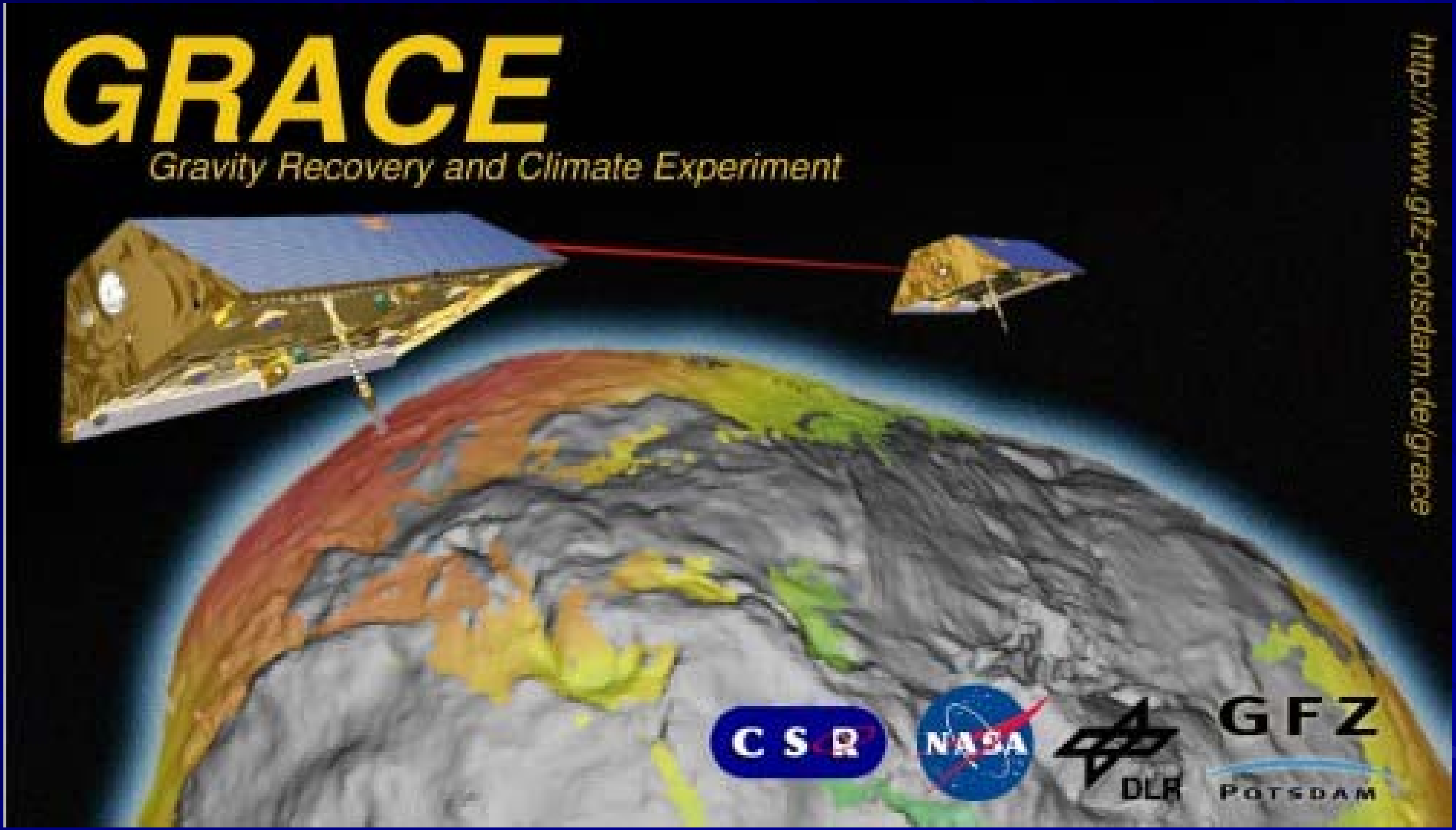




I.C., et al. 1996-1997
(I.C. 1996). (Class.Q.Grav. ...)
Gravity model JGM-3
Obs. period 3.1 years
Result: $\mu \cong 1.1$

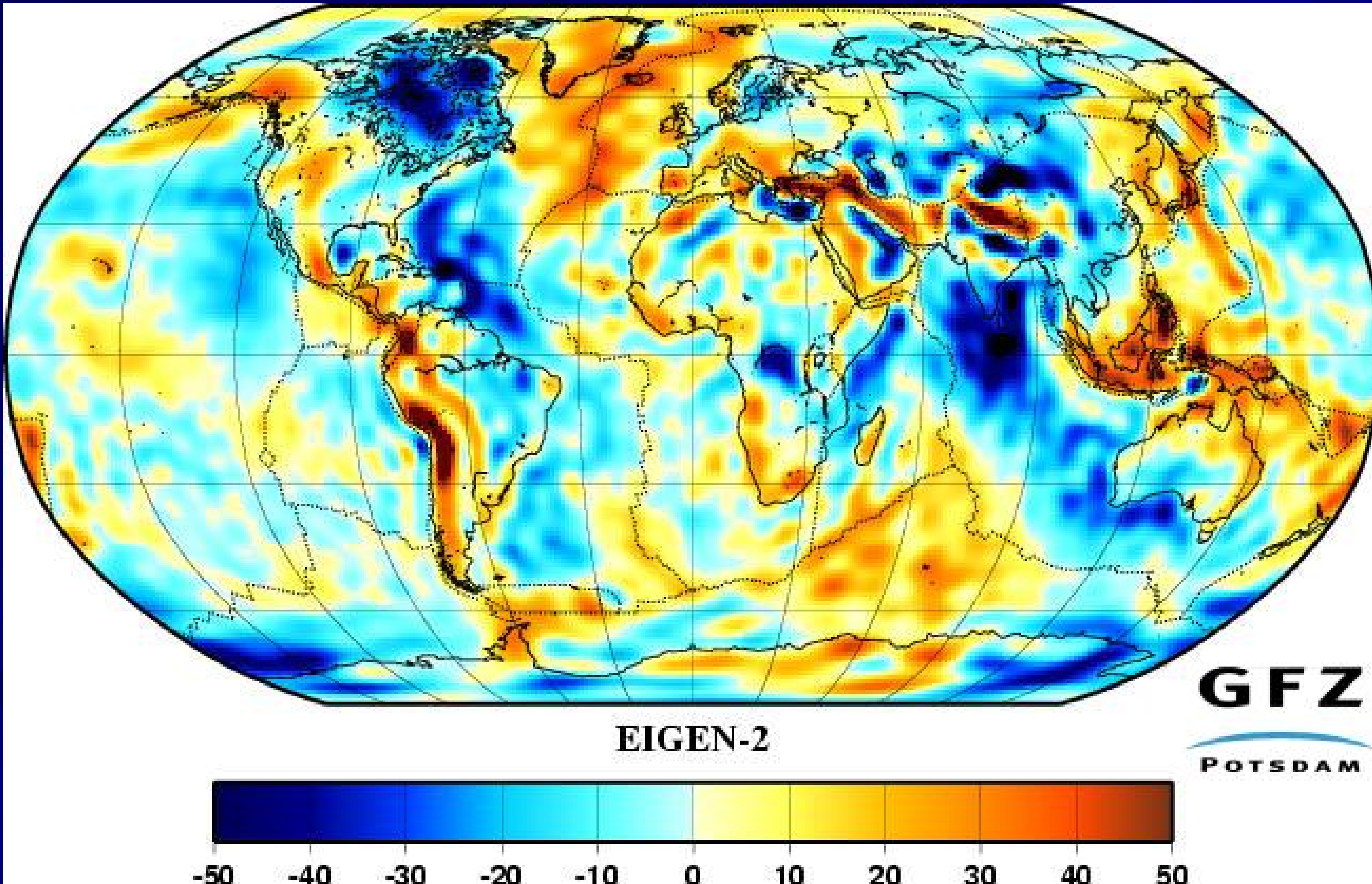
I.C., Pavlis et al. 1998 (Science)
I.C. 2000 (Class.Q.Grav.)
Gravity model EGM-96
Obs. period 4 years
Result: $\mu \cong 1.1$

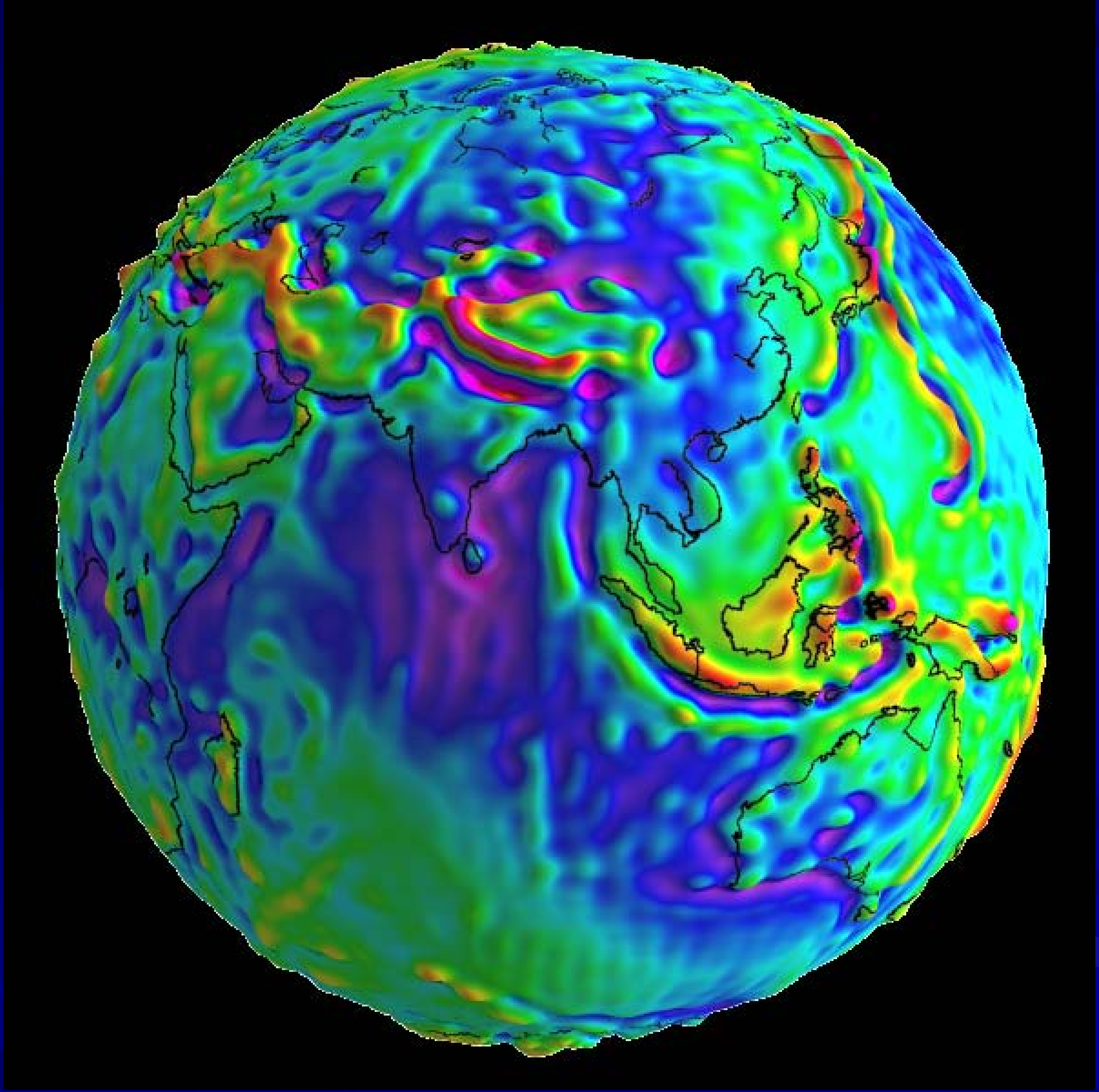
2002



Use of GRACE to test Lense-Thirring at a few percent level:
J. Ries et al. 2003 (1999),E. Pavlis 2002 (2000) [see also Nordtvedt-99]

EIGEN-2 MODEL





EIGEN-GRACE-S (GFZ 2004)

EIGEN-GRACE02S Model and Uncertainties

Even zonals lm	Value $\cdot 10^{-6}$	Uncertainty	Uncertainty on node I	Uncertainty on node II	Uncertainty on perigee II
20	-484.16519788	$0.53 \cdot 10^{-10}$	$1.59\ \Omega_{LT}$	$2.86\ \Omega_{LT}$	$1.17\ \omega_{LT}$
40	0.53999294	$0.39 \cdot 10^{-11}$	$0.058\ \Omega_{LT}$	$0.02\ \Omega_{LT}$	$0.082\ \omega_{LT}$
60	-.14993038	$0.20 \cdot 10^{-11}$	$0.0076\ \Omega_{LT}$	$0.012\ \Omega_{LT}$	$0.0041\ \omega_{LT}$
80	0.04948789	$0.15 \cdot 10^{-11}$	$0.00045\ \Omega_{LT}$	$0.0021\ \Omega_{LT}$	$0.0051\ \omega_{LT}$
10,0	0.05332122	$0.21 \cdot 10^{-11}$	$0.00042\ \Omega_{LT}$	$0.00074\ \Omega_{LT}$	$0.0023\ \omega_{LT}$

Using EIGEN-GRACE02S:
2 main unknowns: δC_{20} and LT
Needed 2 observables:

$\delta\Omega_{I}$,$\delta\Omega_{II}$ (orbital angular momentum vector)

- $\delta\Omega_{I} = K_2 \times \delta C_{20} + K_{2n} \times \delta C_{2n,0} + \mu$ **(31 mas/yr)**
- $\delta\Omega_{II} = K'_2 \times \delta C_{20} + K'_{2n} \times \delta C_{2n,0} + \mu$ **(31.5 mas/yr)**

$$\mu = \delta\Omega_{I} + K * \delta\Omega_{II}:$$

not dependent on δC_{20}

free from non-gravitational errors on the perigee

TOTAL ERROR FROM EVEN ZONALS ρ C40 =
= 3% to 4 % Lense-Thirring

I.C. PRL 1986; I.C. IJMP A 1989;
I.C. NC A, 1996; I.C. Proc. I SIGRAV School, Frascati 2002, IOP.

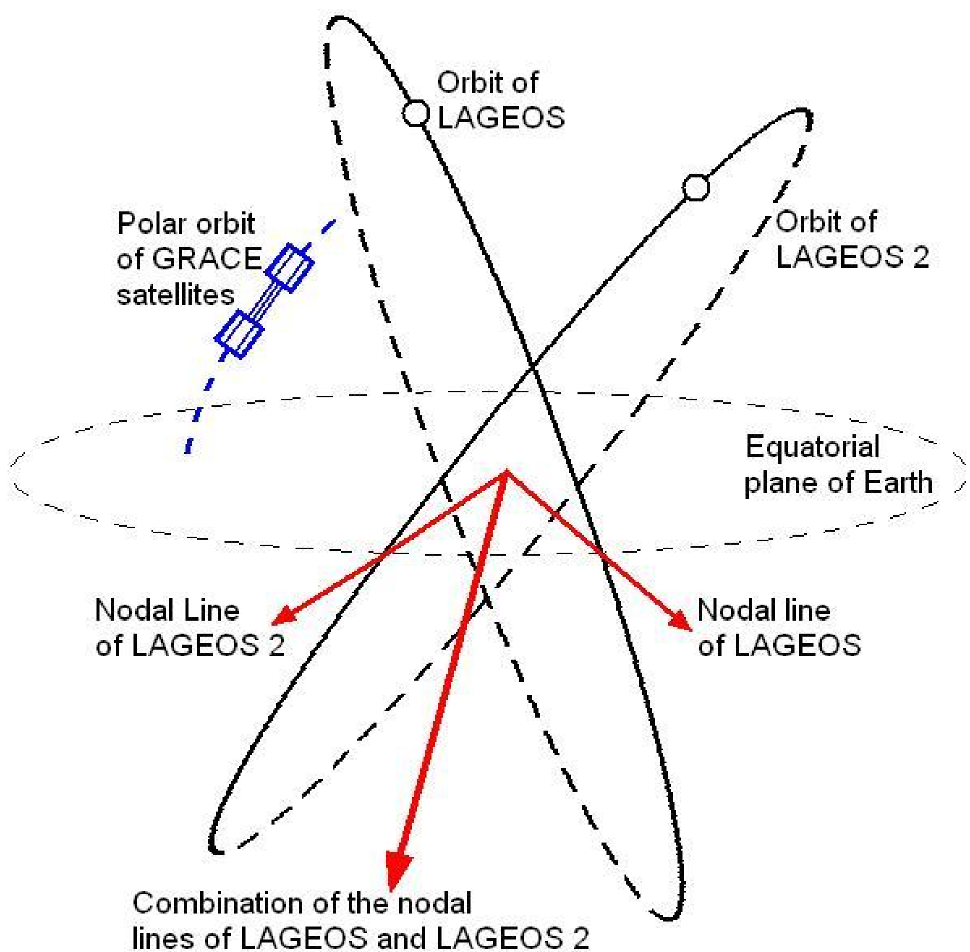
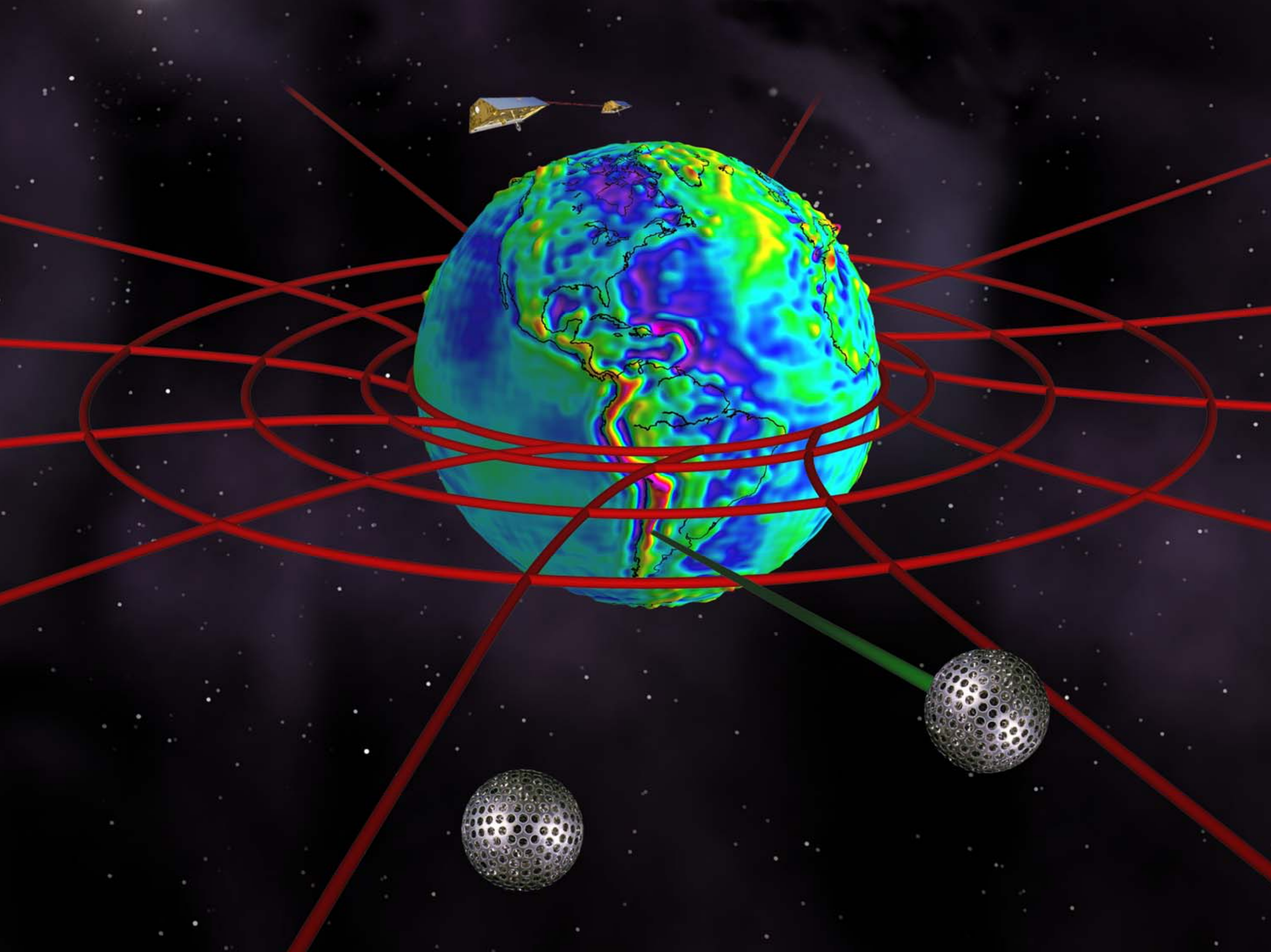

Orbit of
LAGEOS
Polar orbit
of GRACE
satellites
Orbit of
LAGEOS 2
Equatorial
plane of Earth
Nodal Line
of LAGEOS 2
Nodal line
of LAGEOS
Combination of the nodal
lines of LAGEOS and LAGEOS 2

Figure 1



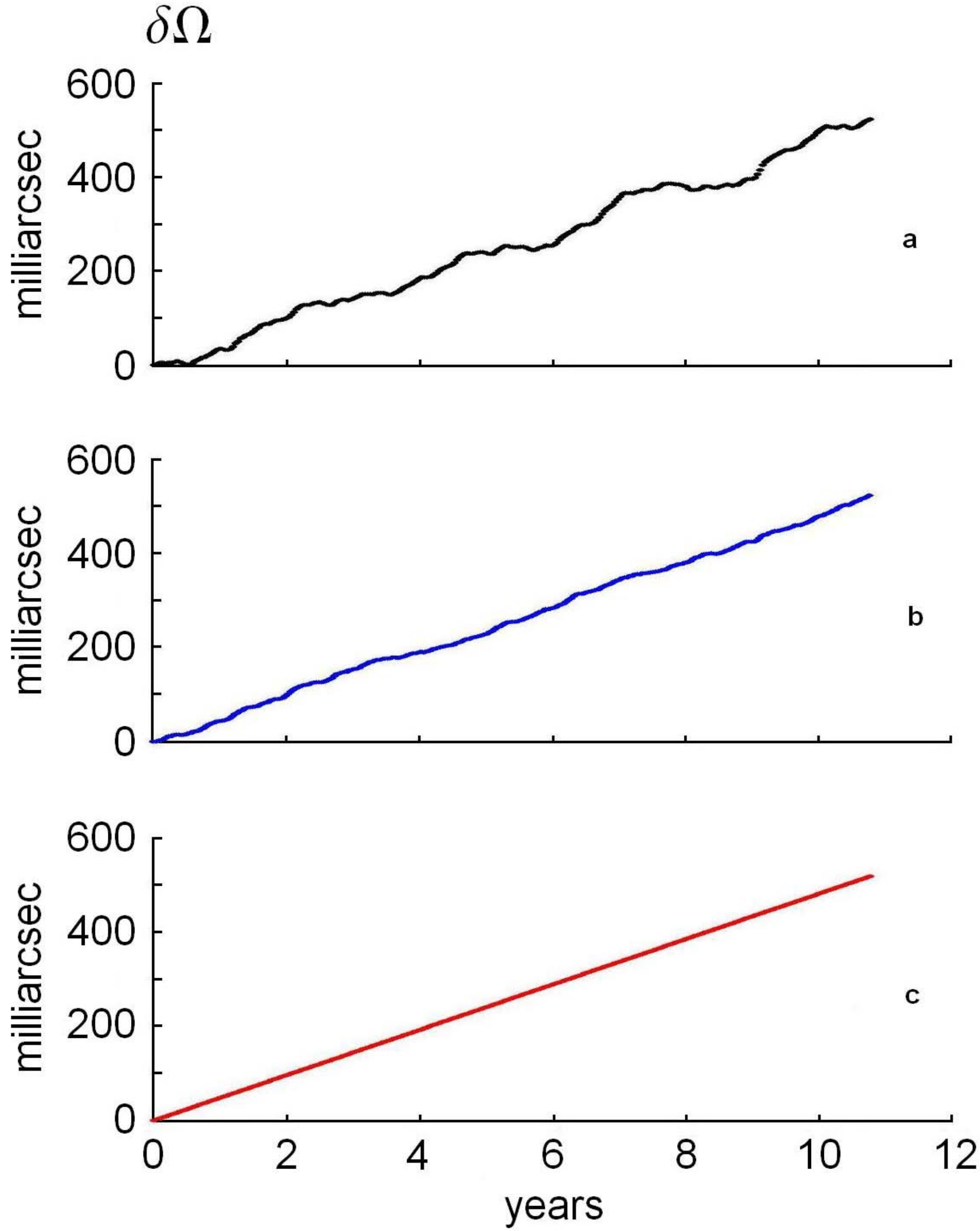


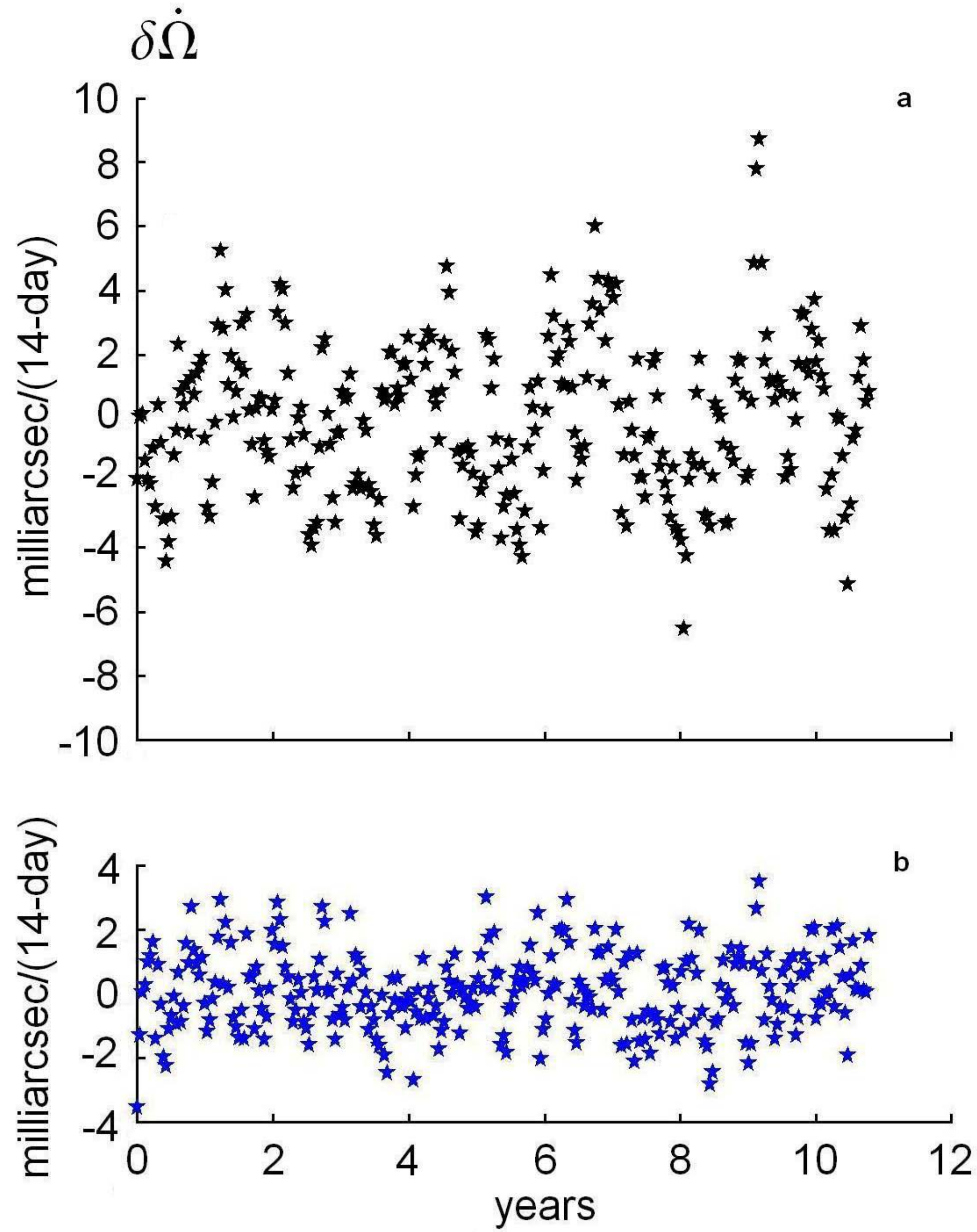
Figure 2

Observed value of Lense-Thirring effect using The combination of the LAGEOS nodes. Fit of linear trend only

Observed value of Lense-Thirring effect = 99% of the general relativistic prediction. Fit of linear trend plus 6 known frequencies

General relativistic Prediction = 48.2 mas/yr

I.C. & E.Pavlis, Letters to NATURE, 431, 958, 2004.



Post-fit residuals:
fit of linear trend only.

Post-fit residuals:
fit of linear trend plus
6 known frequencies

Error budget

- **Static gravitational field (using the EIGEN-GRACE02S uncertainties):**
  3 % to 4 % (the EIGEN-GRACE02S uncertainties include systematic errors) or 6 % to 8 % doubling the uncertainty published with EIGEN-GRACE02S.

  Time dependent gravitational field error:

  2 %
- **Non-Gravitational perturbations:**
  2 % to 3% [most of the modeling errors due to the non-gravitational perturbations are on the perigee, in particular due the Yarkowski effect on the perigee, but with in this combination we only used the nodes]
- 2% error due to random and stochastic errors and other errors

TOTAL: about 10 % (RSS)

I.C., E. Pavlis and R. Peron, New Astronomy (2006).

I.C. and E. Pavlis, New Astronomy (2005).

The 2004 analysis with EIGENGRACE02S:

- Does not use the perigee (i.e., no problems to assess the non-gravitational errors)
- In the error analysis we have summed up the absolute values of the errors due to each individual even zonal harmonic uncertainty: thus we did not use the correlation (anyhow small) among the even zonal harmonic coefficients
- The EIGENGRACE02S model was obtained with the use of GRACE data only and did NOT use any LAGEOS data
- The even zonal harmonics obtained from GRACE are independent of the Lense-Thirring effect (the acceleration of a polar, circular orbit satellite generated by the even zonals is orthogonal to the acceleration generated by the Lense-Thirring effect).

Potentially weak points of the 2004 analysis:

- The analysis was performed with the NASA orbital Estimator GEODYN, but what would happen by Performing it with a different orbital estimator ?

- The 2004 analysis was perfomed with EIGENGRACE02S but what happens if we change the gravity field model (and the corresponding value of the even zonal harmonics) ?

Answer:

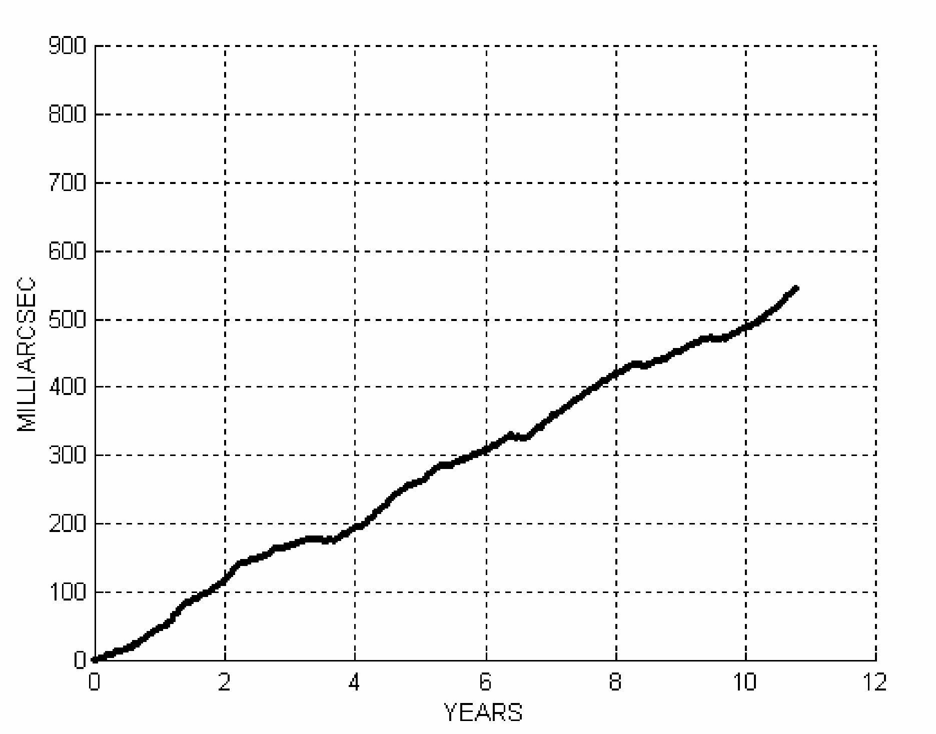
- Let us use the GFZ German orbital estimator EPOS (independent of GEODYN)

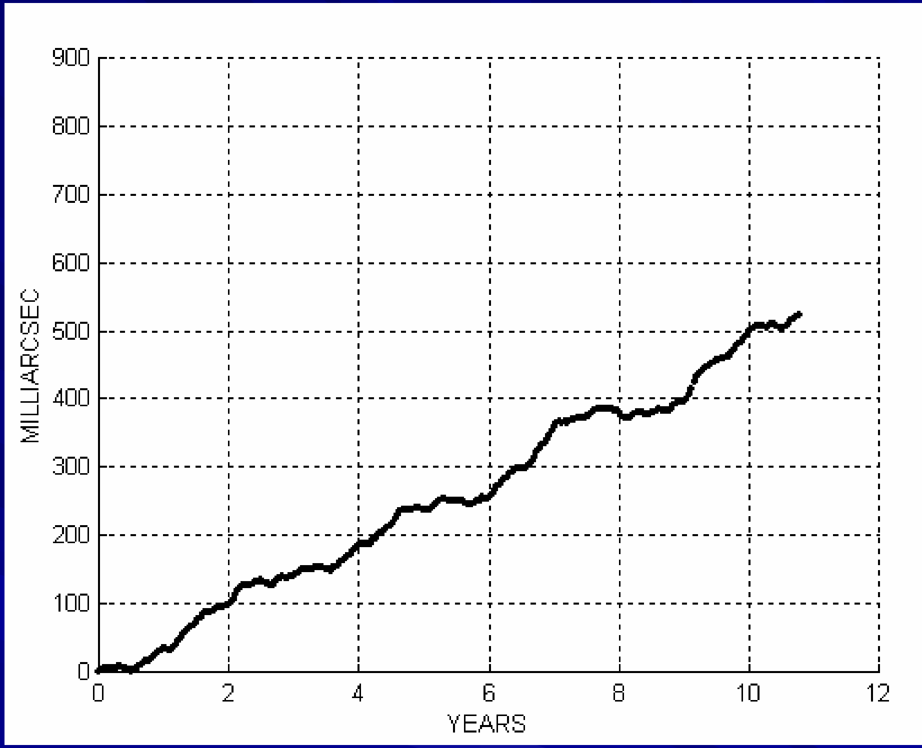
- Let us use different gravity field models obtained using GRACE

IC (Univ. Lecce), E. Pavlis (Univ Maryland Baltimore County),
R. Koenig (GFZ Potsdam),
G. Sindoni and A. Paolozzi (Univ. Roma I),
R. Tauraso (Univ. Roma II),
R. Matzner (Univ. Texas, Austin)

Using GEODYN (NASA) and EPOS (GFZ)

NEW 2006 ANALYSIS OF THE LAGEOS ORBITS USING THE GFZ ORBITAL ESTIMATOR **EPOS**

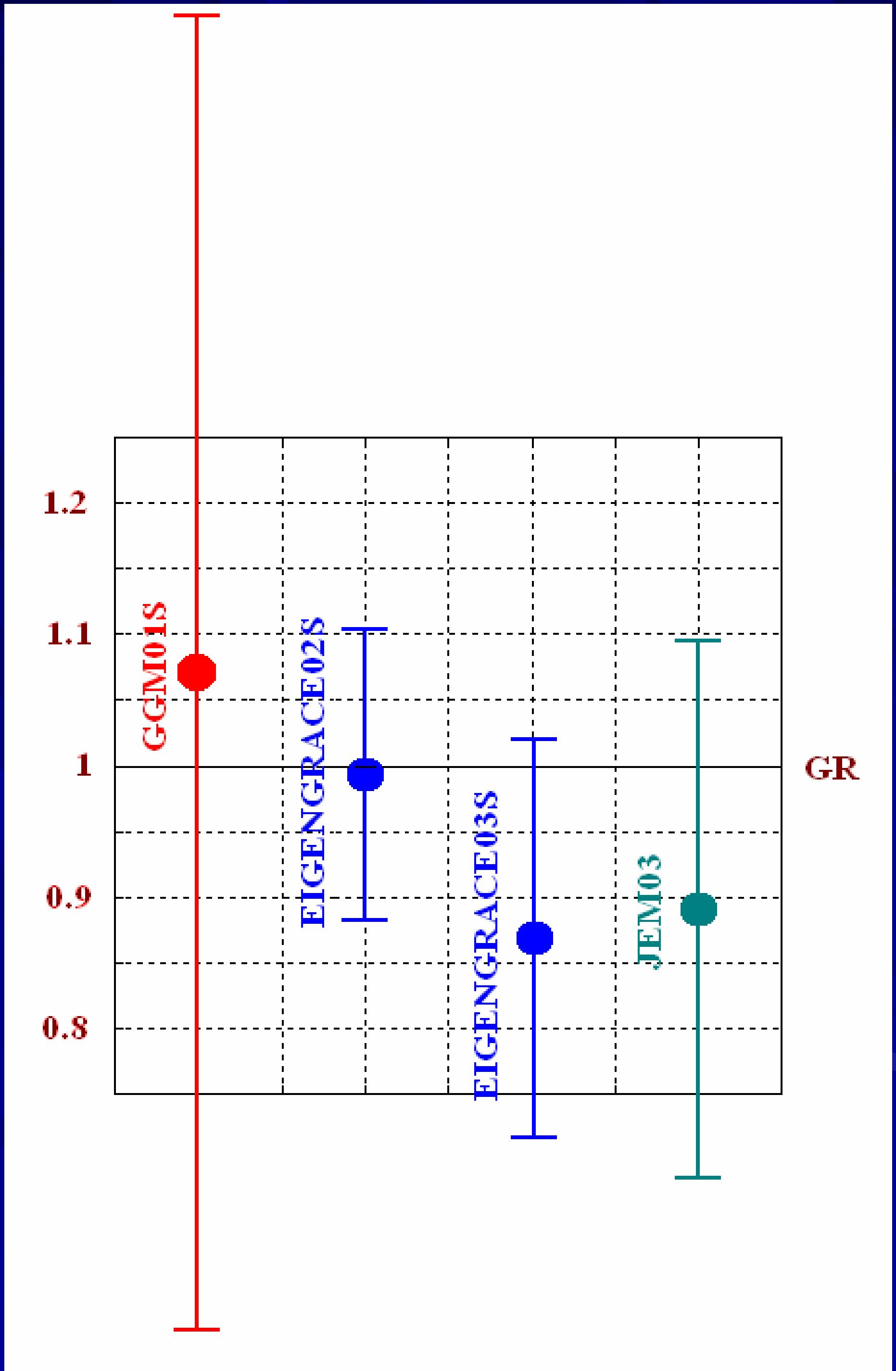


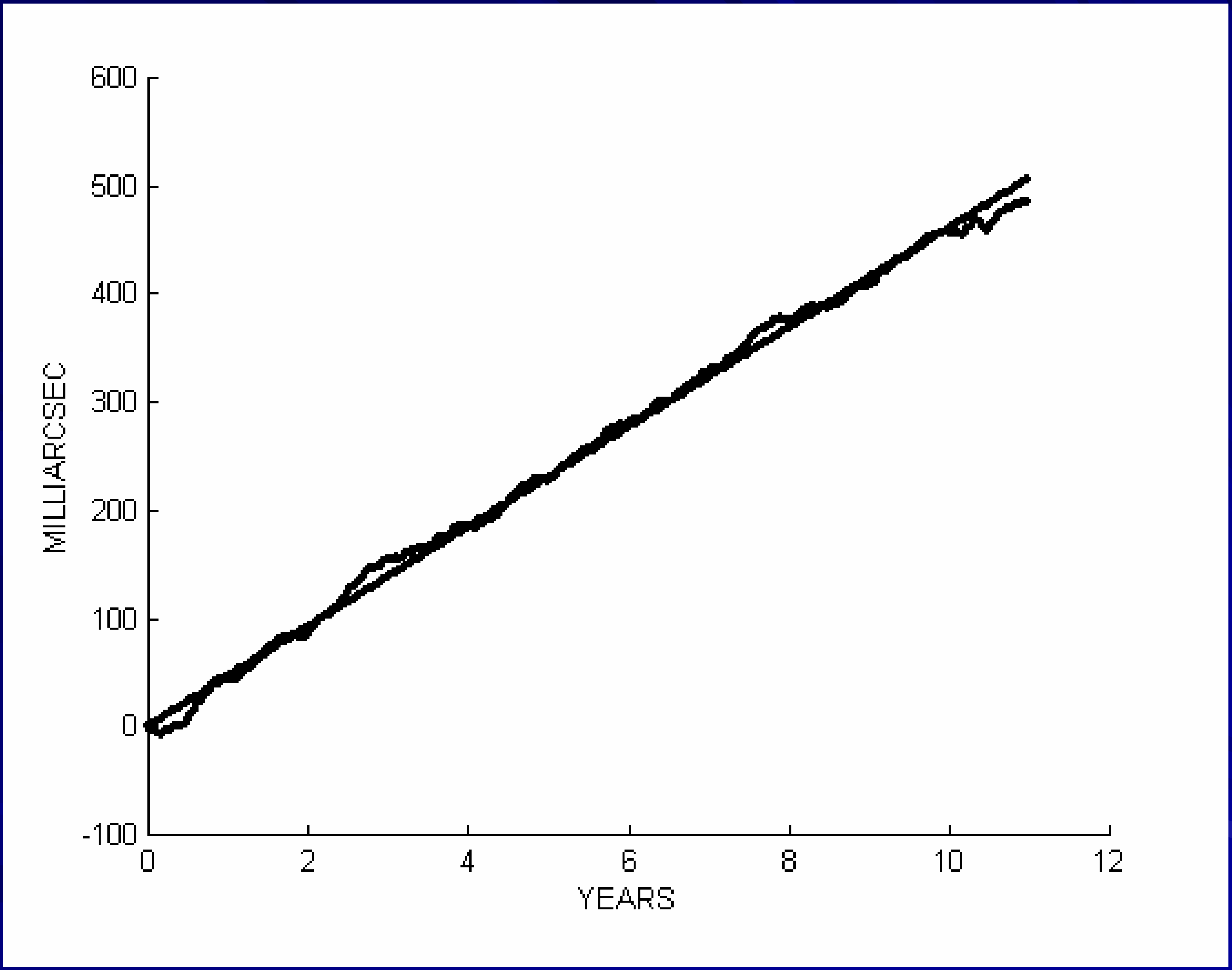


*by subtracting the geodetic precession of the orbital plane of an Earth satellite (not present in the EPOS analyis).

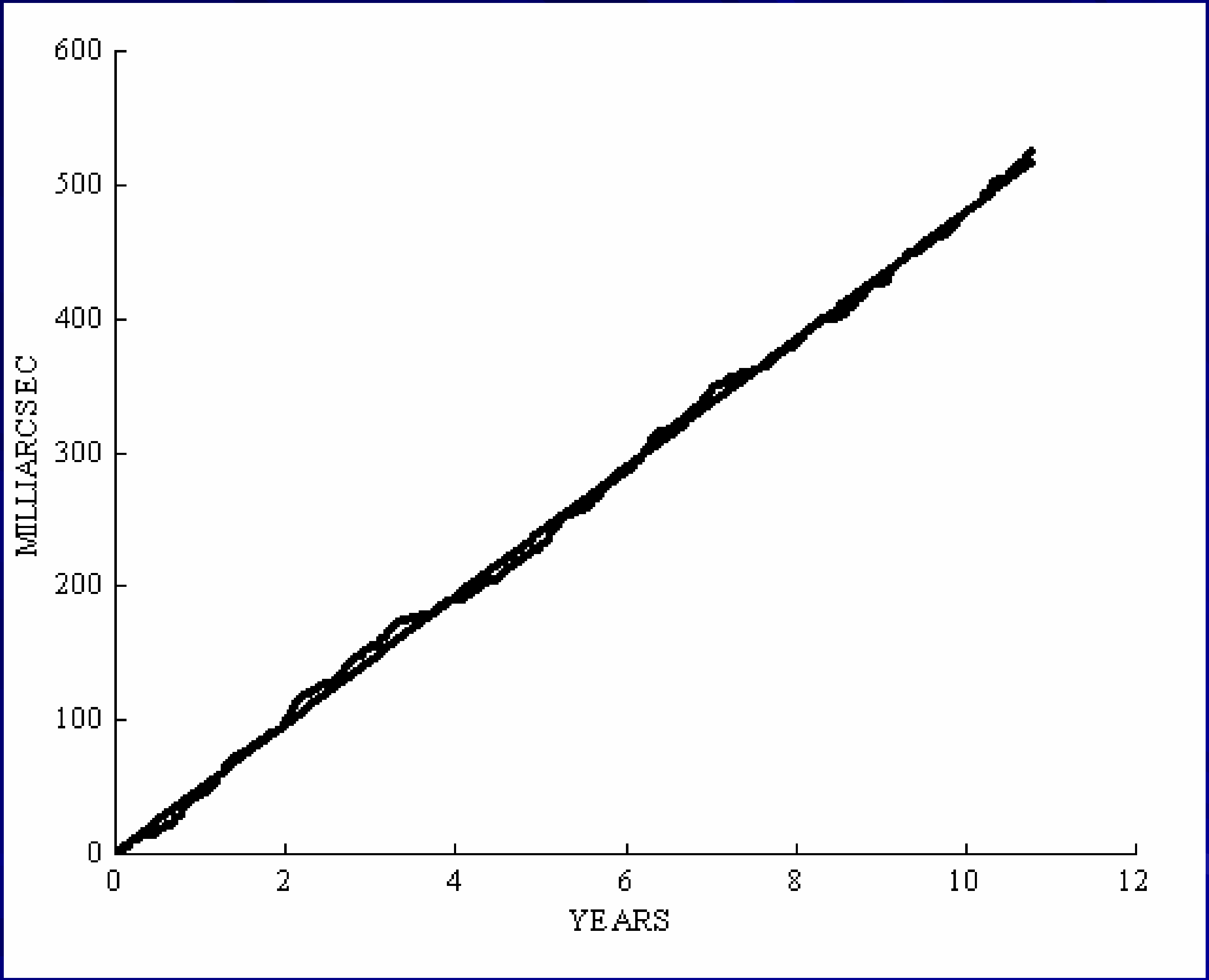
OLD 2004 ANALYSIS OF THE LAGEOS ORBITS USING THE NASA ORBITAL ESTIMATOR **GEODYN**

Comparison of Lense-Thirring effect measured using different Earth gravity field models





JEM03



EIGENGRACE02S

Some conclusions by John Ries of the Center for Space Research of the University of Texas at Austin.

Introduction:

The principal goal was to attempt to validate the earlier published results using a wider variety of GRACE-based gravity models that are now available. This would provide a more confident error assessment. In addition, some sensitivity tests were conducted regarding the modeling of important related effects, and no important limitations were observed. The results show that with the latest generation of GRACE models appear to support a detection of the Lense-Thirring effect at about the 15 percent level. This relativistic test will continue to improve as the the GRACE-based gravity models incorporate more data and the processing methods improve.

Method:

The analysis followed the procedure outlined in Ciufolini et al. 1998 (for the node-node-perigee combination) and Ciufolini and Pavlis (2004) for the node-node combination. LAGEOS-1 and LAGEOS-2 satellite laser ranging (SLR) data covering the span of October 1992 through April 2006.

Several ‘second-generation’ GRACE-based gravity models were tested. These included GGM02S (Tapley et al., 2005), EIGEN-CG02S (Reigber et al., 2005), EIGEN-CG03C (Förste et al., 2005), EIGEN-GL04C (Förste et al., 2006), an unpublished gravity model (JEM04G) from the Jet Propulsion Laboratory based on 626 days of GRACE data (D. Yuan, personal communication, 2006).

Results.

Several points are clear. The LT estimates from the various models are all consistent with the GR prediction to within about 30% maximum or about 17% 1-sigma. The mean across all the models used here agrees with GR to 1%. If we allow some reduction due to averaging across the various solutions, the error is reduced to approximately 7%. Comparing the case where LT was modeled for GGM02S to the case where it was not modeled, the difference is exactly 1.00, indicating that the method is clearly sensitive to the modeling (or lack of modeling) the LT effect. A similar test was conducted regarding the effect of geodesic precession (de Sitter precession). This effect is roughly 50% of the LT effect, and failure to model it leads to a roughly 50% error in the LT estimate. We also note that removing the rates for J3, J4 and J6 from the analysis has a negligible effect, whereas failure to map J4 to a consistent epoch is much more significant (12%).

Finally, we note that the scatter in the estimates for C40 and C60 are significantly larger than the error assigned to these coefficients. In the case of C40, all coefficients were mapped to the same epoch, yet the scatter is larger than even the most pessimistic error estimate. When estimating the expected uncertainty in the LT experiment due to these harmonics, a more pessimistic error estimate should be used rather than those in the gravity model solutions.

I.C. & E.Pavlis,
Letters to NATURE,
21 October, 2004.

I.C., E.Pavlis and R.Peron,,
New Astronomy
2006.