E_{11} from a Supergravity Point of View

Eric Bergshoeff

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based on work in progress with M. de Roo, S. Kerstan, A. Kleinschmidt and F. Riccioni

and with T. Nutma

Supergravity

$$Q_\alpha Q_\beta + Q_\beta Q_\alpha = \left(\gamma^\mu \mathcal{C}^{-1}\right)_{\alpha\beta} P_\mu$$

$$\sharp \text{ Bosons} \quad = \quad \sharp \text{ Fermions}$$

Supergravity (SUGRA)

graviton $g_{\mu\nu}$ and gravitino ψ_μ Spin : 2 + 3/2

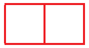
D=11 Supergravity

- describes 128 + 128 degrees of freedom
- maximal supergravity : N=1, D=11 $\Leftrightarrow$ N=8, D=4
- UV properties ?

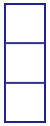
The Lagrangian

$$\mathcal{L} = \sqrt{-g}\,\{\,R - \tfrac{1}{2}\big(F_{(4)}\big)^2 + \cdots\} + C_{(3)} \wedge \partial C_{(3)} \wedge \partial C_{(3)}$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}\,, \quad \delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu \;:$$

 : **44** of SO(9)

$$F_{(4)} = \partial C_{(3)}\,, \qquad \delta C_{(3)} \;=\; \partial \Lambda_{(2)} \;:$$

 : **84** of SO(9)

$C_{(3)} \wedge \partial C_{(3)} \wedge \partial C_{(3)}$: $C_{(3)}$ occurs <u>without</u> derivative

Torus Reduction

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$$\mathcal{L}_{\text{scalars}} = \sqrt{-g}\left[-\tfrac{1}{2}(\partial\phi)^2 + \tfrac{1}{4}\text{Tr}\left(\partial M \partial M^{-1}\right)\right]$$

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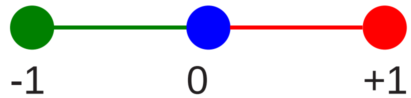
$$\hat{\xi}^\mu = \xi^\mu(x)\,, \qquad \hat{\xi}^m = \lambda^m(x) + \Lambda^m{}_n z^n \;:\qquad SL(n,\mathbb{R}) \times \mathbb{R}^+$$

Some Group Theory

Some Group Theory

SU(2) : angular momentum $\quad [J_3, J_3] = 0\,, \qquad [J_3, J_+] = J_+\,, \qquad [J_+, J_-] = H$



$$3 = 1 + 1_+ + 1_-$$

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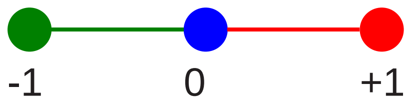
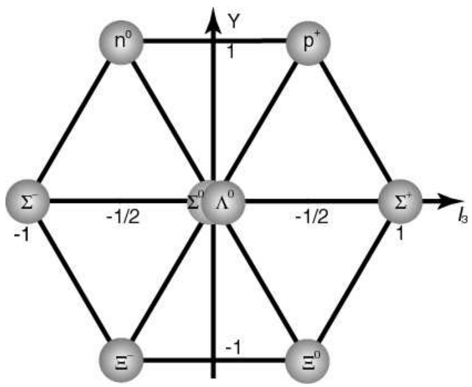


$3 = 1 + 1_+ + 1_-$

SU(3) : “the eightfold way” $8 = 2 + 3_+ + 3_-$ $\quad 3_+ = 2_+ + 1$



Dynkin Diagram $\Leftrightarrow$ Cartan Matrix A

Cartan's Classification

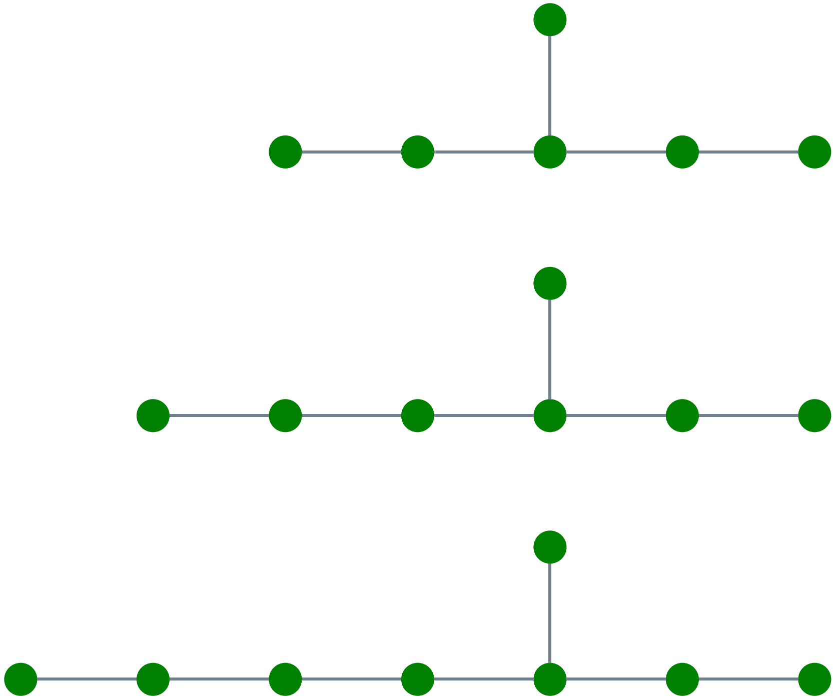
Simple Classical Lie Algebras : A_n , B_n , C_n , D_n

Exceptional Algebras : G_2 , F_4 , E_6 , E_7 , E_8

E_6 :

E_7 :

E_8 :



Nonlinear Symmetries

$$n = 2: \qquad M = e^{\phi}\begin{pmatrix} e^{-2\phi} + \chi^2 & \chi \\ \chi & 1 \end{pmatrix} \quad \Rightarrow$$

$$\mathcal{L}_{\text{scalars}} = \sqrt{-g}\Big[-\tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{2}e^{-\phi}(\partial\chi)^2\Big]:$$ 1 “dilaton” and 1 “axion”

Scalar Coset : $G/H = SL(2,\mathbb{R})/SO(2)$

\# of scalars = dimension of G – dimension of H

Dual Representations

Dual Representations

Examples : $\begin{array}{|c|}\hline \ \\ \hline \ \\ \hline \end{array} \sim \bullet$ in D=4 or $\begin{array}{|c|}\hline \ \\ \hline \ \\ \hline \end{array} \sim \begin{array}{|c|}\hline \ \\ \hline \ \\ \hline \ \\ \hline \ \\ \hline \ \\ \hline \ \\ \hline \end{array}$ in D=10

Dual Representations

Examples :  $\sim$ $\bullet$ in D=4 or  $\sim$  in D=10

$$\boxed{\mathcal{L}^{(1)} \sim F^2_{(p+2)}(A)}$$

$$\mathcal{L}' \sim F^2_{(p+2)} + F_{(p+2)} \wedge \partial B_{(D-p-3)} \quad \longrightarrow \quad F_{(p+2)} \sim {}^\star F_{(D-p-2)}(B)$$

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Condition: A occurs only via $F_{(p+2)}(A)$

Hidden Symmetries

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D	G/H	Dim

from: A. Kleinschmidt

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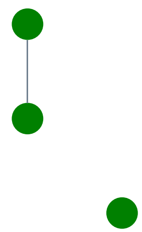

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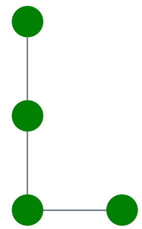

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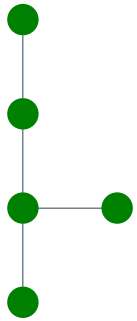

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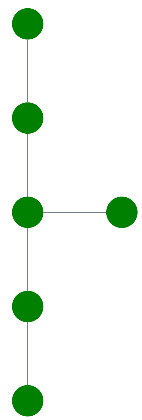

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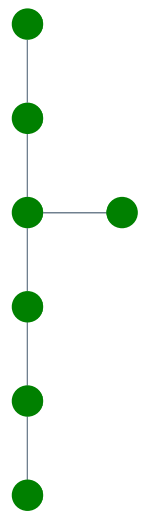

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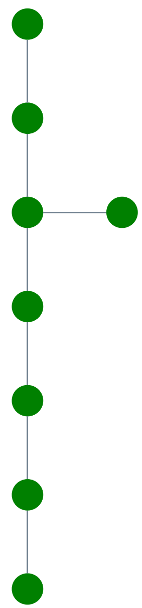

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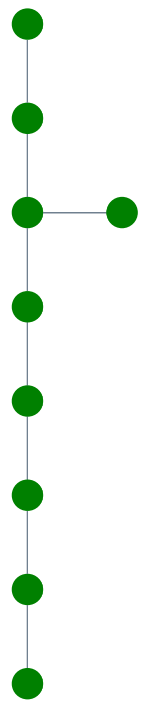

Nicolai 1987

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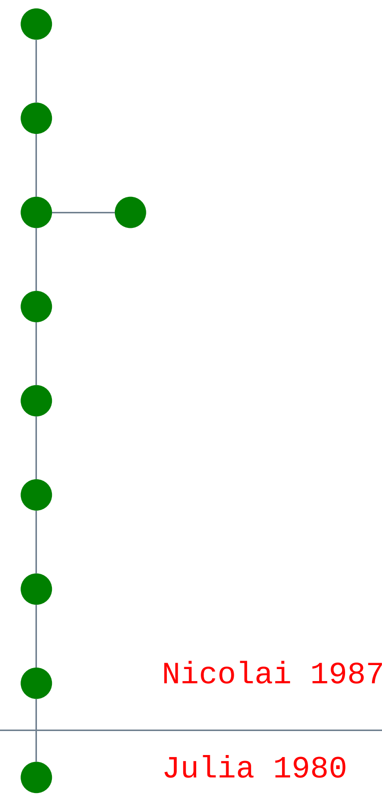

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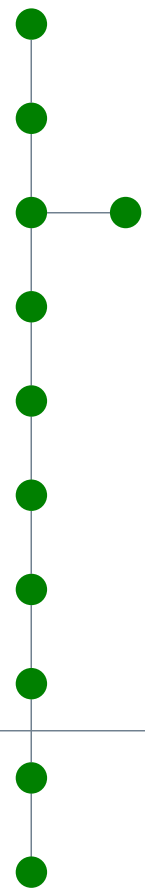
1	$E_{10}/K(E_{10})$?	–
0	$E_{11}/K(E_{11})$?	–



Nicolai 1987

Julia 1980

West 2001

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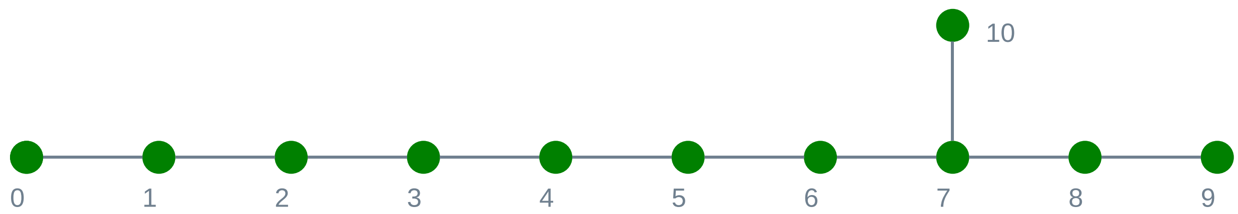

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A_{rs}

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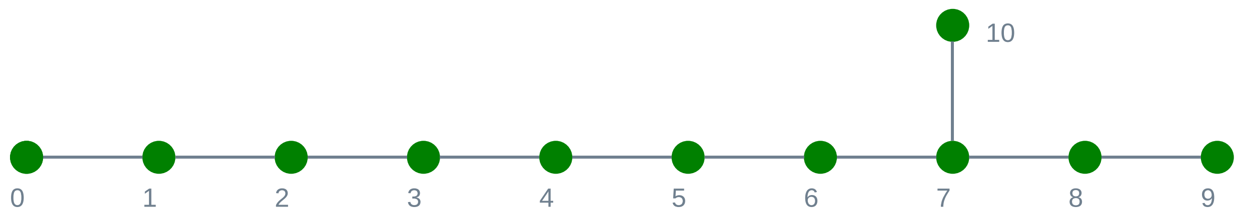

$\Leftrightarrow$ Cartan matrix A_{rs}

- One finds a Lorentzian Cartan matrix that leads to an infinite number of states in the adjoint representation that sofar has not been classified by mathematicians

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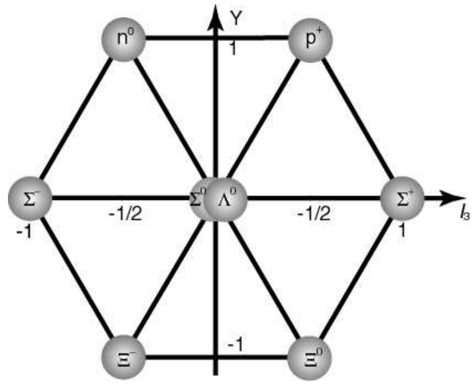

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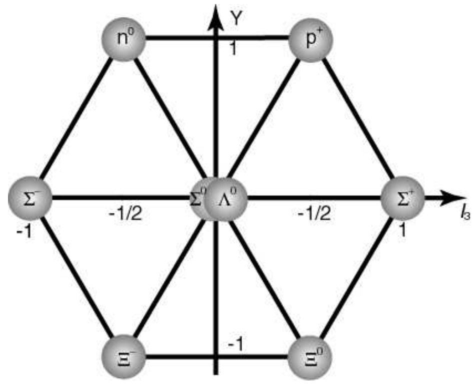

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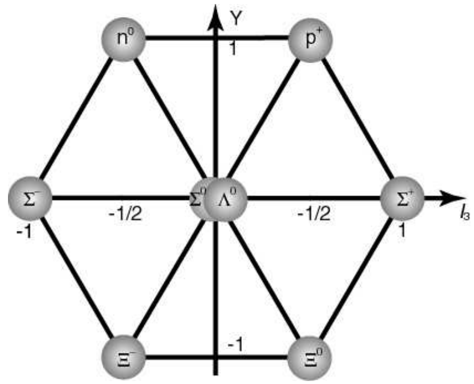

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Borisov, Ogievetsky 1974

level l ≥ 0	0	1	2	3
field	$h_a{}^b$	$C_{(3)}$	$C_{(6)}$?	$D_{(8,1)}$?
Young tableaux				

Six–form Potential

$\mathcal{L}_{11} \sim C_{(3)} \wedge \partial C_{(3)} \wedge \partial C_{(3)}$: $C_{(3)}$ or $C_{(6)}$ <u>plus</u> $C_{(3)}$

$$C_{(3)} \overset{\text{duality}}{\Leftrightarrow} C_{(6)} \qquad \underbrace{\partial C_{(6)} - C_{(3)} \wedge \partial C_{(3)}}_{F_{(6)}} = {}^* \underbrace{\partial C_{(3)}}_{F_{(4)}}$$

$$\delta C_{(3)} = \partial \Lambda_{(2)} \quad \Rightarrow \quad [3,3] = 0$$

$$\delta C_{(6)} = \partial \Lambda_{(5)} + C_{(3)} \wedge \partial \Lambda_{(2)} \quad \Rightarrow \quad \boxed{[3,3] = 6}\,!$$

Dual Gravity

Linearized gravity : $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ $\delta_{\xi_{(1)}} h_{(2)} = \partial \xi_{(1)}$

$$\boxed{S_{\mu_1\cdots\mu_9\nu\rho} = \frac{1}{2}\epsilon_{\mu_1\cdots\mu_9\alpha\beta}\, R_{\nu\rho}{}^{\alpha\beta}\big(\omega(h)\big)}$$

Hull, West

$$S_{\mu_1\cdots\mu_9\nu\rho} = \partial_{[\nu} Y_{\mu_1\cdots\mu_9\rho]} \qquad Y_{\mu_1\cdots\mu_9\rho} = 9\,\partial_{[\mu_1} D_{\mu_2\cdots\mu_9]\rho} \qquad D_{[\mu_1\cdots\mu_8\rho]} = 0$$

$$\delta_{\Lambda_{(7,1)}} D_{(8,1)} = \partial \Lambda_{(7,1)} \qquad \delta_{\Sigma_{(8)}} D_{(8,1)} = \partial \Sigma_{(8)}$$

Dual Gravity with Matter

de Roo, Kerstan, Kleinschmidt, Riccioni + E.B., work in progess

Require D=11 susy $\Rightarrow$ $Y = \star\omega + C_{(3)} \wedge \partial C_{(6)}$ terms

$\delta_{\Lambda_{(2)}} D_{(8,1)} \neq 0$ and $\delta_{\Lambda_{(5)}} D_{(8,1)} \neq 0 \Rightarrow$

$$\left[\delta_{\Lambda_{(2)}}, \delta_{\Lambda_{(5)}}\right] D_{(8,1)} = \delta_{\Lambda_{(7,1)}} D_{(8,1)} + \delta_{\Sigma_{(8)}} D_{(8,1)}$$ or

$$\boxed{[3\,,6] = (8,1)}\,!$$

Dual Representations

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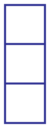
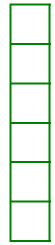
spin 1 : 

dual spin 1 : 

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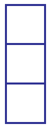
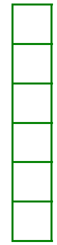
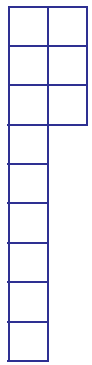
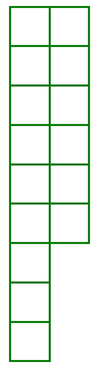
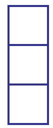
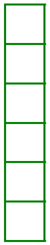
spin 1 : 

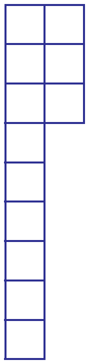
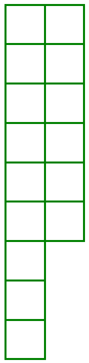
dual spin 1 : 

more dual spin 1 :  ,  , etc.

Hull

Dual Representations

spin 1 :  dual spin 1 : 

more dual spin 1 :  ,  , etc. Hull

3 families of reps. : spin 1, dual spin 1 and dual spin 2

Non-dual Representations

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D=10 Field	# d.o.f.	Name	Interpretation
IIA : $C_{(9)}$	0	de form	cosmological constant
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$$\partial_\lambda\left(\sqrt{-g}F^{\lambda\mu_1\cdots\mu_9}(C)\right) = 0 \qquad \Rightarrow \qquad F_{\mu_1\cdots\mu_{10}}(C) = \frac{1}{\sqrt{-g}}\epsilon_{\mu_1\cdots\mu_0} m$$

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IIA : $C_{(9)}$	0	de form	cosmological constant
IIB : $C_{(10)}$	0	top form	Type I string

$$\partial_\lambda\left(\sqrt{-g}F^{\lambda\mu_1\cdots\mu_9}(C)\right) = 0 \qquad \Rightarrow \qquad F_{\mu_1\cdots\mu_{10}}(C) = \frac{1}{\sqrt{-g}}\epsilon_{\mu_1\cdots\mu_0} m$$

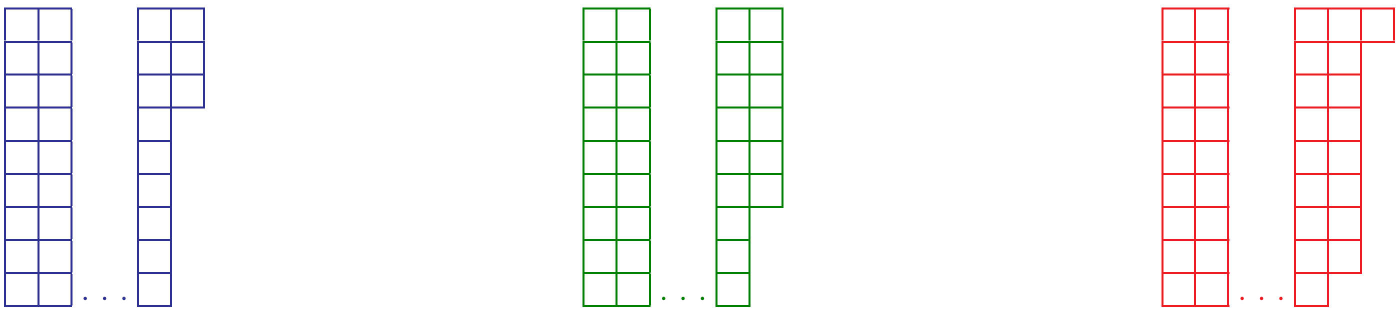
$\delta C_{(10)} = \partial\Lambda_{(9)}$: $C_{(10)}$ is not a scalar !

D=11 Supergravity and E_{11}

Riccioni,West, hep-th/0612001

The adjoint representation of E_{11} contains the following fields with $3l$ indices :

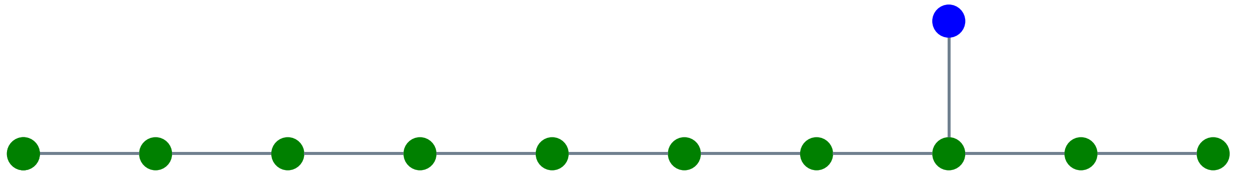
- 3 families of dual representations



- Many non-dual representations with 10 or 11 antisymm. indices

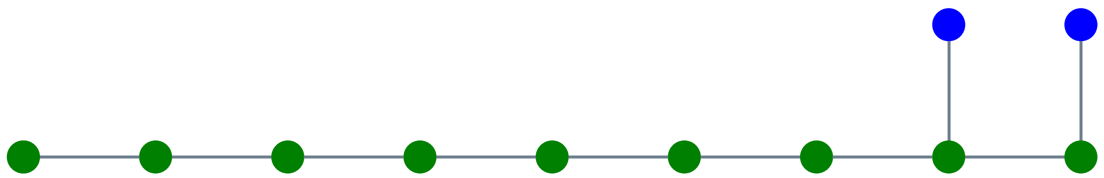
Lower Dimensions

D=11



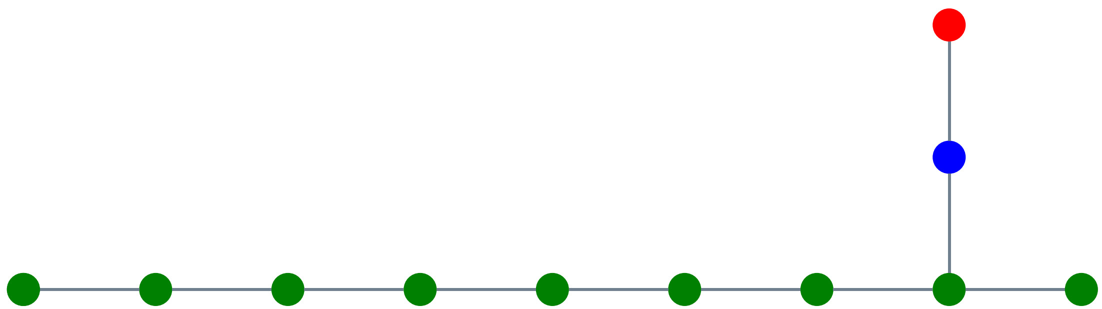
Lower Dimensions

D=10, IIA



Lower Dimensions

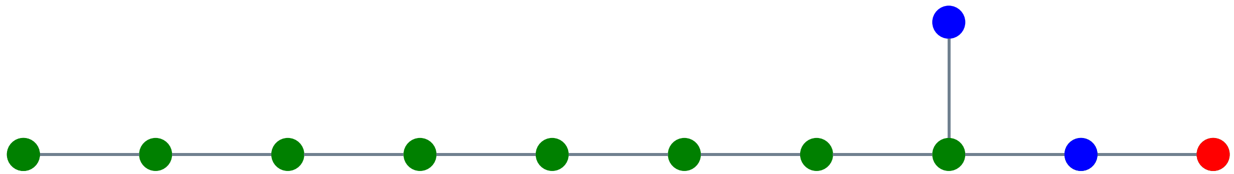
D=10, IIB



$SL(2,\mathbb{R})$

Lower Dimensions

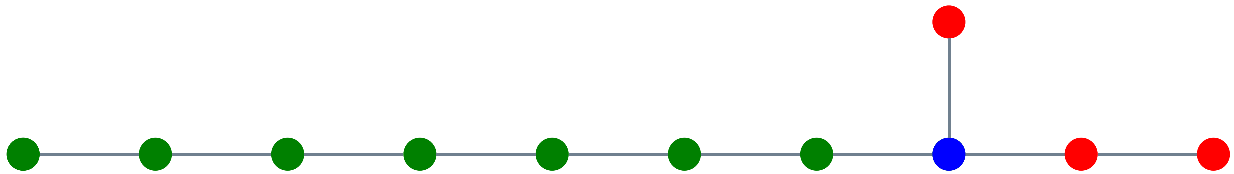
D=9



$SL(2, \mathbb{R})$

Lower Dimensions

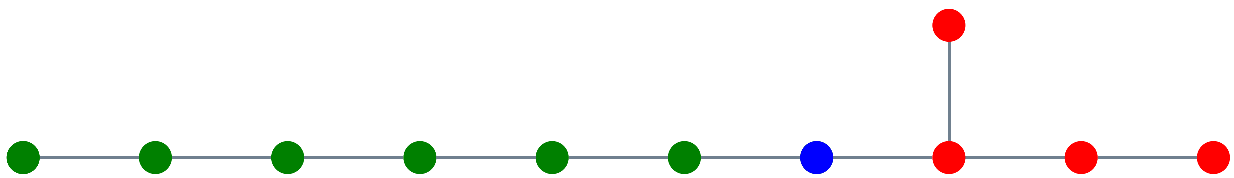
D=8



$$SL(2,\mathbb{R}) \times SL(3,\mathbb{R})$$

Lower Dimensions

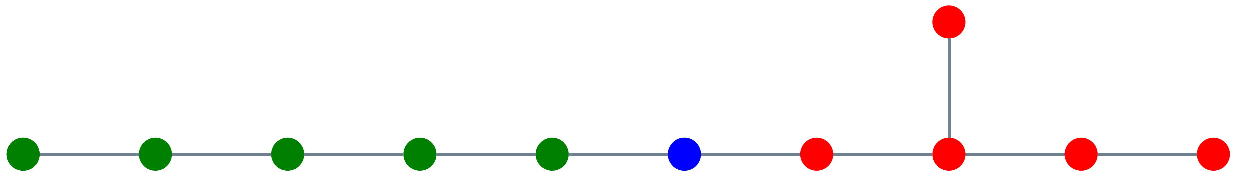
D=7



$SL(5, \mathbb{R})$

Lower Dimensions

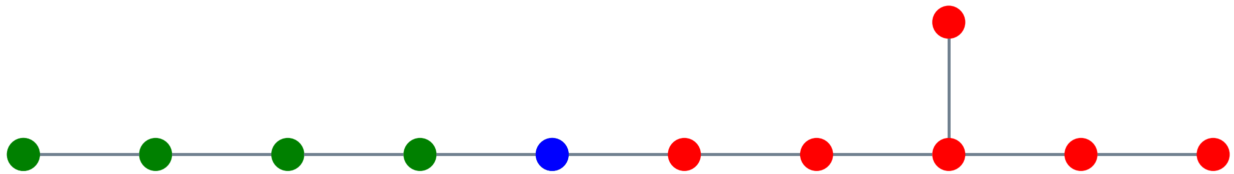
D=6



$SO(5,5)$

Lower Dimensions

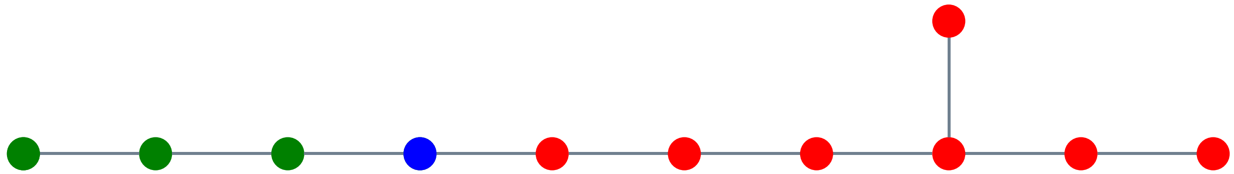
D=5



E_6

Lower Dimensions

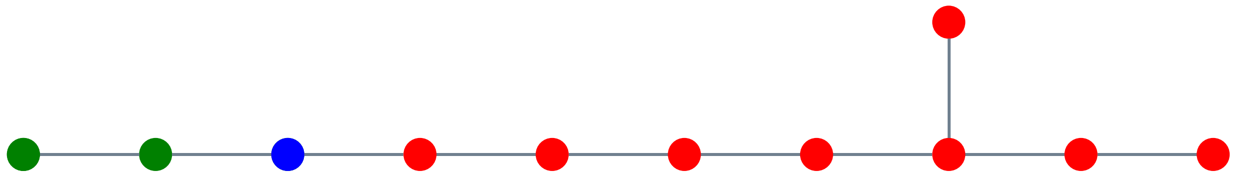
D=4



E_7

Lower Dimensions

D=3



E_8

D9–branes

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Dimension	D-form gauge field
D=11	–
D=10,IIA	1+1
D=10,IIB	**4 + 2**

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`M. de Roo, S. Kerstan, T. Ortín, F. Riccioni, 2006`

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Supergravity calculation agrees with E_{11} result!

E_{11} and Massive Supergravity

E_{11} and Massive Supergravity

Dimension	Cosmological Constant
D=11	–
D=10,IIA	1
D=10,IIB	–
D=9	**3** + **2**

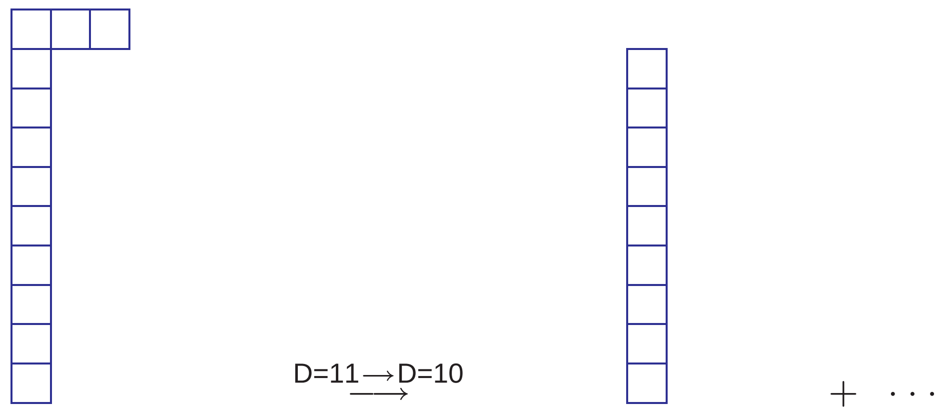
Romans

E_{11} and Massive Supergravity

Dimension	Cosmological Constant
D=11	–
D=10,IIA	1
D=10,IIB	–
D=9	**3 + 2**

Romans

"D=11 cosmological constant" :



E_{11} and Massive Supergravity

Dimension	Cosmological Constant
D=11	–
D=10,IIA	1
D=10,IIB	–
D=9	**3 + 2**

Romans

3 : SO(2), SO(1,1) and $\mathbb{R}^+$ Gauged Supergravity

2 : New Massive Maximal Supergravities ?

Remarks

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- E_{11} gives non-trivial information about (non-linear) supergravities in D<11 dimensions ?

work in progress