Different approaches to Dark Energy include amongst many:

- A true cosmological constant -- but why this value?
- Solid –dark energy such as arising from frustrated network of domain walls.
- Time dependent solutions arising out of evolving scalar fields -- Quintessence/K-essence.
- Modifications of Einstein gravity leading to acceleration today.
- Anthropic arguments.
- Perhaps GR but Universe is inhomogeneous.

Over 1800 papers on archives since 1998 with dark energy in title.

Early evidence for a cosmological constant type term.

1987: Weinberg argued that anthropically ρ_{vac} could not be too large and positive otherwise galaxies and stars would not form. It should be not be very different from the mean of the values suitable for life which is positive, and he obtained $\Omega_{vac} \sim 0.6$

1990: Observations of LSS begin to kick in showing the standard $\Omega_{CDM} = 1$ struggling to fit clustering data on large scales, first through IRAS survey then through APM (Efstathiou et al)

1990: Efstathiou, Sutherland and Maddox - Nature (238) -- explicitly suggest a cosmology dominated today by cosmological constant with $\Omega_{vac} < 0.8$!

1998: Type Ia SN show evidence of cosm const.

The problem with the cosmological constant

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} - \lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Einstein (1917) -- static universe with dust

Not easy to get rid of it, once universe found to be expanding.

Anything that contributes to energy density of vacuum acts like a cosmological constant

$$< T_{\mu\nu} > = < \rho > g_{\mu\nu}$$

Lorentz inv

$$\lambda_{eff} = \lambda + 8\pi G < \rho >$$

or

$$\rho_V = \lambda_{eff} / 8\pi G$$

Effective cosm const

Effective vac energy

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho + \lambda - \frac{k}{a^2}$$

$$H_0 \simeq 10^{-10} yr^{-1} : \frac{|k|}{a_0^2} \leq H_0^2 : |\rho - < \rho > | \leq \frac{3H_0^2}{8\pi G}$$

Age

Flat

Non-vac matter

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho + \lambda - \frac{k}{a^2}$$

$$H_0 \simeq 10^{-10} yr^{-1} : \frac{|k|}{a_0^2} \leq H_0^2 : |\rho - <\rho>| \leq \frac{3H_0^2}{8\pi G}$$

Hence: $$\lambda_{eff} \leq H_0^2 \; or |\rho_V| \leq 10^{-29} gcm^{-3} \simeq 10^{-47} GeV^4$$

Problem: expect $<\rho>$ of empty space to be much larger. Consider summing zero-point energies ($\hbar\omega/2$) of all normal modes of some field of mass m up to wave number cut off Λ>>m:

$$<\rho> = \int_0^\Lambda \frac{4\pi k^2 dk}{2(2\pi)^3}\sqrt{k^2+m^2} \simeq \frac{\Lambda^4}{16\pi^2}$$

For many fields (i.e. leptons, quarks, gauge fields etc...):

$$< \rho > = \frac{1}{2} \sum_{\text{fields}} g_i \int_0^{\Lambda_i} \sqrt{k^2+m^2}\,\frac{d^3k}{(2\pi)^3} \simeq \sum_{\text{fields}} \frac{g_i \Lambda_i^4}{16\pi^2}$$

where g_i are the dof of the field (+ for bosons, - for fermions).

Imagine just one field contributed an energy density $\rho_{cr} \sim (10^{-3}\,\text{eV})^4$. Implies the cut-off scale Λ<0.01 eV -- well below scales we understand the physics of.

Planck scale: $\Lambda \simeq (8\pi G)^{-1/2} \rightarrow <\rho> \simeq 2 \times 10^{71} GeV^4$

But: $|\rho_V| = |<\rho> + \lambda/8\pi G| \leq 2 \times 10^{-47} GeV^4$

Must cancel to better than 118 decimal places.

Even at QCD scale require 41 decimal places!

Very unlikely a classical contribution to the vacuum energy density will cancel this quantum contribution to such high precision

Not all is lost -- what if there is a symmetry present to reduce it? Supersymmetry does that. Every boson has an equal mass SUSY fermion partner and vice-versa, so their contributions to $<\rho>$ cancel.

However, SUSY seems broken today - no SUSY partners have been observed, so they must be much heavier than their standard model partners. If SUSY broken at scale M, expect $<\rho> \sim M^4$ because of breakdown of cancellations. Current bounds suggest M~1TeV which leads to a discrepancy of 60 orders of magnitude as opposed to 118 !

Still a problem of course -- is there some unknown mechanism perhaps from quantum gravity that will make the vacuum energy vanish ?

A few issues over the cosmological constant:

Is the observed dark energy really representing the energy of the vacuum or is it just that we have not yet reached it and it is a dynamical process?

The cosmological constant is the simplest addition, requires nothing other than one more fundamental constant and requires no modification of GR or addition of new fields.

How does it relate to early universe inflation? That lasted a finite time, perhaps this will implying there is nothing special about our vacuum.

Maldacena has shown stable QG vacuum of negative vacuum energy can exist (AdS/CFT), as can vacuum of zero energy (include SUSY). No one has shown a stable positive vacuum energy is possible in theories of QG. [Witten 2008]

This would imply our Universe is unstable - perhaps a bit drastic!

Quintessence and M-theory -- where are the realistic models?

`No go' theorem: forbids cosmic acceleration in cosmological solutions arising from compactification of pure SUGR models where internal space is time-independent, non-singular compact manifold without boundary --[Gibbons]

Why? : 1. acceleration requires violation of strong energy condition.

i.e $R_{00} \leq 0$

2. Strong energy condition not violated by either 11D SUGR or any of the 10D SUGR theories

3. For any compactification described above, if higher dim stress tensor satisfies SEC then so does the lower dimensional stress tensor.

Recent extension: forbids four dimensional cosmic acceleration in cosmological solutions arising from warped dimensional reduction --[Wesley 08]

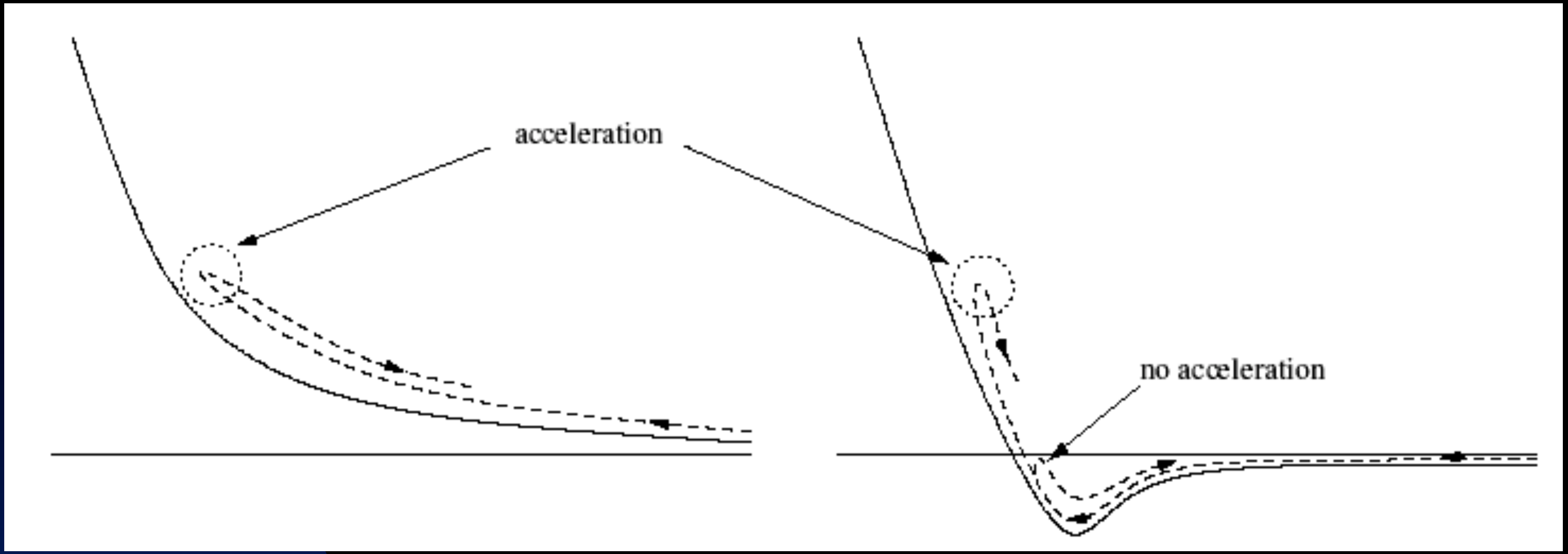
To avoid no-go theorem need to relax conditions of the theorem.

1. Drop condition that internal space is compact, but not so realistic -- Townsend

2. Allow internal space to be time-dependent, analogue of time-dependent scalar fields -- Lukas et al, Kaloper et al, Townsend & Wohlfarth, Emparan & Garriga.

Compactified spaces are hyperbolic and lead to cosmologies with transient accelerating phase. Four dimensional picture, solutions correspond to bouncing the radion field off its exponential potential.

Acceleration occurs at the turning point where the radion stops and potential energy momentarily dominates.



Emparan & Garriga

- Field starts at large positive values, with large kinetic energy.
- At turning point, energy is pot dominated and acceleration.
- Left picture, two positive potentials, right picture, sum of positive and negative potentials.

Problems:

Current realistic potentials are too steep

These models have kinetic domination, not matter domination before entering accelerated phase.

Four form Flux and the cosm const: [Bousso and Polchinski]

Effective 4D theory from $M^4 x S^7$ compactification

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2\kappa^2} R + \Lambda_b - \frac{1}{2 \cdot 4!} F_4^2 \right)$$

Negative bare cosm const: $-\Lambda_b$

EOM: $\nabla_\mu \left(\sqrt{-g} F^{\mu\nu\rho\sigma} \right) = 0$ $\rightarrow F^{\mu\nu\rho\sigma} = c\epsilon^{\mu\nu\rho\sigma}$

Eff cosm const: $\Lambda = -\Lambda_b - \frac{1}{48} F_4^2 = -\Lambda_b + \frac{c^2}{2}$

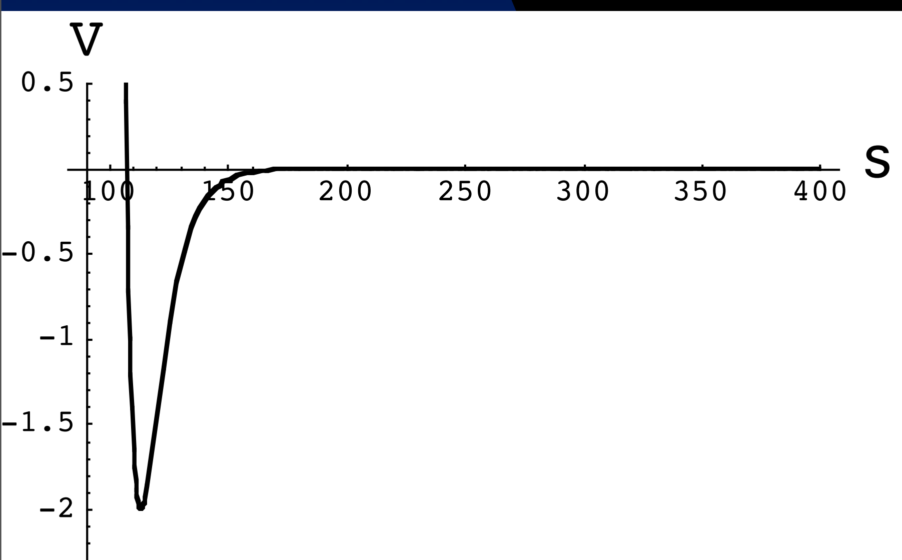
Quantising c and considering J fluxes $\Lambda = -\Lambda_b + \frac{1}{2} \sum_{i=1}^{J} n_i^2 q_i^2$

Observed cosm const with J~100

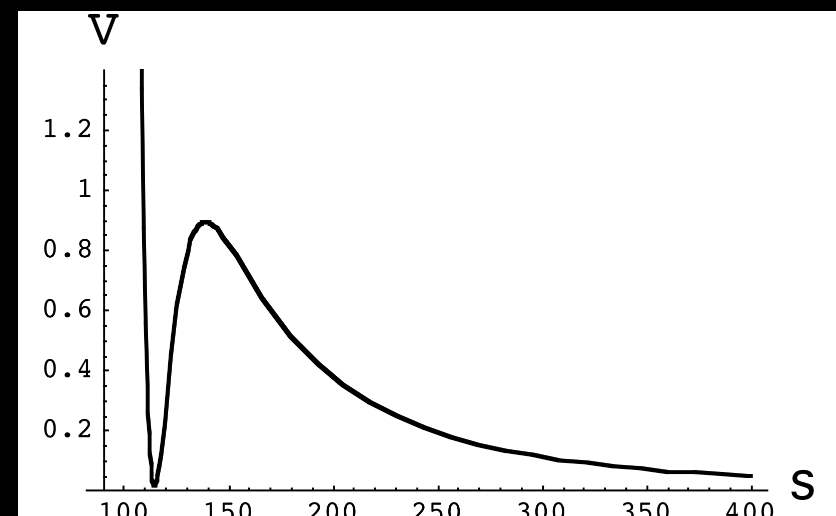
Still needed to stabilise moduli but opened up way of obtaining many de Sitter vacua using fluxes -- String Landscape in which all the vacua would be explored because of eternal inflation.

Example of stabilised scenario: Metastable de Sitter string vacua in TypeIIB string theory, based on stable highly warped IIB compactifications with NS and RR three-form fluxes. [Kachru, Kallosh, Linde and Trivedi 2003]

Metastable minima arises from adding positive energy of anti-D3 brane in warped Calabi-Yau space.



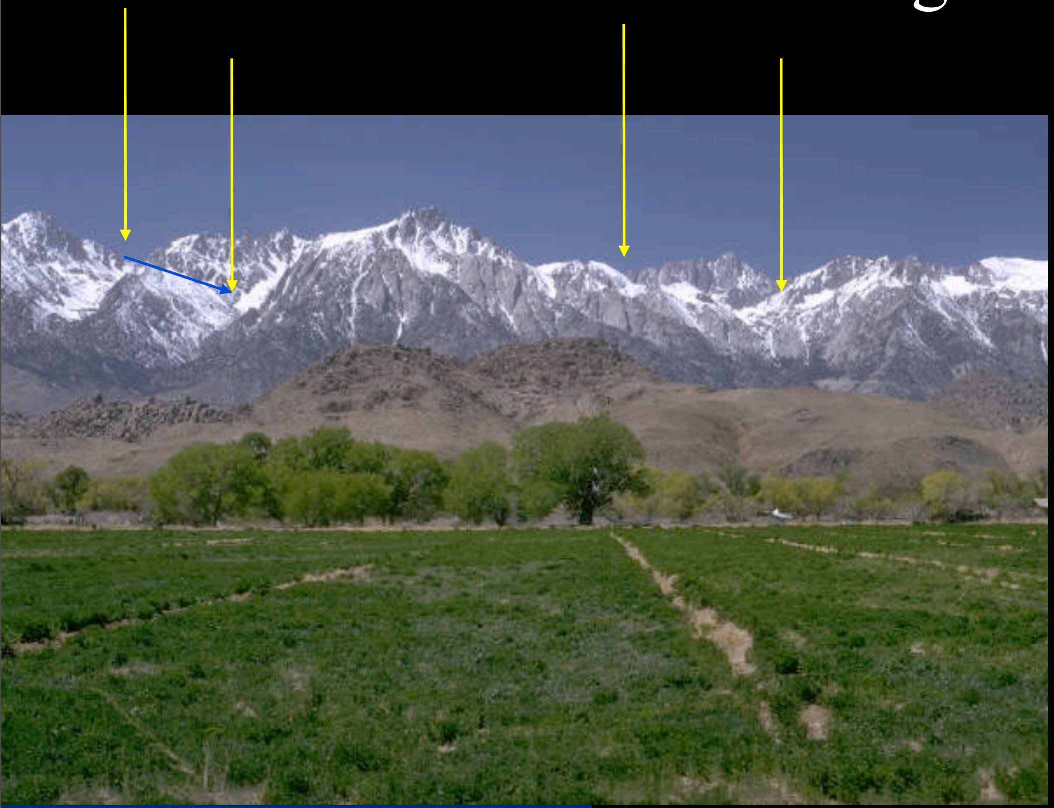
AdS minimum



Metastable dS minimum

$$V_{\mathrm{KKLT}} = V_{\mathrm{AdS}} + \frac{D}{\sigma^2}$$

1.The String Landscape approach



Type IIB String theory compactified from 10 dimensions to 4.

Internal dimensions stabilised by fluxes.

Many many vacua ~ 10^{500} !

Typical separation ~ $10^{-500}\ \Lambda_{pl}$

Assume randomly distributed, tunnelling allowed between vacua --> separate universes .

Anthropic : Galaxies require vacua < $10^{-118}\ \Lambda_{pl}$ [Weinberg] Most likely to find values not equal to zero!

Some Landscape predictions

1. Most likely our local universe born in tunnelling event from neighbouring vacuum leading to open FRW with small negative spatial curvature $\Omega_k<0$ [Freivogel et al 05] .

2. Including dynamics on probability distribution of landscape vacua. Starting from generic initial conditions, most fluxes are dynamically driven to different and narrower range of values than expected from landscape statistics alone. [Bousso and Yang (2007)]

In particular they argue cosmological evolution accesses a tiny fraction of vacua with a small cosmological constant.

Some Landscape predictions continued

3. Landscape and the weak anthropic principle [Ellis and Smolin (2009)]

Argue can make falsifiable prediction for String Landscape.

WAP - existence of life can be explained by random selection from an ensemble of universes with different properties.

If infinitely more vacua with one sign of parameter over another, within anthropically allowed range, then under weak assumptions about the probability measure, a firm prediction is obtained favouring that sign of the parameter.

Applied to the current understanding of the Lanscape it implies a negative cosmological constant is predicted.

This then requires either an infinite discretum of anthropically allowed vacua for $\Lambda>0$, or the reduction of the infinite number of $\Lambda<0$ solutions to a finite number.

[Witten 2008]

Landscape gives a realisation of the multiverse picture.

There isn’t one true vacuum but many so that makes it almost impossible to find our vacuum in such a Universe which is really a multiverse.

So how can we hope to understand or predict why we have our particular particle content and couplings when there are so many choices in different parts of the universe, none of them special ?

This sounds like bad news, we will rely on anthropic arguments to explain it through introducing the correct measures and establishing peaks in probability distributions.

Or perhaps, it isn’t a cosmological constant, but a new field such as Quintessence which will eventually drive us to a unique vacuum with zero vacuum energy -- that too has problems, such as fifth force constraints, as we will see.

For a critique of interpreting and using multiverse see talk by George Ellis at Emmanuel College Nov 07

For a defence of the Landscape and its predictive power see Polchinski - hep/th/0603249

2. Λ from a self-tuning universe [Feng et al 2001].

Λ relaxes through nucleation of branes coupled to gauge potential, the particular branes depending on the compactification assumed. Need rapid relaxation from high energy scales but remains stable over age of universe today.

Leads to constraint $M^2_{\mathrm{SUSY}} \leq (10^{-3}\mathrm{eV})(M_{\mathrm{Planck}})$

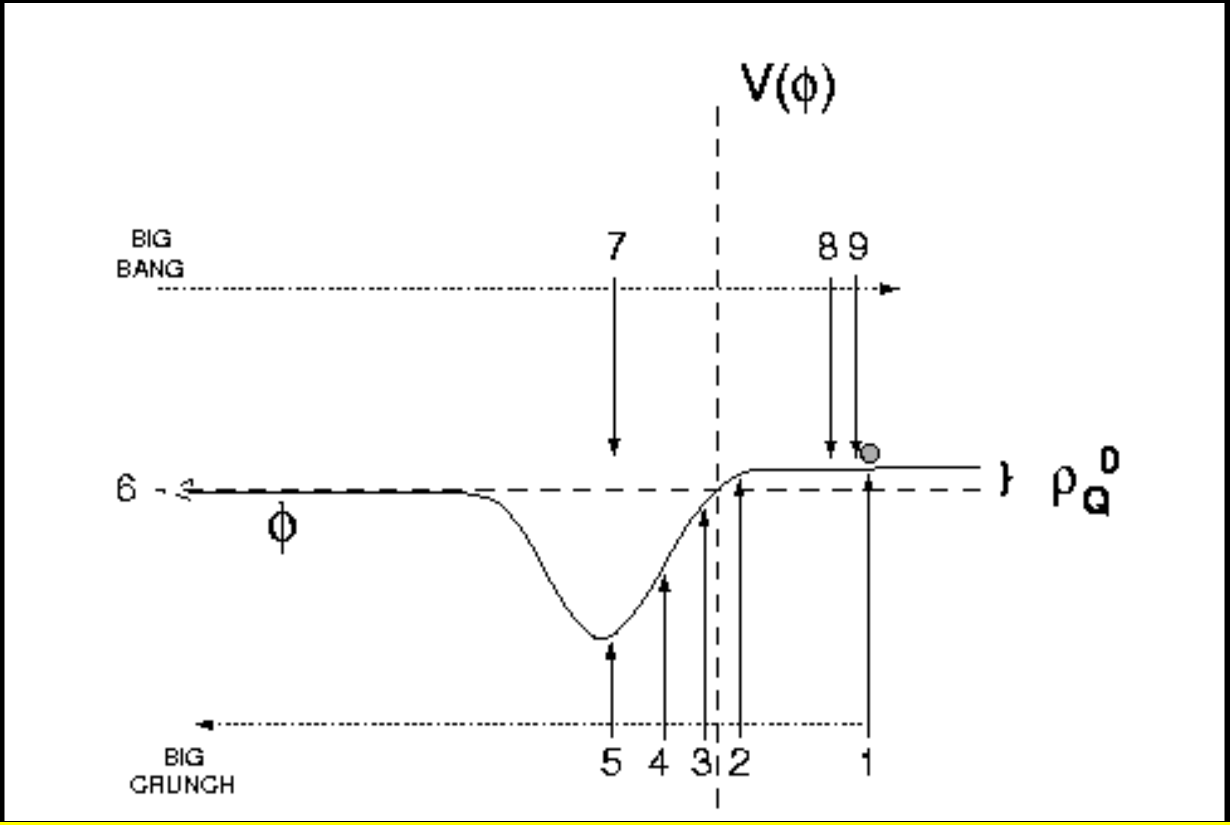
3. Relaxation of Λ [Kachru et al 2000, Arkani Hamad et al 2000].

Relies on presence of extra dimension to remove the gravitational effect of the vacuum energy.

3 brane solns in 5D eff theories leads to standard model vacuum energy warping the higher dimensional spacetime while preserving 4D flatness with no cosm constant. Quantum treatment of standard model implies result stable against quantum loops and changes to standard model couplings. Problems with evolving constants and singularities [Nilles et al]

4. Λ from the Cyclic Perspective [Steinhardt and Turok 2002, 2006].

1. Quintessence (trillion years)
2. Decelerated expansion (billion yrs)
3. H=0, contraction begins.
4. Density flucns on observed scales.
5. KE dom
6. Bounce and reversal.
7. End of KE dom
8. RD begins (10^{-25} s)
9. MD begins (10^{10}s)
10. Pot dominates, field turns round, back to (1)



$$S = \int d^4x\, a^4 \left[\frac{1}{2} R + \frac{1}{2} (\nabla\phi)^2 - V(\phi) + \beta^4(\phi) \rho_R \right]$$

Require: $\alpha\beta \rightarrow \text{const as } a \rightarrow 0$

for finite energy density at crunch.

Require exponential potential for scale-inv fluctuations during contraction

Propose dynamical mechanism based on earlier approach of Abbott, that automatically relaxes the value of Λ, including contribution to vacuum density at all scales.

Relaxation time grows exponentially as vac den decreases, so almost all space spends majority of time at the stage when Λ is small and positive.

Key feature, because many cycles and each cycle lasts a trillion years, universe today is much older than today's Hubble time, so Λ has had long time to reduce to the observed value today.

5. Supersymmetric Large Extra Dims and Λ [Burgess et al].

Solutions to 6D Supergravity

In more than 4D, the 4D vacuum energy can curve the extra dimensions instead of the observed 4 dimensions [Carroll and Guica; Aghababaie et al]

Proposal: Physics is 6D above 10^{-2} eV scale with supersymmetric bulk. We live in 4D brane with 2 extra dim.

Integrate out brane physics leads to large 4D vacuum energy, but it is localised in extra dimensions.

Integrate out classical contributions in bulk and find tensions cancel between bulk and brane.

Static and time dependent solutions exist, most of them runaway with rapid growing or shrinking dimensions.

Albrecht-Skiordis type quintessence evolution leads to late time acceleration and testable predictions.

6. Anthropic selection of Λ [Weinberg, Linde, Vilenkin, Efstathiou …].

Weinberg pointed out that once Λ dominates energy density, structure formation stops because density perturbations cease to grow. Need structure formation to complete before this otherwise no observers today. Leads to

$$\rho_\Lambda < 500\rho_m^{(0)}$$

Two orders of magnitude out.

What if Λ differs in different parts of universe? [Efstathiou et al (1990) , Garriga and Vilenkin (2000)].

Intro conditional prob density

$$\mathrm{d}\mathcal{P}(\rho_\Lambda) = \mathcal{P}_*(\rho_\Lambda) n_G(\rho_\Lambda) \mathrm{d}\rho_\Lambda$$

$n_G(\rho_\Lambda)$ Ave number of galaxies that can form per unit vol

$\mathcal{P}_*(\rho_\Lambda)$ A Priori probability density distribution on Λ

For a flat a priori probability density distribution it has been shown that peaks around $\mathcal{P}(\rho_\Lambda)$

$$\rho_{\rm vac} \sim 8\rho_m^{(0)}$$

[Martel et al (1998)]

Two important aspects to Anthropic argument:

1. Prediction of a priori probability
2. Assuming Λ takes on diff values in diff parts of universe.

How are we going to determine the a priori probability?

See also [Garriga. et al (2005,2007), Linde (2007), Bousso et al (2007), Gibbons and Turok (2007), Easther et al (2005), Vanchurin (2007)…]

A great deal of work going on trying to determine possible measures on the multiverse and the Landscape as a manifestation of that -- no definitive conclusion yet.

Testing for a true Λ -- [Zunckel and Clarkson (2008)]

Proposed neat way of determining whether w=const based on luminosity distance.

$$H_0 d_L(z) = \frac{1+z}{\sqrt{-\Omega_k}} \sin\left(\sqrt{-\Omega_k}\int_0^z dz' \frac{H_0}{H(z')}\right) = (1+z)D(z)$$

$$\mathbf{H^2(z) = H_0^2\left(\Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_{de}\exp\left(3\int_0^z \frac{1+w(z')}{1+z'}dz'\right)\right)}$$

$$w(z) = \frac{2(1+z)(1+\Omega_k D^2)D'' - [(1+z)^2\Omega_k D'^2 + 2(1+z)\Omega_k DD' - 3(1+\Omega_k D^2)]D'}{3((1+z)^2[\Omega_k + (1+z)\Omega_m]D'^2 - (1+\Omega_k D^2))D'}$$

Rather than fit for w(z) from d_L which relies on accurate knowledge of Ω_k and Ω_m, instead look for consistency of flat ΛCDM where:

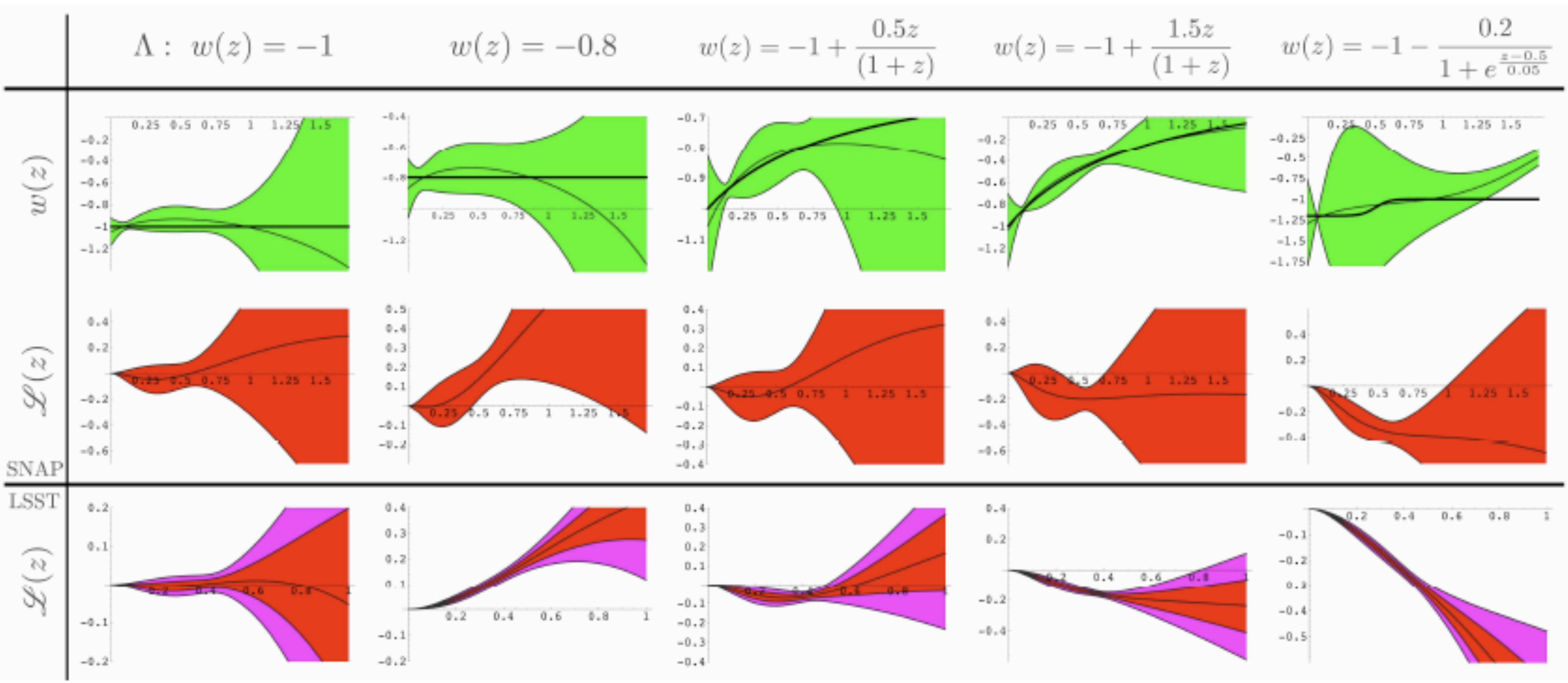
$$\Omega_m = \frac{1 - D'^2(z)}{[(1+z)^3 - 1]D'^2(z)} = \Omega_m(z)$$

$$\mathcal{L}(z) = \Omega_m'(z) = 0$$

the final equation being independent of Ω_m!

Take parameterised form of D(z) and fit to data, RHS should be indep of z.

If L(z) outside of n-σ error bars have n-σ evidence of deviation from Λ



[Zunckel and Clarkson (2008)]