Supersymmetric string backgrounds: from bottom to top

Diederik Roest

Departament Estructura i Constituents de la Materia
Universitat de Barcelona

Galileo Galilei Institute, Firenze
April 10, 2007

Outline

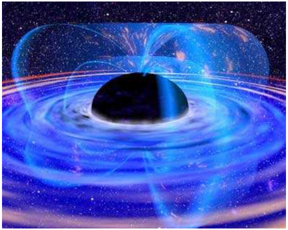
Supersymmetric string backgrounds

Supersymmetric solutions of supergravity, i.e. some given metric with accompanying fluxes, play an important role in string/M-theory:

- entropy matching,
- flux compactifications,
- AdS/CFT correspondence,
- and many many more!

Supersymmetry is a very powerful tool!

Desirable to have a full classification of all supersymmetric backgrounds!



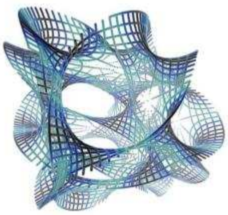
Supersymmetric string backgrounds

Supersymmetric solutions of supergravity, i.e. some given metric with accompanying fluxes, play an important role in string/M-theory:

- entropy matching,
- flux compactifications,
- AdS/CFT correspondence,
- and many many more!

Supersymmetry is a very powerful tool!

Desirable to have a full classification of all supersymmetric backgrounds!



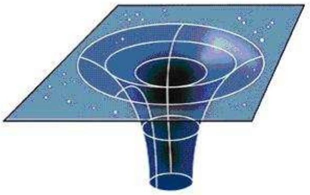
Supersymmetric string backgrounds

Supersymmetric solutions of supergravity, i.e. some given metric with accompanying fluxes, play an important role in string/M-theory:

- entropy matching,
- flux compactifications,
- AdS/CFT correspondence,
- and many many more!

Supersymmetry is a very powerful tool!

Desirable to have a full classification of all supersymmetric backgrounds!



Supersymmetric string backgrounds

Supersymmetric solutions of supergravity, i.e. some given metric with accompanying fluxes, play an important role in string/M-theory:

- entropy matching,
- flux compactifications,
- AdS/CFT correspondence,
- and many many more!

Supersymmetry is a very powerful tool!

Desirable to have a full classification of all supersymmetric backgrounds!

Supersymmetry transformations

Supersymmetry transforms fermions into bosons:

$$\delta_\epsilon \psi_\mu = D_\mu \epsilon \,, \qquad \delta_\epsilon \lambda = A \epsilon \,,$$

and bosons into fermions. The supersymmetry parameter ϵ is a fermion.

Supercovariant derivative D_μ and algebraic expression A are given in terms of the bosons, e.g. for IIB:

$$\begin{aligned} D_\mu = & (\partial_M + \tfrac{1}{4}\Omega_{M,PQ}\Gamma^{PQ} - \tfrac{i}{2}Q_M + \tfrac{1}{48} i \Gamma^{N_1 \ldots N_4} F_{MN_1 \ldots N_4}) + \\ & - \tfrac{1}{96}(\Gamma_M{}^{N_1 N_2 N_3} G_{N_1 N_2 N_3} - 9 \Gamma^{N_1 N_2} G_{MN_1 N_2}) C * \,, \\ A = & P_N \Gamma^N C * + \tfrac{1}{24} G_{N_1 N_2 N_3} \Gamma^{N_1 N_2 N_3} \,. \end{aligned}$$

Contractions of spin connection Ω and fluxes P, G, F with Γ-matrices.

Supersymmetry conditions

Consider bosonic background of some space-time g with fluxes P, G, F. When is this supersymmetric?

Supersymmetry variations of the fields have to vanish:

$$\delta_\epsilon \psi_\mu = D_\mu \epsilon = 0\,, \qquad \delta_\epsilon \lambda = A\epsilon = 0\,.$$

Differential and algebraic Killing spinor equations (KSE).

Solutions ϵ are Killing spinors. There can be $0 \leq N \leq 32$ of these.

Requiring a background to admit Killing spinors imposes strong conditions.

Question: what are the possible backgrounds for any number of supersymmetries N?

Supersymmetry conditions

Refinement: what are the possible backgrounds for N Killing spinors with common stability subgroup G in the Lorentz group?

Important since lead to G-structure.

For example, in IIB a single spinor can have stability subgroup [a]

$$G_2\,, \quad Spin(7) \ltimes \mathbb{R}^8\,, \quad SU(4) \ltimes \mathbb{R}^8\,.$$

(Compare with time-like, null or space-like vector.)

For $N \geq 2$ there are more possibilities, depending on the embeddings of the separate stability subgroups. E.g. two spinors can have trivial stability subgroup.

[a]: Gran, Gutowski, Papadopoulos ’05.

Classifications

In which theories are there classifications of all supersymmetric solutions?

Many, many partial results.

Full classifications, i.e. for all possible fractions of susy, for

- $D = 4$: minimal $\mathcal{N} = 2$ [a], coupled to vectors [b],
- $D = 5$: minimal $\mathcal{N} = 1$ [c]
- $D = 6$: minimal $\mathcal{N} = 1$ [d]

Theories with 8 supersymmetries and solutions with $N = 4, 8$.

Different techniques:

- Newman-Penrose ('82, '83),
- spinor bilinears ('02,'03,'06),

[a]: Gibbons, Hull '82, Tod, '83, [b]: Meessen, Ortín '06, [d]: Gauntlett et al '02, [e]: Gutowski, Martelli, Reall '03,

Maximal Supergravity in 11D / 10D

More difficult since more supersymmetries. Few systematic results:

- purely gravitational solutions [a],
- $N = 32$ (top): AdS $\times$ S and Penrose limits (Hpp-wave or Minkowski) [b],

  |
- $N > 24$: homogeneous spaces [c],

  |
- $N = 1$ (bottom):
  11D: two cases with $SU(5)$ or $(Spin(7) \ltimes \mathbb{R}^8) \times \mathbb{R}$ structure [d],
  IIB: three cases with G_2, $Spin(7) \ltimes \mathbb{R}^8$ or $SU(4) \ltimes \mathbb{R}^8$ structure [e],

We will focus on IIB and its truncation to type I, but the story generalises to other supergravities.

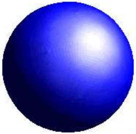
[a]: Figueroa-O'Farrill '99, Bryant '00, [b]: Figueroa-O'Farrill, Papadopoulos '02, [c]: Figueroa-O'Farrill, Meessen, Philip '04, Figueroa-O'Farrill, Hackett-Jones, Moutsopoulos '07, [d]: Gauntlett, Pakis '02, Gauntlett, Gutowski, Pakis '03 [e]: Gran, Gutowski, Papadopoulos '05.

IIB maximal Supergravity

All maximally supersymmetric ($N = 32$) backgrounds of IIB are classified [a]:

$AdS_5 \times S^5$ & its Penrose limits the Hpp-wave or $Mink_{1,9}$

$\times$



There are many more examples with less supersymmetry, i.e. between top and bottom:

- fundamental objects with $N = 16$,
- intersections thereof with $N = 1, 2, 4, 8$,
- pp-wave solutions with $N = 20, 24, 28$,
- many more, e.g. including interesting gravity duals.

Only the tip of the iceberg? What are the possible values of N?

Apart from the gravitational and maximally supersymmetric solutions there were no systematic results, i.e. all possibilities for a given N and G.

[a]: Figueroa-O'Farrill, Papadopoulos '02.

Gauge group G

Supersymmetric backgrounds are to be classified up to the use of the gauge group, e.g. for IIB

$$G = Spin(9,1) \times U(1)\,,$$

which consists of the Lorentz and the R-symmetry.

The gauge group is reduced when there are N Killing spinors,

$$G \subset Spin(9,1) \times U(1)\,,$$

where G leaves all Killing spinors invariant.

Holonomy H

The supercovariant derivative squares into the supercurvature

$$[D_M, D_N]\epsilon = \sum_{i=1}^{5} R_{MN,P_1\cdots P_i}\Gamma^{P_1\cdots P_i}\epsilon\,,$$

which takes values in the holonomy group H. IIB has $H = SL(32,\mathbb{R})$. [a]

Due to N Killing spinors the holonomy group is reduced:

$$[D_M, D_N]\epsilon = \sum_{i=1}^{5} R_{MN,P_1\cdots P_i}\Gamma^{P_1\cdots P_i}\epsilon = 0\,, \quad \Rightarrow \quad H \subset SL(32,\mathbb{R})\,.$$

It is the interplay of $G \subseteq H$ that gives rise to complications:

- gravitational solutions, $G = H$,
- adding fluxes, $G \subset H$ in general.

[a]: Papadopoulos, Tsimpis ’03.

Gravitational holonomy

For purely gravitational solutions the Killing spinor equations reduce to

$$D_M \epsilon = (\partial_M + \tfrac{1}{4}\Omega_{M,PQ}\Gamma^{PQ})\epsilon = 0\,,$$

which is the Levi-Civita connection ∇. The corresponding curvature is

$$[D_M, D_N]\epsilon = R_{MN,PQ}\Gamma^{PQ}\epsilon\,,$$

where $R_{MN,PQ}$ is the Riemann tensor.

Holonomy H = gauge group $spin(9,1)$.

All configurations with the same supercurvature are gauge-equivalent.

N Killing spinors ϵ with stability subgroup G

$\Rightarrow$ Riemann tensor $R_{MN,PQ}$ takes values in G

$\Rightarrow$ all G-invariant spinors are Killing,

$\Rightarrow$ $\exists$ a gauge with constant Killing spinors ϵ.

Gravitational solutions

All purely gravitational solutions with special holonomy have been classified:

- Riemannian case: product of metrics with holonomies $SU(D/2)$ (Calabi-Yau's), $Sp(D/4)$, G_2 or $Spin(7)$, [a]
- Lorentzian case: reducible holonomy does not imply product metric. [b]

[a]: Berger '55, [b]: Figueroa-O'Farrill '99, Bryant '00.

More general holonomies

When adding fluxes the holonomy H of the supercovariant connection D is in general extended to $H \subseteq SL(32, \mathbb{R})$:

$$[D_M, D_N]\epsilon = \sum_{i=1}^{5} R_{MN,P_1\cdots P_i} \Gamma^{P_1\cdots P_i}\epsilon\,,$$

with $R_{MN,P_1\cdots P_i}$ given in terms of the spin connection and fluxes.

Possible holonomies:

- purely gravitational: $H \subseteq Spin(9,1)$, $\Rightarrow$ N = 2,4,6,8,16,32,
- only g and F: $H \subseteq GL(16, \mathbb{C})$, $\Rightarrow$ N = 2,4,6,8,10, . . . ,30,32,
- most general backgrounds: $H \subseteq SL(32, \mathbb{R})$, $\Rightarrow$ N = 1,2,3,4,5, . . . ,30,31,32.

Gauge group vs. holonomy

The fact that the gauge group is a (small) subgroup of the holonomy H of the supercurvature is the underlying reason for the difficulty in classifying all supersymmetric solutions.

Not all configurations with the same supercurvature are gauge-equivalent.
Example: maximally supersymmetric solutions with $R = 0$.

Killing spinor ϵ with stability subgroup G
$\not\Rightarrow$ all G-invariant spinors are Killing.

Killing spinors ϵ imply (part of) curvature R is zero,
$\not\Rightarrow$ $\exists$ a gauge with constant Killing spinors ϵ.

More and more complicated cases.

What is the analog of the gravitational special holonomy?

G-structures [a]

N globally well-defined Killing spinors ϵ_i $D\epsilon_i = 0$ isotropy group G	$\Longleftrightarrow$	N^2 globally well-defined diff. forms $\kappa_{ij} = \bar{\epsilon}_i \Gamma^{(p)} \epsilon_j$ diff. relations $\nabla \kappa_{ij} \sim F \kappa_{ij}$ invariant under G

The existence of the global differential forms κ_{ij} implies that the structure group of the frame bundle is reduced: $SO(\mathrm{dim}-1,1) \to G$

Intrinsic torsion: decompose $\nabla \kappa_{ij} \sim \oplus W_i$ - modules (irreps of G)
All $W_i \neq 0$ - most general G-structure (and fluxes), ...
..., all $W_i = 0$ - special holonomy G (no fluxes).

susy-ic solutions without flux $\Leftrightarrow$ manifolds with special holonomy
susy-ic solutions with flux $\Leftrightarrow$ manifolds with G-structures

Different G-structures possible depending on the fluxes. Solve KSEs!

[a]: Gauntlett, Martelli, Pakis, Waldram '02

Solving the Killing spinor equations

Different methods to solve the Killing spinor equations:

Spinor bilinears:

- Uses the N^2 differential relations $\nabla \eta_{ij} \sim F \eta_{ij}$,
- Necessary conditions, check sufficiency by hand,
- 11D: all $N = 1$ [a]

Spinorial geometry:

- Basis in the space of spinors and description in terms of forms,
- Analyses the N Killing spinor equations $D\epsilon_i = 0$ directly,
- Necessary and sufficient conditions,
- 11D: all $N = 1$, some $N = 2, 4, ...$, all $N = 31$ cases [b,c]
- IIB: all $N = 1$, some $N = 2, 4, ...$, all $N = 31$ cases [d]
- Type I: all supersymmetric configurations [e]

[a]: Gauntlett, Pakis '02, Gauntlett, Gutowski, Pakis '03, [b]: Gillard, Gran, Papadopoulos, '04, Gran, Papadopoulos, DR '05, Gran, Gutowski, Papadopoulos, DR '06, [c]: Cariglia, Mac Conamhna '04, Mac Conamhna '05, [d]: Gran, Gutowski, Papadopoulos '05, Gran, Gutowski, Papadopoulos, DR '05, '06, [e]: Gran, Papadopoulos, DR, Sloane '07.

Basis in space of spinors

Spinor in terms of forms [a]:

space of Dirac spinors ϵ of $Spin(9,1)$ dimension 64	$\equiv$	space of forms η spanned by $e_1, \ldots, e_5$ (with compl. coeff.) dimension $2 \cdot 2^5$

Γ-matrices in null and holomorphic basis $M = (-, +, \alpha, \bar{\alpha})$:

$\Gamma_a \eta = \sqrt{2} e_a \wedge \eta$ for $a = (-, \alpha)$ $\Leftrightarrow$ creation operators

$\Gamma_{\bar{a}} \eta = \sqrt{2} e_a \lrcorner \eta$ for $\bar{a} = (+, \bar{\alpha})$ $\Leftrightarrow$ annihilation oper.

Satisfy Clifford algebra $\Gamma_M \Gamma_N + \Gamma_N \Gamma_M = 2 g_{MN}$.

$\Rightarrow$ $\{1, e_a, \ldots, e_{a_1 \cdots a_5}\}$ with $a = (\alpha, 5)$ form a basis for Dirac spinors!
Γ-matrices act as creation/annihilation operators
$\eta = 1$ - Clifford vacuum, $\eta = e_{12345}$ - fully excited state $\Leftarrow$

Weyl spinors $\equiv$ even/odd forms and Majorana spinors $\equiv \eta^* = \Gamma_{6789} \eta$.

[a]: Lawson, Michelsohn '89, Wang '89, Harvey '90

Weyl spinors as forms

Arbitrary Weyl spinor in 10D:

$$\epsilon = f^i \eta_i \,,$$

composed of the sixteen MW basis elements η_i

$$1 + e_{1234}, i(1 - e_{1234}), e_{\alpha\beta} - \tfrac{1}{2}\epsilon_{\alpha\beta}{}^{\gamma\delta} e_{\gamma\delta}, i(e_{\alpha\beta} + \tfrac{1}{2}\epsilon_{\alpha\beta}{}^{\gamma\delta} e_{\gamma\delta}) \,,$$
$$e_{\alpha 5} + \tfrac{1}{6}\epsilon_{\alpha}{}^{\beta\gamma\delta} e_{\beta\gamma\delta 5}, i(e_{\alpha 5} - \tfrac{1}{6}\epsilon_{\alpha}{}^{\beta\gamma\delta} e_{\beta\gamma\delta 5}) \,,$$

with $\alpha = 1, 2, 3, 4$.

Functions f_i dependent on space-time:

- real for type I spinors,
- complex for type IIB spinors.

Killing spinor equations

Supercovariant connection D_μ and algebraic constraint A consists of products of spin connection $\Omega_{M,PQ}$, derivatives of scalars P_M, Q_M and fluxes $G_{M_1M_2M_3}, F_{M_1\ldots M_5}$ with Γ-matrices.

Substitute ϵ and expand in basis (amounts to products of Γ-matrices), and set all coefficients equal to zero [a].

KSE reduces to linear system of equations for scalars, fluxes, spin connection and functions f (and their derivatives) of Killing spinor.

Very complicated linear system for geometry and fluxes due to N arbitrary spinors.

Problem is reduced to parametrising the N Killing spinors. Orbit analysis for every N!

[a]: Papadopoulos et al '04, '05, Mac Conamhna '04, '05.

$N = 1$ orbits of IIB

Bottom-up approach:

Using Lorentz symmetry, any Killing spinor can be brought to one of the three orbit representatives with stability subgroup G:

$G = Spin(7) \ltimes \mathbb{R}^8$ with $\epsilon = (f_1 + if_2)(1 + e_{1234})$
$G = SU(4) \ltimes \mathbb{R}^8$ with $\epsilon = (f_1 + if_2)1 + (h_1 + if_2)e_{1234}$
$G = G_2$ with $\epsilon = f_1(1 + e_{1234}) + ig_1(e_{15} + e_{2345})$

Simple form, plugging into KSE gives (relatively) simple linear system. Gives complicated geometries [a].

For $N \geq 2$ the orbit analysis is much more complicated.up to $N = 31$!

[a]: Gran, Gutowski, Papadopoulos '05.

$N = 31$ orbits of IIB

Top-down approach:

$N = 31$ Killing spinors ϵ_i defining one orthogonal spinor: $< \epsilon_i, \nu >= 0$.

Use Lorentz symmetry to bring ν to one of three orbit representatives.

Alg. KSE of IIB $\Rightarrow$ scalars and three-form field strengths vanish.

$$\Rightarrow \text{holonomy } GL(16, \mathbb{C}) \in SL(32, \mathbb{R}) \Rightarrow N \text{ even} \Rightarrow N = 32.$$

$N = 31$ is not IIB [a]

First constraint on N in type II theories.

[a]: Gran, Gutowski, Papadopoulos, DR '06.

$N = 31$ in IIA and 11D

Same result in IIA using moving G-frame method [a]. Algebraic KSE implies that all fluxes vanish.

In 11D there is no algebraic KSE. However, the $N = 31$ supercurvature vanishes on-shell [b].

No $N = 31$ by quotients of $N = 32$ either [c].

[a]: Bandos, De Azcárraga, Varela ’06, [b]: Gran, Gutowski, Papadopoulos, DR ’06, [c]: Figueroa-O’Farrill, Gadhia ’07.

Type I theory

Truncation of IIB to type I with 16 supercharges and coupled to arbitrary number of vector multiplets.

The supercovariant derivative is given by

$$D_M = \partial_M + \tfrac{1}{4}\Omega_{M,PQ}\Gamma^{PQ} + \tfrac{1}{2}H_{MPQ}\Gamma^{PQ} \,,$$

Three-form flux acts as torsion but does not enlarge the holonomy H.

Same simplifications as in gravitational case. All backgrounds with parallel spinors $D_M\epsilon = 0$ have been classified [a].

But parallel spinors are not enough for supersymmetry:

- gaugino variation $\delta\chi = F_{PQ}\Gamma^{PQ}\epsilon = 0$
  (allows for a Lie-theoretic analysis)
- dilatino variation $\delta\lambda = (\partial_M\phi\Gamma^M - \frac{1}{12}H_{MNP}\Gamma^{MNP})\epsilon = 0$
  (has to be solved explicitly)

What are the type I backgrounds with P parallel spinors of which $N < P$ are Killing? Orbit analysis!

[a]: Gran, Gutowski, Lohrmann, Papadopoulos ’05.

Bottom up approach

Killing spinors can be taken constant due to $G = H$.

Using the Lorentz symmetry, any 10D Majorana-Weyl spinor can be brought to the form

$$\epsilon_1 = 1 + e_{1234} \, ,$$

i.e. there is only one orbit. The remaining gauge symmetry is its stability subgroup $G = Spin(7) \ltimes \mathbb{R}^8$.

For the second spinor, there are two orbits. Using the remaining G, it can be brought to the form

$$\epsilon_2 = i(1 - e_{1234}) \, ,$$

with $G = SU(4) \ltimes \mathbb{R}^8$, or

$$\epsilon_2 = e_{15} + e_{2345} \, ,$$

with $G = G_2$.

For $N \geq 2$, this splits up in chains of orbits with non-compact and compact stability subgroups.

Orbits with non-compact stability subgroups

These are composed of basis elements

$$1 + e_{1234}, \quad i(1 - e_{1234}), \quad e_{ij} - \epsilon_{ij}{}^{kl} e_{kl}, \quad i(e_{ij} + \epsilon_{ij}{}^{kl} e_{kl}),$$

which transform in the fundamental of $SO(8)$. This allows one to bring any spinor to the form of a basis element.

Hence one can choose the following representatives:

- $\epsilon_1 = 1 + e_{1234}$ $\qquad G = Spin(7) \ltimes \mathbb{R}^8$
- $\epsilon_2 = i(1 - e_{1234})$ $\qquad G = SU(4) \ltimes \mathbb{R}^8$
- $\epsilon_3 = e_{12} - e_{34}$ $\qquad G = Sp(2) \ltimes \mathbb{R}^8$
- $\epsilon_4 = i(e_{12} + e_{34})$ $\qquad G = (SU(2) \times SU(2)) \ltimes \mathbb{R}^8$
- $\epsilon_5 = e_{13} + e_{24}$ $\qquad G = SU(2) \ltimes \mathbb{R}^8$
- $\epsilon_6 = i(e_{13} - e_{24})$ $\qquad G = U(1) \ltimes \mathbb{R}^8$
- $\epsilon_7 = e_{14} - e_{23}$ $\qquad G = \mathbb{R}^8$
- $\epsilon_8 = i(e_{14} + e_{23})$ $\qquad G = \mathbb{R}^8$

The stability subgroups are isomorphic to $SO(8 - N)$, acting in the fundamental representation on the remaining $8 - N$ Killing spinors, hence there is only one case per N.

Orbits with compact stability subgroups

One can choose the following representatives:

- $\epsilon_1 = 1 + e_{1234}$ $\qquad G = Spin(7) \ltimes \mathbb{R}^8$
- $\epsilon_2 = e_{15} + e_{2345}$ $\qquad G = G_2$
- $\epsilon_{3-A} = i(1 - e_{1234})$ $\qquad G = SU(3)$
  $\epsilon_{3-B} = i(1 - e_{1234}) + e_{25} - e_{1345}$ $\qquad G = SU(2)$
- $\epsilon_{4-A1} = i(e_{15} - e_{2345})$ $\qquad G = SU(3)$
  $\epsilon_{4-A2} = i(e_{12} + e_{34})$ $\qquad G = SU(2)$
  $\epsilon_{4-A3} = i(e_{15} - e_{2345}) + e_{12} - e_{34}$ $\qquad G = SU(2)$
  $\epsilon_{4-A4} = i\sin(\phi)(e_{15} - e_{2345}) + \cos(\phi)(e_{25} - e_{1345})$ $\qquad G = SU(2)$
  $\epsilon_{4-B} = i\sin(\phi)(e_{15} - e_{2345}) + i\cos(\phi)(e_{12} + e_{34})$ $\qquad G = SU(2)$
  plus a number of unknown orbits with $\qquad G = 1$

All $G \neq 1$ orbits with $N > 4$ can be determined by the top down approach, using the Lorentz symmetry on the normal spinors.

All orbits of type I Killing spinors with $G \neq 1$. [a]

[a]: Gran, Papadopoulos, DR, Sloane '07.

Type I orbits [a]

$G =$ \ $N =$	1	2	3	4	5	6	7	8	16
G_2	–	√	–	–	–	–	–	–	–
$SU(3)$	–	–	√	√	–	–	–	–	–
$SU(2)$	–	–	√	√	√	√	√	√	–
1	–	–	–	?	?	?	?	?	?
$Spin(7) \ltimes \mathbb{R}^8$	√	–	–	–	–	–	–	–	–
$SU(4) \ltimes \mathbb{R}^8$	–	√	–	–	–	–	–	–	–
$Sp(2) \ltimes \mathbb{R}^8$	–	–	√	–	–	–	–	–	–
$(SU(2) \times SU(2)) \ltimes \mathbb{R}^8$	–	–	–	√	–	–	–	–	–
$SU(2) \ltimes \mathbb{R}^8$	–	–	–	–	√	–	–	–	–
$U(1) \ltimes \mathbb{R}^8$	–	–	–	–	–	√	–	–	–
$\mathbb{R}^8$	–	–	–	–	–	–	√	√	–

√: Type I background with N Killing spinors with stability subgroup G.
?: occur but with unknown orbits.
–: do not occur.

[a]: Gran, Papadopoulos, DR, Sloane '07.

Complete classification

For $G \neq 1$ we have [a]:

- determined the different orbits for all values of N,
- solved the corresponding Killing spinor equations.

For $G = 1$ the backgrounds are parallelisable which have been classified [b], include e.g. $AdS_3 \times S^3 \times S^3 \times \mathbb{R}$.

Full classification of all solutions to the Killing spinor equations of type I supergravity!

In some cases, part of the field equations still need to be imposed.

Look for interesting string backgrounds?

[a]: Gran, Papadopoulos, DR, Sloane '07, [b]: Figueroa-O'Farrill, Kawano, Yamaguchi '03.

Summary

- Classification of supersymmetric backgrounds with N Killing spinors with stability subgroup G,
- Spinorial geometry:
  - basis in the space of spinors in terms of forms $e_{a_1 \cdots a_i}$
  - converts KSEs for N arbitrary Killing spinors to linear system
  - use of gauge symmetry $Spin(9,1)$ to determine orbit structure of Killing spinors
  - bottom up and top down approach
- Allows for a full classification of all orbits and the KSE solutions in type I supergravity,
- $N = 1$ and $N = 31$ in IIB supergravity, more complicated orbit structure for $2 \leq N \leq 30$.

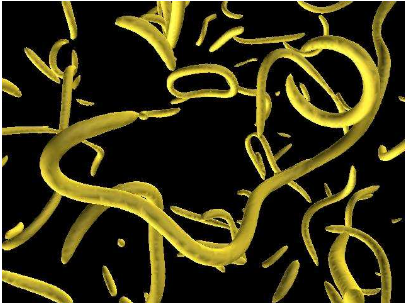
Outlook

- pp-wave with $N = 28$ in IIB. What about $N = 29, 30$, i.e. what is the lowest non-maximal number of supersymmetries?
- Conjecture based on warped product Ansatz [a]:

$$N = 1, 2, 3, 4, 5, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 32,$$

  Consistent with data. Derivation?
- $G = 1$ leads to very strong constraints, e.g. parallelisability in type I. What is the analog in IIB?
- Generic $N = 16$ Killing spinors lead to $P = G = 0$ and hence $N = 32$. What are the remaining possibilities?

[a]: Duff '02.



Thanks for your attention!