Slow-Roll and DBI Inflation with Wilson Lines

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Based on:
Gen.Rel.Grav.39:1203-1234,2007
and
arXiv:0810.5001 [to appear in JCAP]

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Outline

- Inflation & Stringy Models
- Wilson Line Inflation → Interaction Potential
  → Vacuum Energy
- Slow-Roll Approximation [AA, Cremades & Quevedo, hep-th/0606031]
- Full non-linear (DBI) case [AA & Zavala, 0810.5001]
- Summary (model predictions)

Inflation

- Horizon, Flatness, Monopole problems

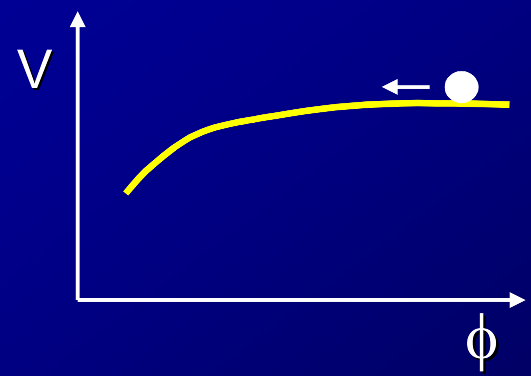
$$\tau = \int_0^t a(t)^{-1} dt \qquad |\Omega - 1| = |k| \big/ a^2 H^2 \qquad \pi_2(G/SM) \supset \pi_1(U(1)) \neq I$$

- Exponential expansion with H~const

  → Comoving horizon shrinks

  → Expansion dilutes curvature and monopoles

- Typical situation: scalar field slowly rolling down a flat potential



- Structure formation

- Where does V(ϕ) come from?

Stringy Inflation

- String Theory Moduli as inflatons
- Break SUSY to lift potential
- Inflaton candidates
  - Closed String: dilaton, Kahler, cx structrure
  - Open String: Brane positions, tachyon, Wilson lines
- All other moduli fields must be fixed, so that they do not interfere with inflationary dynamics & perturbations

Models

- Kahler Moduli

  Racetrack (B-PBCEG-RKLQ 2004, 2006)

  Kahler moduli Inflation (Conlon & Quevedo 2005, Cicoli et al 2008)

- Brane separations

  Brane Inflation (Dvali & Tye 1999)

  Brane-Antibrane (BMNQRZ 2001, DSS 2001, BMQRZ 2001)

  Branes @ angles (G-BRZ 2001, JST 2002, G-BRZ 2002)

  D3-D7 (Hsu, Kallosh & Prokushkin 2003)

  DBI (Silverstein & Tong 2003, AST 2004)

- String Tachyon

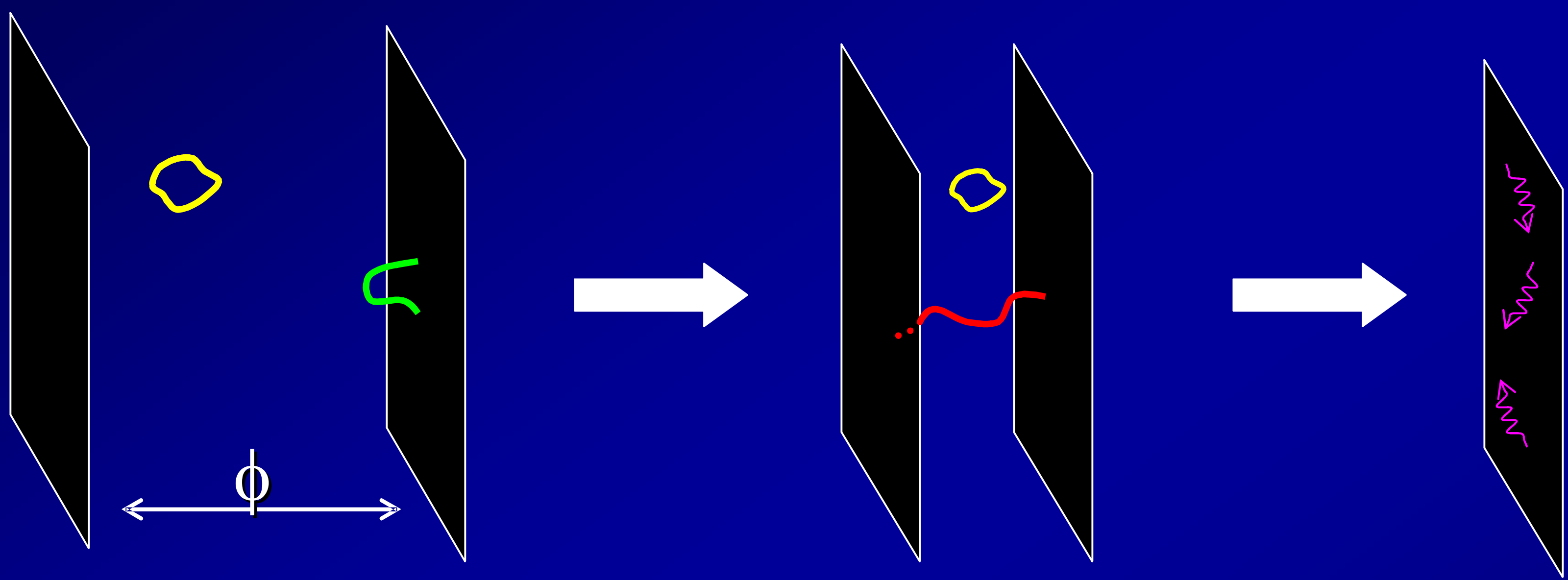
  Warped tachyonic Inflation (Cremades, Quevedo & Sinha, 2005)

  Complex Structure?

  Volume?

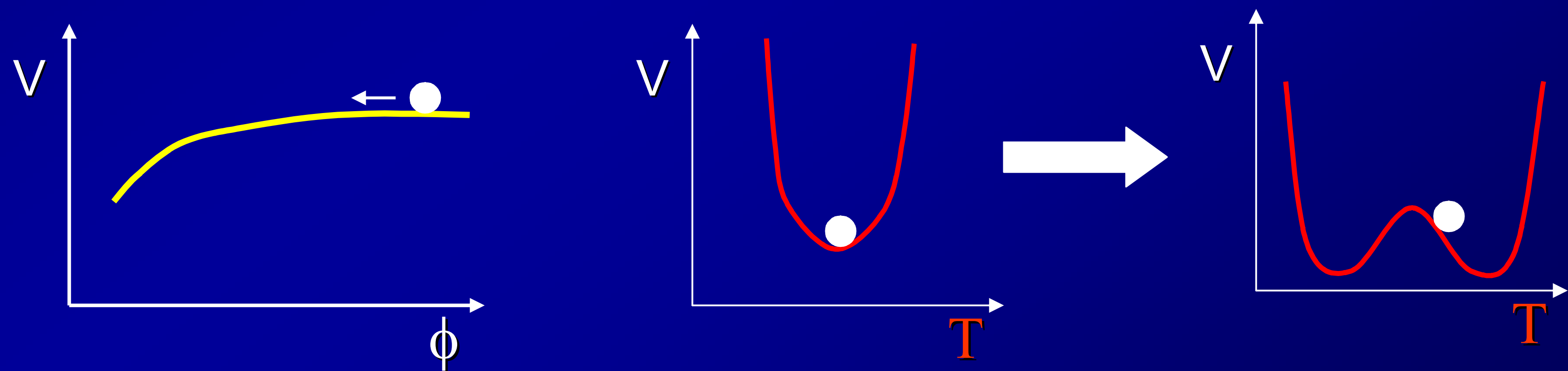
  Wilson Lines?

Brane Inflation



$$V_{\mathrm{int}}(\phi) \approx -\frac{c}{\phi^{d-2}}$$

$$M^2(T) \approx M_s^2\left(\frac{\phi}{\phi_c} - 1\right)$$



Towards Realistic Models

- Older models (pre 2003) assumed moduli fixed by some unspecified mechanism
- After work of GKP 2001, KKLT 2003, it became possible to talk about dynamical stabilisation in IIB
- GKP: Dilaton and cx structure fixed by fluxes
- KKLT: Non-pert effects may fix Kahler moduli T

$$W = W_0 + A\exp^{-aT} \qquad K = -3\log(T + T^*)$$

- Scalar potential $V_F = e^K\left(|DW|^2 - 3|W|^2\right)$

gives SUSY AdS minimum

- Add an anti D3 brane $V = V_F + \delta V$ ⟶ dS minimum

KKLMMT

- KKLMMT, 2003: Add a mobile D3 brane
- GOOD NEWS: Brane-Antibrane Inflation with dynamical moduli stabilisation
- BAD NEWS: Stabilisation mechanism generically spoils inflation → Fine tuning

  Any model of inflation has to address this issue

- Multi-throat (Iizuka & Trivedi 2004, Barnaby, Burgess & Cline 2004) Kuperstein embedding (BDKMS 07) …

Wilson Lines

- Consider gauge field: $F = dA, \quad A \to A + d\chi$
- Wilson Line: $U_\gamma = P\exp \oint_\gamma A$
- If γ contractible, then: $U_\gamma = P\exp \int_C F, \quad \gamma = \partial C$
  and $F = 0 \Rightarrow U = 1$
- However, if $\pi_1(C) \neq I$ one can have: $F = 0, \quad U \neq 1$
- Abuse of terminology $U_\gamma \leftrightarrow A$
- In particular: $A = \lambda\, dx, \quad \lambda = const$

WL Inflation: The Idea

- Consider two parallel D7 branes wrapping a T^4
- Turn on a Wilson line along, say, y^1: $A = \frac{\lambda}{2\pi R_1} dy^1$
- SUSY implies flat potential for λ
- Adding magnetic field or B-field introduces FI term:

$$\xi \propto \int_{T^4} J \wedge (B + 2\pi\alpha' F)$$

Branes @ angles

- T duality
  - Wilson Lines ↔ Brane Separations
  - Magnetic Field ↔ Brane Angles

Setup (IIB on T^6)

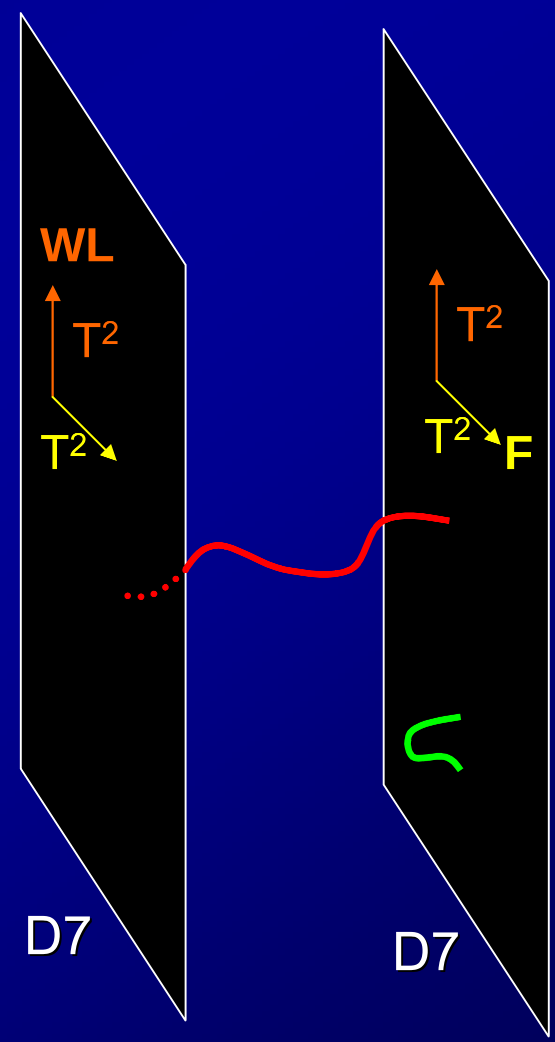
- Parallel D7 branes wrapping a 4-torus $T^4 = T^2 \times T^2$
- Brane1: Turn on a **WL** in one T^2

$$A = \frac{\lambda}{2\pi R_1} dy^1$$

- Brane2: Turn on **F** in the other T^2

$$F = \frac{m}{2\pi R_3 R_4} dy^3 \wedge dy^4$$

- Open Strings: BCs, mode expansion, Virasoro operators, **MASS**



Interaction Potential

- BCs lead to twisted mode expansions (cf Branes @ angles)
- Mass op: $$\alpha' M^2 = \sum_{i=1}^{2} \frac{(y_i + 2\pi w_i)^2 \tilde{R}_i^2}{4\pi^2 \alpha'} + \frac{(\lambda + 2\pi n)^2 \alpha'}{4\pi^2 R_1^2} + N_\nu + \nu(\theta - 1)$$
- Interaction Energy given by Coleman-Weinberg formula:

$$V_{\text{int}} = 2\int \frac{d^4 k}{(2\pi)^4} \int_0^\infty \frac{dt}{t} Tr \exp[-2\pi\alpha' t(k^2 + M^2)]$$

- Leads to: $$V_{\text{int}}(\lambda, y) = -\frac{\sin^2(\theta/2)\tan(\theta/2)}{8\pi^3 \alpha'^2 \|Y,\Lambda\|^2}$$ (cf G-BRZ,2003)

where $$\|Y,\Lambda\|^2 = \sum_{i=1}^{2} \frac{y_i^2 \tilde{R}_i^2}{\alpha'} + \frac{\lambda^2 \alpha'}{R_1^2} \equiv Y^2 + \Lambda^2$$

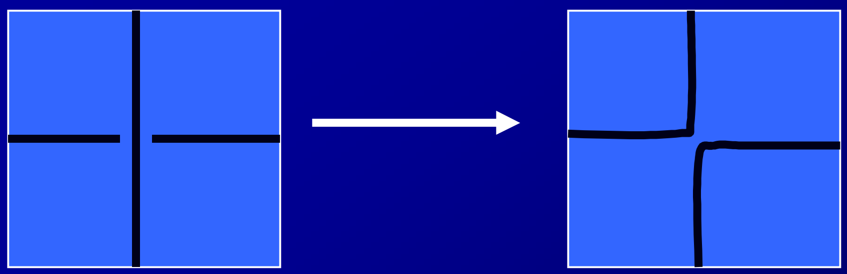
Vacuum Energy I

- Mass of the lowest open string state between the branes:

$$M^2 = \sum_{i=1}^{2} \frac{y_i^2 \tilde{R}_i^2}{4\pi^2 \alpha'^2} + \frac{\lambda}{4\pi^2 R_1^2} - \frac{|\theta|}{2\pi\alpha'}$$

- Can arrange so that at $\lambda = \lambda_{crit}$ this state becomes tachyonic

- T-dual picture: geometric interpretation in terms of branes at angles reconnecting



- Calculate the energy difference in this picture and T-dualise back

Vacuum Energy II

- Consider two D1 branes wrapping (n_1,m_1) and (n_2,m_2) cycles resp. in a T^2

$$E = T\left(\sqrt{n_1^2R_1^2 + m_1^2R_2^2} + \sqrt{n_2^2R_1^2 + m_2^2R_2^2}\right)$$

- Minimum energy state with same charges is the reconnected configuration wrapping (n_1+n_2,m_1+m_2)

$$E_{\min} = T\sqrt{(n_1+n_2)^2R_1^2 + (m_1+m_2)^2R_2^2}$$

- For $n_1=n_2=1$, $m_1=0$, $m_2=m$ we have:

$$\Delta E \approx \frac{1}{4}TR_1\frac{m^2R_2^2}{R_1^2}$$

- T dualising in the R_2 direction:

$$\Delta E \approx \frac{1}{4}TR_1\frac{m^2\alpha'}{R_1^2R_2^2}$$

- Similarly for a D7 wrapping a T^4 with magnetic flux in a T^2 submanifold:

$$\Delta E \approx \frac{1}{4}T_7 Vol(T^4)\frac{(2\pi)^4 m^2\alpha'^2}{V_{flux}^2} \equiv V_0$$

Potential & Slow Roll Parameters

- Total potential: $$V = V_0 + V_{\rm int}(\lambda) = \frac{Vol(T^4)}{8\pi^3\alpha'^4 g_s}\frac{\tan^2(\theta)}{4} - \frac{\sin^2(\theta/2)\tan(\theta/2)}{8\pi^3\alpha'^2(Y^2+\Lambda^2)}$$
- Slow Roll Parameters: $$\varepsilon = \frac{M_p^2}{2}\left(\frac{V'}{V}\right)^2, \quad \eta = M_p^2\frac{V''}{V}$$

$$\varepsilon = 4(8\pi)^2 g_s \frac{\tilde{R}_1\tilde{R}_2\alpha'^4}{R_1^4R_2^2R_3^2R_4^2}\frac{\sin^4(\theta/2)\tan^2(\theta/2)}{\tan^4(\theta)}\frac{\lambda^2}{\|Y,\Lambda\|^8}$$ naturally small

$$\eta = (8\pi)^2\frac{\tilde{R}_1\tilde{R}_2\alpha'}{R_1R_2R_3R_4}\frac{\sin^2(\theta/2)\tan(\theta/2)}{\tan^2(\theta)}\frac{1}{\|Y,\Lambda\|^4}\left(1-\frac{4\alpha'\lambda^2}{R_1^2\|Y,\Lambda\|^2}\right)$$ harder

- Two cases
  - $\Lambda \ll Y$, $V_{int}(\phi) \sim \phi^2$
  - $\Lambda \gg Y$, $-V_{int}(\phi) \sim \phi^{-2}$

Model Predictions

- Λ<<Y: $V(\varphi) \approx V_0 + c\varphi^2$, ε,η +ve

  Potential problem: $n_s \approx 1 - 6\varepsilon + 2\eta > 1$

  Numbers: $\varepsilon \approx 10^{-13}, \eta \approx 3\times 10^{-3}$ $\longrightarrow$ $n_s \approx 1.006$ HZ spectrum!

- Λ>>Y: $V(\varphi) \approx V_0 - \frac{c}{\varphi^2}$, η -ve

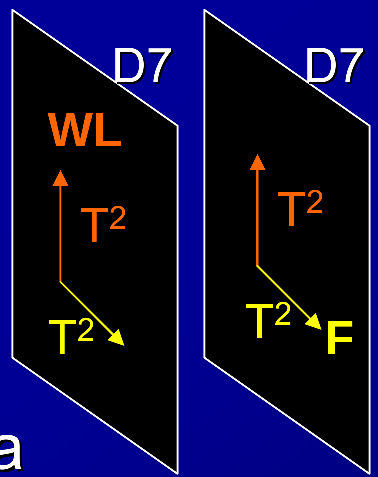
  Now: $\varepsilon \approx 10^{-13}, \eta \approx -1.5\times 10^{-2}$ $\longrightarrow$ $n_s \approx 0.97$

- Hybrid-like exit: $M^2 = \sum_{i=1}^{2} \frac{y_i^2 \tilde{R}_i^2}{4\pi^2 \alpha'^2} + \frac{\lambda}{4\pi^2 R_1^2} - \frac{|\theta|}{2\pi\alpha'}$ strings
  (more fine tuning)

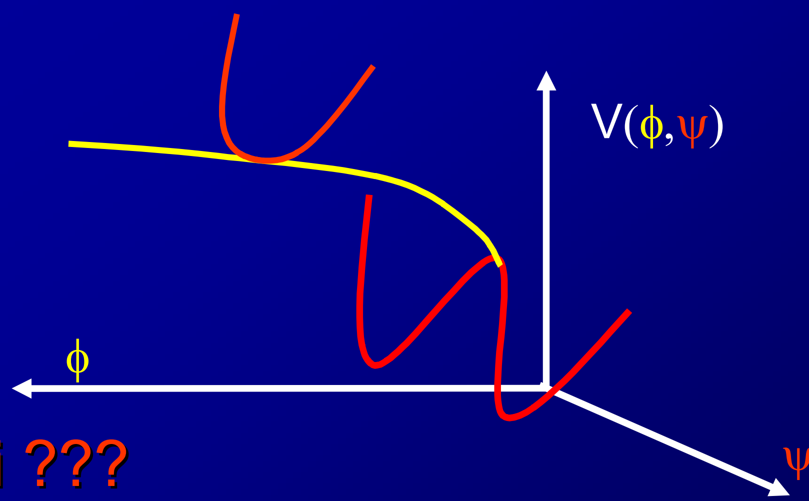
- Should also consider contribution of strings to CMB
  (Battye, Garbrecht, Moss & Stoica 2007; also BHKU 2007)

Quick Recap

- Work in IIB compactified on T^6
- Consider two parallel D7 branes, wrapping a T^4, at a fixed distance apart in the remaining T^2



- Turn on a **WL** in one T^2 (brane1) and **F** in the other T^2 (brane2)
- Generates slow roll phase, ending in a tachyonic instability



Fixing Distance, Moduli ???

Fixing the moduli (closed & open)

Work in a IIB toroidal orientifold, where:

- Dilaton & Complex Structure fixed by RR and NSNS flux
  (Kachru, Schulz & Trivedi 2002)

- Brane Positions fixed by magnetic flux on the brane
  (Gomis, Marchesano & Mateos 2005)

- Kahler moduli: non-perturbative effects

$$K = -\log(T_1 + T_1^*) - \log(T_2 + T_2^* - \phi\phi^*) - \log(T_3 + T_3^*)$$

$$\mathrm{Re}\,T_i \leftrightarrow A_i, \phi$$

Warped Compactifications: DBI Inflation

- Kinetic terms for WL's are of the DBI type
  (Slow roll model is only an approximation)
- In a warped compactification, need to compute full non-linear action:

Gauge Field

$$S_{DBI} = -\mu_p \int d^{p+1}\xi \, e^{-\phi} \sqrt{-\det(\gamma_{ab} + F_{ab})}$$

p+1 worldvol dimensions

Pullback of 10D metric

- For position fields, kinetic term is obvious:

$$\gamma_{ab} = G_{MN} \partial_a X^M \partial_b X^N$$

- For Wilson lines, note that: $F_{ab} \sigma F_{ab} \sigma F_{\mu m} \equiv \partial_\mu A_m$

  and that we shall need to integrate out compact dimensions.

DBI Kinetic Terms for WL's

- In fact:

$$\gamma_{ab} + F_{ab} = \begin{pmatrix} \gamma_{\mu\nu} + B_{\mu\nu} & 2\pi\alpha' e^{-\frac{\phi}{2}} F_{\mu n} \\ 2\pi\alpha' e^{-\frac{\phi}{2}} F_{n\nu} & E_{mn} \end{pmatrix}$$

- So, using the matrix identity

$$\det \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \det(D)\,\det(A - B D^{-1} C)$$

the determinant of $\gamma_{ab} + F_{ab}$ becomes:

$$\det[E_{mn}]\,\det[g_{\mu\lambda}]\,\det[\delta^{\lambda}_{\nu} + h\,\partial_\lambda y^i \partial_\nu y^j g_{ij} + h^{1/2}(2\pi\alpha')^2 e^{-\phi}\,\partial_\lambda A_n \partial_\nu A_m E^{nm}]$$

4D metric

Kinetic term for position fields

Kinetic term for Wilson Lines

Will be absorbed in volume and field redefinitions when we integrate over compact dims

4D Action & Perfect Fluid Description

- Including the Wess-Zumino term, the worldvolume 4D action reads:

$$S_{4D} = \int d^4x \sqrt{-g}\,\left[f(\varphi_0)\sqrt{1 - f(\varphi_0)^{-1}\dot{\chi}^2} + V_\varphi(\varphi_0) + V_\chi(\chi) - qF_p(\varphi_0) \right]$$

(at a fixed canonical position φ_0)

- Can couple to gravity and write down dynamical equations
- Note similarity to relativistic particle with Lorentz factor:

$$\gamma = \frac{1}{\sqrt{1 - f(\varphi_0)^{-1}\dot{\chi}^2}}$$

- Perfect fluid description with eqn of state:

$$w \equiv \frac{p}{\rho} = \frac{-\gamma^{-2} - (V f^{-1} - q f^{-1} F_p)\gamma^{-1}}{1 + (V f^{-1} - q f^{-1} F_p)\gamma^{-1}}$$

- Note important difference to the position DBI case!!!

So what is new???

- First important difference: field normalization

$$\frac{\hat{A}^2}{M_{Pl}^2} = \frac{(2¼)^9}{2}\, g^{\hat{A}\hat{A}} g_s \frac{f_0}{l^2 V_6}\, ¸^2 \longrightarrow \left(\frac{¢\hat{A}}{M_{Pl}}\right)^2 < 2^8 ¼^{11}\, g^{\hat{A}\hat{A}} g_s \frac{f_0}{l^2 V_6}$$

cf ordinary DBI inflation where: $\left(\frac{¢\,'}{M_{Pl}}\right)^2 < \frac{4}{N}$ (small field model)

-allows "large field" ⟶ gravitational waves

-evades Lidsey-Huston constraints of ordinary (position-field) DBI

- Second difference: can fix $'_0$, limiting speed is not evolving

-more tuning freedom

-can avoid backreaction problems of ordinary (position) DBI

[McAllister & Silverstein 2008, Chen 2008]

Position DBI Constraints

- Lyth bound

$$r < \frac{8}{(N_{eff})^2}\left(\frac{\Delta\phi}{M_{Pl}}\right)^2$$

[Lyth 1997]

Combined with the field range constraint yields:

$$\frac{r}{0.009} < \frac{1}{N}\left(\frac{60}{N_{eff}}\right)^2$$

No gravitational waves!

[Baumann & McAllister 2006]

- Lidsey-Huston bounds

-Strong Upper Bound on t-s ratio for DBI inflation: $r < 10^{-7}$

-Lower Bound based on WMAP3 limit on N-G and favoured spectral index:

$$1 - n_s = \frac{r}{4}\sqrt{1 - 3f_{NL}^{equil}} + O(1/f_{NL}^{equil}) \longrightarrow r > 0.001$$

BOUNDS ARE INCONSISTENT!!!!

[Lidsey & Huston 2007]

WL DBI Constraints

- For WL DBI inflation the Lyth bound becomes:

$$r < \frac{(2¼)^{11}}{(N_{eff})^2} \, g^{ÂÂ} g_s \, \frac{f_0}{l^2 V_6}$$

- Lidsey-Huston constraint now becomes a Lower Bound:

$$r > \frac{32¼(N_{eff})^2}{P_S^2} \, \frac{l^2 g_s^3}{g^{ÂÂ} V_6}$$

- Still get 2nd Lidsey-Huston Lower Bound $r > 0{:}001$ (but can now be made consistent with the above)

- Can choose parameters to achieve a s-t ratio in the range:

$$0{:}01 < r < 0{:}24$$

Summary

- Wilson Line Slow Roll Inflation: just another stringy model of inflation (theoretical motivation, η problem)

  Predictions: -small ε (no gravitational waves)
  -HZ or slightly red scalar spectrum
  -Cosmic strings with $G\mu<10^{-7}$

- Wilson Line DBI Inflation: evades the (inconsistent) bounds relevant to position field DBI inflation

  Non-trivial phenomenological implications:
  -(Equilateral) Non-Gaussianity, as in other DBI models
  -Significant tensor perturbations

  (only shared with Monodromy & Large Volume models)

  [Silverstein & Westphal 2008, Cicoli et al 2008]