Last time we saw that there is a particular background of IIB preserving 4 SUSY that can be thought of the backreaction of N D_5 branes wrapping a 2-cycle inside a CY_3 fold.

We mentioned that the background can compute (or encodes) many non-perturbative aspects of $\mathcal{N}=1$ SYM. We discussed just a few (Wilson and t'Hooft loops, β-functions)

So, today, we will discuss a slightly more general problem. Let me motivate.

Consider $\mathcal{N}=1$ SYM

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + i\bar{\lambda}\not{D}\lambda$$

Consider QCD (massless version)

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + i\bar{\Psi}\not{D}\Psi.$$

one may say that the two theories are VERY similar but this is not the case. While $\lambda^a \sim$ adjoint ; Ψ_i^{κ} ← fundamental of $SU(N_c)$.

This makes both theories VERY different (from a dynamical viewpoint)

In the same line, there will be important differences between $\mathcal{N}=1$ SYM and $\mathcal{N}=1$ SQCD.

This is the topic of today's meeting; how to learn about SQCD from String theory

Most of today's discussion elaborates on a paper I wrote with

Roberto Casero and Angel Paredes [Polytechnique] March 2006

But my poor understanding was improved thanks to discussions

with Francesco Bigazzi and Aldo Cotrone [Never to be published?]

and was very improved thanks to a paper I wrote with

Francesco Benini*, Felipe Canoura*, Stefano Cremonesi* and Alfonso Ramallo. [December 2006]

It is unlikely that I will be able to comment on the second paper (due to time constraints). But is a beautiful work!

Let me summarize in two transparencies the outcomes of last time's discussions.

[* People in red above are graduate students looking for Postdoc positions next year. They are all VERY good Physicists]

The background consists of $g_{\mu\nu}$, Φ, $F_{\mu\nu\rho}$ and reads

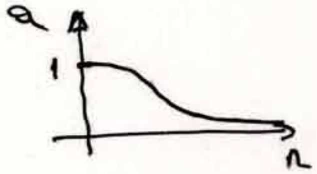
$$ds^2_E = e^{\Phi/2}\Big\{ dx^2_{1,3} + \alpha' g_s N \Big[dr^2 + e^{2h}\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) +$$

$$\frac{1}{4}\left(\tilde{\omega}_1 + a\, d\theta\right)^2 + \frac{1}{4}\left(\tilde{\omega}_2 - a \sin\theta\, d\theta\right)^2 + \frac{1}{4}\left(\omega_3 + \cos\theta\, d\varphi\right)^2 \Big] \Big\}.$$

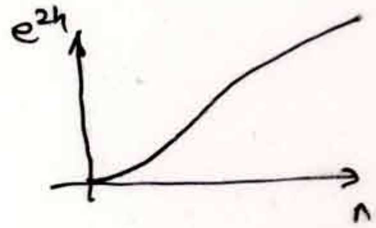
$$F_3 = -\frac{N_c}{4}\Big\{ -\left(\tilde{\omega}_1 + a(r)\, d\theta\right) \wedge \left(\tilde{\omega}_2 - a(r) \sin\theta\, d\varphi\right) \wedge \left(\tilde{\omega}_3 + \cos\theta\, d\varphi\right) +$$

$$a'\, dr \wedge \left(\tilde{\omega}_1 \wedge d\theta + \sin\theta\, d\varphi \wedge \tilde{\omega}_2\right) + \left(1 - a^2\right) \sin\theta\, d\theta \wedge d\varphi \wedge \tilde{\omega}_3 \Big\}$$

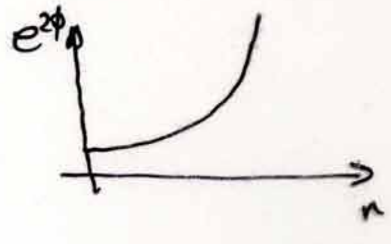
$$a(r) = \frac{2r}{\sinh 2r}$$



$$e^{2h(r)} = r \coth 2r - \frac{r^2}{\sinh^2(2r)} - \frac{1}{4}$$

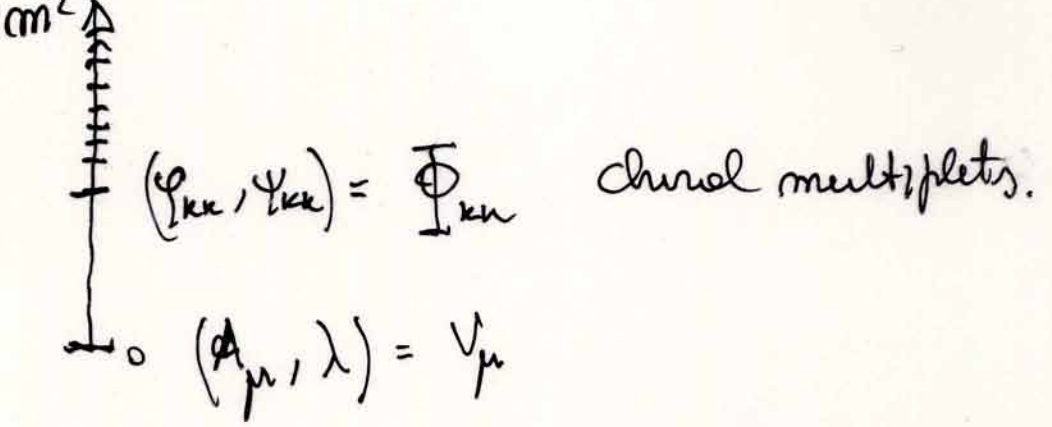


$$e^{2\Phi(r)} = e^{2\Phi_0} \frac{\sinh 2r}{2 e^{h(r)}}$$



In the functions $a(r)$, $e^{2h(r)}$, $\Phi(r)$, many non-perturbative aspects of $\mathcal{N}=1$ SYM are "hidden".

The field theory dual to this background is not pure $\mathcal{N}=1$ SYM but an "UV completion" of it, whose spectrum looks like



In spite of the presence of these extra-modes or "UV completion", many non perturbative aspects have been computed and succesfully checked using this background.

A way of understanding why so many tests have been succesfully passed as a dual to $\mathcal{N}=1$ SYM can be seen in

Gürsoy and Nuñez Nucl Phys B725, 45 (2005)

hep-th/0505100

Now, let us concentrate on the flavored background;

There are many checks that this solution captures very nicely non-perturbative aspects of $\mathcal{N}=1$ SYM:

Confinement of quarks.
Screening of monopoles. Maldacena, Nuñez

$U(1)_R$ symmetry breaking / chiral symmetry breaking Hartnoll Portugues Gursoy
Extension to SQCD Casero, Nuñez, Paredes.

Instantons Maldacena Nuñez, ...
Dipole deformation and dynamics of KK modes Gursoy Nuñez
gluino condensate. Petrini, Zaffaroni; Aharony Loewy Sonnenschein
Domain walls Camara, Merlatti
Beta function. di Vecchia, Lerda, Merlatti; Bertolini-Merlatti, ...
Correlators and holography Berg, Haack, Mück,
Strings tensions. Herzog Klebanov, Hartnoll Portugues

Domain walls. Maldacena, Nuñez, Sonnenschein Loewy.

Finite temperature; Viscosity. Son, Starinets Kovtun Buchel Liu

Addition of flavors; SQCD. quenched; mesonic dynamics.
Nuñez Paredes Ramallo

glueballs ; non susy deformations. Caceres, Nuñez; Pons Talavera.
Baryonic vertex Sonnenschein, Loewy; Ramallo, Anam Aharony Loewy Sonnenschein
Veneziano Yankielowicz - Superpotential Mück Evans Petrini Zaffaroni
Non-commutative version Mateos, Pons, Talavera
Broadening of flux tubes Sonnenschein, Loewy

Addition of flavors

What we do

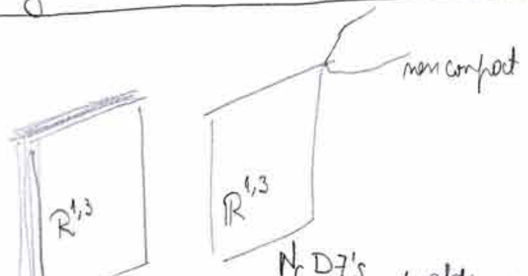
1) we have a background that preserves $\mathcal{N}=1$ SUSY and whose low energy theory is $\mathcal{N}=1$ SYM with an UV completion

The gauge coupling of the 4d theory is $\frac{1}{g_4^2} \sim \frac{\mathrm{Vol}\, S^2}{g_0^2}$

2) we add branes that will wrap a <u>non-compact</u> cycle, so, in the effective 4d theory we will have a brane with zero coupling. This is a <u>flavor group</u>

The general idea with D3/D7 branes

Karch Katz
Kruczenski, Mateos, Myers, Winters } ~ 2003



N_f D7's wrapping a "trivial" or contractible cycle

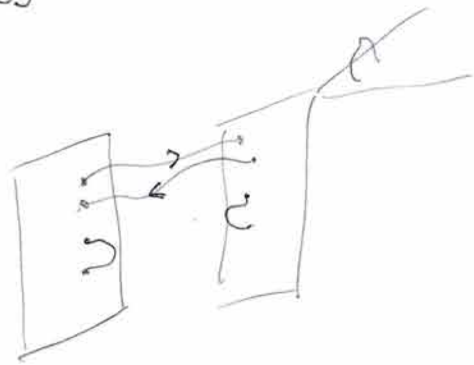
N_c D3

The strings are

3-3 ⇝ $SU(N_c)$

7-7 ⇝ $SU(N_f)$ $\underline{g=0}$

3-7
7→3 ⇝ quarks



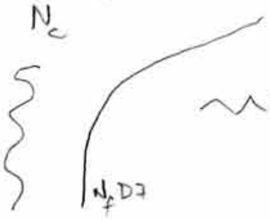
when taking the decoupling limit on the D3

g_s = fixed

$\alpha' \to 0$

The gauge coupling on the D7's $g^2_{YM_8} = g_s \alpha'^2 \to 0.$

So in taking the decoupling limit the N_c D3's are replaced by a background and if $\frac{N_f}{N_c} \sim 0$ one can treat them as "probes"



⇝ write a **BI action** for the N_f D7's in the background $AdS_5 \times S^5$.

using this idea people have worked out using the BI action

different aspects of the dynamics of "few" flavors

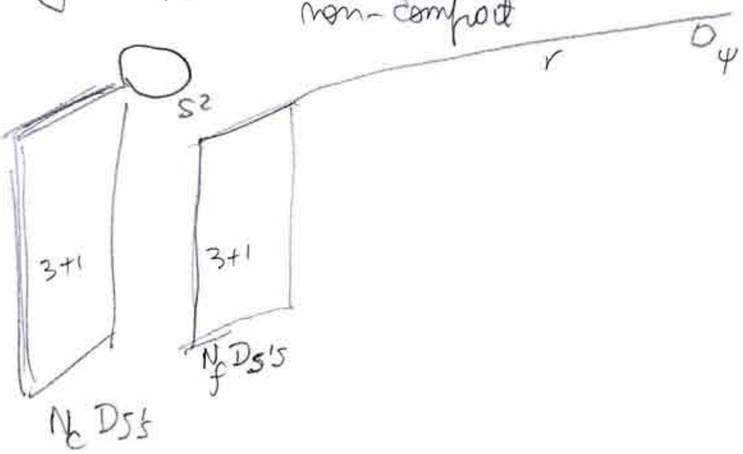
- meson spectrum
- meson interactions
- many different aspects.

Kruczenski Mateos Myers Winters
Erdmenger, Kirsch, Guralnik + ...
Sakai, Sugimoto ...

$\boxed{03\ 11\ 201}$

we have played this game with Paredes and Ramallo and found

many different "surfaces" where to place a D5 brane (in a SUSY way)



Technically, this is done by solving for κ-symmetric "surfaces"

that are a distance away from the color branes (mass of the quarks)

$$\Gamma_\kappa \varepsilon = \varepsilon \qquad \Gamma_\kappa \ \underline{\text{depends on the embedding}}$$

$$\Gamma_\kappa = \frac{1}{6!\sqrt{g}} \varepsilon^{m_1 \ldots m_6} \gamma_{m_1} \ldots \gamma_{m_6}$$

propose an embedding
↓
impose that

$$\Gamma_\kappa \varepsilon = \varepsilon$$

(use the projections of the background)

$\rightsquigarrow$ gives eqs $\partial_m X^\mu$

$$g_{mn} = G_{\mu\nu} \partial_m X^\mu \partial_n X^\nu$$

$$\gamma_m = E^a_\mu \Gamma_a \partial_m X^\mu$$

one finds many different SUSY embedding

for example

$$X^\mu, \qquad \begin{aligned} \theta &= \tilde\theta \\ \varphi &= 2\pi - \tilde\varphi \\ \psi &= m\pi \\ r &\to \infty \end{aligned}$$

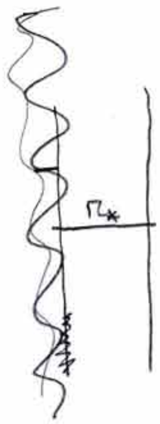
X^μ

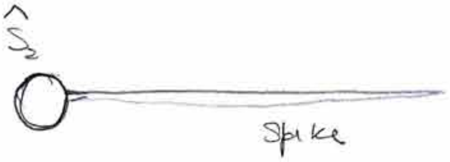
$\Theta = \tilde\theta$

$\varphi = 2\pi - \tilde\varphi$

$\psi = (2n+1)\pi$

$\sinh r = \dfrac{\sinh r_*}{\sin\theta}$



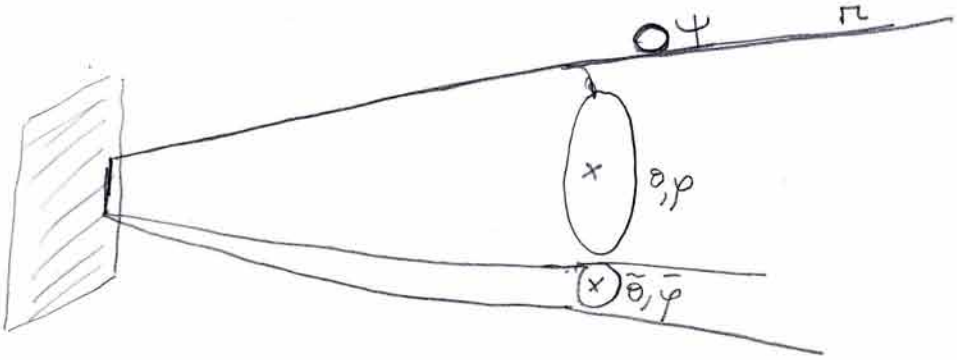


etc

we will be interested in one particular embedding

X^μ, ψ, r

$\theta, \tilde\theta, \varphi, \tilde\varphi \to$ free   A plot of this is the following



Now, suppose one would like to extend previous ideas of probing with a "flavor" brane to backreacting with a flavor brane

So, in this case, one would like to find a new solution to the action

$$S = S_{IIB} - T_5 \int d^6x \sqrt{\det g_6} + T_5 \int C_6$$

$$S = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{g_{10}} \left[R - \frac{1}{2} (\partial_\mu \phi)^2 - \frac{e^\phi}{2} \hat{F}_3^2 \right] + T_5 \sum_1^{N_f} \left[\int -e^{\phi/2} \sqrt{\det \hat{g}_6} + \int C_6 \right]$$

and we will need to solve in a <u>SUSY way</u> the eqs of motion coming from the previous action.

Now, before going into <u>how</u> to do that, let me motivate <u>the problem in Physics</u>

when you have a field theory with flavors and colors $SU(N_c)$ with N_f flavors, you may think about doing what t'Hooft taught us and make $N_c \to \infty$, hence considering as the real coupling $\underline{\lambda = g_{YM}^2 N_c}$. When you do this, keeping N_f fixed

$\frac{N_f}{N_c} \to 0$ There is a <u>great simplification</u> of the Physics

Lattice workers call this the <u>quenched approximation</u>

(it is slightly different!) in lattice is an approximation, for $N_c \to \infty$ is exact

What a lattice person does? (in QCD)

$$Z = \int DA_\mu D\bar{\psi} D\psi \; e^{-\frac{1}{g^2}\int \frac{F_{\mu\nu}^2}{4} + i \bar{\psi}(\not{D} + m)\psi \, d^4x}$$

They integrate out the quarks

$$Z = \int DA_\mu \det(i\not{D} + m) \, e^{-\frac{1}{4g^2}\int F_{\mu\nu}^2} \quad \text{this is QCD}$$

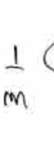
Now, the <u>quenched approximation</u> is basically doing this

$$\det(i\not{D}+m) \sim \det m + \text{correction}$$

<u>Como es esto ?</u>

$$\log\det(i\not{D}+m) = \mathrm{Tr}\log(i\not{D}+m) = \mathrm{Tr}\log\left(m\left[1+\frac{i\not{D}}{m}\right]\right) \sim$$

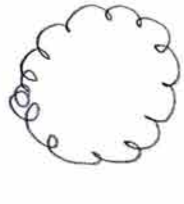
$$\mathrm{Tr}\log m + \mathrm{Tr}\left[\frac{i\not{D}}{m} + \frac{i^2\not{D}\not{D}}{2m^2} + \cdots\right]$$

$= \frac{1}{m}$  $+ \frac{1}{m^2}$ 

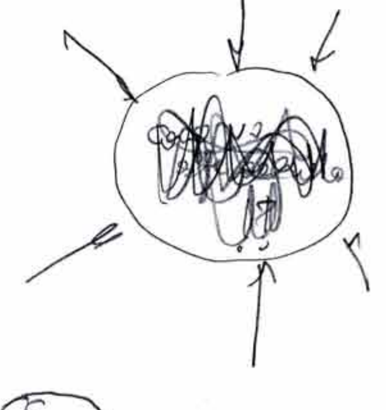
So, we see that ~~compact~~ approximating $\det(i\not{D}+m) \sim \det m$ is good if $m \to \infty$ that is the quarks are quite heavy or in other words they ~~have~~ <u>do not run in loops</u>

This has a very nice ~~perspective~~ manifestation in the t' Hooft language.

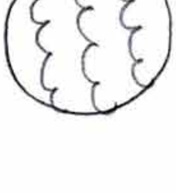
indeed, if $N_c \to \infty$
N_f is fixed

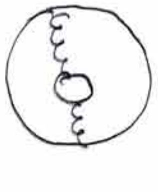
 $\sim N_c^0$

 $\sim \frac{N_f}{N_c}$



 $\sim N_c^0$

 $\sim N_c$

 $\sim \frac{N_f}{N_c}$.

~~...~~ When quenched is good / bad

Static properties → baryon/meson spectrum → good $\boxed{10\%\ \text{accuracy}}$

when light fermions influence / determine the dynamics is bad

Example
Finite temperature → chiral fermions determine universality class
$m=0$ $O(4)$

~~...~~

$A_\mu : N^2-1 \sim N^2$

$q, \tilde{q}, : 2 \times N_f \times N_c \sim 2N_c^2$

in quenched: $2 \cdot N_c \sim N_c$

Deconfinement phase transition → goes bad

$\boxed{T_{deconf} \sim 250\,\text{MeV}\ ,\ T_{\underset{\text{quenched}}{dec}} \sim 170\,\text{MeV}}$

Finite chemical potential → input baryons that are not spectators
→ duplicates of # of fermions

→ goes bad.

~~Finite den...~~

So, the quenched approximation is limited

Now, we see that the t'Hooft expansion produces the same effect of the Quenched approximation (But in a consistent way).

while Quenched is not unitary, at $N \to \infty$, $N_f \to$ fixed is OK

Same, in string theory ~~...~~ using a probe flavor brane is a consistent procedure because one is in the limit $N_c \to \infty$.

Now, one may wonder if something is lost when quenching / considering the t'Hooft scaling.

Indeed Lattice experience indicates that the quenched approximation is good for the mass spectrum (and some other static properties) % 10 respect to QCD

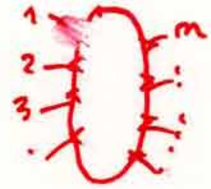
but goes bad for

- Finite temperature → ~~...~~
- Deconfinement phase transition
- Finite chemical potential

So, quenching is a limitation experimentally.

From a more theoretical viewpoint, let us consider different diagrams.

To be concrete, let me pick the formula for scattering of n-mesons from Capella et al Phys Rept 236 (1994)

$\langle B_1 \dots\dots B_n \rangle =$ 

$\sim \left(\frac{N_f}{N_c}\right)^{w} N_c^{\,2-\frac{n}{2}-2h-b}$

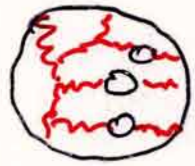
w: internal fermion loops

n: # of mesons

h: non planar handles

b: external fermion loops (boundaries)

So, for a 2-meson scattering

$\langle B_1\, B_2 \rangle =$  $+$  $+$ 

<u>t' Hooft</u> $g^2 N$ fixed $N \to \infty$ N_f: fixed	$N_c^0 \sim 1$	N_c^{-1}	N_c^{-2}
<u>Veneziano</u> $g^2 N_c$ = fixed $N_c \to \infty$ $N_f \to \infty$ $x = \frac{N_f}{N_c}$ = fixed	$N_c^0 \sim 1$	$\frac{N_f}{N_c} \sim 1$	N_c^{-2}

This new expansion might capture <u>New Physics</u>

So, Veneziano proposed in 1974 a different scaling

$$\left.\begin{array}{l} N_c \to \infty \\ N_f \to \infty \\ g^2 N_c = \text{fixed} \\ N_f/N_c = \text{fixed} \end{array}\right\}$$ that captures more Physics

How do we realize this in String theory?

We need to back react with the flavor branes. ⇝ quarks must run in loops!

⇝ we need to find a solution to

$$S = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{g_{10}}\left[R - \frac{(\partial\phi)^2}{2} - \frac{e^{\phi}}{12}F_3^2\right] + \sum_1^{N_f} T_5\left[-\int\sqrt{\det \hat{g}_6} + \int C_6\right]$$

in a SUSY way.

$\boxed{\text{(Point to discuss later: why not a purely IIB solution without BI+WZ action?)}}$

Before going into the solution itself, let me describe why this field theory is interesting.

We start with $\mathcal{N}=1$ SYM + UV completion

$$S_1 = \int d^4x \left\{ -\frac{1}{4} F_{\mu\nu}^2 + i\bar{\lambda} \not{D} \lambda + \underbrace{\sum_k |D_\mu \phi_k|^2 + i \bar{\Psi}_k (\not{D} + m_k) \Psi_k + V(\phi, \lambda, \Psi, \bar{\Psi})}_{\text{KK modes}} \right\}$$

When we add quarks ($Q, \tilde{Q}$ superfields) we propose to do it in this way

$$S = S_1 + \int d^4x \sum_{KK} \tilde{Q} \, \Phi_{KK} \, Q + Q^\dagger e^V Q + \tilde{Q}^\dagger e^{-V} \tilde{Q}$$

this gives usual kinetic terms +

$$|D_\mu \tilde{q}|^2 + |D_\mu q|^2 + \bar{\tilde{\Psi}}_q \not{D} \Psi_q + \bar{\Psi}_q \not{D} \tilde{\Psi}_q$$

but also an interaction $\tilde{Q} \, \Phi_{KK} \, Q$

(mass eigenstates $\begin{cases} \frac{\ell(\ell+1)}{R^2} & \text{massive vector} \\ \frac{\ell^2}{R^2} & \text{massive chiral} \rightsquigarrow \text{lightest} \end{cases}$)

This field theory (with only one KK superfield Φ_{KK}) is very well studied because is $\mathcal{N}=2$ SQCD $\xrightarrow{\text{broken}}$ to $\mathcal{N}=1$ by the mass term for the KK superfield

$\rightsquigarrow$ many things are known about this field theory.

many methods
- FT
- HW
- DV
- SW

- Vacuum structure
- ~~[illegible]~~

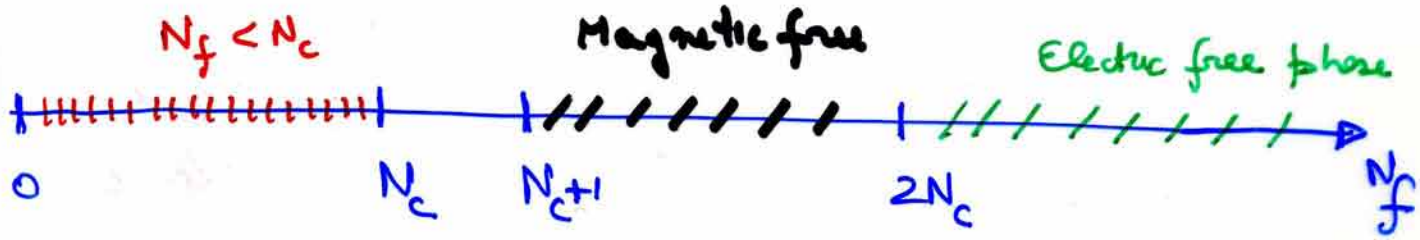
$N_f < 2N_c$
$N_f > 2N_c$
$N_f = 2N_c$

gauge th moduli space

This field theory that is basically $\mathcal{N}=2 \longrightarrow \mathcal{N}=1$ by the presence of the mass term $\int \mu \Phi^2 d^2\theta$ was VERY much studied using field theory ⊕ Seiberg Witten ⊕ Hanany Witten ⊕ Dijkgraaf-Vafa Techniques. Most of the results refer to F term Physics.

Let me summarize some salient points of this analysis

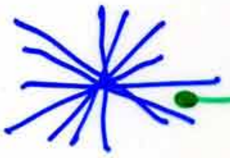
Moduli Space (<u>Quantum theory</u>)



Let me describe the main features of the superpotential in each of these regions

- $N_f=0 \longrightarrow$ the theory is $\mathcal{N}=1$ SYM.

$$U(1)_R \underset{\text{anomaly}}{\longrightarrow} Z_{2N_c} \underset{\langle\lambda\lambda\rangle \neq 0}{\longrightarrow} Z_2 \longrightarrow 2N_c \text{ vacua} \rightarrow \text{domain walls}$$



The supergravity solution describes the physics in <u>one</u> particular of these vacua. for example this one

The region

$N_f < N_c$: here we have apart from the tree level

superpotential

$$\boxed{M = \tilde{Q} Q}$$

$$\mathcal{W}_0 = \frac{1}{2\mu}\left[\mathrm{tr}(M)^2 - \frac{1}{N_c}(\mathrm{tr}\, M)^2\right] ,$$

an induced superpotential $\mathcal{W}_{ADS} = (N_c - N_f)\left(\frac{\Lambda^{3N_c - N_f}}{\det M}\right)^{\frac{1}{N_c - N_f}}$

So,

$$\boxed{\mathcal{W} = \mathcal{W}_0 + \mathcal{W}_{ADS}}$$

when studying the F term eqs for these $\mathcal{W}$ one finds

a very interesting structure.

$$M = \begin{bmatrix} \overbrace{m_1 \cdots m_1}^{N_f - n} & & & \\ & & \overbrace{m_2 \cdots m_2}^{n} & \end{bmatrix}$$

- M : meson matrix ($N_f \times N_f$)
- can be diagonalized
- has at most two different eigenvalues

the F-term eq reads

$$\boxed{\frac{\Lambda^{3N_c - N_f}}{\det M} = \frac{1}{\mu}\left(M^2 - \frac{1}{N_c}(\mathrm{Tr}\, M)\, M\right)} \rightsquigarrow$$

from here we get expression for m_1, m_2

Notice

1) $n = 0 \rightsquigarrow m_1 = m_2 = \mu \Lambda_{\mathcal{N}=2}\left(\frac{N_c}{N_c - N_f}\right)^{\frac{N_c - N_f}{2N_c - N_f}} \rightsquigarrow \boxed{2N_c - N_f \text{ vacua}}$

2) $m = 0$ is NOT a solution (Same for $m_1 = 0$ or $m_2 = 0$)

The case $N_f = N_c$

This case is <u>very</u> interesting since the moduli space is deformed quantum mechanically by a constraint $\boxed{\det M - B\tilde{B} - \Lambda^{2N_c} = 0}$

$$\boxed{W = X\left(\det M - B\tilde{B} - \Lambda^{2N_c}\right) + \frac{1}{2\mu}\left[(\mathrm{Tr}\, M^2) - \frac{1}{N_c}(\mathrm{Tr}\, M)^2\right]}$$

Lagrange Multiplier — Baryons — tree level superpot.

Now the F-term eqs read;

$$\det M - B\tilde{B} - \Lambda^{2N_c} = 0 \qquad (\text{eq for } X)$$

$$XB = X\tilde{B} = 0 \qquad (\text{eq } B, \tilde{B})$$

$$M^2 - \frac{1}{N_c}(\mathrm{Tr}\, M)\, M = -\mu X \det M \qquad (\text{eq } M).$$

<u>Notice</u>: There are many different solutions

$\{X = 0,\ B = 0,\ M \neq 0\}$ or $\{X \neq 0, B = 0 = \tilde{B};\ M \neq 0\}$ or $\left\{\begin{matrix} X = 0 \\ B \neq 0 \\ M = 0 \end{matrix}\right\}$; etc

mesonic branch I — mesonic branch II — Baryonic branch.

Let me concentrate on the mesonic branch I

$m_1 = m_2 = \Lambda^2$

But is important to notice the richness that the system develops and the possibility of having non mesonic branches ($M = 0$)

For the case $N_f \geq N_c+1$

things again are very interesting. We are in a "free magnetic phase. We can apply the Seiberg ideas of duality and move into a "magnetic" IR free description

$q_{mag}, \tilde{q}_{Mag}, \tilde{B}_{Mag}, B_{Mag}, M_{mag}$ (are the magnetic degrees of freedom)

There are relations between them and the electric ones (in some cases non-linear ones!)

The bottomline is that one can write a superpotential

$$W = \frac{1}{\mu}\tilde{q}_m M_e q_m + (N_c-N_f)\left[\frac{\Lambda^{3N_c-N_f}}{\det M}\right]^{\frac{1}{N_c-N_f}} + \frac{1}{2\mu}\left[\mathrm{Tr} M^2 - \frac{1}{N_c}(\mathrm{Tr} M)^2\right]$$

notice that now $\underline{N_c-N_f<0}$

Here, again, there are different branches
- baryonic $\langle q\rangle, \langle\tilde{q}\rangle \neq$
- mesonic $\langle q\rangle = \langle\tilde{q}\rangle = 0$

In the mesonic case $q=\tilde{q}=0$ one has a structure identical to the case $N_f<N_c$

$$M = \begin{bmatrix} m_1 & & & & \\ & \ddots & & & \\ & & m_1 & & \\ & & & m_2 & \\ & & & & \ddots \\ & & & & & m_2 \end{bmatrix}$$

r branch; with $2N_c-N_f$ vacua

but now, one possible solution is $\boxed{M=0}$

Finally, in the case

$$N_f = 2N_c$$

the theory is argued to be

conformal $\longrightarrow$ $\beta = 0$

$\longrightarrow$ $U(1)_R$ is unbroken

$\longrightarrow$ Self-Seiberg dual.

$\Big\}$ G^4 is marginal

So, summarizing these moduli space study (Hori, Ooguri, Oz); (Carlino, Konishi, Murayama); (Balasubramanian, Feng, Naqvi); (Argyres, Plesser, Seiberg) + ...

Beta function $\quad \beta = -\frac{3}{2}(2N_c - N_f) g^3 \quad$ confines if $2N_c > N_f$

R symmetry $\quad U(1)_R \longrightarrow \mathbb{Z}_{2N_c - N_f} \xrightarrow{?} \mathbb{Z}_2$ (argued)

Quartic Operator $\quad \mathrm{Tr}\, M^2 - \frac{1}{N_c}(\mathrm{Tr}\, M)^2$ is dangerously irrelevant $N_f < 2N_c$

irrelevant if $N_f > 2N_c$

and the case $N_f = 2N_c$ is "conformal"

$\beta = 0$, $U(1)_R$ unbroken; line of fixed points

To finish with this introduction, let me mention a couple of maps that are useful to study vacua.

Multiplication map: relates vacua of (Balasubramanian, Feng, Huang, Naqvi)

$SU(N_c)$ N_f, in $r=0$ branch $\longleftrightarrow$ $SU(N_c t)$, $t N_f$, $r=0$ branch

and $\boxed{x = \frac{N_f}{N_c}}$ is an invariant of this map.

Addition map: relates vacua of

$SU(N_c)$, N_f, r-branch with $SU(N_c')$, N_f', r' if

$$\left.\begin{array}{l} N_c - r = N_c' - r' \\ N_f - 2r = N_f' - 2r' \end{array}\right\} \to \boxed{2N_c - N_f} \text{ is an invariant}$$

—o—

~~Notice~~ Now let me concentrate on part of our work with Roberto Casero and Angel Paredes

hep-th/0602027

Phys Rev D 73, 086005, (2006)

Also, material developed with Bigazzi and Cotrone will be mentioned

Let me describe the solutions to the eqs of motion of IIB + BF we have found (SUSY solutions)

We have 3 sets of solutions whose ~~sing~~ singularity structure is such that can be given a gauge theory interpretation

$N_f < 2N_c$ (type I)

$$ds^2_E = e^{\phi/2}\Big\{ dx_{1,3}^2 + e^{2k}d\rho^2 + e^{2h}(d\theta^2+\sin^2\theta\, d\varphi^2) + \frac{e^{2g}}{4}\big[(\tilde\omega_1 + a\, d\theta)^2 + (\tilde\omega_2 - a\sin\theta\, d\varphi)^2\big]$$
$$+ \frac{e^{2k}}{4}(d\psi + \cos\theta\, d\varphi + \cos\tilde\theta\, d\tilde\varphi)^2$$

$$F_3 = \frac{N_c}{4}\Big[-(\tilde\omega_1 + b\, d\theta)\wedge(\tilde\omega_2 - b\sin\theta\, d\varphi)\wedge(\tilde\omega_3+\cos\theta\, d\varphi) + b'\, d\rho\wedge(\tilde\omega_1\wedge d\theta + \sin\theta\, d\varphi\wedge\tilde\omega_2)$$
$$+ (1-b^2)\sin\theta\, d\theta\wedge d\varphi\wedge(\tilde\omega_3+\cos\theta\, d\varphi)\Big] + \frac{N_f}{4}\sin\theta\, d\theta\wedge d\varphi\wedge(\tilde\omega_3)$$

$\phi(\rho)$

$$\boxed{dF_3 = \frac{N_f}{4}\sin\theta\, d\theta\wedge d\varphi\wedge \sin\tilde\theta\, d\tilde\theta\wedge d\tilde\varphi}$$

$N_f \geq N_c$ (type II)

$$ds^2 = e^{\phi/2}\Big[dx_{1,3}^2 + e^{2k}d\rho^2 + e^{2h}(d\theta^2+\sin^2\theta\, d\varphi^2) + \frac{e^{2g}}{4}(d\tilde\theta^2+\sin^2\tilde\theta\, d\tilde\varphi^2) + \frac{e^{2k}}{4}(\tilde\omega_3+\cos\theta\, d\varphi)^2\Big]$$

$$F_3 = \frac{N_c}{4}\big[-\tilde\omega_1\wedge\tilde\omega_2\wedge\tilde\omega_3 + \sin\theta\, d\theta\wedge d\varphi\wedge\tilde\omega_3\big] + \frac{N_f}{4}\sin\theta\, d\theta\wedge d\varphi\wedge\tilde\omega_3$$

$\phi(\rho)$

$N_f = 2N_c$

$$ds^2 = e^{\frac{\phi(\rho)}{2}}\Big[dx_{1,3}^2 + N_c\, d\rho^2 + \frac{N_c}{\xi}(d\theta^2+\sin^2\theta\, d\varphi^2) + \frac{N_c}{4-\xi}(d\tilde\theta^2+\sin^2\tilde\theta\, d\tilde\varphi^2) + \frac{N_c}{4}(\tilde\omega_3 + \cos\theta\, d\varphi)^2\Big]$$

$$F_3 = \frac{g_s\alpha' N_c}{4}\big[\sin\theta\, d\theta\wedge d\varphi + \sin\tilde\theta\, d\tilde\theta\wedge d\tilde\varphi\big]\wedge(\tilde\omega_3+\cos\theta\, d\varphi)$$

$$\phi = \phi_0 + \rho$$

Let me tell you that for the functions

$e^{2h}, e^{2g}, e^{2\phi}, e^{2k}, a(\rho)$

we did not find exact solutions, but we found a series near $\rho\sim 0$ another near $\rho\sim\infty$ and a smooth numerical interpolation (this is as good as an exact solution)

For solutions type I and $N_f < 2N_c$

$\rho \sim 0$ $\qquad$ $X = N_f/N_c$ $\qquad$ $\rho\sim\infty$

$a \sim 1-2\rho^2$ $\qquad$ $a \sim e^{-2\rho}(4-2x)\rho + \dots$

$e^{2k} \sim N_c c_2 \rho^2 + \dots$ $\qquad$ $e^{2k} \sim N_c + \dots$

$e^{2h} \sim \frac{N_c}{3c_1}(4-2x)\rho + \dots$ $\qquad$ $e^{2h} \sim \frac{N_c}{2}(2-x)\rho$

$e^{2g} \sim \frac{2N_c}{3c_1}(2-x)\frac{1}{\rho} + \dots$ $\qquad$ $e^{2g} \sim N_c + \dots$

$e^{2\phi} \sim e^{2\phi_0}\left(1 + \frac{3c_1 x}{2(2-x)}\rho\right) + \dots$ $\qquad$ $e^{2\phi} \sim \frac{e^{2\rho}}{\sqrt{\rho}} + \dots$

c_1, c_2 : 2 constants to be determined, for example by matching the domain wall tension.

Notice also that the fact that two solutions exist for $N_f \geq N_c$

is the two possible solutions to the vacuum eqs if $N_f > N_c$

$\langle M \rangle \neq 0$ and $\langle M \rangle = 0$ $\to$ without the need of a baryonic branch.

Now, let me concentrate on different tests of these solutions

that is, how do the solutions help us learn about non perturbative

aspects of $\mathcal{N}=1$ SQCD $W = (\tilde{Q}Q)^{4}$.

$\boxed{\text{There are } \underline{\text{many}} \text{ more things to do}}$

Many non-perturbative aspects of $\mathcal{N}=1$ SQCD (on the versions by a softly broken $\mathcal{N}=2$) have been developed.

- Wilson loop and Screening ✱
- Anomaly matching between dual theories
- Beta function computation
- Moduli space matching ✱ . $N_f = 2N_c$ properties
- Instantons.
- Seiberg duality. ✱
- R-symmetry breaking . Domain walls.
- Finite temperature effects → Viscosity → Jet quenching. Bigazzi Cotrone Bertoldi Edelstein (2007)
- Approach to SUSY by Metastability (Hirano)
- Exact duality in the $N_f = 2N_c$ case → marginal deformation ✱ → χS preserved → $\beta = 0$

Today I will (in the interest of time) comment on the points with ✱

Computation of the Wilson loop.

Here we followed a very nice theorem by

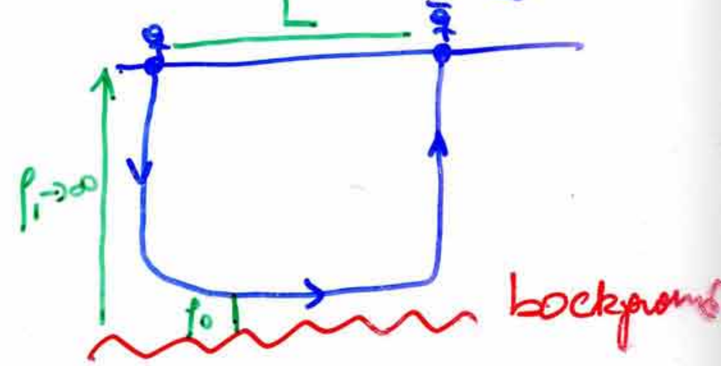
Brandhuber, Itzhaki, Sonnenschein and Yankielowicz (1998)
Kinar, Schreiber

They studied the expression for the Wilson loop computed in a general background. One obtains expressions for the $q\bar{q}$ separation and the $q\bar{q}$ energy that read in our flavored background

$$L(\rho_0) = 2\int_{\rho_0}^{\infty} \frac{e^{k+\phi(\rho_0)}}{\sqrt{e^{2\phi}-e^{2\phi(\rho_0)}}}\,d\rho$$

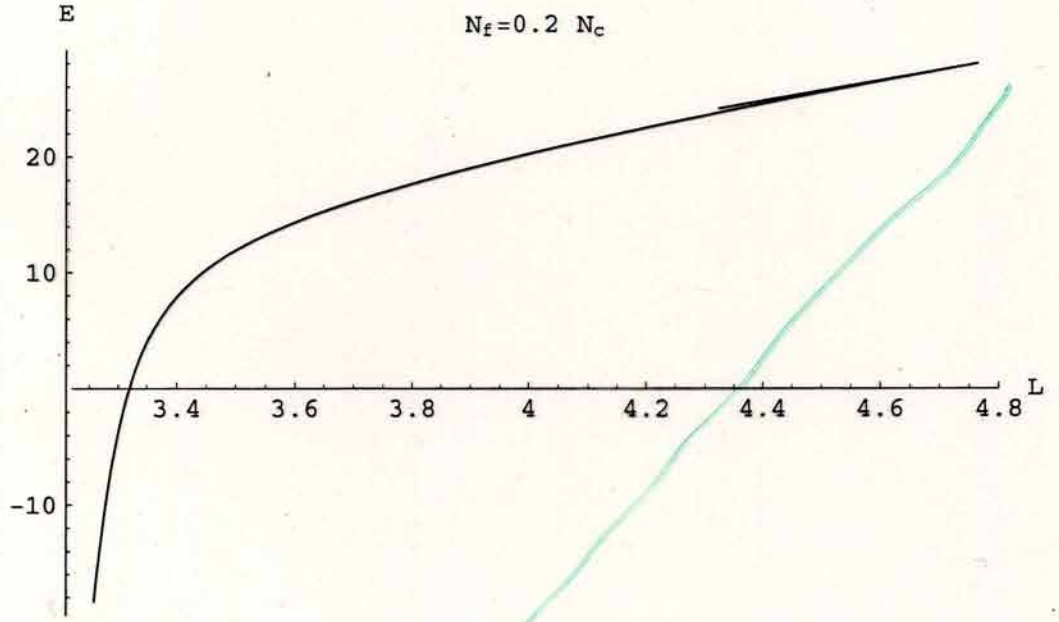
$$E(\rho_0) = \frac{1}{2\pi\alpha'}\, 2\left[\int_{\rho_0}^{\infty} \frac{e^{2\phi+k}}{\sqrt{e^{2\phi}-e^{2\phi(\rho_0)}}}\,d\rho - \int e^{\phi+k}\,d\rho\right]$$

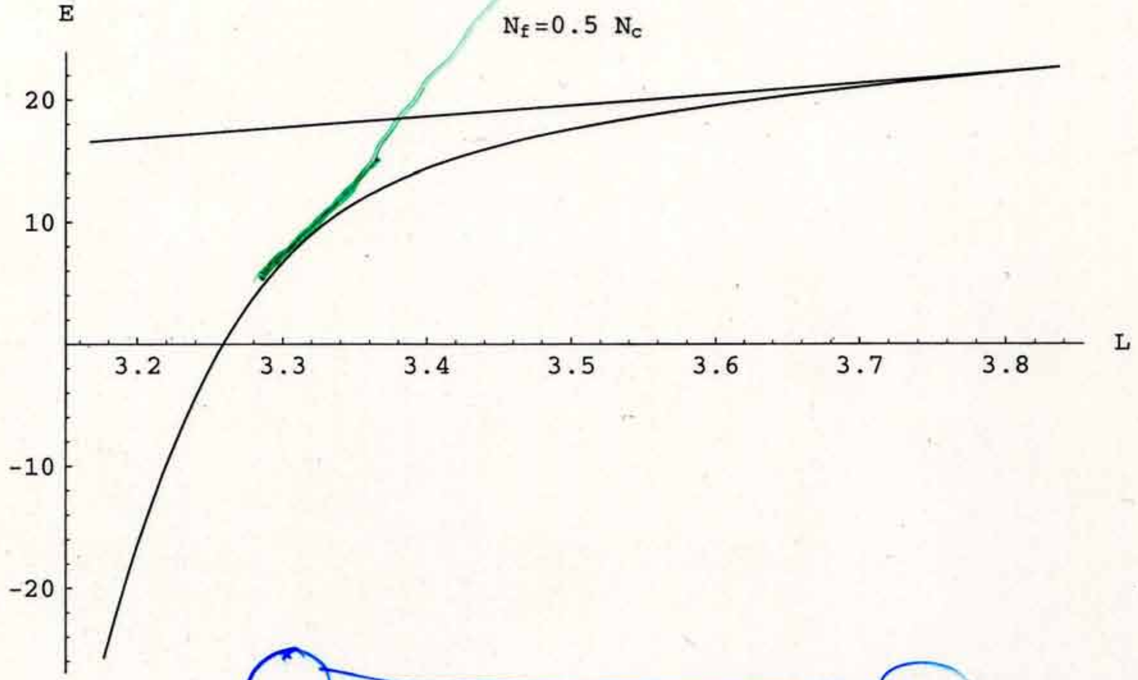
The picture to keep in mind is

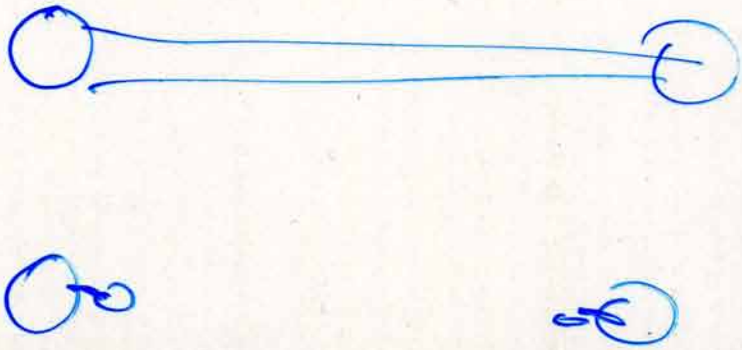


We plot $E_{q\bar{q}}$ in terms of L

So, the confining behavior leads to a sort of pair creation ⇝ the background "knows" there are massless dynamical quarks







Pair creation
is suppressed
$\mathcal{O}\left(\frac{N_f}{N_c}\right)$

A curious duality

Let me start by writing the background found above for the $\mathcal{N}=1$ field theory with N_c colors and N_f flavours.

$$ds^2_E = e^{\phi/2}\Big\{ dx^2_{1,3} + dr^2 + e^{2h}(e_1^2+e_2^2) + \frac{e^{2g}}{4}\big[(\tilde{\omega}_1 + a e_1)^2 + (\tilde{\omega}_2 + a e_2)^2\big] + \frac{e^{2k}}{4}\hat{\omega}_3^2 \Big\}.$$

(A)

$$F_3 = \frac{N_c}{4}\Big\{ -\tilde{\omega}_1\wedge\tilde{\omega}_2\wedge\tilde{\omega}_3 - b(r)(e_1\wedge\tilde{\omega}_2 - e_2\wedge\tilde{\omega}_1)\wedge\hat{\omega}_3 - b'(r)\, dr\wedge(e_1\wedge\tilde{\omega}_1 + e_2\wedge\tilde{\omega}_2) + (x-1)e_1\wedge e_2\wedge\hat{\omega}_3 \Big\}$$

Now, we might want to find a solution for the field theory with N_f flavours and $N_f - N_c = \bar{N}_c$ colors. So, we will propose a "similar" solution, but in terms of functions $(e^{2\bar{h}}, e^{2\bar{g}}, e^{2\bar{\phi}}, \bar{a}, \bar{b}, \bar{x})$

$$ds^2_E = e^{\bar{\phi}/2}\Big\{ dx^2_{1,3} + dr^2 + e^{2\bar{h}}(e_1^2+e_2^2) + \frac{e^{2\bar{g}}}{4}\big[(\tilde{\omega}_1 + \bar{a} e_1)^2 + (\tilde{\omega}_2 + a e_1)^2\big] + \frac{e^{2k}}{4}\hat{\omega}_3^2 \Big\}$$

(B)

$$F_3 = \frac{\bar{N}_c}{4}\Big\{ -\tilde{\omega}_1\wedge\tilde{\omega}_2\wedge\hat{\omega}_3 - \bar{b}(r)(e_1\wedge\tilde{\omega}_2 - e_2\wedge\tilde{\omega}_1)\wedge\hat{\omega}_3 - \bar{b}'\, dr\wedge(e_1\wedge\tilde{\omega}_1 + e_2\wedge\tilde{\omega}_2) + (\bar{x}-1)e_1\wedge e_2\wedge\hat{\omega}_3 \Big\}$$

Now, is this a solution? It is if:

$$e^{2\bar h} = \frac{e^{2g}}{4}\left(1 - \frac{a^2 e^{2g}}{4e^{2h}+a^2e^{2g}}\right)$$

$$e^{2\bar g} = 4e^{2h} + a^2 e^{2g}$$

$$\bar a = a\,\frac{e^{2g}}{4e^{2h}+a^2e^{2g}} \quad ; \quad \bar b = \frac{b}{1-x} \; ; \; \bar x = \frac{x}{x-1}$$

Now, let us compare the functions $\{e^{2g}, e^{2h}, a\}$ with $\{e^{2\bar g}, e^{2\bar h}, \bar a\}$

It is nice to see using the series expansions presented before that they agree up to third order (near $\rho \simeq 0$) but differ strongly in the UV. ($\rho \to \infty$)

$$e^{2h} = e^{2\bar h} + \mathcal{O}(\rho^4) \quad ; \quad e^{2\bar g} = e^{2g} + \mathcal{O}(\rho^2)$$

$$a = \bar a + \mathcal{O}(\rho^3).$$

So, we might think that we have two solutions that describe the field theory with $SU(N_c)$, N_f and $SU(N_f - N_c)$; N_f whose metric and 3 Form functions do coincide in the I.R. ($\rho \to 0$)

So, this looks like a "gravity version" of Seiberg duality.

Now, what is the relation (or the duality); in other words, are these solutions related and how?

Indeed; one can see that taking the solution (A)

So, if one takes the background (A) written above and changes

$$
\left.\begin{array}{lcl}
e_1 \longleftrightarrow \tilde{e}_1 & & \\
e_2 \longleftrightarrow \tilde{e}_2 & \text{or, in other words} & \\
\omega_1 \longleftrightarrow \tilde{\omega}_1 & & \theta \longleftrightarrow \tilde{\theta} \\
\omega_2 \longleftrightarrow \tilde{\omega}_2 & & \varphi \longleftrightarrow \tilde{\varphi}
\end{array}\right\} \rightsquigarrow \text{one gets the Background (B)}
$$

$\rightsquigarrow$ this operation is Seiberg duality.

what we are doing, in other words

$$
\begin{array}{ll}
x = e^{2h(\rho)} & \alpha = e^{2\bar{h}(\rho)} \\
y = e^{2g(\rho)} & \beta = e^{2\bar{g}(\rho)} \\
z = a(\rho) & \gamma = \bar{a}(\rho)
\end{array}
\left\}\begin{array}{l}
\alpha = \dfrac{xy}{4x+z^2y} \; ; \; x = \dfrac{\alpha\beta}{4\alpha+\gamma^2\beta} \\
\beta = 4x+z^2y \; ; \; y = 4\alpha+\gamma^2\beta \\
\gamma = \dfrac{zy}{4x+z^2y} \; ; \; z = \dfrac{\gamma\beta}{4\alpha+\beta\gamma^2}
\end{array}\right.
$$

$\rightarrow$ The transformation is nilpotent.

It also has some fixed points and fixed surfaces and many nice properties.

One might wonder, how anomalies in the backgrounds (A) and (B) [global anomaly matching] are computed on each side of the duality

Let us see if there is $U(1)_R$ anomaly matching

for this we have to reproduce the computation done before in each background (A) and (B)

Anomaly matching ($U(1)_R$)

I will be computing $\langle J_R J_R J_R \rangle$. For this one picks

C_2 on the cycle $\quad \theta = \tilde{\theta}$

$$\varphi = 2\pi - \tilde{\varphi}$$

In the A theory

$$C_{2A} = (\psi - \psi_0)\, N_c\, (2 - x)\, \sin\theta\, d\theta \wedge d\varphi$$

In the B theory

$$C_{2B} = (\psi - \psi_0)\, \bar{N}_c\, (2 - \bar{x})\, \sin\theta\, d\theta \wedge d\varphi$$

As before, we impose $\quad e^{i\int C_2} = 1$ under $\psi \to \psi + \varepsilon$

(A) $\quad \varepsilon_A = \dfrac{2m\pi}{2N_c - N_f} \quad ; \quad \varepsilon_B = \dfrac{2k\pi}{N_f - 2N_c} \quad \rightsquigarrow$ anomaly matching

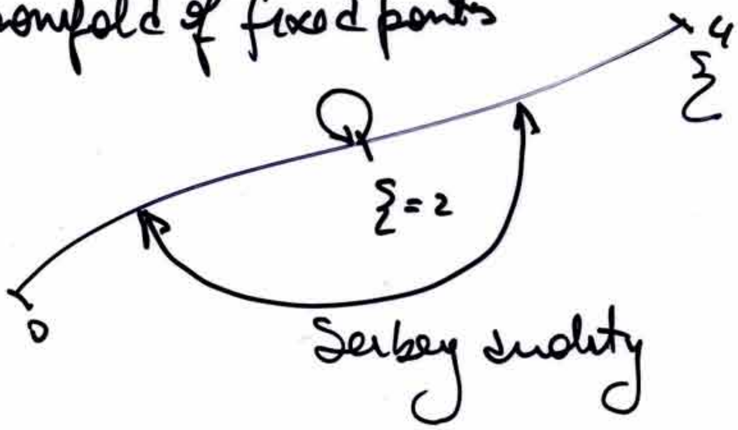
To be sure that the mapping we propose is Seiberg duality, we should study mesons, couplings, glueballs, moduli spaces, on both sides of the duality (this is in progress)

Now, let us move to a last check, the controversial β function

When $N_f = 2N_c$ the solution is very simple; in terms of a parameter $\xi \in [0,4]$

One can see that $\xi = 2$ is self dual point and that Seiberg duality interchanges $\xi \leftrightarrow 4-\xi$.

There is a manifold of fixed points



$\rightsquigarrow$ ξ is a "marginal operator" $\rightsquigarrow \{ \tilde{Q} Q \tilde{Q} Q$

This suggests that the gravity version to compute the quartic coupling should involve $\dfrac{\mathrm{vol}\, S^2}{\mathrm{vol}\, \hat{S}^2} = k \Big\{$

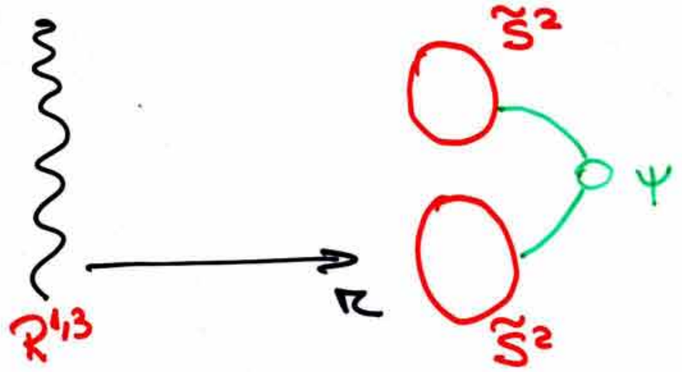
You can see that $U(1)_R \to$ unbroken

Di Vecchia Lerda Merlatti + Bertolini Beta function $= 0$
+ other interesting things

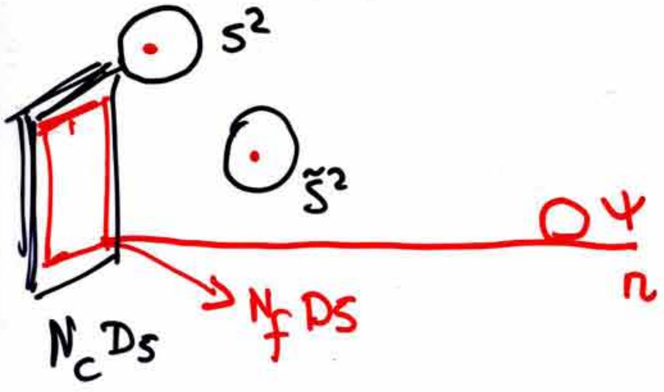
How to construct the solutions?

We start from the $N_f=0$ background

$$ds^2 \sim Mink_4 \times \mathbb{R} \times S^2 \times S^2 \times U(1)$$



and place N_f D5 branes on $R^{1,3} \times r \times \Psi$

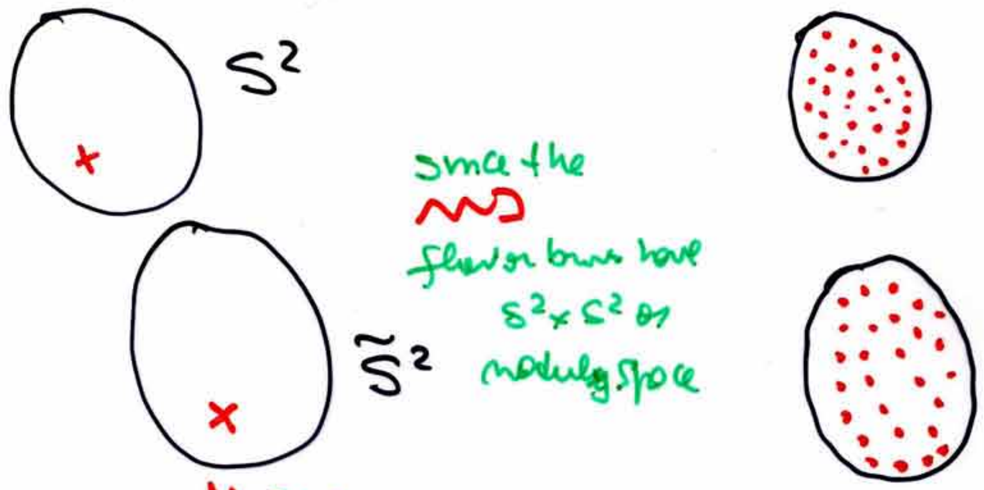


obviously this configuration breaks the $SO(3) \times SO(3)$ of the $S^2 \times \tilde{S}^2$

⇝ The solution in this situation would depend on $\theta, \tilde{\theta}$ and be quite complicated.

So, one may think about "reg~~r~~ouping" these isometries to make the resulting solution easier to find

In other words, we look for a flavored solution with the same isometries as the unflavored one how to reconstruct these geometries?



since the flavor branes have $S^2 \times S^2$ as moduli space

$\times$ N_f D5's

In the N_f finite limit these are fuzzy spheres

When $N_f \to \infty$ these are spheres. In the IIB action

$$S = S_{IIB} + \sum_{1}^{N_f} - T_5 \int e^{\phi_{/2}} \sqrt{\det g}\, d^6x + T_5 \int C_6\, d^6x$$

$$S = S_{IIB} - \frac{T_5}{(4\pi)^2} \int d^{10}x\, e^{\phi_{/2}} \sqrt{\det g_6} + \frac{T_5}{(4\pi)^2} \int C_6 \wedge \underbrace{Y_4}$$

volume element of the $S^2 \times S^2$ space.

So the action we have is

$$S = S_{IIB} - \frac{T_5}{(4\pi)^2} \int d^{10}x\, e^{\frac{\phi}{2}} \sqrt{\det g_6} + \frac{T_5}{(4\pi)^2} \int C_6 \wedge Y_4$$

The eqs of motion read :

The eqs of motion are

$$\frac{1}{\sqrt{-g_{10}}}\partial_\mu\left[\sqrt{g_{10}}\,g^{\mu\nu}\partial_\nu\phi\right]=\frac{e^{\phi}}{12}F_3^{\,2}+\frac{N_f}{8}e^{\phi/2}\frac{\sqrt{-g_6}}{\sqrt{g_{10}}}\sin\theta\sin\tilde\theta$$

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=\frac{1}{2}\left(\partial_\mu\phi\,\partial_\nu\phi-\frac{1}{2}g_{\mu\nu}(\partial\phi)^2\right)+\frac{e^{\phi}}{12}\left(3F_{\mu..}F_\nu^{\;..}-\frac{g_{\mu\nu}}{2}F^2\right)+T_{\mu\nu}^{\text{flavor}}$$

$$T_{\mu\nu}^{\text{flavor}}=\frac{2\kappa_{10}}{\sqrt{g_{10}}}\frac{\partial\mathcal{L}_{\text{flavor}}}{\partial g^{\mu\nu}}=-\frac{N_f}{\text{(8)}}\sin\theta\sin\tilde\theta\,e^{\phi/2}\,\delta_\mu^\alpha\delta_\nu^\beta\,g_{6\alpha\beta}\frac{\sqrt{g_6}}{\sqrt{g_{10}}}$$

$$d\left[*e^{\phi}F_3\right]=0$$

$$dF_3=N_f\,\mathrm{Vol}\,Y_4 \rightsquigarrow \text{indicates the presence of the flavor branes.}$$

This eqs can be solved in a SUSY way

$\delta\psi_\mu=\delta\lambda=0$ proposing a "deformed" background with functions to be determined.

one can also find a Superpotential from which the same BPS eqs are derived