F(R) gravity from scalar-tensor theory and inhomogeneous EoS dark energy

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FLRW cosmology

Flat FLRW metric

$$ds^2 = -dt^2 + a^2(t)\left(dx^2 + dy^2 + dz^2\right) \quad (1)$$

Friedmann equations

From Einstein equations:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa^2 T_{\mu\nu} \longrightarrow \begin{cases} H^2(t) = \frac{\kappa^2}{3}\sum_i \rho_i \\ \dot{H}(t) = -\frac{\kappa^2}{2}\sum_i(\rho_i + p_i) \end{cases} \quad (2)$$

Equation of state for each perfect fluid that fills our Universe: $p_i = w_i\rho$. Acceleration parameter:

$$\frac{\ddot{a}}{a} = -\frac{\kappa^2}{6}\sum_i(\rho_i + 3p_i) \quad (3)$$

Current observations

Constraints on EoS parameter for dark energy

Since 1998 the observations suggest that $\frac{\ddot{a}}{a} > 0$, then the effective EoS has to be:

$$w_{eff} < -1/3 \Longrightarrow \text{We need some new form of energy:}$$

Dark Energy or a **modification of the law of gravity.**
Restrictions on Dark energy:

$$w_{DarkEnergy} \sim -1 \ , \quad \text{even less than -1(phantom case)}$$

$$\Omega_{DE} \sim 0.73 \tag{4}$$

Action for F(R) gravity and field equations

Field equations

Action:

$$S=\int d^4x\sqrt{-g}\left(f(R)+L_m\right) \ ,$$

$$R_{\mu\nu}f'(R)-\frac{1}{2}g_{\mu\nu}f(R)+g_{\mu\nu}\Box f'(R)-\nabla_\mu\nabla_\nu f'(R)=\frac{\kappa^2}{2}T^{(m)}_{\mu\nu} \ . \quad (5)$$

Modified Friedmann equations:

$$\frac{1}{2}f(R)-3(H^2+\dot{H})f'(R)+18f''(R)(H^2\dot{H}+H\ddot{H})=\frac{\kappa^2}{2}\rho_m \ ,$$

$$\frac{1}{2}f(R)-(3H^2+\dot{H})f'(R)-\Box f'(R)=-\frac{\kappa^2}{2}p_m \ . \quad (6)$$

F(R) gravity in terms of a scalar field

F(R) gravity and scalar-tensor theory

We start from a Brans-Dicke-like theory given by an scalar field ϕ:

$$S = \int d^4x\sqrt{-g}\,(P(\phi)R + Q(\phi) + L_m)\ , \tag{7}$$

Then, the Friedmann equations take the form:

$$\begin{aligned} 3H\frac{dP(\phi)}{dt} + 3H^2P(\phi) + \frac{1}{2}Q(\phi) - \frac{\rho_m}{2} = 0\ , \\ 2\frac{d^2P(\phi)}{dt^2} + 4H\frac{dP(\phi)}{dt} + (4\dot{H} + 6H^2)P(\phi) + p_m = 0\ . \end{aligned} \tag{8}$$

And the scalar field equation:

$$P'(\phi)R + Q'(\phi) = 0\ \to \phi = \phi(R) \to F(R) = P(\phi(R))R + Q(\phi(R)) \tag{9}$$

Solving the field equations

Solution in absence of matter

For simplicity, we consider the above equations in absence of matter, then the following solutions are found :

$$\phi = t \quad \text{and} \quad H(t) = g(t) \ , \quad \text{where}$$

$$g(\phi) = -\sqrt{P(\phi)} \int d\phi \frac{P''(\phi)}{2P^{2/3}(\phi)} + kP(\phi) \tag{10}$$

Thus, the solution for the Hubble parameter in a flat FLRW Universe is found. The F(R) function can be recovered. Hence, any cosmology can be expressed in terms of an F(R) theory derived from a scalar-tensor theory.

Some F(R) cosmology examples

Examples

Example 1. For $P(\phi) = \phi^{\alpha}$,

$$g(\phi) = k\phi^{\alpha/2} + \frac{\alpha(\alpha-1)}{\alpha+2}\frac{1}{\phi} \implies H(t) = kt^{\alpha/2} + \frac{\alpha(\alpha-1)}{\alpha+2}\frac{1}{t} . \tag{11}$$

Then,

$$\boxed{f(R) = [R - 6(k(k+1)+5/2)]\,\frac{R - 2k \pm \sqrt{R(1-2k)}}{2} + const} . \tag{12}$$

Example 2. For $P(\phi) = A(t_s - \phi)^{\alpha_+} + B(t_s - \phi)^{\alpha_-}$,

$$g(\phi) \sim \frac{H_0}{t_s - \phi} \quad \to H(t) \sim \frac{H_0}{t_s - t} \quad \text{and} \quad \frac{\ddot{a}}{a} \sim \frac{H_0(H_0+1)}{(t_s - t)^2} . \tag{13}$$

$$\implies \boxed{f(R) \sim R^{1-\alpha_-/2}} . \tag{14}$$

F(R) gravity as a "Dark fluid"

EoS for F(R)

We consider $f(R)$ as a perfect fluid contribution in the Friedmann Equations:

$$3H^2 = \kappa^2 \rho_m + \rho_{F(R)} \ , \quad 3H^2 + 2\dot{H} = -\kappa^2 p_m - p_{F(R)} \ , \qquad (15)$$

where,

$$\rho_{F(R)} = -\frac{1}{2}F(R) + 3(H + \dot{H})F'(R) - 18F''(R)(H^2\dot{H} + H\ddot{H}) \ ,$$
$$p_{F(R)} = \frac{1}{2}F(R) - (3H^2 + \dot{H})F'(R) - \Box F'(R) \ . \qquad (16)$$

The EoS parameter can be defined as:

$$w_{F(R)} = \frac{\frac{1}{2}F(R) - (3H^2 + \dot{H})F'(R) - \Box F'(R)}{-\frac{1}{2}F(R) + 3(H + \dot{H})F'(R) + \Box F'(R) - \nabla_0 \nabla^0 F'(R)} \ . \qquad (17)$$

Problems and possible solutions in F(R) gravity

Troubles in F(R) gravity

Some F(R) models contain such inconsistencies as:

1. No flat solutions
2. Violation of local gravity tests
3. Matter instability
4. Singularities

Some viable models $f(R) = R + F(R)$

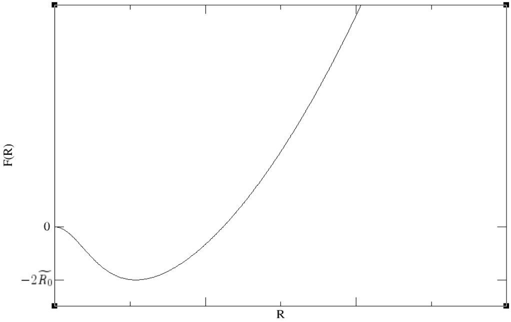
Recently some viable F(R) functions have been propossed as:

$$F(R) = -\frac{m^2 c_1 (R/m^2)^n}{c^2 (R/m^2)^n + 1} \quad \text{(Hu-Sawicki)}$$

$$F(R) = \frac{\alpha R^{m+1} - \beta R^n}{1 + \gamma R^l} \quad \text{(Nojiri-Odintsov)} \tag{18}$$

Example for a viable F(R) gravity

We consider $F(R) = \frac{R^2(\alpha R^2 - \beta)}{1+\gamma R^2}$,



For $R_0 \sim \left(\frac{\beta}{\alpha\gamma}\right)^{1/4}$, $F'(R_0) = 0$, and $F(R_0) = -2\tilde{R}_0 \sim -\frac{\beta}{\gamma}$

Solution for redshifts close to zero

For simplicity, we study such solutions for $z \sim 0$ and around zero, where $F(R_0) \sim -2\tilde{R}_0$ behaves as an effective cosmological constant,

$$H^2(z) = \frac{\tilde{R}_0}{3}\left(A^2(1+z)^{3(w_m+1)} + 1\right) . \tag{19}$$

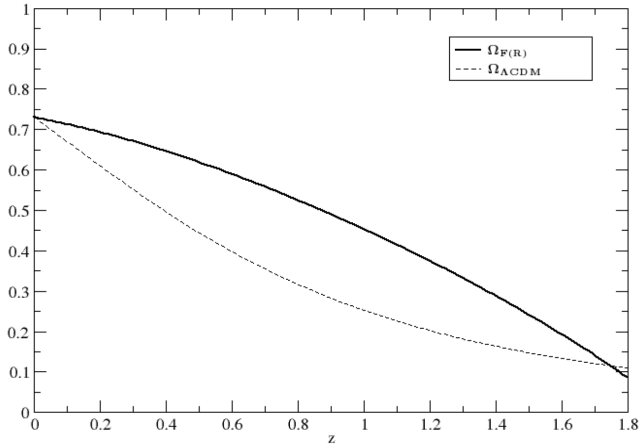
The perturbations around $z = 0$ are neglible. Cosmological parameter for the "energy density" of the modified gravity term,

$$\Omega_{F(R)}(z) = \frac{\rho_{F(R)}}{3H^2(z)} =$$

$$= -\frac{F(R)}{6H^2(z)} + \left[1 + \frac{\dot{H}(z)}{H^2(z)}\right] F'(R) - 18F''(R)\left[\dot{H}(z) + \frac{\ddot{H}(z)}{H(z)}\right] , \tag{20}$$

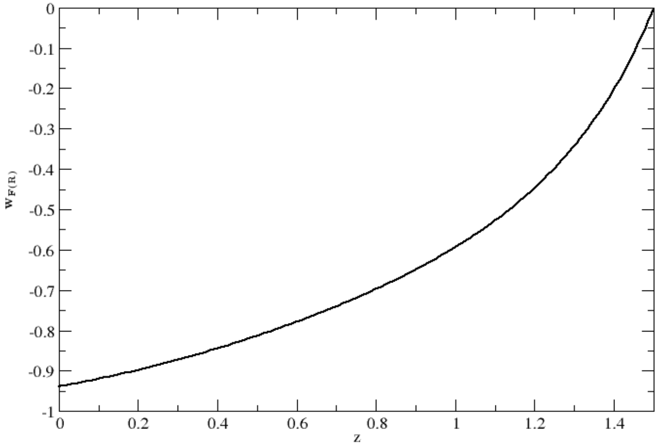
Cosmological evolution: F(R) gravity vs. ΛCDM model

$H_0 = 100h$ km s^{-1} Mpc^{-1} with $h = 0.71 \pm 0.03$, $\Omega_m^0 = 0.27 \pm 0.04$



Evolution for the EoS parameter $w_{F(R)}$

$$w_{F(R)} = \frac{\frac{1}{2}F(R) - (3H^2 + \dot{H})F'(R) - \Box F'(R)}{-\frac{1}{2}F(R) + 3(H + \dot{H})F'(R) + \Box F'(R) - \nabla_0 \nabla^0 F'(R)}$$



Conclusions and future work

1. Any comological solution in scalar-tensor gravity can be expressed in terms of a F(R) theiry, which gives an alternative and satisfactory solution to the mechanism which produces the current accelerated expansion observed, and even the early accelerated epoch, inflation.
2. The problems of local gravity tests and instabilities are avoided in models like the ones presented above, where in some limit GR is recovered.
3. Current work is trying to connect dark matter to F(R) gravity investigating if the rotational curves of galaxies can be explained with these theories.
4. The cosmological evolution of density perturbations is also currently studied, and it may gives some fundamental differences to compare with other dark energy models.

Grazie mille!!!

Background references

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