20-1-2009

TRISPECTRUM FROM γ_{ij} EXCHANGE with Seery / Sloth 0811.3934

Single field inflation : bispectrum

Slow-roll $\Rightarrow$ weak-coupling $\Rightarrow$ Gaussian fluctuations

- The variation of the parameter $\eta \equiv M_p^2 \frac{V''}{V}$ over an Hubble time must be small:

$$\Delta\eta_H \sim M_p^2 \frac{V'''}{V} \frac{\dot\phi}{H} \sim \xi \ll 1 \Rightarrow \frac{V'''}{H} \ll \frac{H^2}{\dot\phi} \sim 10^{-5}$$

- The leading source of NG is given by the inflaton coupling to gravity:

We perturb a FRW universe with a scalar pert:

- Flat gauge: $\delta\phi(t,\vec x)$, $\zeta(t,\vec x) \equiv H\frac{\delta\phi}{\dot\phi} + \ldots$
- Uniform field gauge: $\delta\phi = 0$, $g_{ij} = a^2(t) e^{2\zeta(t,\vec x)} \delta_{ij}$

- $\mathcal{L}_2 = \frac{a^3}{2}\left(\delta\dot\phi^2 - \frac{\partial_i\delta\phi\,\partial^i\delta\phi}{a^2}\right) \sim \epsilon H^2 \zeta^2$, $\epsilon = \frac{\dot\phi^2}{M_p^2 H^2}$

- $\mathcal{L}_3^{grav} = a^3\left(-\frac{\dot\phi}{H}\delta\phi\,\delta\dot\phi^2 + \ldots\right) \sim \epsilon^2 H^2 \zeta^3$

- $\mathcal{L}_3^{self} = -a^3 \frac{1}{3!} V''' \delta\phi^3 \sim \epsilon\,\xi\,H^2\zeta^3 \ll \mathcal{L}_3^{grav}$

$$NG \sim \frac{\mathcal{L}_3^{grav}}{\mathcal{L}_2} \sim \epsilon\,\zeta \sim \epsilon\,10^{-5}$$

two suppressions

$\langle \zeta_{k_1} \zeta_{k_1'} \rangle = (2\pi)^3 \delta(k_1 + k_1') P_{k_1}$

We want to compute (and test)

$$\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle = (2\pi)^3 \delta(\textstyle\sum_i \vec{k}_i) \frac{6}{5} f_{NL}(\vec{k}_1, \vec{k}_2, \vec{k}_3) \left[P_{k_1} P_{k_2} + 2 \text{ cyclic} \right]$$

This is motivated by the local definition

$$\zeta(x) = \zeta_g(x) + \frac{3}{5} f_{NL}^{\text{local}} \zeta_g^2(x) \quad \text{where } \zeta_g \text{ is gaussian}$$

$$\text{NG.} \sim \frac{\mathcal{L}_3}{\mathcal{L}_2} \approx \frac{\langle \zeta\zeta\zeta \rangle}{\langle \zeta\zeta \rangle^{3/2}} \sim f_{NL} \cdot \zeta \sim f_{NL} \cdot 10^{-5}$$

Single-field inflation : $f_{NL} \sim \mathcal{O}(\epsilon, \eta)$ (Maldacena 2002)

Trispectrum :

$$\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \zeta_{k_4} \rangle = (2\pi)^3 \delta(\textstyle\sum_i \vec{k}_i) \Big\{ \tau_{NL} \left[P_{k_{13}} P_{k_3} P_{k_4} + 11 \text{ cyclic} \right] + \frac{54}{25} g_{NL} \left[P_{k_2} P_{k_3} P_{k_4} + 3 \text{ cyclic} \right] \Big\}$$

This is motivated by the local definition

$$\zeta = \zeta_g + \frac{1}{2} \left(\tau_{NL}^{\text{local}} \right)^{1/2} \zeta_g^2 + \frac{9}{25} g_{NL}^{\text{local}} \zeta_g^3$$

A local NG is generated by gravitational evolution subsequent to horizon crossing

For NG generated by quantum interference at Hubble crossing, there need be no relationship such as

$$\tau_{NL}^{\text{local}} = \left(\frac{6}{5} f_{NL}^{\text{local}} \right)^2 \quad !$$

We do not necessarily expect $\tau_{NL} \sim \mathcal{O}(\epsilon^2, \eta^2, \epsilon\eta)$

- As for the bispectrum, the leading source of NG is given by the inflaton coupling to gravity:

$$\mathcal{L}_4 = a^3\left(-2\,\delta\dot\phi\,\partial_j\delta\phi\,\partial^{-4}(\partial_j\delta\dot\phi\,\partial^2\delta\phi)+\dots\right)\sim \epsilon^2 H^2 \zeta^4$$

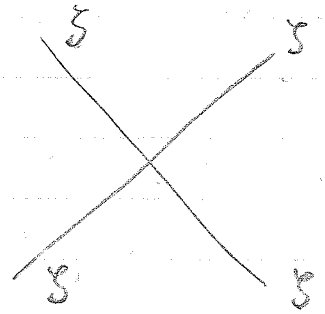
$$NG \sim \frac{\mathcal{L}_4}{\mathcal{L}_2} \sim \epsilon\,\zeta^2 \sim \epsilon\,10^{-10}$$

$$\sim \frac{\langle\zeta\zeta\zeta\zeta\rangle}{\langle\zeta\zeta\rangle^2} \sim (\tau_{NL}, g_{NL})\,10^{-10}$$

Seery, Lidsey, Sloth 2006

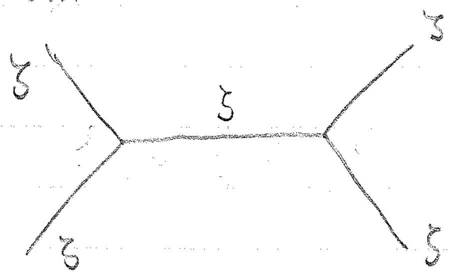
$$\tau_{NL}, g_{NL} \sim \epsilon$$

- <u>NOTE</u>: that $\tau_{NL} \neq f_{NL}^2 \sim O(\epsilon, \eta)$. The contribution that they computed comes from the diagram



$\tau_{NL} \sim \epsilon$

However, from



one would expect a different answer:

$$NG \sim \left(\frac{\mathcal{L}_3}{\mathcal{L}_2}\right)^2 \sim \epsilon^2\,\zeta^2 \;\Rightarrow\; \tau_{NL} \sim \epsilon^2$$

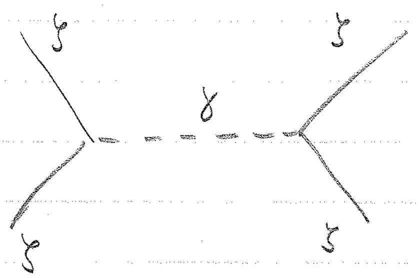
However, there is another diagram contributing to the trispectrum. Indeed, gravity waves are ubiquitously produced by second order scalar perturbations through scattering

$$g_{ij} = a^2 e^{2\zeta} e^{2\gamma_{ij}}$$

$$\partial_i \gamma_{ij} = \gamma_{ii} = 0$$

$$\mathcal{L}_{\gamma\zeta\zeta} = \frac{1}{2} a^3 \gamma^{ij} \partial_i \delta\phi \, \partial_j \delta\phi \sim H^2 \gamma \zeta \zeta$$

New contribution to the Trispectrum coming from



$$NG \sim \left(\frac{\mathcal{L}_{\gamma\zeta\zeta}}{\mathcal{L}_2}\right)^2 \sim \gamma^2 \sim r \cdot 10^{-10} \Rightarrow \tau_{NL} \sim r$$

Contribution to the trispectrum of the same order of magnitude as the contact interaction one. Indeed, it generically dominates the trispectrum. Important to compute it if we want to get the correct numbers!

Relevance & Motivations

Observations : $|\tau_{NL}| \lesssim 10^5$ CMB

$|\tau_{NL}| \lesssim 10$ 21 cm

Cooray, Li, Melchiorri 2008

Models producing a large τ_{NL} and a small (i.e. undetected) f_{NL} are rare and/or contrived :

Eg : - Engel, Lee, Wise 2008 for $P(X, \phi)$

- Byrnes, Choi, Hall 2008 for multi-fields

Thus, the importance of τ_{NL} seems to be smaller than that of f_{NL} ...

- However, experimental constraints can always evolve and if one sees a large τ_{NL} one can always say that inflation is excluded.
- We need to theoretically correct the previous estimate
- First time of a calculation of an exchange diagram leading to some observable.

Computing the graviton exchange diagram :

Using the so called IN-IN formalism

$$\langle Q(t)\rangle = \langle 0|\left[\bar{T} e^{i\int_{-\infty}^{t} H_I(t)\,dt}\right] Q^I(t) \left[T e^{-i\int_{-\infty}^{t} H_I(t)\,dt}\right]|0\rangle$$

interaction part of the Hamiltonian

product of ζ in the interaction picture

time ordering

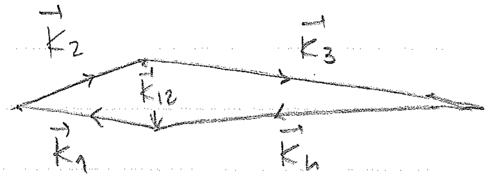
$$H_I = -\frac{1}{2}\int dx^3\, a^3 \gamma^{ij}\partial_i\zeta\, \partial_j\zeta$$

$$\langle \zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\zeta_{k_4}\rangle = \int_{-\infty}^{\eta_*} d\eta \int_{-\infty}^{\eta} d\eta' \frac{1}{H^4\eta^2\eta'^2}\Big[\langle\gamma^{ij}\gamma^{lm}\rangle\langle\partial_i\zeta\,\zeta\rangle\langle\partial_j\zeta\,\zeta\rangle \langle\partial_l\zeta\,\zeta\rangle\langle\partial_m\zeta\,\zeta\rangle\Big] + \text{perms}$$

$$= (2\pi)^3\,\delta\Big(\sum_i \vec{k}_i\Big)\,(P_K)^3\, \underbrace{r}_{\frac{P_T}{P_\zeta}} \sum_{s=+,\times} \underbrace{\epsilon^s_{ij}}_{(\vec{k}_{12})} k_1^i k_2^j\, \underbrace{\epsilon^s_{lm}}_{(-\vec{k}_{12})} k_3^l k_4^m \times \text{scale dep function}$$

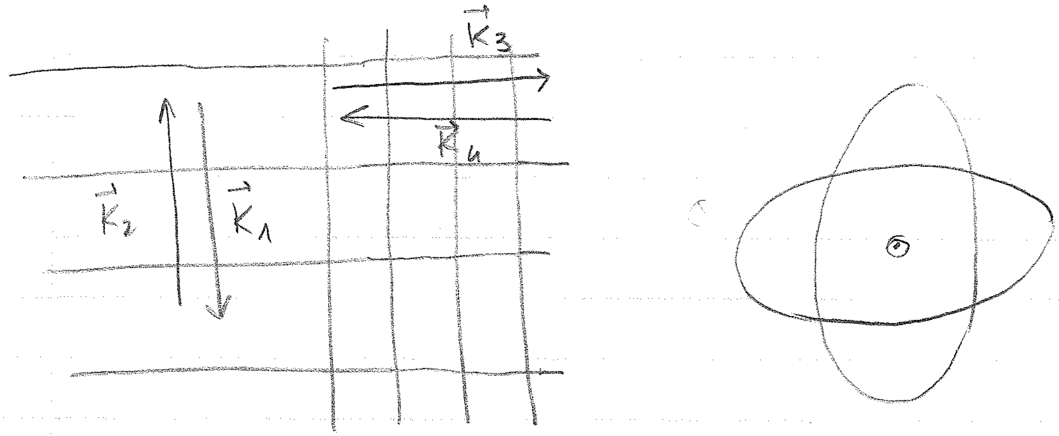
Consistency check : (inspired by Maldacena 2002)

Consider the case of $|\vec{k}_1 + \vec{k}_2| \ll k_1 \sim k_2\,,\ k_3\sim k_4$



$\vec{k}_{12} = \vec{k}_1 + \vec{k}_2$ is the momentum associated to the gravitational wave.

We are considering the effect of a very long wavelength gravity wave on four modes



the gravity wave will distort the lengths of the projections of the modes $\vec{k}_i$ on the plane perpendicular to $\vec{k}_{12}$.

Since it exited the Hubble radius much before the other 4 modes, it acts as a classical distortion of the geometry:

$$g_{ij} = a^2 e^{2\gamma_{ij}} \simeq a^2(\delta_{ij} + \gamma_{ij})$$

$$k_\gamma^2 = k_0^2 - \gamma^{ij} k_i k_j$$

When the 4 modes k_i are frozen, they feel the distorted geometry. The values of $\langle \zeta_{k_1} \zeta_{k_2} \rangle$ and $\langle \zeta_{k_3} \zeta_{k_4} \rangle$ will be changed

$$\langle \zeta_{k_1} \zeta_{k_2} \rangle_\gamma = \langle \zeta_{k_1} \zeta_{k_2} \rangle_0 + \frac{3}{2} \gamma^{ij} \frac{k_i k_j}{k^2} \langle \zeta_{k_1} \zeta_{k_2} \rangle_0$$

$$P_{k_1} \propto \frac{1}{k_1^3}$$

If we neglect other correlations among the 4 modes

$$\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \zeta_{k_4} \rangle = \langle \langle \zeta_{k_1} \zeta_{k_2} \rangle_\gamma \langle \zeta_{k_3} \zeta_{k_4} \rangle_\gamma \rangle =$$

$$= (2\pi)^3 \delta(\Sigma_i k_i) \frac{9}{4} r \, \epsilon_{ij} \frac{k_1^i k_1^j}{k_1^2} \epsilon_{\ell m} \frac{k_3^\ell k_3^m}{k_3^2} P_{k_{12}} P_{k_1} P_{k_3}$$

which matches with the very complicated expression that we have found.

It turns out that the contact point interaction is subdominant in the $k_{12} \to 0$ limit with respect to the exchange diagram.

In this limit the trispectrum becomes local like

$$\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \zeta_{k_4} \rangle = (2\pi)^3 \delta(\Sigma_i \vec{k}_i) \, 4 \tau_{NL}^{local} P_{k_{12}} P_{k_1} P_{k_3}$$

for $k_{12} \to 0$

$$\tau_{NL}^{local} = \frac{9}{64} r \cos 2 \chi_{12,34}$$

Numerical values

Seery, Lidsey, Sloth : $\tau_{NL}^{CI} \sim \frac{r}{50}$ $(g_{NL} \approx 0)$

However $\tau_{NL}^{CI} \sim \frac{r}{34}$

New contribution can be $\tau_{NL}^{GE} \sim r$ so in general dominates over the other one, but it is probably still unobservable.