Slow-Roll and DBI Inflation with Wilson Lines

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Based on:
and
Outline

- Inflation & Stringy Models
  - Interaction Potential
  - Vacuum Energy
- Wilson Line Inflation
- Slow-Roll Approximation
- Full non-linear (DBI) case
- Summary (model predictions)

[AA, Cremades & Quevedo, hep-th/0606031]
[AA & Zavala, 0810.5001]
**Inflation**

- Horizon, Flatness, Monopole problems
  
  \[ \tau = \int_0^t a(t)^{-1} dt \quad |\Omega - 1| = \frac{|k|}{a^2 H^2} \quad \pi_2(G/SM) \supset \pi_1(U(1)) \neq I \]

- Exponential expansion with \( H \sim \text{const} \)

- Typical situation: scalar field slowly rolling down a flat potential

- Structure formation

- Where does \( V(\phi) \) come from?

- Comoving horizon shrinks

- Expansion dilutes curvature and monopoles
Stringy Inflation

- String Theory Moduli as inflatons

- Break SUSY to lift potential

- Inflaton candidates

  - Closed String: dilaton, Kahler, cx structure

  - Open String: Brane positions, tachyon, Wilson lines

- All other moduli fields **must be fixed**, so that they do not interfere with inflationary dynamics & perturbations
Models

- Kahler Moduli
  - Racetrack (B-PBCEG-RKLQ 2004, 2006)
  - Kahler moduli Inflation (Conlon & Quevedo 2005, Cicoli et al 2008)

- Brane separations
  - Brane Inflation (Dvali & Tye 1999)
  - Branes @ angles (G-BRZ 2001, JST 2002, G-BRZ 2002)
  - D3-D7 (Hsu, Kallosh & Prokushkin 2003)
  - DBI (Silverstein & Tong 2003, AST 2004)

- String Tachyon
  - Warped tachyonic Inflation (Cremades, Quevedo & Sinha, 2005)

Complex Structure?
Volume?
Wilson Lines?
Brane Inflation

\[ V_{\text{int}}(\phi) \approx -\frac{c}{\phi^{d-2}} \]

\[ M^2(T) \approx M_s^2 \left( \frac{\phi}{\phi_c} - 1 \right) \]
Towards Realistic Models

- Older models (pre 2003) assumed moduli fixed by some unspecified mechanism
- After work of GKP 2001, KKLT 2003, it became possible to talk about dynamical stabilisation in IIB

- GKP: Dilaton and cx structure fixed by fluxes
- KKLT: Non-pert effects may fix Kahler moduli $T$

Scalar potential

$$ W = W_0 + A \exp^{-aT} $$

$$ K = -3 \log(T + T^*) $$

$$ V_F = e^K \left( |DW|^2 - 3|W|^2 \right) $$

gives SUSY AdS minimum

- Add an anti D3 brane

$$ V = V_F + \delta V $$

dS minimum
KKLMMT, 2003: Add a mobile D3 brane

GOOD NEWS: Brane-Antibrane Inflation with dynamical moduli stabilisation

BAD NEWS: Stabilisation mechanism generically spoils inflation \( \rightarrow \) Fine tuning

Any model of inflation has to address this issue

- Multi-throat (Iizuka & Trivedi 2004, Barnaby, Burgess & Cline 2004)
- Kuperstein embedding (BDKMS 07) …
Wilson Lines

- Consider gauge field: \[ F = dA, \quad A \to A + d\chi \]

- Wilson Line: \[ U_\gamma = P \exp \oint_A \]

- If \( \gamma \) contractible, then: \[ U_\gamma = P \exp \int_C F, \quad \gamma = \partial C \]
  and \[ F = 0 \Rightarrow U = 1 \]

- However, if \( \pi_1(C) \neq 1 \) one can have: \[ F = 0, \quad U \neq 1 \]

- Abuse of terminology \[ U_\gamma \leftrightarrow A \]

- In particular: \[ A = \lambda \, dx, \quad \lambda = \text{const} \]
WL Inflation: The Idea

- Consider two parallel D7 branes wrapping a $T^4$
- Turn on a Wilson line along, say, $y^1$:
  \[ A = \frac{\lambda}{2\pi R_1} dy^1 \]
- SUSY implies flat potential for $\lambda$
- Adding magnetic field or B-field introduces FI term:
  \[ \xi \propto \int_{T^4} J \wedge (B + 2\pi\alpha'F) \]

Branes @ angles

- T duality
  - Wilson Lines ↔ Brane Separations
  - Magnetic Field ↔ Brane Angles
Setup (IIB on $T^6$)

- Parallel D7 branes wrapping a 4-torus $T^4 = T^2 \times T^2$

- Brane1: Turn on a $WL$ in one $T^2$
  \[ A = \frac{\lambda}{2\pi R_1} \, dy^1 \]

- Brane2: Turn on $F$ in the other $T^2$
  \[ F = \frac{m}{2\pi R_3 R_4} \, dy^3 \wedge dy^4 \]

- Open Strings: BCs, mode expansion, Virasoro operators, MASS
**Interaction Potential**

- BCs lead to twisted mode expansions (cf Branes @ angles)

- Mass op:

\[
\alpha' M^2 = \sum_{i=1}^{2} \frac{(y_i + 2\pi w_i)^2 \tilde{R}_i^2}{4\pi^2 \alpha'} + \frac{(\lambda + 2\pi n)^2 \alpha'}{4\pi^2 R_1^2} + N_v + \nu (\theta - 1)
\]

- Interaction Energy given by Coleman-Weinberg formula:

\[
V_{\text{int}} = 2 \int \frac{d^4 k}{(2\pi)^4} \int_0^\infty \frac{dt}{t} \text{Tr} \exp[-2\pi \alpha' t(k^2 + M^2)]
\]

- Leads to:

\[
V_{\text{int}}(\lambda, y) = -\frac{\sin^2(\theta/2) \tan(\theta/2)}{8\pi^3 \alpha'^2 \|Y, \Lambda\|^2}
\]

**(cf G-BRZ,2003)**

where

\[
\|Y, \Lambda\|^2 = \sum_{i=1}^{2} \frac{y_i^2 \tilde{R}_i^2}{\alpha'} + \frac{\lambda^2 \alpha'}{R_1^2} \equiv Y^2 + \Lambda^2
\]
Vacuum Energy I

- Mass of the lowest open string state between the branes:

\[ M^2 = \sum_{i=1}^{2} \frac{y_i^2 \tilde{R}_i^2}{4\pi^2\alpha'^2} + \frac{\lambda}{4\pi^2 R_1^2} - \frac{|\theta|}{2\pi\alpha'} \]

- Can arrange so that at \( \lambda = \lambda_{\text{crit}} \) this state becomes tachyonic

- T-dual picture: geometric interpretation in terms of branes at angles reconnecting

- Calculate the energy difference in this picture and T-dualise back
Consider two D1 branes wrapping \((n_1,m_1)\) and \((n_2,m_2)\) cycles resp. in a \(T^2\)

\[
E = T \left( \sqrt{n_1^2 R_1^2} + m_1 R_2^2 + \sqrt{n_2^2 R_1^2} + m_2 R_2^2 \right)
\]

Minimum energy state with same charges is the reconnected configuration wrapping \((n_1+n_2,m_1+m_2)\)

\[
E_{\text{min}} = T \sqrt{(n_1 + n_2)^2 R_1^2 + (m_1 + m_2)^2 R_2^2}
\]

For \(n_1=n_2=1, m_1=0, m_2=m\) we have:

\[
\Delta E \approx \frac{1}{4} T R_1 \frac{m^2 R_2^2}{R_1^2}
\]

\(T\) dualising in the \(R_2\) direction:

\[
\Delta E \approx \frac{1}{4} T R_1 \frac{m^2 \alpha'}{R_1^2 R_2^2}
\]

Similarly for a D7 wrapping a \(T^4\) with magnetic flux in a \(T^2\) submanifold:

\[
\Delta E \approx \frac{1}{4} T_7 \text{Vol} (T^4) \frac{(2\pi)^4 m^2 \alpha'^2}{V_{\text{flux}}^2} \equiv V_0
\]
Potential & Slow Roll Parameters

- Total potential:
  \[ V = V_0 + V_{\text{int}}(\lambda) = \frac{\text{Vol}(T^4)}{8\pi^3\alpha'^4 g_s} \tan^2(\theta) \frac{\sin^2(\theta/2) \tan(\theta/2)}{4} - \frac{\sin^2(\theta/2) \tan(\theta/2)}{8\pi^3\alpha'^2(Y^2 + \Lambda^2)} \]

- Slow Roll Parameters:
  \[ \varepsilon = \frac{M_p^2}{2} \left( \frac{V'}{V} \right)^2, \quad \eta = M_p^2 \frac{V''}{V} \]

\[ \varepsilon = 4(8\pi)^2 g_s \frac{\tilde{R}_1 \tilde{R}_2 \alpha'}{R_1^4 R_2 R_3^2 R_4^2} \sin^4(\theta/2) \tan^2(\theta/2) \frac{\lambda^2}{\tan^4(\theta) \|Y, \Lambda\|^8} \]

\[ \eta = (8\pi)^2 \frac{\tilde{R}_1 \tilde{R}_2 \alpha'}{R_1 R_2 R_3 R_4} \frac{\sin^2(\theta/2) \tan(\theta/2)}{\tan^2(\theta)} \frac{1}{\|Y, \Lambda\|^4} \left( 1 - \frac{4 \alpha' \lambda^2}{R_1^2 \|Y, \Lambda\|^2} \right) \]

- Two cases
  \[ \Lambda \ll Y, \ V_{\text{int}}(\phi) \sim \phi^2 \]
  \[ \Lambda \gg Y, \ -V_{\text{int}}(\phi) \sim \phi^{-2} \]
Model Predictions

\( \Lambda << Y: \quad V(\varphi) \approx V_0 + c\varphi^2 \), \( \varepsilon, \eta \) +ve

Potential problem: \( n_s \approx 1 - 6\varepsilon + 2\eta > 1 \)

Numbers: \( \varepsilon \approx 10^{-13}, \eta \approx 3 \times 10^{-3} \) \( \rightarrow n_s \approx 1.006 \) HZ spectrum!

\( \Lambda >> Y: \quad V(\varphi) \approx V_0 - \frac{c}{\varphi^2} \), \( \eta \) -ve

Now: \( \varepsilon \approx 10^{-13}, \eta \approx -1.5 \times 10^{-2} \) \( \rightarrow n_s \approx 0.97 \)

Hybrid-like exit: (more fine tuning)

\[ M^2 = \sum_{i=1}^{2} \frac{y_i^2 \tilde{R}_i^2}{4\pi^2 \alpha'^2} + \frac{\lambda}{4\pi^2 R_1^2} - \frac{|\theta|}{2\pi \alpha'} \]

Strings

Should also consider contribution of strings to CMB

(Battye, Garbrecht, Moss & Stoica 2007; also BHKU 2007)
Quick Recap

- Work in IIB compactified on $T^6$
- Consider two parallel D7 branes, wrapping a $T^4$, at a fixed distance apart in the remaining $T^2$
- Turn on a **WL** in one $T^2$ (brane1) and **F** in the other $T^2$ (brane2)
- Generates slow roll phase, ending in a tachyonic instability

Fixing Distance, Moduli ???
Fixing the moduli (closed & open)

Work in a IIB toroidal orientifold, where:

- **Dilaton & Complex Structure** fixed by RR and NSNS flux  
  (Kachru, Schulz & Trivedi 2002)

- **Brane Positions** fixed by magnetic flux on the brane  
  (Gomis, Marchesano & Mateos 2005)

- **Kahler moduli**: non-perturbative effects

\[
K = - \log(T_1 + T_1^*) - \log(T_2 + T_2^* - \phi \phi^*) - \log(T_3 + T_3^*)
\]

\[
\text{Re} T_i \leftrightarrow A_i, \phi
\]
Kinetic terms for WL’s are of the DBI type
(Slow roll model is only an approximation)

In a warped compactification, need to compute full non-linear action:

\[ S_{DBI} = i \frac{1}{p} \int d^{p+1} \epsilon \sqrt{1 - \epsilon} \det(\mathcal{G}^{ab} + F_{ab}) \]

For position fields, kinetic term is obvious:

\[ \mathcal{G}^{ab} = G_{MN} \cdot \partial_a x^M \cdot \partial_b x^N \]

For Wilson lines, note that:

\[ F_{ab}^{\frac{3}{4}} \quad F_{ab}^{\frac{3}{4}} \quad F_{1m} \]

and that we shall need to integrate out compact dimensions.
In fact:

\[ \phi_{ab} + F_{ab} = \\
\begin{pmatrix}
\phi_{10} + B_{10} \\
2\frac{1}{4} e^{-\frac{i}{2} A} F_{1n} \\
2\frac{1}{4} e^{\frac{i}{2} F_{n0}} \\
\end{pmatrix} \]

So, using the matrix identity

\[
\det \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \det(D) \det(A \ B D i^{-1} C)
\]

the determinant of \( \phi_{ab} + F_{ab} \) becomes:

\[
\det[E_{mn}] \det[g_{ij}] \det[\pm i \partial_{A_n} \partial_{A_m} E^{nm}] + h \partial_{y^i} \partial_{y^j} g_{ij} + h^{1=2} (2\frac{1}{4} e^{-\frac{i}{2} A} \partial_{A_n} \partial_{A_m} E^{nm})
\]

Will be absorbed in volume and field redefinitions when we integrate over compact dims
4D Action & Perfect Fluid Description

- Including the Wess-Zumino term, the worldvolume 4D action reads:
  \[ S_{4D} = \int d^4x \left( \frac{1}{g} f('0)^p \right) \frac{1}{\left( f('0) \right)^{1/2}} + V_i('0) + V_A(\tilde{A})_i q F_p('0) \]

  (at a fixed canonical position '0)

- Can couple to gravity and write down dynamical equations

- Note similarity to relativistic particle with Lorentz factor:
  \[ \rho = p \frac{1}{1 + f('0)i^{1/2} \tilde{A}_i^{1/2}} \]

- Perfect fluid description with eqn of state:
  \[ w = \frac{\rho}{\rho + p} = \frac{\rho}{\rho + p} \]

- Note important difference to the position DBI case!!!
So what is new???

First important difference: field normalization

\[
\frac{\hat{A}^2}{M_{Pl}^2} = \left(\frac{2^{3/4}}{2}\right)^9 g^\hat{A} \hat{g}_s \frac{f_0}{l^2 V_6} \lesssim 2 \quad \text{cf ordinary DBI inflation where:}
\]

\[
\left(\frac{\phi}{M_{Pl}}\right)^2 < 2^{81/4} \left(\frac{g^\hat{A} \hat{g}_s f_0}{l^2 V_6}\right)
\]

- allows “large field” gravitational waves
- evades Lidsey-Huston constraints of ordinary (position-field) DBI

Second difference: can fix \(\phi_0\), limiting speed is not evolving

- more tuning freedom
- can avoid backreaction problems of ordinary (position) DBI

[McAllister & Silverstein 2008, Chen 2008]
Position DBI Constraints

- **Lyth bound**
  \[ r < \frac{8}{(N_{\text{eff}})^2} \left( \frac{\phi'}{M_{\text{Pl}}} \right)^2 \]  
  \[ \text{[Lyth 1997]} \]
  
  Combined with the field range constraint yields:
  \[ \frac{r}{0.009} < \frac{1}{N} \left( \frac{60}{N_{\text{eff}}} \right)^2 \]
  
  No gravitational waves! \[ \text{[Baumann & McAllister 2006]} \]

- **Lidsey-Huston bounds**
  - Strong **Upper Bound** on t-s ratio for DBI inflation:
    \[ r < 10^7 \]
  - Lower Bound based on WMAP3 limit on N-G and favoured spectral index:
    \[ n_s = \frac{r}{4} \left( \frac{q}{1} \right) 3 f_{N_{\text{L}}}^{\text{equil}} + 0 \left( 1 \equiv f_{N_{\text{L}}}^{\text{equil}} \right) \]
    \[ r > 0.001 \]

**BOUNDS ARE INCONSISTENT!!!!** \[ \text{[Lidsey & Huston 2007]} \]
For WL DBI inflation the Lyth bound becomes:

\[ r < \frac{(2^{1/4})^{11}}{(N_{\text{eff}})^2} g_s^2 \left[ \frac{l^2}{V_6} \right] \]

Lidsey-Huston constraint now becomes a Lower Bound:

\[ r > \frac{32^{1/4}(N_{\text{eff}})^2}{P_s^2} \frac{l^2 g_s^3}{\bar{g}^2 V_6} \]

Still get 2\textsuperscript{nd} Lidsey-Huston Lower Bound \( r > 0.001 \)

(but can now be made consistent with the above)

Can choose parameters to achieve a s-t ratio in the range:

\[ 0.01 < r < 0.24 \]
Summary

Wilson Line Slow Roll Inflation: just another stringy model of inflation (theoretical motivation, $\eta$ problem)

Predictions: -small $\varepsilon$ (no gravitational waves)
- HZ or slightly red scalar spectrum
- Cosmic strings with $G_{\mu}<10^{-7}$

Wilson Line DBI Inflation: evades the (inconsistent) bounds relevant to position field DBI inflation

Non-trivial phenomenological implications:
- (Equilateral) Non-Gaussianity, as in other DBI models
- Significant tensor perturbations

(only shared with Monodromy & Large Volume models)

[Silverstein & Westphal 2008, Cicoli et al 2008]