GGI – Workshop
Firenze, April 12th, 2007

Flux Compactifications (Clearing the Swampland)

Gianguido Dall'Agata

(Padua University / INFN)

References

- **Flux compactifications in string theory: A Comprehensive review.**
  Mariana Grana (Ecole Normale Superieure & Ecole Polytechnique, CPHT) . LPTENS-05-26, CPHT-RR-049-0805, Sep 2005. 85pp.
  Published in **Phys.Rept.423:91-158,2006**.
  e-Print: **hep-th/0509003**

- **Flux compactification.**
  Michael R. Douglas (Rutgers U., Piscataway & IHES, Bures-sur-Yvette) , Shamit Kachru (Stanford U., Phys. Dept. & SLAC & Santa Barbara, KITP) . SLAC-PUB-12131, Oct 2006. 68pp.
  Submitted to Rev.Mod.Phys.
  e-Print: **hep-th/0610102**

Outline

- Part I: An overview of flux compactifications
  - Setup, problems and solutions
  - Properties of the effective theories
- Part II: Twisted tori (and geometric fluxes)
- Part III: Effective theories for general backgrounds
- Part IV: Non-geometric backgrounds
  (Every supergravity from string theory?!)

Part I: Overview of Flux Compactifications

Part I: Overview

- String Theory as the ultimate unified theory: **no dimensionless free parameters**
- **But**: lives in 10 (or 11) dimensions.
- Low energy theory: supergravity
- Standard approach to obtain sensible phenomenology from string theory: compactification
- Field fluctuations in the extra dimensions are seen as masses and couplings in 4d.
- Hence: ***low energy properties depend on high energy choices***

Compactification Ansatz to 4 dimensions:

$$M_{10} = M_4 \times Y_6$$

$$ds^2(x,y) = \mathrm{e}^{2A(y)} ds_4^2(x) + \mathrm{e}^{-2A(y)} ds_{Y_6}^2(y)$$

Other fields proportional to 4d volume (or independent)

Minimal setup: pure geometry

If all fluxes are set to zero F =0, the only non-trivial equation of motion is the Einstein equation

$$R_{MN} = 0$$

Compactification Ansatz to 4 dimensions:

$$M_{10} = M_4 \times Y_6$$

$$ds^2(x,y) = \mathrm{e}^{2A(y)} ds_4^2(x) + \mathrm{e}^{-2A(y)} ds_{\mathrm{Y}_6}^2(y)$$

Minimal setup: pure geometry

$$R_{MN}(x,y) = 0$$

M_4 x Y_6 is a direct product

$$A(y) = 0$$

The internal space is Ricci-flat

$$R_{mn}(y) = 0$$

- Ansatz $M_{10} = M_4 \times Y_6$
- Supersymmetry $\Longleftrightarrow \exists\, \eta \mid \delta\psi_m = \nabla_m \eta = 0$
- Integrability

$$\nabla^2 \eta = R^{ab} \gamma_{ab} \eta = 0$$

Reduced holonomy Ricci flatness

- Result: Y_6 is a *special holonomy* manifold

This is a general result for any geometric reduction

$$M_D \Rightarrow M_d \times Y_{D-d}$$

Special holonomy manifolds were classified by Bergèr (1955):

D-d	Y_{D-d}
6	Calabi-Yau (H=SU(3))
7	G_2-manifolds
8	Spin(7)

- Special-holonomy manifolds specify the vacuum
- The lower dimensional effective theories describe the dynamics of the fluctuations around these backgrounds
- Example: metric fluctuations

$$g_{MN}(x,y) = g^0_{MN}(y) + \delta g_{MN}(x,y)$$

The background is not changed if:

$$R_{MN}\left(g^0_{MN} + \delta g_{MN}\right) = 0$$

This forces:

$$m_{\delta g_{mn}}{}^2 = 0$$

MODULI FIELDS

MODULI SPACE
(SPACE OF DEFORMATIONS)

$$V\left(\phi^i\right) = 0$$

Problem: HUGE VACUUM DEGENERACY

- For minimal supersymmetry

Y_6 has SU(3) holonomy = *Calabi-Yau*

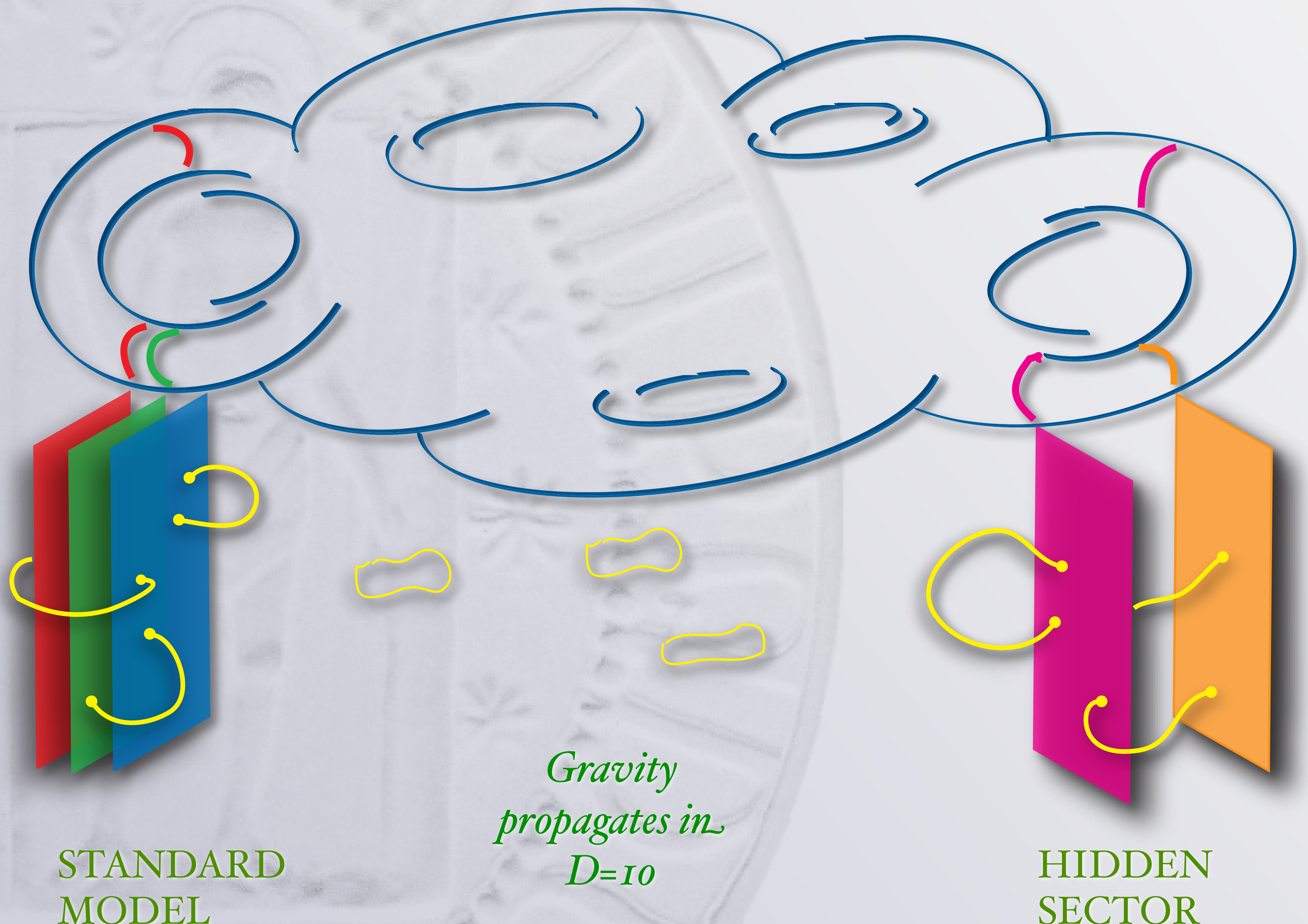
- Since '86 Standard-Model like vacua have been searched

- Heterotic string theory has large gauge groups partially broken by compactification

- Huge number of CY manifolds

- Moduli ϕ^i related to the size and shape of Y_6 have flat potential

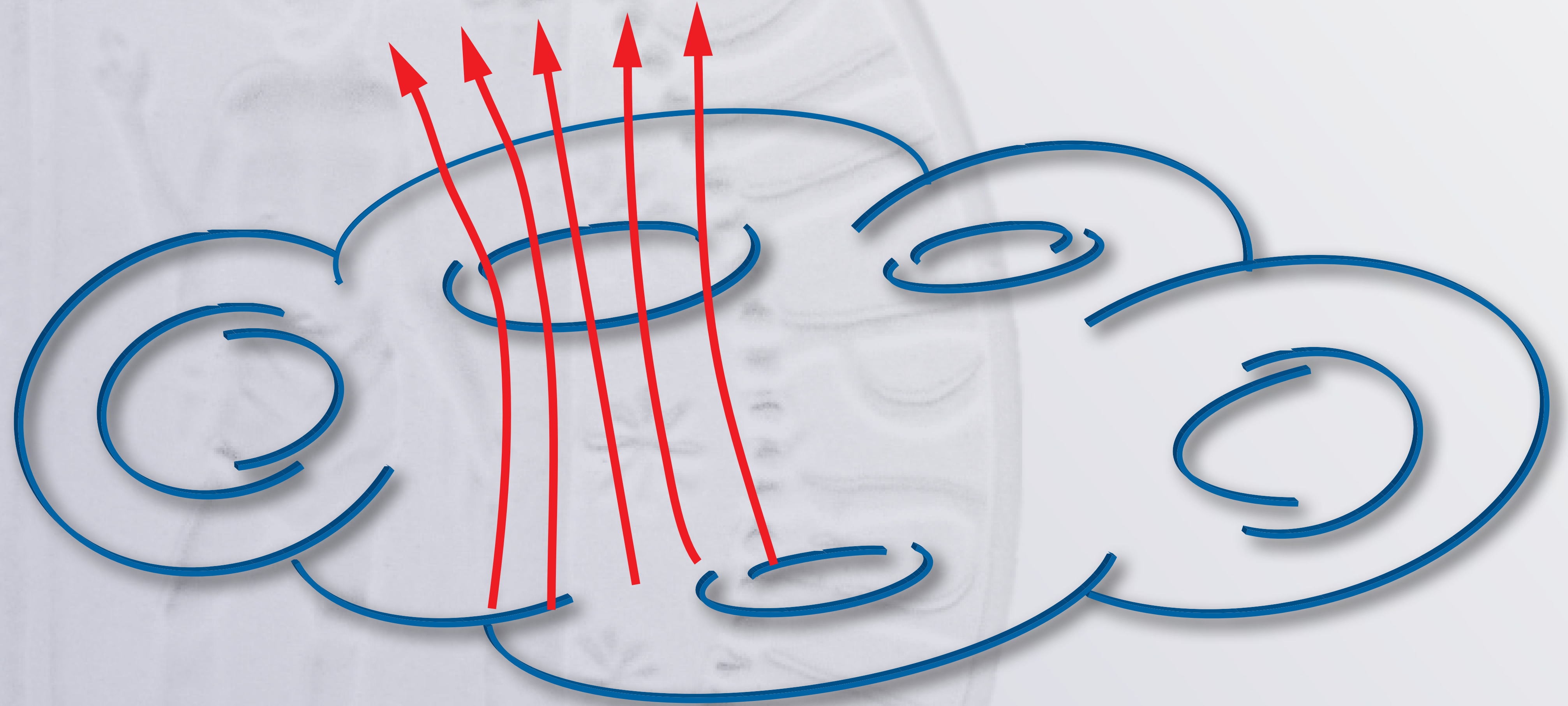
More modern approach: Intersecting Brane Worlds



Can we remove the vacuum degeneracy?

- Add non perturbative effects (difficult to compute and control)
- Add Neveu-Schwarz and Ramond-Ramond fluxes!

- Introducing fluxes constrains the system



- Furthermore: introducing fluxes means adding energy to the system
- Effective theory is deformed
- Vacuum degeneracy may be lifted

BUT

No-go theorem forbids this!

The No-Go Theorem (Assumptions)

- Standard action (no higher curvature corrections)

$$\alpha' R^2 + \ldots$$

- All massless fields have positive kinetic energy
- Semi-negative definite potential: $V_D \leq 0$
- Smooth solution
- Warped product Ansatz:

$$ds^2(x,y) = \mathrm{e}^{2A(y)} \left(ds_4^2(x) + ds_{Y_6}^2(y) \right)$$

THE NO-GO THEOREM

The trace of the Einstein equation on the space-time indices becomes an equation for the warp factor:

$$(D-2)^{-1}\mathrm{e}^{(2-D)A}\nabla^2\mathrm{e}^{(D-2)A} = R_4 + \mathrm{e}^{2A}\tilde{T}$$

For a p-form F_p respecting Poincaré invariance

$$\widetilde{T} = -F_{\mu\nu\rho\sigma m_1\ldots m_{p-4}}F^{\mu\nu\rho\sigma m_1\ldots m_{p-4}} + \frac{d}{D-2}\left(1-\frac{1}{p}\right)F^2$$

Integrating by parts (r.h.s positive definite for M_4 or dS)

$$\int_{Y_6}\left(\nabla\mathrm{e}^{(D-2)A}\right)^2 \leq 0 \Rightarrow A = const$$

THE NO-GO THEOREM

The trace of the Einstein equation on the space-time indices becomes an equation for the warp factor:

$$0 = R_4 + e^{2A}\tilde{T}$$

For a p-form F_p respecting Poincaré invariance

$$\widetilde{T} = -F_{\mu\nu\rho\sigma m_1 \ldots m_{p-4}} F^{\mu\nu\rho\sigma m_1 \ldots m_{p-4}} + \frac{d}{D-2}\left(1 - \frac{1}{p}\right) F^2$$

We are left with 2 options:

1) Minkowski ($R_4 = 0$) and NO fluxes

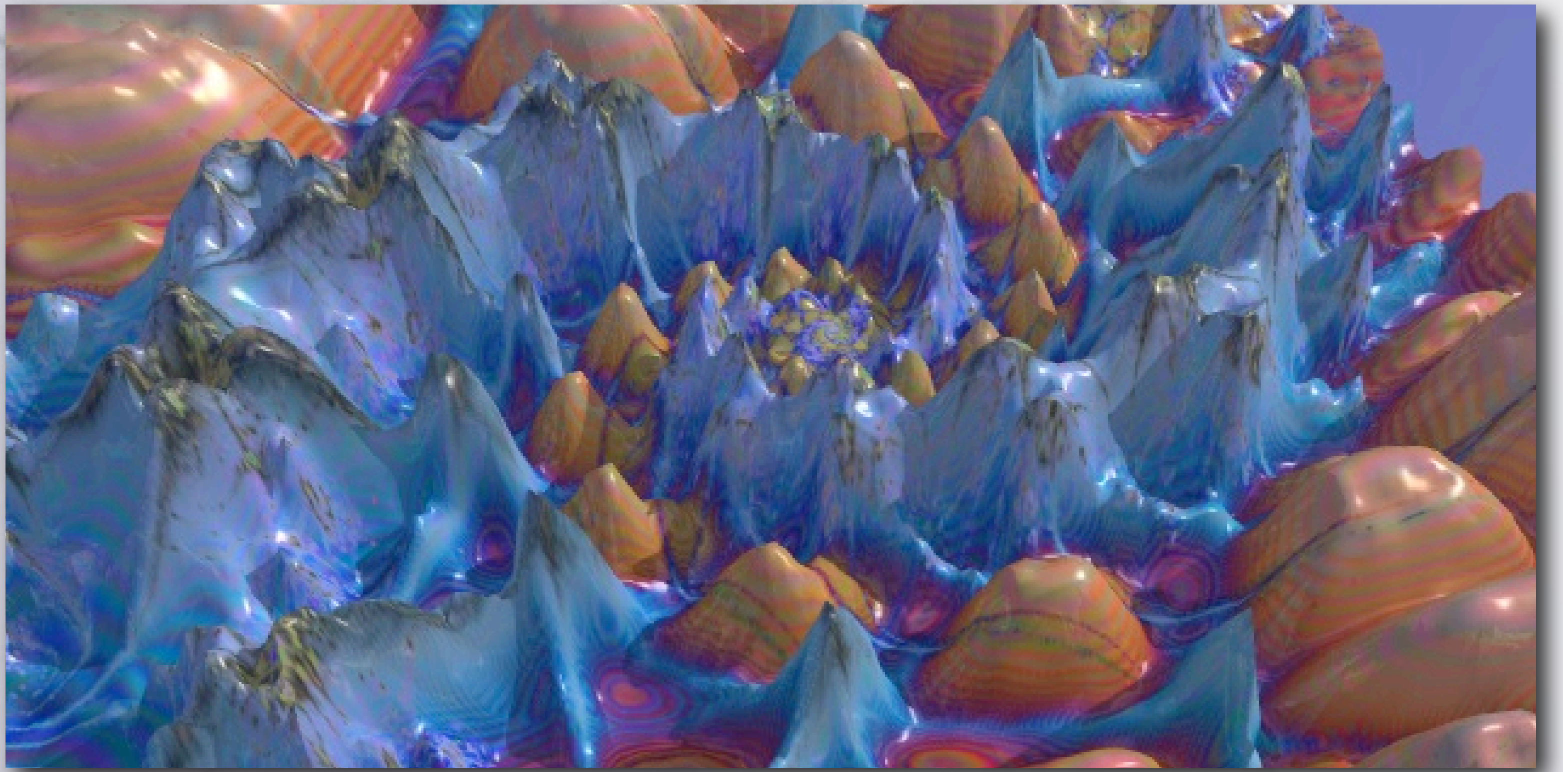
2) Anti-de Sitter spacetime ($R_4 < 0$) with flux

THE NO-GO THEOREM

String theory can avoid (naturally) these constraints.

- Exotic theories (Type * theories)
- Use *non-compact* manifolds
- Introduce sources (D-branes and O-planes)
  - Must produce negative tension (O-planes)
- Higher derivative terms (stringy corrections)
  Natural in Heterotic theory for anomaly cancellation

Moduli Space
(Space of Deformations)



$$V\left(\phi^i\right)$$

This is **"The Landscape of Flux Compactifications"**

What do we do with this?

- Determine the number of different vacua
- Determine their properties (classify them)
- Extract phenomenology (Λ, α,...distribution)
- Measure? (anthropic vs. entropic selection)
- Dynamical selection

These Lectures' Approach:

Effective Theories
(Bottom-Up)

Part I: Overview

- What kind of effective theories do we get?
- How much can we believe these theories?
- Which 4d supergravities have a stringy origin and which ones have not?
  - Can we realize any 4d sugra from some 10d construction?
- Equivalence classes?

Fluxes generate a potential for the moduli fields:

Let us give a v.e.v. to the common sector 3-form

$$\langle H_{IJK}(x,y)\rangle = h_{IJK}$$

The 3-form kinetic term becomes a scalar potential in 4d

$$\begin{aligned}\int_{M_{10}} H\wedge\star H &= \int d^4x\,\left(h_{abc}g^{ad}(x)g^{be}(x)g^{cf}(x)h_{def}+\ldots\right)\\ &= V(g_{ab})\end{aligned}$$

But there is more...

- Fluxes determine (non abelian) gauge couplings:

$$\int H \wedge \star H \;=\; \int d^4x \sqrt{-g_4}\, \Big(\partial_\mu B_\nu{}^a \partial^\mu B^{\nu b} g_{ab} + \partial_\mu B_\nu{}^a g^{\mu b} g^{\nu c} h_{abc} + \ldots\Big)$$

- Vector fields from the metric $g_{\mu I}$ and tensors $B_{\mu I}$

- *Gauged SUGRA* (couplings and potential) fixed by the gauge group (and symplectic embedding)

- **Jacobi** identities = 10/11d **Bianchi** identities

A Lightning Review of Gauged Supergravities

- Standard supergravity has a scalar manifold $\mathcal{M}$ describing their σ-model

- A subgroup of its isometries are realised as global symmetries

- Deformation

  global symmetries

  ∂_μ μ

- This process modifies

Remarkably:

No need of O(g^3) terms to consistently close the action

Lagrangean		Susy rules
O(g) mass terms	O(g^2) potential	O(g) fermion shifts

Explicit realization in 4d: $\{g_{\mu\nu}, \psi^i_\mu, A^I_\mu, \lambda^A, \phi^a\}$

Consider the isometries $\delta\phi^a = \epsilon^\alpha k^a_\alpha(\phi)$

A subgroup can be gauged by the vector fields

$$D_\mu \phi^a = \partial_\mu \phi^a + $$

Geometric relations

$$D_a S_{ij} = \mathcal{N}_i^A e^a_{Aj} + k^a_I f^I_{ij}$$

Modified SUSY rules

$$\delta\psi^i_\mu \quad = \quad D_\mu \epsilon^i + h_I(\phi)\, F^{I\,\nu\rho}\gamma_{\mu\nu\rho}\epsilon^i + g\,\gamma_\mu\, S^{ij}\epsilon_j$$

$$\delta\lambda^A \quad = \quad e^A_{ai}(\phi)\, \not{D}\phi^a\, \epsilon^i + f^A_I(\phi)\,\gamma^{\mu\nu} F^I_{\mu\nu}\epsilon^i + g\, N^A_i \epsilon^i$$

Scalar potential: $\mathcal{V} = N^i_A g^A{}_B N^B_i - \mathrm{tr}\ S^2$

- Introducing fluxes generates a backreaction on Y_6 (and its moduli space)

- The 4d effective theory has a potential

Moduli acquire mass

- Which modes should we keep?

"Small fluxes" approximation

"Small fluxes" approximation

For zero fluxes the geometry is given $M_{10} = M_4 \times Y_6$

Turning on fluxes
$$\begin{aligned} d \star H &= \ldots \\ R_{mn} &= H_m{}^{ij} H_{nij} + \ldots \end{aligned}$$

Linear approx. = No backreaction
(H small compared to the cu

We also need to impose flux

Good supergravity approximation!

$$\frac{1}{2\pi\alpha'} \int_{C_3} H = N \quad \Rightarrow \quad H \sim \frac{\alpha'}{t^3} << \frac{1}{t} \quad \Rightarrow \quad t^2 >> \alpha'$$

- Other issues:
  - Consistent truncations vs. Effective theories
    (do we actually need a vacuum?)
  - Effective potentials may not contain all the 10d information

An Example: IIB on Calabi-Yau + Fluxes
($T^6/Z_2 x Z_2$ with O-planes)

Reminder of IIB action and Bianchi

$$S = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}e^{-2\Phi}\left(R + 4\partial_\mu\Phi\partial^\mu\Phi - \frac{1}{2}H^2\right)$$
$$- \frac{1}{4\kappa_{10}^2}\int d^{10}x\sqrt{-g}\left(F_1^2 + \tilde{F}_3^2 + \frac{1}{2}\tilde{F}_5^2\right)$$
$$- \frac{1}{4\kappa_{10}^2}\int C_4 \wedge H_3 \wedge F_3$$

$$\tau \equiv C_0 + ie^{\Phi} \qquad G_3 \equiv F - \tau H$$

$$d\tilde{F}_5 = H_3 \wedge F_3$$
$$d\tilde{F}_3 = H_3 \wedge F_1$$

An Example: IIB on Calabi-Yau + Fluxes
(T^6/Z_2xZ_2 with O-planes)

- A very simple (singular) Calabi–Yau manifold is the Z_2xZ_2 orbifold of T^6
- The orbifold action is

	4	5	6	7	8	9
Z_2	–	–	–	–	+	+
Z_2	+	+	–	–	–	–
Z_2 x Z_2	–	–	+	+	–	–

- This results in a factorized $(T^2)^3$
- Each torus has one complex structure modulus U^i and one Kæhler modulus T^i

The complex structure moduli are

$$U^i = \frac{1}{g^i_{11}}\left(\sqrt{\det g^i} + i\, g^i_{12}\right) \text{ where } g^i = \begin{pmatrix} g^i_{11} & g^i_{12} \\ g^i_{12} & g^i_{22} \end{pmatrix}$$

The Kæhler moduli are

$$T^i = c^i + i\sqrt{\det g^i}$$

They also follow as deformation parameters of the complex structure

$$\Omega = \alpha_\Lambda X^\Lambda(U) - \beta^\Lambda F_\Lambda(U) \quad \text{with } \alpha, \beta \in H^3(M, \mathbb{R})$$

$$\star_6 C_4 + iJ = T^i \omega_i \quad \text{with } \omega \in H^2(M, \mathbb{R})$$

We can actually write the surviving basis of 3-forms

$$dy^4 \wedge dy^6 \wedge dy^8 \qquad dy^5 \wedge dy^7 \wedge dy^9$$
$$dy^5 \wedge dy^6 \wedge dy^8 \qquad dy^4 \wedge dy^7 \wedge dy^9$$
$$dy^4 \wedge dy^7 \wedge dy^8 \qquad dy^5 \wedge dy^6 \wedge dy^9$$
$$dy^4 \wedge dy^6 \wedge dy^9 \qquad dy^5 \wedge dy^7 \wedge dy^8$$

$$\Omega = \left(dy^4 + U^1 dy^5\right) \wedge \left(dy^6 + U^2 dy^7\right) \wedge \left(dy^8 + U^3 dy^9\right)$$

And of 2-forms

$$dy^4 \wedge dy^5 \qquad dy^6 \wedge dy^7 \qquad dy^8 \wedge dy^9$$

$$\star_6 C_4 + iJ = T^1 dy^4 \wedge dy^5 + T^2 dy^6 \wedge dy^7 + T^3 dy^8 \wedge dy^9$$

The moduli describe Kæhler manifolds with potentials

$$K_{cs} = -\log\left[i\int \Omega \wedge \overline{\Omega}\right]$$

$$K_k = -\log\frac{4}{3}\left[\int J \wedge J \wedge J\right]$$

plus a factor for the axio/dilaton S

$$K_S = -\log\left(S - \bar{S}\right)$$

Before introducing fluxes the solution is flat

$$ds^2_{10} = \mathrm{e}^{2A(y)}(-dy_0^2 + dy_1^2 + dy_2^2 + dy_3^2) + \mathrm{e}^{-2A(y)}\left(\sum_{i=4}^{9}(dy_i)^2\right)$$

We can turn on 3-form fluxes on the 3-cycles

$$G = H_{NS} - SF_{RR} = \left(h^\Lambda - Sf^\Lambda\right)\alpha^\Lambda - \left(h_\Lambda - Sf_\Lambda\right)\beta^\Lambda$$

The backreaction on the geometry generates only a warping in the geometry

$$\nabla^2 \mathrm{e}^{4A} = \mathrm{e}^{2A}\frac{G_{mnp}\overline{G}^{mnp}}{6i(\bar{S}-S)} + \frac{\mathrm{e}^{-6A}}{4}\partial_m\alpha\partial^m\alpha + \rho^{loc}$$

Consistency further imposes that the flux gives a solution to the equations of motion only if it is of type imaginary self dual (ISD), i.e.

$$G + i \star G = 0$$

Supersymmetry further restricts to type (2,1) and primitive, i.e.

$$G^{(3,0)} = G^{(0,3)} = G^{(1,2)} = 0$$

and

$$J \wedge G = 0$$

Let us now describe the deformation to the effective action

- No-backreaction approximation justified because full solution is warped Calabi-Yau:

$$ds^2 = e^{2A(y)}\left(-dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2\right) + e^{-2A(y)} ds^2_{CY}(y)$$

- For "small" fluxes $e^{2A(y)} \sim 1$

  - Light fields are CY moduli

- Masses are related to the warp-factor

$\phi^i \sim$ *harmonic forms* $\qquad A(y) \Rightarrow m^2_{\phi^i} \neq 0$

The scalar potential follows from reduction of the 3-form kinetic term

$$V = -\frac{1}{2}\int_{Y_6} \frac{G \wedge \star\overline{G}}{i(\bar{S} - S)} = -\frac{1}{2}\int_{Y_6} \frac{G^+ \wedge \star\overline{G}^+}{i(\bar{S} - S)} + \int_{Y_6} \frac{G \wedge \overline{G}}{(\bar{S} - S)}$$

4d potential   topological term

$$G^+ \equiv \frac{1}{2}(G + i \star G)$$

The topological term is used to cancel tadpoles

$$dF_5 = -iG \wedge \overline{G} + \rho^{loc}$$

Integrated ⇒ $e \times m + Q^{loc}(N_{D3}, N_{O3}, N_{D7}, \ldots) = 0$

For small fluxes G^+ can be expanded on the basis of harmonic 3-forms and we get the N=1 potential

$$V = e^K \left(g^{a\bar{b}} D_a W \overline{D_{\bar{b}} W} - 3|W|^2 \right)$$

for the famous superpotential

$$W = \int G \wedge \Omega$$

which depends only on the axio/dilaton and complex structure moduli

$$W = c_i U^i + d_i S U^i$$

- The consequences are:
  - We cannot stabilize all the moduli (no Kæhler dependence)
  - The potential is positive definite (no-scale model)
  - The only supersymmetric vacua are Minkowski

We can use the effective superpotential to describe the 10d vacua by minimizing W:

$$D_{U^i} W \equiv \partial_{U^i} W + \partial_{U^i} K\, W = \int G \wedge \chi_i^{(2,1)}$$

$$D_{T^i} W \equiv \partial_{T^i} W + \partial_{T^i} K\, W = \partial_{T^i} K \int G \wedge \Omega$$

$$D_S W \equiv \partial_S W + \partial_S K\, W = \frac{1}{\bar{S} - S} \int \bar{G} \wedge \Omega$$

$$\Rightarrow \quad G^{(3,0)} = G^{(0,3)} = G^{(1,2)} = 0$$

No conditions on $G_{NP}^{(2,1)}$